Assessing Eddy Heat Flux and its Parameterization: A Wavenumber Perspective from a 1/10° Ocean Simulation

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Abstract

Diffusivities diagnosed from eddy heat fluxes in eddying models tend to show unphysically large positive and negative values and it has been suggested that this is due to rotational components that do not influence the heat budget. We diagnose the rotational component of the horizontal eddy heat flux, as measured by its curl, in the Southern Ocean of the 1/10° Parallel Ocean Program as a function of averaging lengthscale. At the scales at which most of the eddy heat flux energy occurs, the rotational component accounts for more than 95% of the total flux, increasing to more than 99% with increasing lengthscale. Hence, rotational components are still dominant when eddy fluxes are averaged over scales larger than the eddy scales. The rotational component, approximated by that part that is along temperature variance contours, only accounts for a fraction of the total rotational component as measured by the curl, leaving a residual that is still more rotational than divergent.

The eddy heat flux can be parameterized as a function of averaging lengthscale using coherence analysis. The meridional eddy heat flux is most coherent with its parameterizations on lengthscales larger than 50°, but this coherence is due to the rotational component. The divergence of the eddy heat flux is most coherent with the divergence of the parameterizations on scales less than 4°, where the curl of the eddy heat flux has a minimum. The coherence for the divergence is low since it is associated with very small and noisy features no matter how large the averaging lengthscale.

Diffusivities estimated from the eddy heat flux show a high wavenumber dependence and are as high as 10000 m²s⁻¹, reflecting the presence of the rotational component. Using the divergence, more realistic, less wavenumber dependent values are estimated, ranging from 500 m²s⁻¹ within the ACC to 3000 m²s⁻¹ in the near surface ocean in the Aghulas Retroreflection region.

Key words: eddies, parameterization, divergence, curl, wavenumber

PACS:

1. Introduction

Significant challenges arise when computing diffusivities by correlating the eddy fluxes and parameterizations from eddying models: it is the aim of this paper to explore and address some of them. The spatial structure of eddy fluxes and their relation to mean quantities is complex: eddy fluxes can be up, down, and along mean gradients of the tracer, and they act on different spatial scales and have rotational components that bias analyses. When diffusivities are diagnosed from eddying models, correlations are typically low and the distributions are unphysical and noisy (Rix and Willebrand, 1996; Roberts and Marshall, 2000; Nakamura and Chao, 2000; Eden et al., 2007a,b).

Here we assess whether correlations of eddy fluxes with their parameterizations and diffusivity distributions are improved when the eddy fluxes are averaged spatially. More specifically, this paper addresses two main questions:

(1) How large is the rotational part of the eddy heat flux, and can it be reduced by averaging?
(2) Are diffusive parameterizations appropriate, and can we identify averaging lengthscales that lead to less noisy distributions and more meaningful diffusivities?

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Preprint to be submitted to Elsevier 2 June 2008
Eddy fluxes of active or passive tracers, $\overline{u'q'}$, in coarse resolution models are parameterized assuming flux-gradient relationships: they are related to mean quantity gradients $\overline{q}$ by an eddy diffusion tensor $K$ (e.g. Griffies, 1997)

$$\langle u'q' \rangle = K \nabla \overline{q}. \quad (1)$$

Here, the overbar is an average in time, the prime denotes the deviation from the time mean. The brackets $\langle \rangle$ are an average in space and may represent the average over the grid cell of a coarse resolution model which is attempting to parameterize the effects of the eddies. The subgrid scale processes that need to be parameterized are all the processes with time and space scales smaller than the scales over which the large scale variables were averaged.

One reason for low correlations of the eddy fluxes with their parameterizations and unphysically large positive and negative values of the diffusivities is that the raw eddy fluxes in equation (1) are comprised of rotational and divergent components (Lau and Wallace, 1979; Marshall and Shutts, 1981, henceforth MS). The term that enters the actual tracer balance equation is the divergence of the eddy flux

$$\partial_t \overline{q}_{eddies} = \nabla \cdot \overline{u'q'} = \nabla \cdot (K \nabla \overline{q}), \quad (2)$$

which specifies how the eddy fluxes influence the large scale quantity. The rotational component of the eddy flux does not contribute to the local heat budget since it transports as much heat into a region as out of it (Jayne and Marotzke, 2002) and does not play any role in equation (2).

Only a limited number of studies have evaluated the parameterization of the divergence of the eddy fluxes directly (Gille and Davis, 1999; Bryan et al., 1999; Solovev et al., 2002; Tanaka et al., 2007). As equation (2) shows, the diffusivity appears within the divergence operator and its determination is not unique. Even though the rotational component is eliminated by considering the divergence of the eddy flux, patterns of diffusivities are noisy with no clear sign or order of magnitude for the diffusivities (Nakamura and Chao, 2000). In some studies, spatial averaging or sectioning of the divergence improved the results (Bryan et al., 1999; Tanaka et al., 2007), but correlations of the divergence of the eddy flux with the parameterization were still found to be low (Gille and Davis, 1999; Bryan et al., 1999; Solovev et al., 2002).

On the other hand, if diffusivities or correlations are computed from the raw eddy fluxes, then the presence of the rotational flux biases the analyses (Eden, 2006; Eden et al., 2007a,b). Rotational and divergent fluxes can be separated using a Helmholtz decomposition (e.g. Lau and Wallace, 1979; Roberts and Marshall, 2000; Drijfhout and Hazeleger, 2001; Jayne and Marotzke, 2002). However, the Helmholtz decomposition depends on the choice of boundary conditions and is therefore not unique (Fox-Kemper et al., 2003). It is also computationally fairly expensive. Alternatively, a rotational component can be identified by considering the eddy variance equation (MS, Medvedev and Greatbatch, 2004; Eden et al., 2007a). The objective of this approach is not to construct a completely irrotational residual flux. Rather it is to define a rotational flux that is associated with the advection of eddy variance while the residual diffusive part is associated with the irreversible removal of eddy variance, but need not be completely rotation-free. Regardless of the choice of decomposition, the divergence of the parameterizations is the same, even though the spatial distributions of the resulting diffusivities might be completely different. Ideally, the diffusivities will be smooth and positive and therefore usable in climate models. Eden et al. (2007a)’s decomposition, a generalization of the MS and Medvedev and Greatbatch (2004) approaches, will potentially lead to residual fluxes that best represent the potential to kinetic energy conversion terms in the eddy energy equations and therefore lead to the most physical diffusivities. However, the decomposition poses challenges for the use in point observations or high resolution models because they require sufficiently high quality data to allow sufficiently accurate computations.

The intention here is to consider relatively simple approaches for isolating a meaningful divergent component of the eddy heat flux. Instead of focussing on the subtraction of rotational components, to address question (1), we pursue the simple concept that even though there is not necessarily a strong association locally between mean gradients and the raw eddy fluxes, the raw eddy fluxes averaged over large enough scales may be expected to have a net component down the mean gradient resulting in the release of potential energy. Hence they may be correlated with a diffusive parameterization. No study has yet systematically investigated the effect of that averaging scale on the presence of the rotational component, correlations and diffusivities. We use the curl and divergence of the eddy heat fluxes as unique measures of the rotational and divergent part. Our results will show that the rotational component dominates over all length scales and hence cannot be removed by averaging.

We then follow the ideas of MS and assess the extent to which eddy fluxes are directed along temperature variance contours. We expect that if most of the eddy heat flux can be explained locally by the entirely rotational flux along temperature variance contours, there should be no connection between total eddy heat flux and its diffusive parameterization. However, we will argue that the rotational part cannot be removed using the MS method. The rotational component makes up more than 95% of the total eddy heat flux. Therefore, small errors in computing the rotational flux using the MS method can overwhelm the divergent flux, while the residual is still dominated by the curl.

To address the second question of whether diffusive parameterizations are still appropriate and for which length scales, we examine spatial coherence between two pairs of quantities: (a) the eddy heat fluxes and their downgradient parameterizations, and (b) the divergence of the heat flux and the divergence of the parameterizations. We will show that coherence is low and that averaging in space does not improve the coherence.
In this study we use the 1/10° horizontal resolution Parallel Ocean Program (POP) in the Southern Ocean domain (90°S - 35°S). The circumpolar uniqueness of the Southern Ocean allows us to investigate length scales up to 360°. Many studies have shown the important role that Southern Ocean eddies play in the zonally or streamline averaged picture of the ACC and the associated Southern Ocean overturning cell (Karsten et al., 2002; Marshall and Radko, 2003; Olbers and Visbeck, 2005; Lee et al., 2006). In a zonally-averaged framework, the rotational part of the meridional eddy heat flux cancels out, and many aspects of the three-dimensional eddy field are masked. In particular, Radko, 2003; Olbers and Visbeck, 2005; Lee et al., 2006).

In this paper, whether meaningful rotational parts can be removed by averaging over appropriate scales or by taking into account the physics of the system into account. Section 5 examines the second question and provides the spatial coherence analysis of the eddy heat flux and its divergence with the parameterizations. Section 6 presents the discussion and conclusions.

2. Model description and eddy heat fluxes

2.1. Model description

We use the 1/10° POP model, which has one of the highest horizontal resolutions currently used in global modeling. The model is sufficiently eddy resolving in the sense that between about 50°S and 50°N it resolves the first baroclinic Rossby radius, which is smaller than typical eddy length scales (Smith et al., 2000). The model successfully reproduces the mesoscale sea surface height variance of the Southern Ocean (McCLean et al., 2006). It uses a Mercator grid in the Southern Hemisphere with a latitudinal grid spacing of 1/10°cos(φ), where φ is latitude, leading to horizontal resolutions between 4 and 9 km for the Southern Ocean. There are 40 depth levels in the vertical, ranging from 10 m at the surface to 250 m at depth, with a maximum depth of 5500 m. Details of the bathymetry can be found in Maltrud and McClean (2005).

The horizontal diffusion of tracers and momentum is biharmonic with coefficients varying with the cube of the average grid length:

\[ \nu = \nu_0 \left( \frac{d\ell}{d_0} \right)^3; \quad \kappa = \kappa_0 \left( \frac{d\ell}{d_0} \right)^3, \]

where \( d\ell = \sqrt{A} \), and \( A \) is the area of the grid cell. Equatorial values are \( \nu_0 = -27 \times 10^6 \text{ m}^2\text{s}^{-1} \) and \( \kappa_0 = 9 \times 10^6 \text{ m}^4\text{s}^{-1} \). The biharmonic mixing is scale selective and acts to dampen short scales and noise. For vertical mixing in the model, the K-profile parameterization of Large et al. (1994) is used, with background mixing coefficients ranging from 0.1 \( \times 10^{-4} \text{ m}^2\text{s}^{-1} \) near the surface to 1 \( \times 10^{-4} \text{ m}^2\text{s}^{-1} \) at depth, to parameterize mixing associated with breaking of internal waves. Viscosity values are an order of magnitude higher. Large values of diffusivity and viscosity are used to simulate convection.

Tracer advection is discretized with a centered difference scheme that does not introduce any numerical diffusion. The tracer time step is 6.3 min.

Maltrud and McClean (2005) discuss the forcing and initial conditions. The model was spun up for 14 years (1979 - 1993) and then run post-spinup from 1994 - 2003. Wind, air-sea heat fluxes and evaporation are calculated from NCEP/NCAR reanalysis products (the 6-hour fields are averaged to one day) in combination with monthly data from the International Satellite Cloud Climatology project (for shortwave flux and cloud fraction). Precipitation comes from the monthly microwave sounding unit (Spencer, 1993) and the Xie and Arkin (1997) dataset. Fields are then interpolated to the tracer time step. Since no sea ice model is used, under-ice sea surface temperature and salinity are additionally restored to climatology (Steele et al., 2001) in regions known to be ice covered. In the open ocean, only sea surface salinity is restored to climatological data with a restoring time scale of 6 months to avoid model drift.

2.2. Meridional heat fluxes

Figure 1 shows the vertically and zonally integrated meridional eddy heat transport (PW) over each grid cell in the Southern Ocean as calculated from the deviation of the annual 1998 mean heat flux. The spatial extent of the ACC is indicated in Figure 1 by the two circumpolar contours of annual 1998 mean surface temperature. The eddy heat flux in different regions of the Southern Ocean (Figure 1) are characterized by different eddy heat flux intensities. In this study, four regions are chosen that are characterized by different eddy heat flux regimes. Region 360 encircles the Southern Ocean and largely encloses the ACC. Region AGR includes the intense eddy activity in the Agulhas Retractification. Region SSP in the Southern South Pacific is within the ACC and also shows relatively high eddy activity. Region SPA is relatively quiescent and is in the South Pacific subtropics outside the ACC. Region 360 has 3600 gridpoints in longitude and 50 gridpoints in latitude. Regions AGR, SSP and SPA have 600 points in longitude and 100 points in latitude. There are no bathymetric features in any subregion above the depth of 918 m. In Regions 360, AGR, SSP and SPA monthly averages of all fields from model year 1998 were archived. Region WDR west of Drake Passage has 200 grid points in lon-
of the eddy heat flux. Sloyan and Rintoul, 2001; Ganachaud and Wunsch, 2000; Thompson, 1993; Jayne and Marotzke, 2002). Observational estimates of eddy heat fluxes are sparse and difficult to interpret. Analyses based on hydrographic data obtain maximum heat transports between 0.3 and 0.7 PW (deSzoeke and Levine, 1981; Macdonald and Wunsch, 1996; Sloyan and Rintoul, 2001; Ganachaud and Wunsch, 2000; Gille, 2003). From altimeter data, Kefee and Holloway (1988) obtained 0.7 PW at about 53°S and Stammer (1998) obtained a maximum of about 0.3 PW at 40°S.

3. Wavenumber dependence of eddy heat flux components and their parameterization

3.1. Candidate parameterizations and spatial distribution of the eddy heat flux

We introduce here the three parameterizations that are assessed to parameterize the horizontal eddy heat flux. These are contained in the tensor devised by Griffies (1997) for z-coordinate ocean models. The tensor parameterizes eddy tracer fluxes as a sum of two components: as along-isopycnal diffusion of the tracer down the mean tracer gradient (Redi, 1982) plus the advective effects of the eddies, represented by the antisymmetric components of the tensor (Gent and McWilliams, 1990; Gent et al., 1995). We assume eddy fluxes are adiabatic, neglecting any diapycnal flux. While the diapycnal mixing is important for the large-scale ocean circulations and energy budgets (Wunsch and Ferrari, 2004) it is likely much smaller in the oceanic interior than the isopycnal and horizontal mixing considered here (Tandon and Garrett, 1996; Eden and Greatbatch, 2008).

In z coordinates, and under the approximation of small isopycnal slopes, the horizontal eddy heat flux is then given by

$$\overline{u_h T} = -A \left( \nabla_h T + S \partial_z T \right) + \kappa_g m S \partial_z T$$

(4)

with $S = -\nabla_h \rho / \partial_z \rho$ (Griffies, 1997). Here, we consider the individual terms on the right hand side of equation (4) separately as candidate parameterizations of the horizontal eddy heat flux: (1) the first term represents simple horizontal downgradient diffusion of mean temperature, (2) the first two terms together are the diffusion of temperature along isopycnals and across mean temperature contours, involving the diffusion coefficient $A$ (Redi, 1982, henceforth Redi) and (3), the third term represents the Gent and McWilliams (henceforth GM) stirring with $\kappa_g m$ as its coefficient. We restrict our analyses to the parameterization of the horizontal fluxes which are an order of magnitude larger than vertical fluxes.

Only the divergence of the horizontal eddy heat flux plays a role in the horizontal heat budget:

$$\nabla_h \cdot \overline{\left( u_h T^2 \right)} = -A \left( \nabla^2 T + \nabla_h \cdot \left( S \partial_z T \right) \right) + \kappa_g m \nabla_h \cdot \left( S \partial_z T \right)$$

(5)

If isopycnal surfaces are parallel to surfaces of constant temperature, the first two terms on the right hand side of equation (4) will cancel each other and the eddy heat flux is represented solely by the GM stirring.

Figure 3a and Figure 3c show the meridional eddy heat transport and the divergence of the eddy heat transport and Figures 3b and 3d their respective GM parameterizations (last terms in equations (4) and (5)) for Region AGR (see Figure 1) at a depth of 318 m. The eddy heat flux (Figure 3a) alternates between positive and negative values and locally appears to show little correlation with the parameterization, which indicates predominantly southward flow over the region (Figure 3b). However, the meridional eddy heat flux zonally averaged over the region is southward for latitudes south of 37°S. The pattern of positive and negative meridional heat flux cannot be explained solely as an artifact of the rotational flux since the divergence of the eddy heat flux also alternates between positive and negative values, indicating net local heating and cooling respectively (Figure 3c). However, there is evidence that net local heating by the eddy heat flux also corresponds to a local positive divergence of the parameterization (Figure 3d), and conversely cooling corresponds to negative divergence of the parameterization in many places. Note that the horizontal diffusion term (first term in equation (4)) is very similar to the GM term (third term in equation (4)), and therefore the Redi diffusion (first plus second term in equation (4)) is small.

Decorrelation scales indicate that the typical length scales for eddies in the model Southern Ocean are 50–150 km, or approximately 0.8-2.4° at 55°S (McClean et al., 2008). For Region AGR, the length scales for the meridional eddy heat fluxes are around 1–2° (Figure 3a), while for the divergence they are about 0.5° (Figure 3c). The characteristic length scales for both the divergence of the eddy heat flux (Figure 3c) and the divergence of the parameterization (Figure 3d) are smaller than for the meridional eddy heat flux (Figure 3a) and its parameterization (Figure 3b). This is to be expected: taking an extra derivative in the space domain is equivalent to multiplying by the wavenumber in the wavenumber domain and therefore shifts energy to
higher wavenumbers and whitens the spectrum.

3.2. Wavenumber spectra

Figure 3 reveals a greater similarity in scales between the divergence of the eddy heat flux (3c) and its parameterization (3d) than between the raw fluxes (3a) and their parameterization (3b). On the other hand, the raw eddy heat flux averaged over the whole region does show a net southward transport as does the parameterization. Prior to examining coherence in section 5 we wish to understand at which wavenumbers the eddy heat flux and its parameterizations occur and to quantify any differences in length-scales between the eddies and their parameterizations using wavenumber spectra. We concentrate here on zonal wavenumber space, since this allows us to investigate scales as long as 360°. We expect results to be translatable to meridional wavenumber space.

Figures 4f-j show zonal wavenumber spectra for eddy heat fluxes and their derivatives and Figures 4a-e represent terms used for parameterizations of the eddy fluxes; thick solid lines are the parameterization as horizontal diffusion of mean temperature, thin solid lines are the parameterizations as GM eddy advection and dashed lines are the sums of these quantities, corresponding to the Redi isopycnal diffusion. Spectra are shown for Region AGR at a depth of 318 m and represent averages over 100 latitudes. The eddy heat flux terms at any given wavelength can be interpreted as the eddy heat flux that needs to be parameterized by the quantities in the left-hand panels of Figure 4 in a coarse resolution model of a horizontal resolution corresponding to that wavelength.

Most of the meridional eddy heat flux (Figure 4h) occurs between lengthscales of 1° and 10° longitude, longer than the first baroclinic Rossby radius (which is < 0.2° at 60°S), with a pronounced peak at about 80° longitude. The spectrum for the zonal eddy heat flux (Figure 4f) is more red with a substantial part of the energy occurring on lengthscales between 1° and 100° longitude, defined here as the mesoscale, where most of the eddy heat transport takes place. The spectrum for the divergence of the eddy heat transport (Figure 4j) has most energy at wavelengths shorter than 10° longitude, as do its components \( \partial_x (\overline{uT}) \) and \( \partial_y (\overline{vT}) \) (Figures 4g,i). However, the spectral amplitudes of the divergence of the eddy heat flux (Figure 4j) only about 10% of the spectral amplitudes of the zonal and meridional derivatives (Figures 4g,i). This indicates that their values in the space domain are similar in magnitude but opposite in sign: if eddy velocities are along temperature contours, then \( \overline{uT} \propto \partial_x T \), \( \overline{wT} \propto \partial_z T \), and therefore \( \nabla \cdot (\overline{uT}) = \partial_x (\overline{uT}) + \partial_y (\overline{vT}) \simeq 0 \), implying that most of the eddy heat transport is geostrophic and therefore rotational. The two components of the horizontal curl of the eddy heat flux \( (\partial_y \overline{vT}, -\partial_x \overline{uT}) \) have magnitudes similar to the components of the divergence. However, their sum (dashed line in Figure 4j), plotted as the Fourier transform of \( 0.1 \times (\partial_x \overline{vT} - \partial_y \overline{uT}) \) has a similar wavenumber distribution but is about an order of magnitude larger than the divergence.

The spectra of the parameterizations (Figures 4a-e) look quite different from their eddy flux term counterparts (Figures 4f-j). In fact, the wavenumber distributions of the zonal derivatives of temperature or density (Figures 4a-i) are similar to the wavenumber distribution of the meridional eddy heat flux (Figure 4i), rather than the zonal eddy heat flux (Figure 4f). The zonal eddy heat flux has a similar wavenumber distribution as the meridional derivatives of temperature and density (Figure 4c). This is again a result of the importance of the geostrophic relation where flows are perpendicular to tracer gradients. In comparisons of the wavenumber distributions for the different kinds of parameterizations, the Redi diffusion terms are always smaller than the GM and horizontal diffusion terms. In the parameterization of the eddy flux (Figures 4a and 4c), the GM and horizontal diffusion terms cancel each other leaving only a very small residual Redi diffusion flux, illustrating the extent to which isotherms are parallel to isopycnals. The divergence of the parameterizations and their components however (Figures 4b,d,e) indicate that the divergence of the Redi flux is still about half of the divergence of the GM flux.

The spectra in Figure 4 are representative for the depth of 318 m and similar in all regions, except region SSP, where Antarctic Intermediate Water formation occurs. There, the vertical temperature gradient is positive in the upper 500 m and the vertical density gradient still negative, such that the GM and horizontal diffusion terms add up rather than cancel each other leaving the Redi term as the dominant one.

At a deeper depth level (918 m), the spectra of the divergence of the parameterizations reveals much variability at high wavenumbers > 1 cycle/° longitude, especially for the GM terms, which involve multiple derivatives and ratios of derivatives, leading to noisy distributions. Eddy flux and eddy flux divergence terms have very similar wavenumber distributions at different depth levels.

4. Can we reduce the rotational component?

4.1. Reduction by averaging fluxes?

In this section we address the first of the two main questions in this paper, namely, how large is the rotational part of the horizontal eddy heat flux and whether it can be reduced by averaging. In section 4.2 we will briefly outline the challenges arising when rotational components based on physical considerations are removed. Figure 4j showed that for the spatial scales containing most of the eddy heat flux energy, the curl of the eddy heat flux exceeds the divergence by an order of magnitude. When averaged over larger scales, both terms are very small.

Consider an area \( A \) that is circumpolar in the Southern
Ocean and bounded to the south by the Antarctic continent and to the north by a latitude circle at latitude $y_n$ (Figure 5). It is easy to see that the meridional eddy heat flux at latitude $y_n$ into (or out of) the Southern Ocean, averaged over a full latitude circle, as in Figure 2, does not contain a rotational part (see Appendix A). If the area is not circumpolar, but is instead bounded by two lines of longitude $x_w$ to the west and $x_e$ to the east (Figure 5), there are net zonal transports through the lateral sides of $A$ and the zonal integral of the meridional heat transport through the northern boundary no longer represents the net divergent meridional transport.

We now ask whether an average of the eddy heat fluxes over certain spatial scales would reduce the rotational part in the sense that the curl of the averaged quantities will be smaller than the divergence. The eddy heat flux can be decomposed into divergent and rotational components, involving non-uniquely defined scalar potentials $\Phi_{\text{div}}$ and $\Theta_{\text{rot}}$,

$$\overline{u_h^r T} = k \times \nabla_h \Theta_{\text{rot}} + \nabla_h \Phi_{\text{div}}$$

(Appendix B). A zonal average of the meridional eddy heat flux then yields

$$\int_{x_w}^{x_e} \overline{u_h^r T} \, dx = \Theta_{\text{rot}}(x_w) - \Theta_{\text{rot}}(x_e) + \int_{x_w}^{x_e} \partial_x \Phi \, dx$$

(see Figure 5). One might imagine that if the zonal scale $x_w - x_e$ is sufficiently long compared with the typical zonal eddy scale, then (7) would be dominated by the divergent term, and the rotational term might be effectively zero.

In order to evaluate this idea, in Figure 6 we have plotted the spectra of the curl of the eddy heat flux normalized by the sum of the spectra of the curl and divergence of the eddy heat transport. This ratio measures the relative importance of the net divergent eddy heat flux in or out of the region $A$ considered and the rotation of the eddy heat flux around region $A$ as a function of wavenumber. More than 95% of the eddy heat flux is rotational at all depth levels examined in Region 360 (Figure 6a) and for all regions at 318 m depth (Figure 6b). The divergent eddy heat transport is largest for scales smaller than 10° but amounts to less than 2% for Region 360 and no more than 5% for Region SPA at a depth of 318 m. It is below 1% for Region SPA at 318 m. The divergent eddy heat transport decreases with increasing length scale and approaches zero for length scales larger than about 25°.

Similar results are obtained for the one-dimensional spectra in the meridional direction and for a two dimensional Fourier transform (not shown).

### 4.2. Removal of rotational components based on physical considerations

Given that there is no lengthscale at which one can expect the divergent component to dominate $\overline{u_h^r T}$, we ask now if meaningful rotational components can be subtracted from the raw fluxes, based on the considerations of MS.

For this purpose we use the model output from Region WDR west of Drake Passage (see Figure 1) for which daily fields are available. In the previous sections we calculated the eddy heat fluxes as a deviation from the 1998 annual mean. In order to minimize possible sensitivities to the duration of the averages, we here used the deviation from a three-year mean for the years 1998-2000. Time averaging proves not to influence the findings shown here and results based on one year are not qualitatively different from the results derived from three-year averages.

#### 4.2.1. Background

Jayne and Marotzke (2002) solved for the divergent part by assuming zero flux through the boundaries and carrying out a matrix inversion for the global ocean (Appendix B). The method breaks down for very high resolution models for which the matrix inversion is computationally expensive or for cases where zero flux boundary conditions may not be appropriate.

As an alternative, MS suggested that one can choose the rotational streamfunction $\Theta_{\text{rot}}$ based on physical considerations (see also Illari and Marshall, 1983; Cronin and Watts, 1996; Eden et al., 2007b): if eddies are predominantly geostrophic and equivalent barotropic and if we neglect the influence of salinity, the eddy velocities are aligned with temperature contours $u_h^r \propto k \times \nabla_h T$ and

$$\overline{u_h^r T} = \gamma \frac{1}{2} \nabla_h T^2.$$  

Thus, we can choose $\Theta_{\text{rot}}$ in equation (B.1) to be $\gamma T^2/2$ and the rotational component is directed along the contours of eddy variance $T^2$. The factor $\gamma$ needs to be horizontally uniform for the rotational eddy heat flux to be divergence-free. Equation (8) specifies the amount of geostrophic eddy heat flux when $\gamma$ is determined directly from the correlation of the eddy heat flux with gradients of eddy variance under an $f$-plane approximation, and neglecting salinity. MS chose $\gamma$ based on consideration of the steady-state eddy variance equation in the quasigeostrophic approximation, assuming no external sources or sinks of heat and neglecting the eddy advection of temperature. The horizontal eddy heat flux can be written as the sum of the rotational flux and a residual flux

$$\overline{u_h^r T_{\text{res}}} = \overline{u_h^r T} - k \times \nabla_h \Theta_{\text{rot}}.$$  

MS then approximated the mean flow to be along mean temperature contours. If $\Theta_{\text{rot}} = \left[ \frac{\partial \Psi}{\partial z} \right] T^2/2$, where $\Psi$ is the mean velocity streamfunction, the residual horizontal eddy heat flux $\overline{u_h^r T_{\text{res}}}$ is related to the conversion of eddy potential energy to eddy kinetic energy $(w T \partial_z T)$. This motivates the choice to parameterize $\overline{u_h^r T_{\text{res}}}$ as a diffusive process $\kappa \nabla_h T$. For positive $\kappa$, eddies release potential energy, and $w T \partial_z T$ is negative. The MS rotational eddy heat flux is purely horizontally non-divergent. It is associ-
ated with the spatial growth and decay of the eddies and, under the above approximations, balances the mean flow advection of eddy potential energy. It is parallel to the eddy potential energy contours and circulates around the mean temperature contours. The residual flux $\overline{u_0T_{res}}^{MS}$ however, is not rotation-free.

4.2.2. How much of the flux is geostrophic?

We now examine how much of the eddy heat flux is along contours of eddy variance and compare this estimate to the rotational component diagnosed uniquely from the curl and divergence using coherence analysis.

The squared coherence between two fields represents the fraction of the spectral energy of one field attributable to a second field through a linear relation. Translated to the space domain this would mean that the two fields were perfectly correlated when bandpass filters were applied that retain variability at that specific scale. If there is no correlation, then coherence is zero, or below the significance level. As an example, if the fields are highly correlated in space when they are simply averaged over 10 degrees in longitude, then, in the wavenumber domain, the integral of coherence will be high over wavelengths greater than 10 degrees. In section 5 we apply coherence analysis to the eddy heat flux and its parameterizations.

Figure 7 shows the zonal squared coherence of the eddy heat transport with the gradient of eddy variance as a function of wavenumber at 318 m (Figure 7a) and 918 m (Figure 7b) in region WDR. Coherence tells us the extent to which there is a linear relation between the eddy heat flux and the gradients of eddy variance at every wavenumber and hence the amount of rotational flux as defined by equation (8). Coherence has a minimum around a lengthscale of 1° and increases with increasing lengthscale over the mesoscale (Figure 4h). At lengthscales of 2° or larger the rotational contribution as defined by equation (8) seems to level off at about 60%-70% of the total flux at a depth of 318 m (Figure 7a). Deeper, at 918 m (Figure 7b), where geostrophy is a better approximation, 70%-90% of the total flux can be explained by the rotational component as defined by equation (8). The gray line in Figure 7 indicates the amount of rotational flux measured uniquely by the normalized curl of the eddy heat flux (section 4.1). Clearly, the residual eddy heat flux as defined by equation (9) still makes up about 20% of the total eddy heat flux, whereas the rotation-free component of the eddy heat flux, as measured by the curl and divergence, makes up less than 1% of the total eddy heat flux (gray line in Figure 7).

Since the rotation-free component of the heat flux is small compared with the residual heat flux determined by the MS method, we cannot expect the residual flux to represent much improvement over the raw fluxes. The analyses in this section illustrate the difficulty of subtracting a rotational component that captures most of the rotational eddy heat flux, since the residual will likely still have a large rotational component that dominates over the divergent part. Hence, one cannot expect the residual to be correlated any better with the mean temperature gradients or yield physically more reasonable diffusivities than the raw eddy fluxes, at least when fluxes are decomposed with the MS method. The refinement of Eden et al. (2007a) will only lead to an improvement over the MS method, when the residual fluxes can be calculated with sufficient accuracy.

5. Can the eddy heat flux be parameterized?

Our second main question asks whether the eddy heat fluxes can be parameterized despite the dominance of the rotational part or alternatively whether we can parameterize the divergence of the eddy heat flux instead. Diffusivities calculated locally for every grid cell from the ratio of the divergence and the divergence of the parameterization (e.g. Figures 3c and d), reveal extremely noisy and physically unrealistic diffusivities (see also Nakamura and Chao, 2000). Tanaka et al. (2007) averaged over boxes of 4° latitude by 2° longitude. Coherence analysis in this section aims to estimate in a systematic way the dependence of the correlation between the eddy heat flux and its parameterization on averaging length scales.

We assess the coherence for two different cases: Case F (for Flux) considers the meridional eddy heat flux $\overline{v_0T}$ and the parameterization as either horizontal downgradient diffusion ($-\partial_x T$), GM advection ($S_y \partial_y T$) or Redi diffusion ($-\partial_y T - S_y \partial_y T$); Case D (for Divergence) considers the divergence of the eddy heat flux $\nabla_h \cdot (\overline{u_0T})$ and the divergence of the parameterizations. In Case F the rotational part of the eddy heat flux is dominant and we expect coherence to be very low at all scales. By definition, Case D does not include the rotational component and represents the terms as they appear in the temperature balance equations. Comparing the two cases allows us to assess the extent to which the rotational part of the eddy heat flux plays a role for the parameterization as a function of length scale.

5.1. Zonal coherence analysis for different regions and depth levels

Figure 8 shows the zonal one-dimensional squared-coherence for Case F (Figures 8a,c) and Case D (Figures 8b,d,e-h) for the three different parameterizations in selected regions and depth levels. Coherence was computed by Fourier transforming 360° long records at each of 50 latitudinal grid points and then averaging over the 50 records and band-averaging over 9 zonal wavenumbers. Wavenumber 0.25, indicated by a vertical line in Figure 8, corresponds to 4° longitude, a characteristic scale at which coherence reaches a minimum for most cases.

The Case F coherence as shown in Figures 8a,c is very similar for all regions, depth levels and parameterizations. The coherence is small for short wavelengths, increases
somewhat for wavelengths larger than about 4° until a sudden increase occurs at wavelengths of about 50° or larger for all parameterizations, except the GM parameterization at 318 m. More than 40% of the total eddy heat flux can be represented as a diffusive process when fields are filtered in space over length scales of at least 50°, for depth levels below the surface. However, at these large length scales eddy heat flux is essentially rotational (section 4) and the eddies are unlikely to be coherent baroclinic eddies, but rather behave like basin scale Rossby waves.

In contrast, Case D coherence is not significant for length scales larger than 50° (Figures 8b,d) and reaches instead the most significant values for length scales smaller than 4° (Figures 8b,d,e,f,h). The length scale of 4° can be identified to be a characteristic length scale around which Case D coherence reaches a minimum for most cases and there are two wave bands with significant coherence: (1), the wave band between about 10° and 50° wavelength with a small significant coherence of 0.1 and (2), the wave band with wavelengths smaller than about 4° with significant coherence of up to 0.4.

In the first wave band, Case D coherence is the same (and significant for all depth levels except the surface) for the horizontal (Figures 8e,f) and GM (Figures 8g,h) parameterizations for all regions, consistent with the fact that in that wave band and for depth levels below the surface, isothermals and isolynals are aligned. There is no significant coherence for the Redi parameterization in this wave band (Figures 8b,d).

In the second wave band, high coherence up to 0.4 is reached with the horizontal diffusion parameterization (Figures 8b,d,e,f). The coherence for the GM parameterization is very similar but there is no significant coherence for GM at 918 m. (Figure 8g). At that depth, GM has a large high wavenumber component (section 3.2). Coherence for the Redi parameterization is generally very low for depth levels below 318 m.

There is evidence that high Case D coherence corresponds to the scales where the curl has a minimum (Figures 6a,b and Figure 7) and that the minimum in Case D coherence around 4° longitude also corresponds to a maximum of the curl. Hence, even though the curl dominates the divergence of the eddy heat flux over all length scales, significant Case D coherence appears when there is a local minimum in the curl.

To help interpret the coherence, the phase of the coherence is shown for Case F in Region 360 and the three different parameterizations in Figure 9a and for Case D in Region AGR and two different depth levels for the horizontal diffusion parameterization in Figure 9b. Again, a vertical line indicates wavenumber 0.25 or wavelength 4° longitude. The two wave bands with significant coherence have quite distinctly different phases independent of case, parameterization, region or depth; at scales of 4° and smaller, the phase for Case D coherence is π/2. Likely, this is because Case D relates the first zonal derivative of temperature with the second derivative of temperature, if for the scales smaller than 4° the zonal velocity is constant with wavenumber and dominant over the meridional velocity. For scales larger than about 4° and for both cases F and D, the phase is zero, meaning that the eddy flux is down the mean temperature gradient.

There are a few caveats to this discussion. When considering the parameterizations as a function of wavenumber, we are looking at the filter of the whole terms rather than filtering temperature and density first and then computing the derivatives, as a coarse resolution model would effectively do. For the horizontal diffusion parameterization, this does not make a difference since the same result is achieved when filtering before or after taking the derivatives. For the GM and Redi parameterizations, the spectra have more energy at wavenumbers larger than the filter wavenumber when temperature and density are filtered first and then the derivatives are taken. Keep in mind that we compare the full eddy flux and its divergence with the different parameterizations rather than splitting the flux into advective and diffusive components. The relative importance and direction of the advective and diffusive eddy heat transport depends on the position of the isopycnal and isothermal layers to each other (e.g. Lee et al., 2006). Here, the Redi flux is small compared to the GM flux below the surface (see Figure 4) and hence coherence between the full flux or divergence of the flux is small for the Redi term.

5.2. Diffusivities as a function of wavenumber

Now that we have shown the filter scales and parameterizations that have significant coherence we investigate the diffusivities and how much they depend on the filter scale or wavenumber. Wavenumber dependent diffusivities may be an indication that coarse resolution numerical models with different resolutions may need different diffusivities in their parameterizations.

One dimensional diffusivities as a function of wavenumber can be computed from

$$\kappa = \frac{\sum_{i}^{N_x} X X^*}{\sum_{i}^{N_y} X Y^*},$$

where $X$ and $Y$ are zonal Fourier transforms and $N_y$ is the number of meridional grid points over which results are averaged ($N_y = 50$ for Region 360 and 100 for all other regions). Figure 10a shows diffusivities for Case F as computed with $X = \mathbf{w}^T$ and $Y$ as the horizontal diffusion (black), GM (red) and Redi (blue) parameterization of the meridional eddy heat flux for Region 360 at a depth of 918 m. Figure 10b shows the same but for case D and $X = \nabla_h \cdot (\mathbf{u}^H \mathbf{T}^H)$ and $Y$ as the parameterizations of the divergence of the eddy heat flux. Case D diffusivities for different regions at a depth 918 m for the horizontal parameterization are shown in Figure 10c and case D diffusivities for different depth levels in Region AGR and for the horizontal parameterization in Figure 10d.
Case F diffusivities are very wavenumber dependent reaching maxima for wavenumbers between 0.1 and 1 cycles/°longitude, corresponding to wavelengths between 100 and 1° longitude, and reaching values of up to 10000 m²s⁻¹ (Figure 10a). This wavenumber band corresponds to the waveband where most of the energy of the meridional eddy heat flux lies (see Figure 4h) and the unrealistically high diffusivities are due to the rotational component.

Diffusivities for Case D are less wavenumber dependent (Figure 10b-d) with highest values between 1000 and 2000 m²s⁻¹ and up to 3000 m²s⁻¹ for the surface in Region AGR with its high eddy activity. In general, the GM and horizontal parameterizations yield the same diffusivities, except for wavenumbers larger than about 1, where the GM parameterization has much more energy than the horizontal parameterization, and diffusivities for GM are smaller. The Redi parameterization on the other hand always yields higher diffusivities than the GM and horizontal parameterizations. This might be misleading since we assume that all of the eddy flux is represented by the Redi parameterization while that parameterization should only parameterize that part of the flux that is temperature downdraft.

Regions with higher eddy activity yield higher diffusivities (such as region AGR). Region SPA with very low eddy activity has diffusivities below about 200 m²s⁻¹ (Figure 10c). Also, diffusivities decrease with depth (Figure 10d).

The coherence analysis assumes that the diffusivities are constant within each region. Computing diffusivities by comparing the eddy flux with its parameterization for each grid cell yields extremely high variability in space. Instead of assuming constant diffusivities, we recomputed coherence by specifying spatially variable diffusion coefficients as in Visbeck et al. (1997) and Bryan et al. (1999), dependent on the growth rate of Eady waves and on appropriate eddy length scales. The diffusivities estimated in this way do not improve the coherence estimates.

6. Conclusions

The goal of this paper has been to assess eddy heat transport, its downgradient diffusive parameterization and their relation as a function of filtering length scale in a high resolution ocean model. One intention here was to consider relatively simple approaches for isolating a meaningful divergent component of the eddy heat flux. We have pursued a simple hypothesis that the rotational part might dominate on eddy scales but there might be a length scale over which we can filter that would reduce the rotational part. To assess filter scales, rather than attempting a hierarchy of filter lengths, we analyzed output in Fourier transformed wavenumber space.

We used the curl and divergence of the eddy heat flux as a unique measure of the rotational and divergent components respectively. The divergent component accounts for no more than 5% of the total eddy heat transport decreasing towards zero with increasing length scale. Hence, the rotational component dominates all length scales and cannot be eliminated simply by averaging over any length scale. Bryan et al. (1999) also found that the curl of the eddy buoyancy flux is larger than the divergence. This is probably no surprise for flows strongly constrained by rotation.

We used the ideas of MS to separate a rotational component from the total eddy heat flux. This rotational component again constituted a very large portion of the total heat flux and increased with depth, reflecting the extent to which eddies are along contours of eddy variance. This rotational part of the flux increased with increasing wavelength, just as did the curl, but it accounted for no more than 80% of the total eddy heat flux, leaving 20% of the flux as a residual. This residual is not purely divergent and it is 4 to 5 times larger than we would expect the divergent component to be. We conclude that the residual still appears to be dominated by the curl. Ideally, the general decomposition in rotational and residual components of Eden et al. (2007a) should lead to an improvement over the MS method in the sense that the residual flux better approximates the part that balances the conversion term \(wT^\gamma\partial_t T\).

However, our results suggest that the advective terms in the eddy variance equation are so dominant that it is difficult to extract the part that balances the conversion term. This is in agreement with Wilson and Williams (2004), who diagnosed fluxes of potential vorticity in an idealised eddy resolving model and found that they are controlled by the advection of the mean flow as well as the eddy advection. This result is not surprising. In fact, at present, it is difficult to measure the vertical velocity needed to compute the conversion term from observations.

In the coherence analysis for Cases F and D, we identified a characteristic length scale of 4° or wavenumber 0.25 around which coherence for both cases and most depth levels and regions had a minimum. This corresponds to a length scale below which most of the eddy heat flux and its divergence occurs. This is also the scale around which the curl of the eddy heat transport had a local maximum. Given the dominance of the curl, the significant coherence for case F on large scales must be due to the rotational component and is therefore not meaningful. The largest coherence for Case D was reached for scales smaller than 4°, where the curl tended to have a minimum. These are the scales that need to be parameterized in typical coarse resolution numerical ocean models. The variance explained of about 0.4 is consistent with other studies (Gille and Davis, 1999; Roberts and Marshall, 2000; Solovev et al., 2002). Some studies have speculated that low correlation between the eddy heat flux and its parameterization might be the result of poor eddy statistics with insufficient averaging in time (e.g. Rix and Willebrand, 1996). Here, we do not find any sensitivity of the coherence to averages over longer periods of time, and we suggest that the main reason for noisy patterns and low correlations is simply that the divergence is small and noisy which makes obtaining significant corre-
lations with a diffusive parameterization difficult. Unfortunately, averaging in space does not improve the correlation since the relative role of the divergence decreases with increasing lengthscale as compared to the curl. The hypothesis that large-scale averaging will lead to a more diffusive behaviour turns out to not be valid.

Diffusivities were estimated as a function of wavenumber. Diffusivities as estimated from Case D were mildly wavenumber dependent, whereas Case F diffusivities had a large wavenumber dependence with unphysical maximum diffusivities of up to 10000 m$^2$s$^{-1}$ close to the surface. Case D seems to give physically plausible diffusivities that are consistent with recent estimates using the divergence of the eddy buoyancy flux in a high resolution ocean model (Tanaka et al., 2007). Diffusivities of about 500 m$^2$s$^{-1}$ were found within the ACC west of Drake Passage and were highest in Region AGR, being up to 3000 m$^2$s$^{-1}$ close to the surface. The lowest diffusivities of about 200 m$^2$s$^{-1}$ were found for Region SPA, north of the ACC. Diffusivities decreased with depth for all regions. However, coherence is low in Case D, and very little of the eddy heat flux can be explained with parameterizations using these diffusivities.

Our results demonstrate that one needs to be cautious when interpreting correlations between eddy heat flux and temperature gradients. Neither high correlations nor diffusivity estimates need be associated with the relevant divergent component, even when fluxes are averaged over large spatial scales. Our findings underscore the importance of using the divergence of the eddy heat flux, as in (for example, Tanaka et al., 2007), despite its noisiness.
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Appendix A. Divergence Theorem

The integral over an area $A$ of the divergence of the eddy heat flux is equal to the net divergent flux through the boundaries of the area $A$ (see e.g. Fox-Kemper et al., 2003)

$$\int \int \nabla_h \cdot \left( \overline{u_h^T T'} \right) \, dA = \oint \overline{u_h^T T'|}_{div} \, d\mathbf{x} \quad (A.1)$$

Thus, the rotational part of the fluxes is eliminated in the integral around a closed contour, but not in the flux through any of the boundaries alone (see Figure 5).

Appendix B. Helmholtz decomposition

The eddy heat flux can be separated into rotational and divergent components through a Helmholtz decomposition (see e.g. Morse and Feshbach, 1953; Jayne and Marotzke, 2002; Fox-Kemper et al., 2003). The total horizontal eddy heat flux $\overline{u_h^T T'}$ is given by

$$\overline{u_h^T T'} = \overline{u_h^T T'_{div}} + \overline{u_h^T T'_{rot}}, \quad (B.1)$$

where the divergent heat flux is the gradient of a scalar potential $\Phi$

$$\overline{u_h^T T'_{div}} = \nabla_h \Phi, \quad (B.2)$$

and the rotational heat flux $\overline{u_h^T T'_{rot}}$ can be written with a streamfunction $\Theta$:

$$\overline{u_h^T T'_{rot}} = k \times \nabla_h \Theta, \quad (B.3)$$

Taking the divergence of the total horizontal eddy heat flux (B.1), and using (B.2) and (B.3) gives:

$$\nabla_h^2 \Phi = \nabla_h \cdot \overline{u_h^T T'} \quad (B.4)$$

and

$$\nabla_h^2 \Theta = k \times \nabla_h \cdot \overline{u_h^T T'}. \quad (B.5)$$

One can solve for the potentials by applying a Laplacian inverter (e.g. Jayne and Marotzke, 2002) using appropriate boundary conditions for both rotational and divergent parts.