1 Lagrangian and Eulerian diffusivities

1.1 Lagrangian

Lagrangian diffusivities are calculated in bins (5° in latitude and 10° in longitude) as in equation 1 (after Davis, 1987, 1991; Poulain et al., 1996; McClean et al., 2002): Each daily drifter observation for each bin is treated as a deployment point $P_0$ (at $x, t_0$) and diffusivities as a function of time lag are calculated following float trajectories arriving at that point (positive time lag) and leaving from that point (negative time lag). Here, a timelag of ±45 days is used. The Lagrangian average $\langle \rangle_L$ below then is an average over all deployment points for each bin. Here, a minimum number of 250 points is used.

$$\kappa_{ij}(\tau) = -\langle v'_i(t_0|x, t_0)r'_j(t_0+\tau|x, t_0)\rangle_L$$

(1)

where $r'$ is the displacement and the velocity $v'$ is calculated as follows: The Eulerian annual mean velocity over 1998 for each grid point is interpolated to the drifter locations and then subtracted from the drifter velocities.

1.2 Eulerian

We are looking at the simple downgradient parameterization

$$\overline{v' T'} = \kappa_y \partial_y \overline{T}$$

(2)

The Eulerian "Case FG" (for Flux and Gradient) meridional diffusivities are calculated as follows: The meridional 1998 annual mean temperature gradient and meridional eddy heat flux (as calculated from the deviation of the 1998 annual mean heat flux) are interpolated to the drifter locations and then averaged over each bin. Then diffusivities are calculated from the ratio. One can distinguish the "Case FG" diffusivities from the ones calculated from the divergence of the eddy heat flux and the Laplacian of mean temperature ("Case DL" for Divergence and Laplacian), since these are the terms that appear in the tracer balance equations. The divergence of the eddy heat flux eliminates its rotational
part. The Case FG diffusivities incorporate a large contribution of the rotational part that
does not influence the mean heat budget.

2 Comparison

We are looking here at the deployments for PATCH3. The upper right panel of Figure 1
shows Lagrangian meridional diffusivities for Patch 3, 800 m-deployment 1, as calculated
from the maximum of the diffusivities in the 0-45 day timelag frame, which gives some
sort of upper boundary. The lower left panel of figure 1 shows the Lagrangian meridional
diffusivities at the maximum timelag of 45 days, which are smaller (see figures 3 and 4
for diffusivities as a function of timelag). The lower right panel shows the Eulerian ”Case
FG” diffusivities. They are small (in the order 10-1000 m² s⁻¹, and they can be negative,
too): meridional eddy heat transport is relatively small within the ACC (it is high on
the northward flanks) and meridional temperature gradients are large, therefore their ratio
becomes small. The scatter plot in figure 2 compares the Eulerian Case FG diffusivities and
Lagrangian ones at 45 day timelag for the three different deployment depths (300m, 800m,
1500m). Eulerian diffusivities are about 2 times smaller than the Lagrangian ones, which
means that Lagrangian diffusivities overestimate the model meridional eddy heat transport.
It remains to be figured out to what extent the Lagrangian diffusivities reflect the rotational
component of the eddies. A next step is to look at the divergence of the eddy heat flux and
its parameterization:

\[ \nabla \cdot (uT) = \partial_x (\kappa_{xx} \partial_x T) + \partial_y (\kappa_{yy} \partial_y T) + \partial_x (\kappa_{xy} \partial_x T) + \partial_y (\kappa_{yx} \partial_y T) \]  \hspace{1cm} (3)

One can insert the Lagrangian diffusivities in the RHS of equation 3 (the two last terms
are probably small) and compare with the LHS.

2.1 Diffusivities as a function of timelag for each bin

Figures 3 and 4 show how well (or not) meridional and zonal diffusivities converge for
some of the bins of the 800m and 1500m deployments.
3 Questions / Issues

Dependence on bin-size, exclude floats that reach the surface/mixed layer, bin in depth, dependence on what mean to subtract (1998 mean vs three-year-mean vs ..?), which eddy heat flux to compare to (deviation from 1998 annual mean, deviation from three-year-mean ..?), longer timelags ...

etc.
Figure 1: Patch 3, deployment 1, deployment depth 800m Upper Left: Number of float trajectory points in bin. Upper Right: Lagrangian meridional diffusivities from the maximum in the 45 day timelag frame. Lower Left: Lagrangian meridional diffusivities at maximum timelag 45 days. Lower Right: Case FG, Eulerian meridional diffusivities from ratio of 1998 mean meridional eddy heat flux and meridional mean temperature gradient averaged over each bin.
Figure 2: Patch 3, deployment 1: Scatter plot of “Case FG” Eulerian diffusivities and Lagrangian diffusivities at maximum timelag for the three different deployment depths.
Figure 3: Patch 3, deployment 1, deployment depth 800m: Meridional (red) and zonal (blue) diffusivities as a function of time lag and number of points in the bin, i.e. the number over which Lagrangian average is taken (NaN means less than 250 points). BINS 240°E - 300°E.
Figure 4: Patch 3, deployment 1, deployment depth 1500m: Meridional (red) and zonal (blue) diffusivities as a function of time lag and number of points in the bin, i.e. the number over which Lagrangian average is taken (NaN means less than 250 points). BINS 180°E - 240°E.
References


