GFD Homework 1: all about Coriolis

DUE 26 Jan at the start of class

General comments: Feel free to work together in groups, but please write up your own work. If you have a question or need a hint, make an appointment to come see me. If there are lots of you with questions, I might try to schedule a particular set of office hours in which we can all discuss things. I can also hold an office hour to go over the solutions later if there's interest.

1. Consider a steady wind stress imposed by a mesoscale cyclonic atmospheric storm given by

$$\begin{aligned} \vec{\tau} &= [\tau^x, \tau^y] \\ \tau^{\mathrm{x}} &= -y\,A\,e^{-(r/\lambda)^2} \\ \tau^{\mathrm{y}} &= x\,A\,e^{-(r/\lambda)^2} \end{aligned}$$

   where $r^2 = x^2 + y^2$, and A and λ are constants.

   (a) Sketch the wind stress on the ocean surface as seen from above.

   (b) Calculate and sketch the Ekman transport (you can use either Cartesian or cylindrical coordinates, whatever you prefer)

   (c) Calculate the vertical Ekman pumping velocity, w_E, that results from convergences or divergences in the Ekman transport

   (d) At what value of r is there maximum upward w_E? How about maximum downward w_E?

2. In class we derived the oceanic response to a constant wind stress. Now we'd like to expand that analysis to the more realistic case of a time-varying wind stress. Let's start with the vertically averaged equations,

$$\frac{d\bar{u}}{dt} = f\bar{v} + \frac{\tau^x(t)}{\rho_0 H} \tag{1}$$

$$\frac{d\bar{v}}{dt} = -f\bar{u} + \frac{\tau^y(t)}{\rho_0 H} \tag{2}$$

   where $(\bar{u}, \bar{v})$ are the vertically averaged eastward and northward velocities and H is the vertical extent of the layer you've averaged over. Start by using complex velocity and defining the velocity as the sum of inertial and Ekman parts:

$$\tilde{u} = \tilde{u_E} + \tilde{u_I}$$

   Now we're going to say that the Ekman transport, u_E, has the same form as the steady state solution we derived in class, but we're now allowing the wind stress to vary in time. So we get something like this

$$\tilde{u}_E = \frac{-i\tilde{\tau(t)}}{\rho_0 f H}$$

   Note that this is physically a bit funny, since it takes a finite period of time for the Ekman spiral and transport to be set up. But take it as a mathematical decomposition and bear with me for a moment.

   (a) Plug this decomposition for total $\tilde{u}$ into the governing equations (1-2 above) and rearrange to get an equation for the time evolution of the inertial response, $\frac{d\tilde{u_I}(t)}{dt} = ...$

(b) Imagine an impulsive wind stress that imparts some initial velocity, $\tilde{u_{I0}}$. Write an equation for the velocity as a function of time **after** the wind becomes steady ($d\tau/dt = 0$). For a typical $|\tilde{u_{I0}}| = 5$ cm/s, integrate to find the radii of inertial circles at 1 and 60 degrees latitude - how far does a parcel move?

(c) Now bring back a time-dependent wind stress. In particular, consider a quasi-pathological case where the wind stress is rotating as $\tilde{\tau} = \tau_0 e^{-ift}$. Assume the inertial response can be written as the product of a slowly varying amplitude and an inertial term, e.g. $\tilde{u_I} = A(t)e^{-ift}$. Plugging this into your answer to part (a), what's the equation for the evolution of $A(t)$?

(d) After 3 days, what is a typical magnitude of the inertial current compared to the Ekman current? (for typical parameters use $\tau_0 = 0.1\text{Nm}^{-2}, \rho_0 = 1025\text{kg}\,\text{m}^{-3}\,, f = 10^{-4}\,s^{-1}, \text{H} = 50\text{m}$).

(e) Consider the response of the ocean to the passage of a hurricane in which wind is rotating CCW in space (in the NH). Do you expect a stronger inertial response on one side of the hurricane wake than the other? (think about how the wind stress evolves in time at a particular location as the hurricane passes overhead).

3. Lenn and Chereskin paper: As with all the papers we'll read in this class, the goal is to start getting familiar with how people approach active areas of research and how messy the real ocean an atmosphere can be, compared to the simple equations we study in class. To that end, try to get the overall gist of the paper without worrying excessively about details or terminology you don't understand. That being said, if you'd like to come by to ask questions about things you're not familiar with or are interested in, stop by anytime. To help focus your reading, please turn in relatively brief ($\sim 1-2$ paragraphs each) answers to the following questions.

   (a) Discuss some of the difficulties in observing an Ekman spiral in the real world.

   (b) Describe some of the ways the Ekman flow observed here differs from what you'd expect from the equations we derived in class, and comment on why that might be.

   (c) Suggest some methods (either further observations, numerical simulations or theoretical work, depending on what you're personally interested in) that you might use to answer some of the open questions presented here.