GFD Homework 2

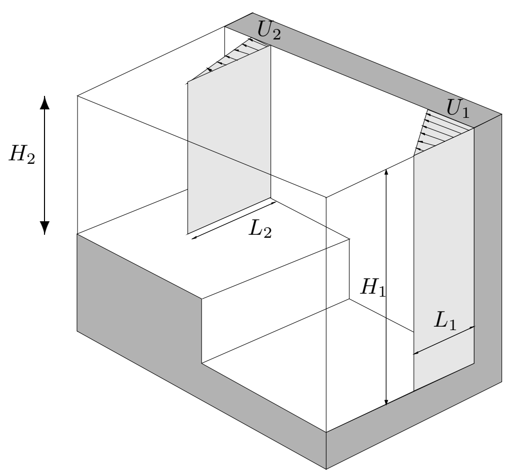
DUE 11 Feb in class

1. Balanced flows:

   (a) Show that a purely zonal flow [u=u(y), v=0] in geostrophic balance with a meridional pressure field [$\eta = \eta(y)$] is an exact solution of the steady inviscid *fully nonlinear* shallow water equations.

   (b) Consider a circular pressure field $\eta = \eta(r)$, where $r = \sqrt{x^2 + y^2}$. Show that one can find an azimuthal velocity field $u_a(r)$ such that η and u_a satisfy steady, inviscid nonlinear shallow water equations under rotation. Discuss the balance of forces in this case. Such a balance is called cyclostrophic. [hint - it's easiest if you switch to cylindrical coordinates]

   (c) Does cyclostrophic balance cause the flow to go around faster or slower than geostrophic balance for the same pressure distribution? Consider both positive and negative pressure cases



2. As depicted in the figure above (from C-R Chapter 7), a vertically uniform but laterally sheared northern hemisphere coastal current must climb a bottom escarpment. Assuming that the jet velocity still vanishes offshore, use conservation of the *nonlinear shallow water PV* (e.g. DQ/Dt=0 following a water parcel) to determine the velocity prole and the width of the jet downstream of the escarpment. Use $H_1 = 200$ m, $H_2 = 160$ m, $U_1 = 0.5$ m/s, $L_1 = 10$ km and f = 10^{-4} s^{-1}. What would happen if the downstream depth were only 100 m?

3. Two-layer geostrophic adjustment. Consider a two-layer fluid with resting depths H_1, H_2 and densities ρ_1, ρ_2. We will discuss the setup for this problem in class on Tuesday, or you can start by yourself. Let's consider the baroclinic version of the "dam break problem"- in particular assume that at time=0 both layers are at rest and the surface height (η) and the interface height (h) are given by:

$$\eta(t=0) = 0 \quad (1)$$
$$h(t=0) = h_0 \quad x <= 0 \quad (2)$$
$$h(t=0) = -h_0 \quad x > 0 \quad (3)$$

(a) Making a rigid lid assumption ($\eta << h$), and assuming a small density difference ($\rho_1/\rho_2 \approx 1$) and assume that PV is conserved within each layer (we'll go over this in class Tuesday, but it's also straightforward to just extrapolate what you have in your notes from 1 to 2 layers), derive a single differential equation that governs the interface height once the system has reached a steady-state solution. It's easier to do this for $x > 0$ and $x < 0$ separately.

(b) Assuming the initial displacement is small $h_0 << H_1, H_2$, rewrite this equation in terms of the baroclinic Rossby radius: $a'^2 = g' H_{\text{eff}}/f^2$, $H_{\text{eff}} = (H_1 H_2)/(H_1 + H_2)$

(c) Solve for $h(x)$ in the entire domain and sketch the solution. You'll need to apply reasonable matching conditions at x=0.

(d) What are typical values of the barotropic and baroclinic rossby radii of deformation? (use $H_1 = 100m, H_2 = 1000m, f = 10^{-4}s^{-1}$)