GFD Homework 3

DUE 8 March in class

1. Topographic Rossby waves. Think back to the QG equation for a single, homogenous layer of fluid. Now let's say that the lower boundary is sloped, as would happen, for example, when we're near the coast. Our local coast runs roughly north/south, and we can approximate the bottom slope linearly as αx where α is small, at which point the total water depth becomes

$$H_0 + \eta - \alpha x$$

The QGPV equation becomes:

$$\frac{Dq}{Dt} = 0 \qquad\qquad q = \zeta + \beta y - \frac{f}{H_0}(\eta - \alpha x)$$

where η is still potentially a function of both x and y.

(a) Write out the linearized form of the QG equation in terms of the streamfunction Ψ

(b) Assuming a wave-like solution for the streamfunction, find the dispersion relation (e.g. $\omega = ...$)

(c) Compute the along-isobath phase speed. In the Northern Hemisphere, do topographic waves propagate with the shallower water on their right or left?

(d) Now let's compare a purely topographic Rossby wave ($\beta = 0, \alpha \neq 0$) with a traditional Rossby wave ($\beta \neq 0, \alpha = 0$) . How steep does the bottom slope have to be for a topographic wave (ignoring beta) to have a comparable magnitude frequency as a traditional Rossby wave? (assume a latitude of 45N and $H_0 = 2000$ m).

(e) Compare your answer to typical steepnesses of real continental slopes.

2. In class we derived equations that govern the vertical mode shapes for both Rossby and internal gravity waves. For Rossby waves, for example, if we assume N varies slowly with depth the equation looks like:

$$\frac{d^2\hat{\Psi}}{dz^2} = \frac{1}{f_0^2}\left(k_H^2 + \frac{k\beta}{\omega}\right) N^2(z)\Psi(\hat{z})$$

And the solutions are a *quantized* set of vertical modes. Show that, even without assuming N to be constant, the modes are **orthogonal** to each other in the particular sense that

$$\int_0^H N^2(z)\hat{\Psi}_n(z)\hat{\Psi}_m(z) = 0 \text{ (if m} \neq \text{n)}$$

This means that you can formally project **any** initial perturbation onto a complete set of modes, which then propagate away at their own speeds.

3. In class Thursday we will derive the dispersion relation for two-layer QG flow with a background mean sheared flow that can lead to baroclinic instability when ω has an imaginary component. We will mention that for the limiting case of $\beta = 0$, the dispersion relation reduced to a simple limit in which there was an instability for ALL values of U, no matter how small. For the full dispersion relation,

$$\omega = \frac{-k\beta}{k_h^2 + k_i^2}\left[1 + \frac{k_i^2}{2k_h^2} \pm \frac{k_i^2}{2k_h^2}\sqrt{1 + \frac{4k_h^4(k_h^4 - k_i^4)}{k_\beta^4 k_i^4}}\,\right]$$

where $k_h = \sqrt{(k^2 + l^2)}$.

(a) For given values of β and k_i, what's the minimum value of U (let's call it U_{min}) that allows growing instability for any wavelength? In other words, what's the smallest U for which there is at least SOME instability possible.

(b) For $U = 2U_{\text{min}}$ and l=0, instability will be possible for a range of wavelengths. What is the k of the motion that grows fastest in time?

(c) Now we'll compare these results with the observed results of baroclinic instability in the atmosphere and ocean. To do so, let's say we're at a mid-latitude of 40N, g'=0.5 (atmosphere), g'=0.01 (ocean), and characteristic "layer" heights are 5 km (atmosphere) and 500 m (ocean). The former is approximately half the tropopause height while the later is a characteristic length scale of the ocean pycnocline. Plug in numbers to get the fastest growing wavelength for the ($U = 2U_{\text{min}}$) case in each fluid.

(d) What is the e-folding time-scale of growth, in days?

(e) Using your no-doubt excellent web-surfing skills, compare these numbers to typical sizes of both mid-latitude atmospheric storms and ocean eddies (look in the Gulf Stream extension or Agulhas retroflection for good examples). Are they similar?