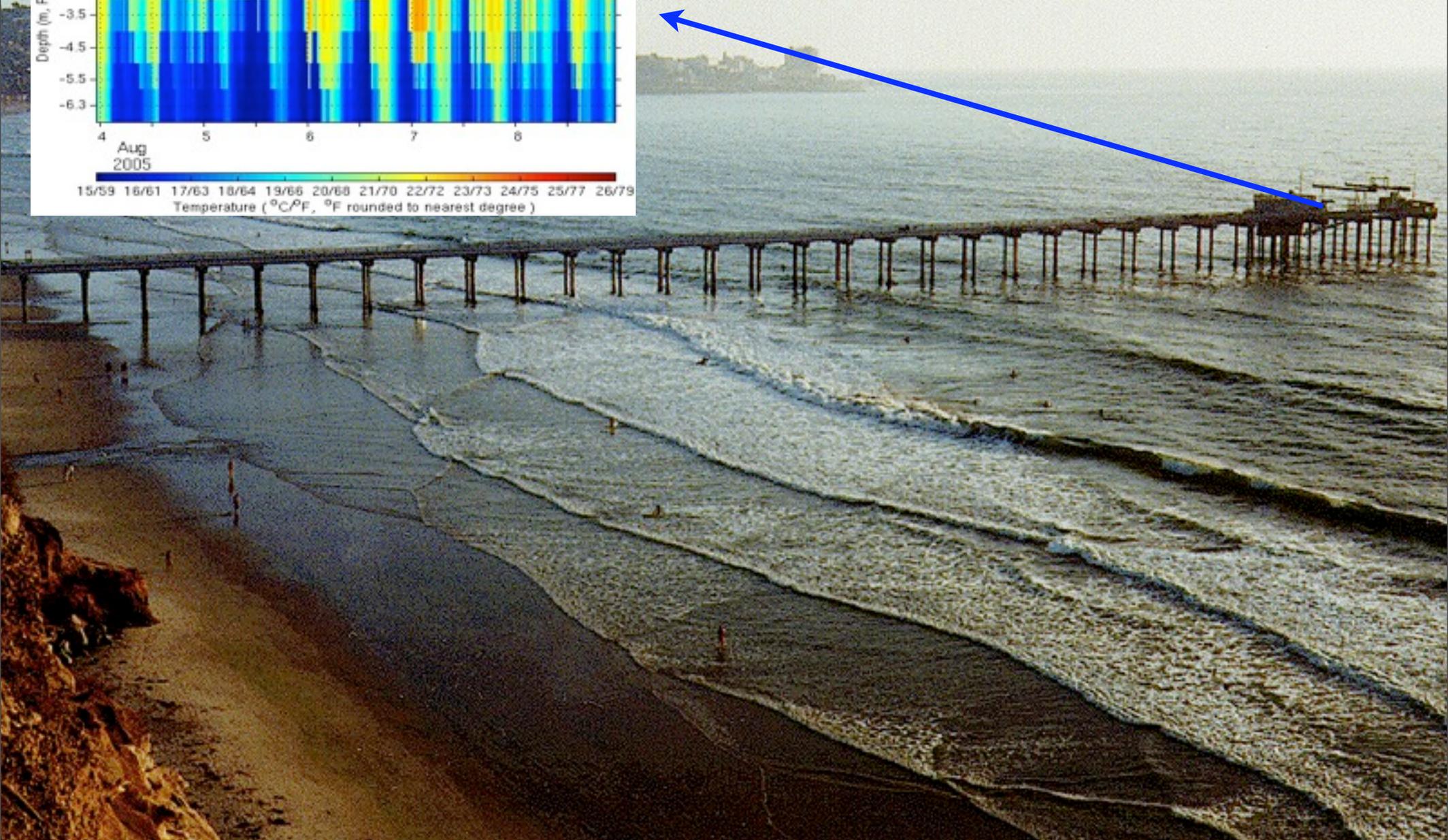
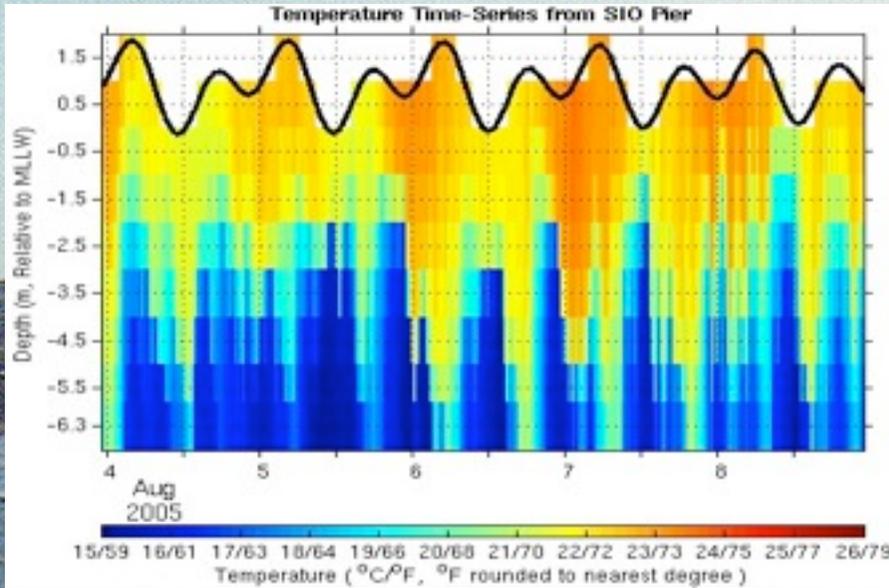


# Erratic internal waves at SIO Pier

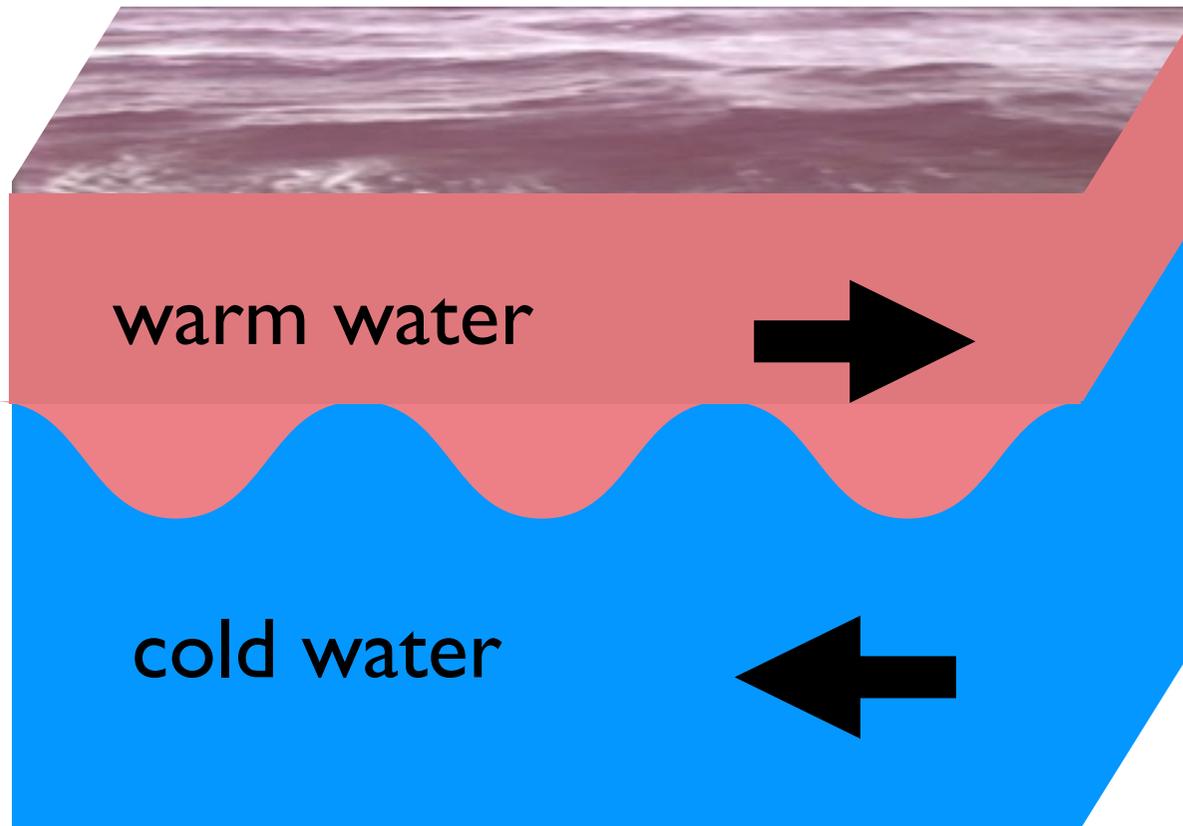
data and wavelet analysis courtesy of E. Terrill, SIO



# Surface Waves



# Internal Waves



- Slow

$$f \leq \omega \leq N$$

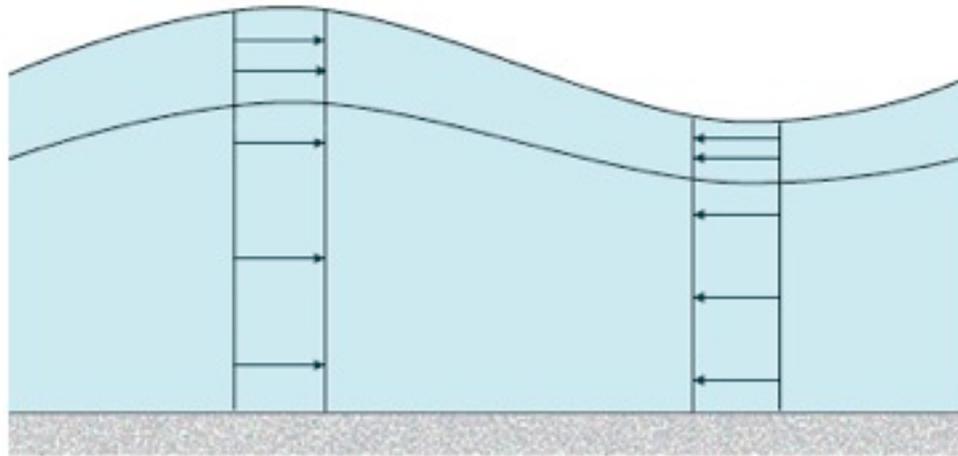
~1 day

~hours

- Big

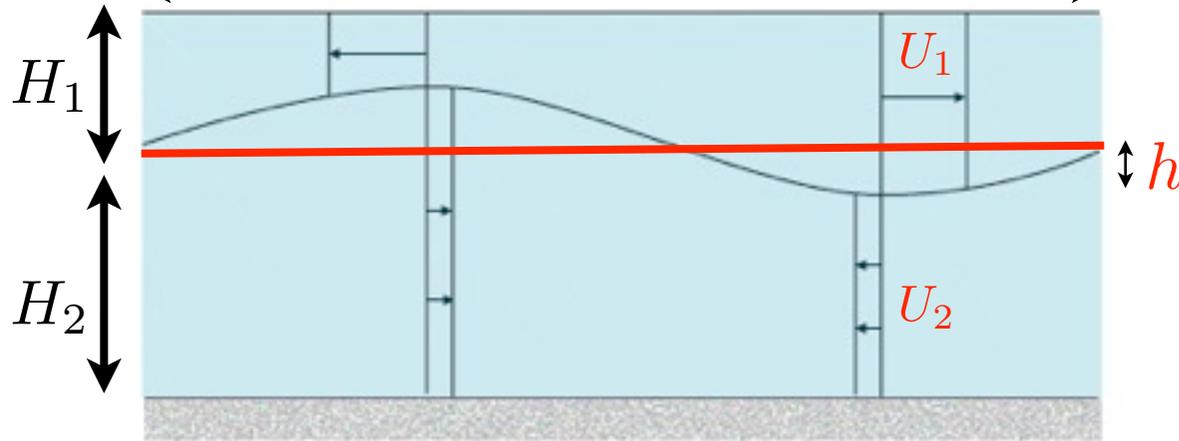
tens of meters

# Simple interfacial internal wave



Barotropic Wave

$2\pi/k =$  one wavelength



Baroclinic Wave

$$h = -h_0 \cos(kx - \omega t)$$

$$U_1 = \frac{\omega h_0}{H_1 k} \cos(kx - \omega t)$$

$$U_2 = -\frac{\omega h_0}{H_2 k} \cos(kx - \omega t)$$

after Gill,  
*Atmosphere-Ocean Dynamics*

# Internal wave equations

## Linearize equations of motion

$$\frac{\partial u}{\partial t} = -\vec{u} \cdot \nabla u + fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u$$

$$\frac{\partial v}{\partial t} = -\vec{u} \cdot \nabla v - fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v$$

$$\frac{\partial w}{\partial t} = -\vec{u} \cdot \nabla w - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w - g$$

$$\frac{\partial \rho}{\partial t} = -\vec{u} \cdot \nabla \rho + \kappa \nabla^2 \rho$$

$$\nabla \cdot \vec{u} = 0$$

# Internal wave equations

## Linearize equations of motion

$$\begin{aligned}\frac{\partial u}{\partial t} &= -\vec{u} \cdot \nabla u + fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ \frac{\partial v}{\partial t} &= -\vec{u} \cdot \nabla v - fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \\ \frac{\partial w}{\partial t} &= -\vec{u} \cdot \nabla w - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w - g \\ \frac{\partial \rho}{\partial t} &= -\vec{u} \cdot \nabla \rho + \kappa \nabla^2 \rho \\ \nabla \cdot \vec{u} &= 0\end{aligned}$$

# Internal wave equations

## Linearize equations of motion

$$\begin{aligned}\frac{\partial u}{\partial t} &= -\vec{u} \cdot \nabla u + fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ \frac{\partial v}{\partial t} &= -\vec{u} \cdot \nabla v - fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \\ \frac{\partial w}{\partial t} &= -\vec{u} \cdot \nabla w - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w - g \\ \frac{\partial \rho}{\partial t} &= -\vec{u} \cdot \nabla \rho + \kappa \nabla^2 \rho \\ \nabla \cdot \vec{u} &= 0\end{aligned}$$

Try a solution of the form

$$u(x, y, z, t) = \hat{u} e^{-i[kx + ly + mz - \omega t]}$$

Get polarization and dispersion relationships

$$\omega^2 = \frac{(k^2 + l^2) * N^2 + m^2 * f^2}{k^2 + l^2 + m^2}$$

# Internal wave equations

## Linearize equations of motion

$$\begin{aligned}\frac{\partial u}{\partial t} &= -\vec{u} \cdot \nabla u + fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ \frac{\partial v}{\partial t} &= -\vec{u} \cdot \nabla v - fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \\ \frac{\partial w}{\partial t} &= -\vec{u} \cdot \nabla w - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w - g \\ \frac{\partial \rho}{\partial t} &= -\vec{u} \cdot \nabla \rho + \kappa \nabla^2 \rho \\ \nabla \cdot \vec{u} &= 0\end{aligned}$$

Try a solution of the form

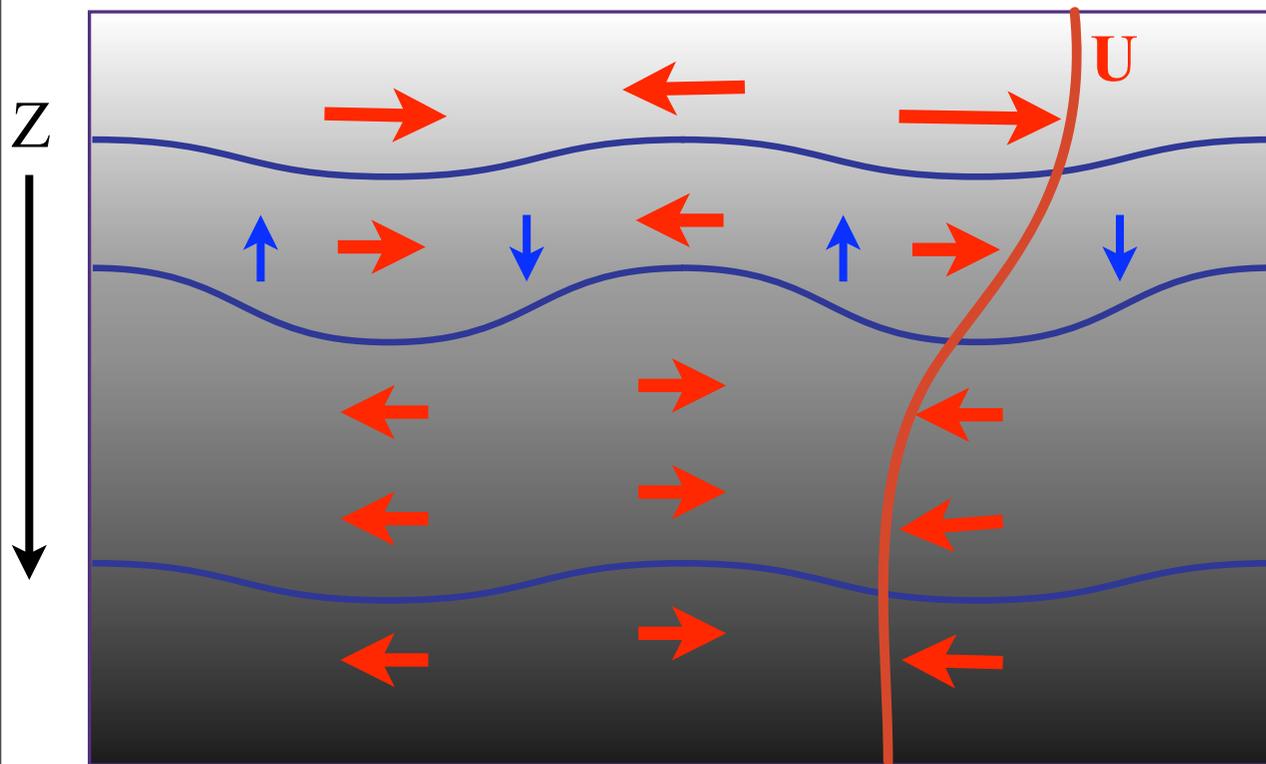
$$u(x, y, z, t) = \hat{u} e^{-i[kx + ly + mz - \omega t]}$$

Get polarization and dispersion relationships

$$\omega^2 = \frac{(k^2 + l^2) * N^2 + m^2 * f^2}{k^2 + l^2 + m^2}$$

(Glenn Flierl)

# Continuous stratification



Mode-1 wave  
(approx two-layer)

$$U = \Psi(z)\cos(kx - \omega t)$$

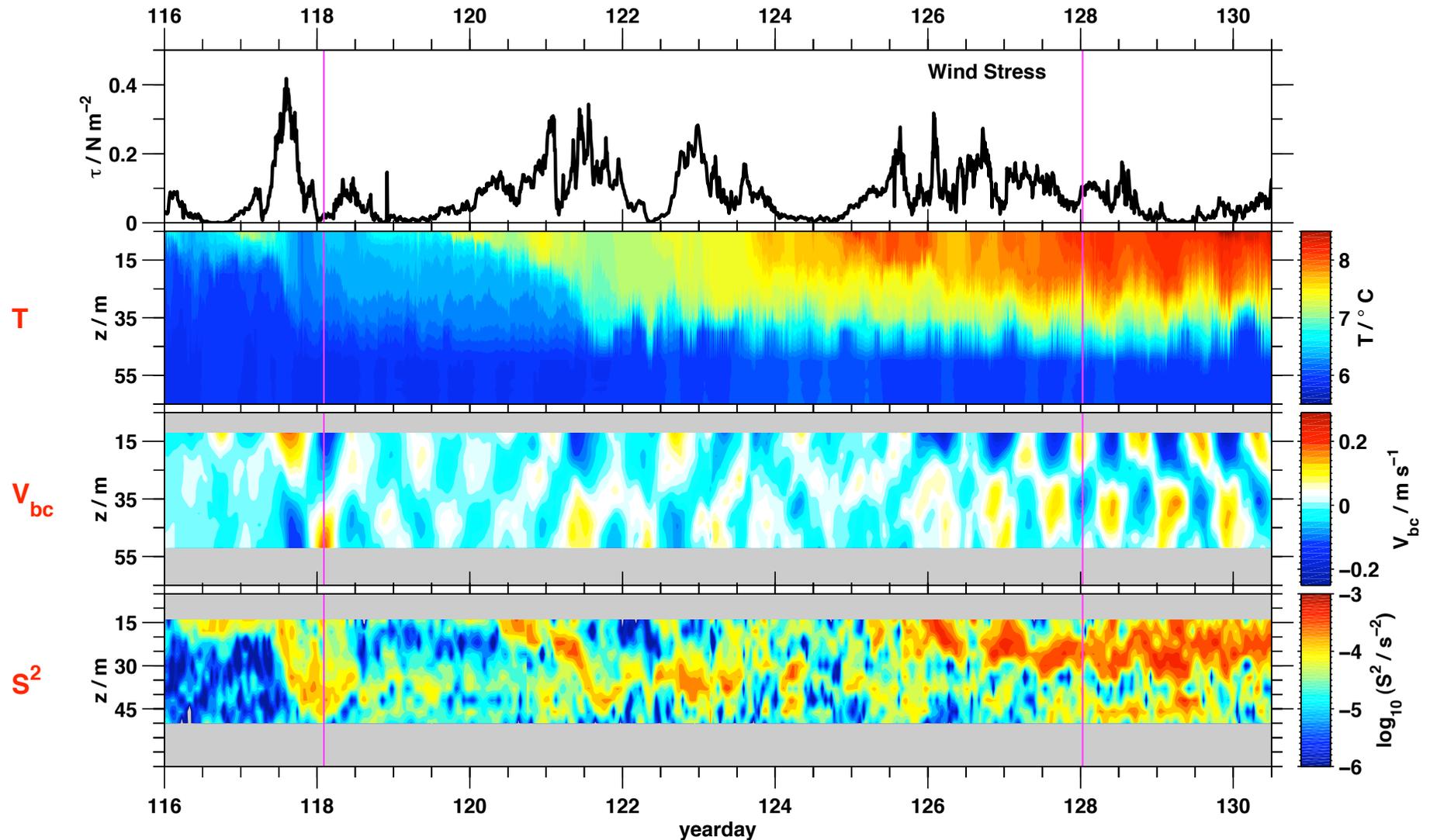
Allowable frequency range

$$f \leq \omega \leq N$$

days to minutes

# What generates internal waves?

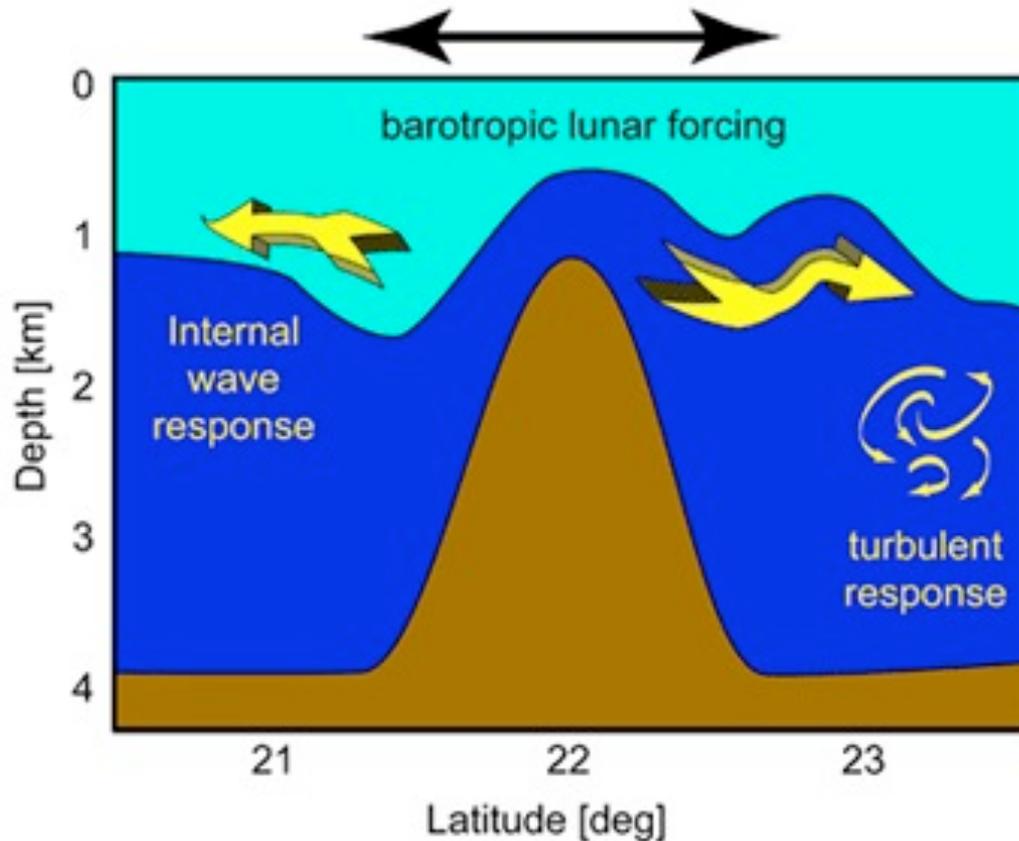
## 1) Wind makes near-inertial internal waves



(MacKinnon and Gregg, JPO, Dec 05)

# What generates internal waves?

## 2) Barotropic tide sloshing over topography

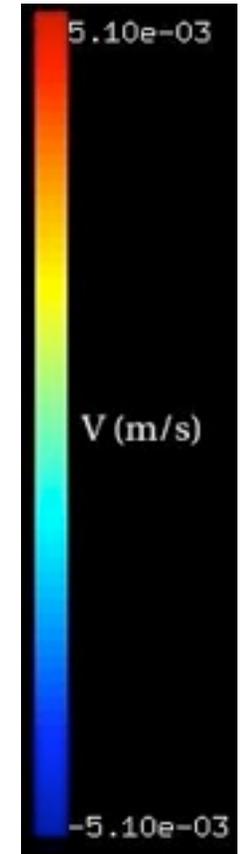
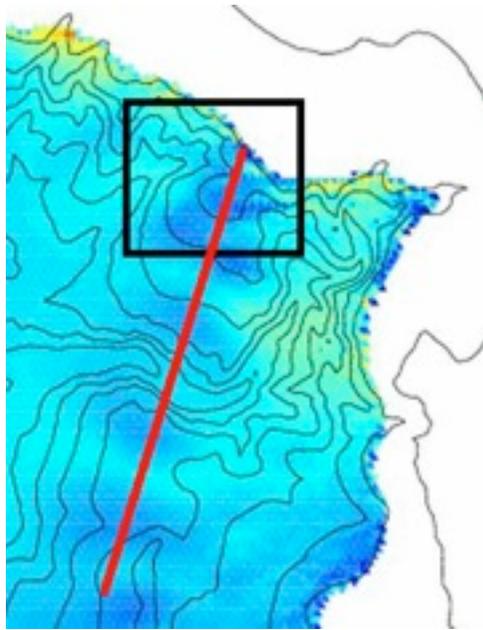


**Internal Tide:** An internal wave with a tidal frequency, usually once in 12.4 hours = M2

Often generated at the continental shelf break, with waves propagating both on and off shore.

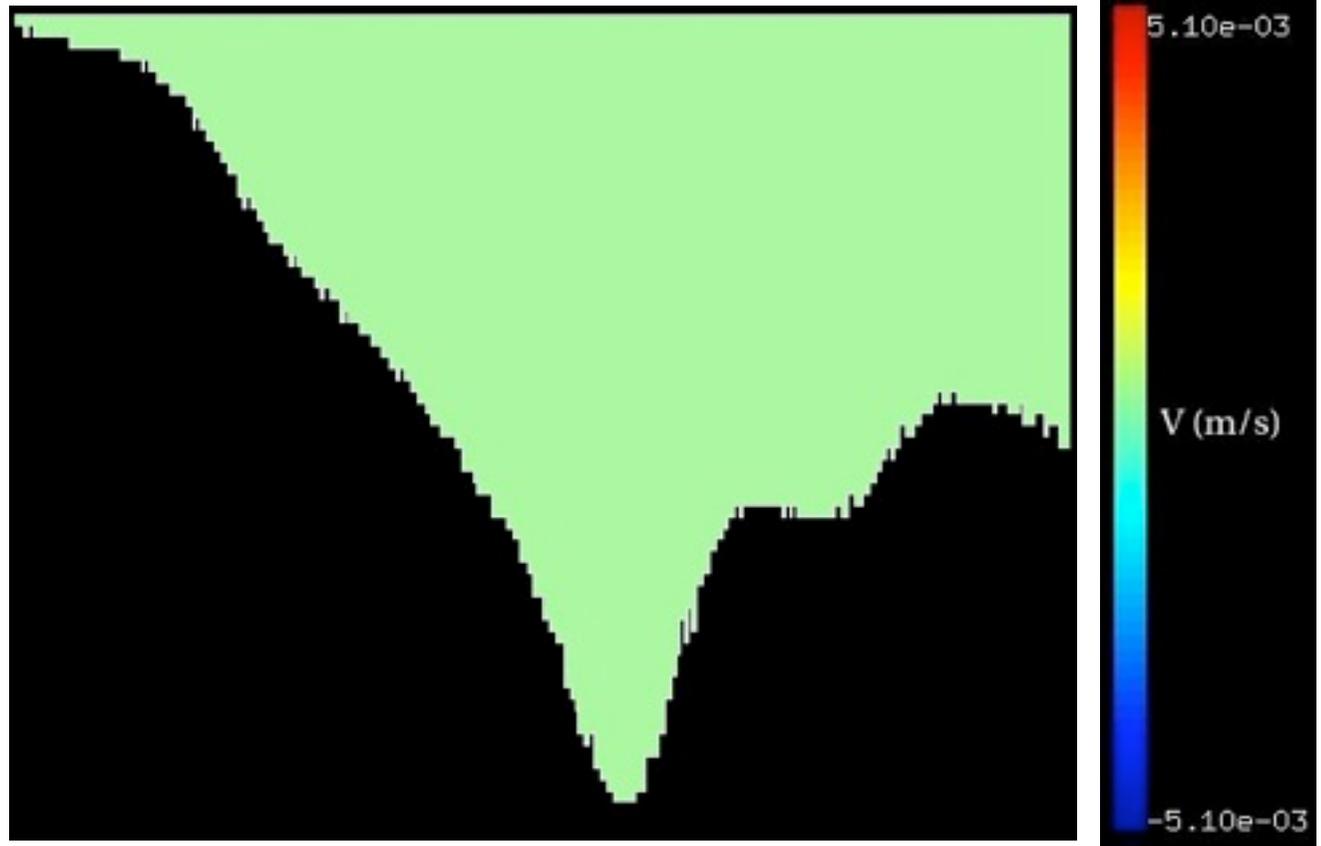
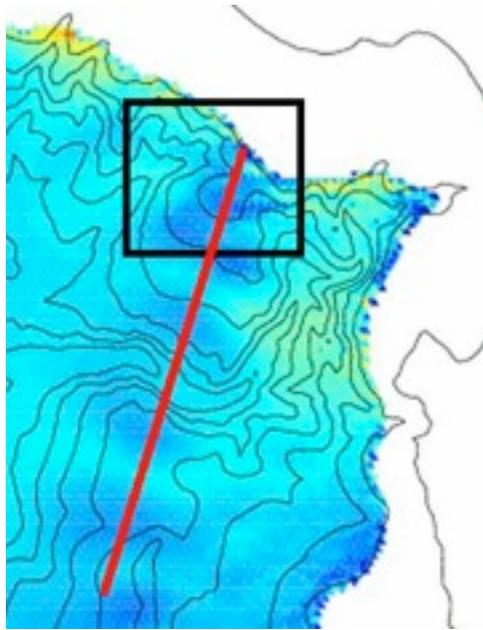
(J. Nash)

# Internal-tide generation in Monterey Bay



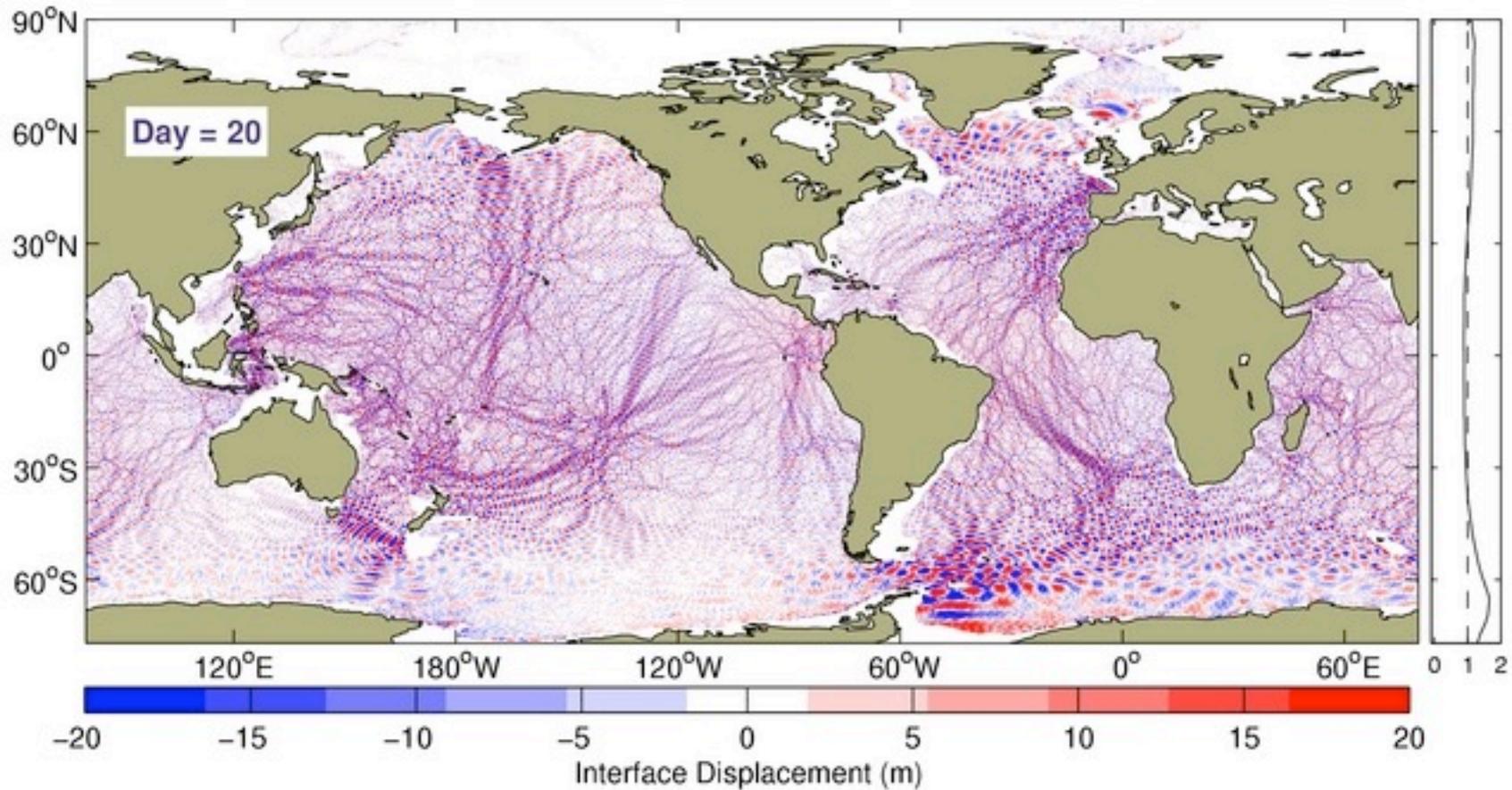
courtesy of  
Oliver Fringer

# Internal-tide generation in Monterey Bay



courtesy of  
Oliver Fringer

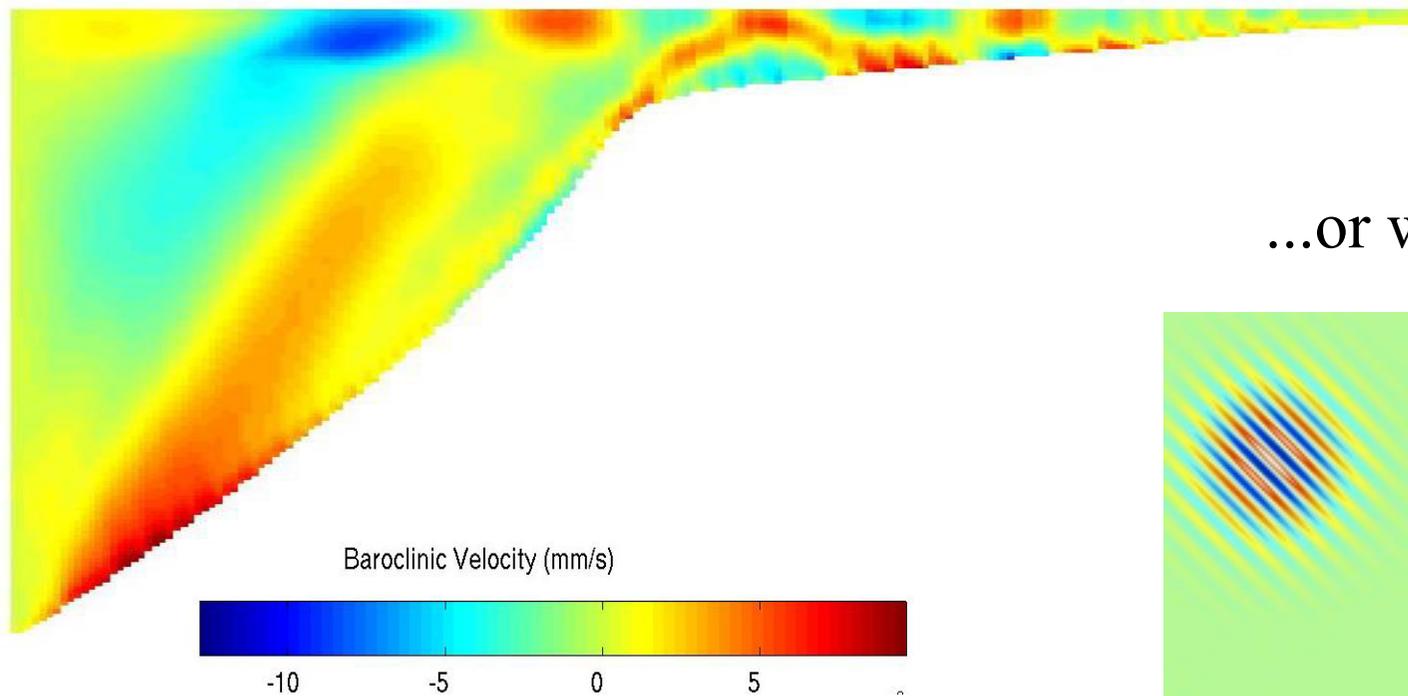
# Global pattern of internal tides



Simmons et al 2004

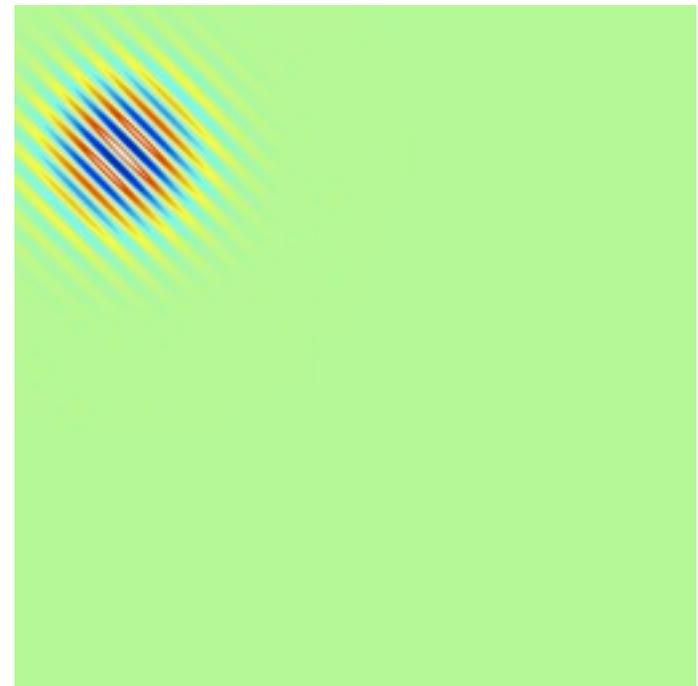
# Complicating factors: higher-mode waves

Waves propagate in beams...



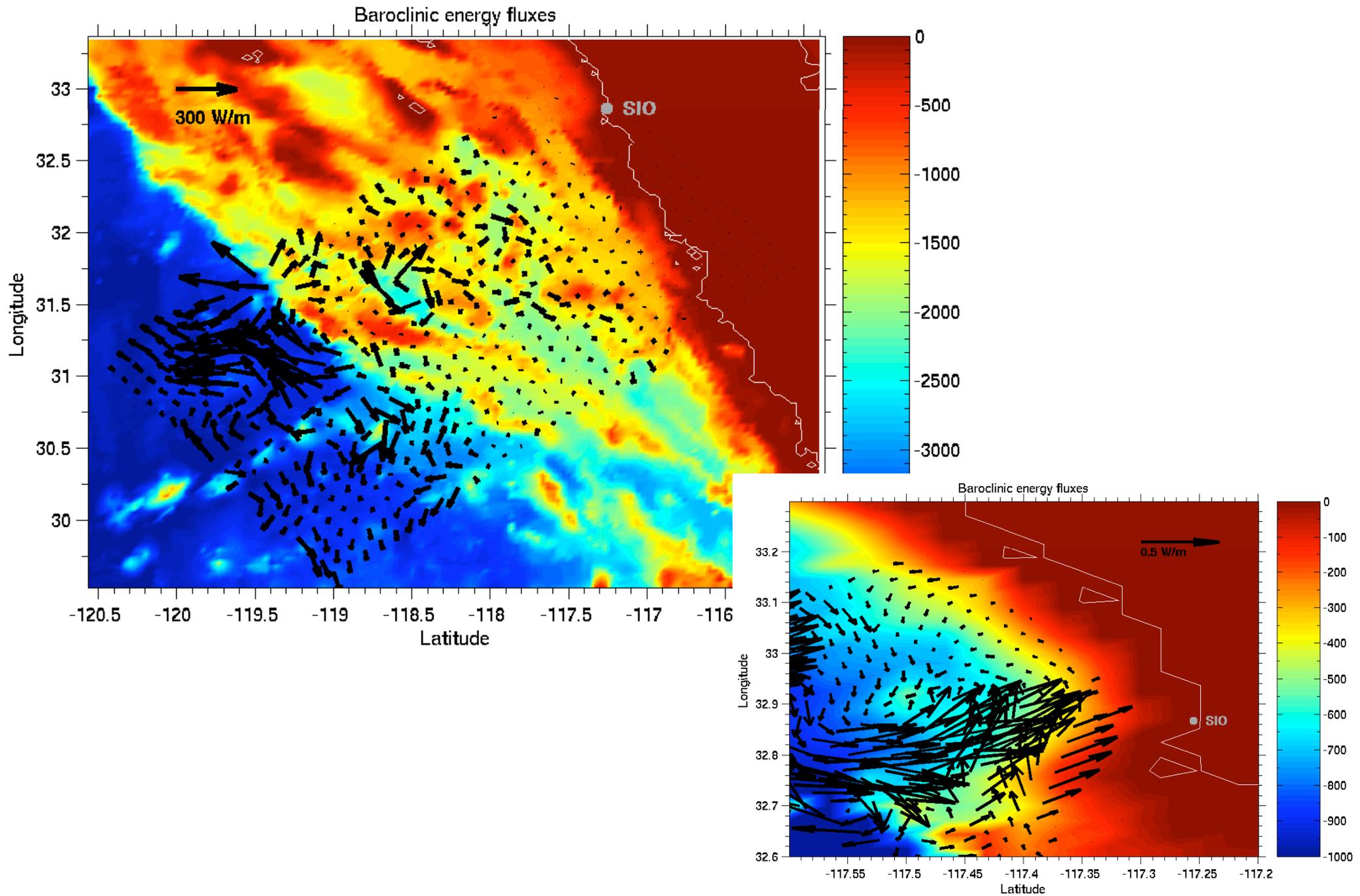
(Oliver Fringer)

...or wave packets



(Glenn Flierl)

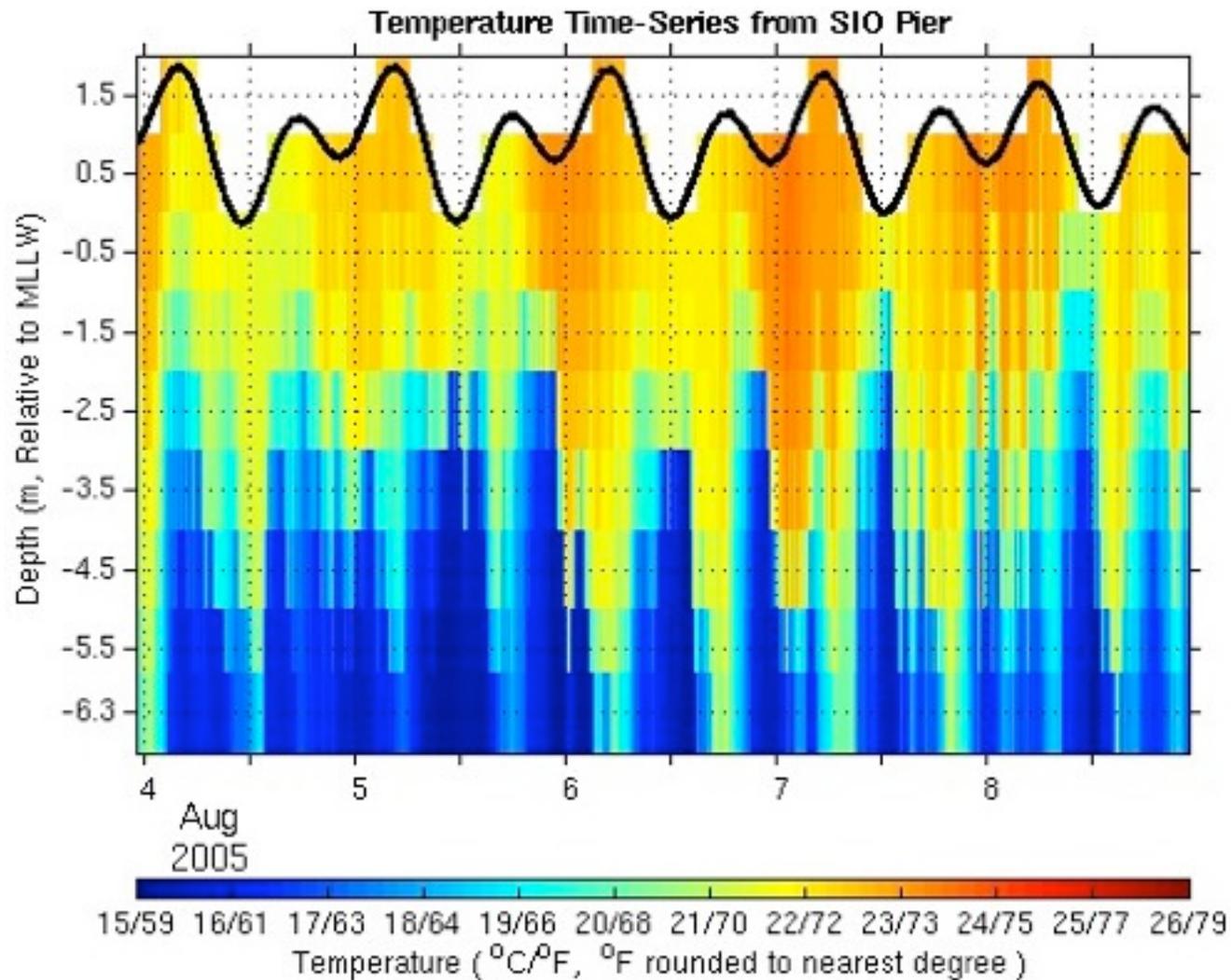
# Complicating factors: complex topography





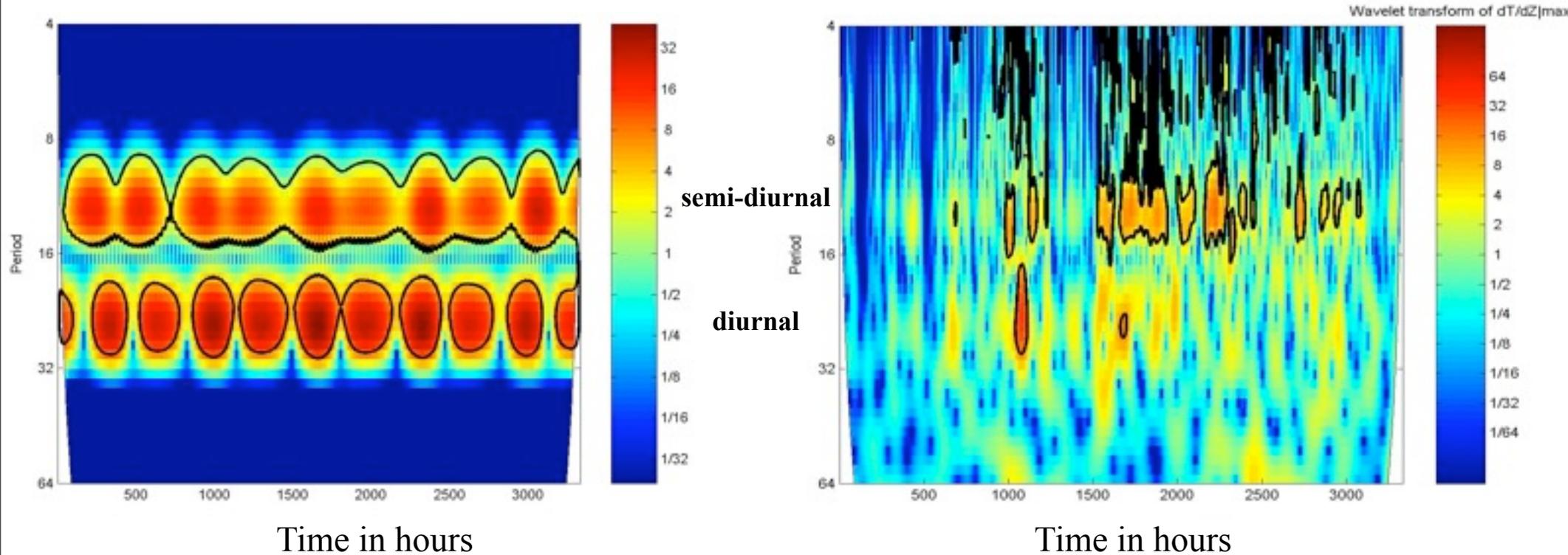
Monday, November 16, 2009

# SIO Pier temperatures



# Strength of surface and internal tide (SIO pier)

Eric Terill



## Barotropic tide:

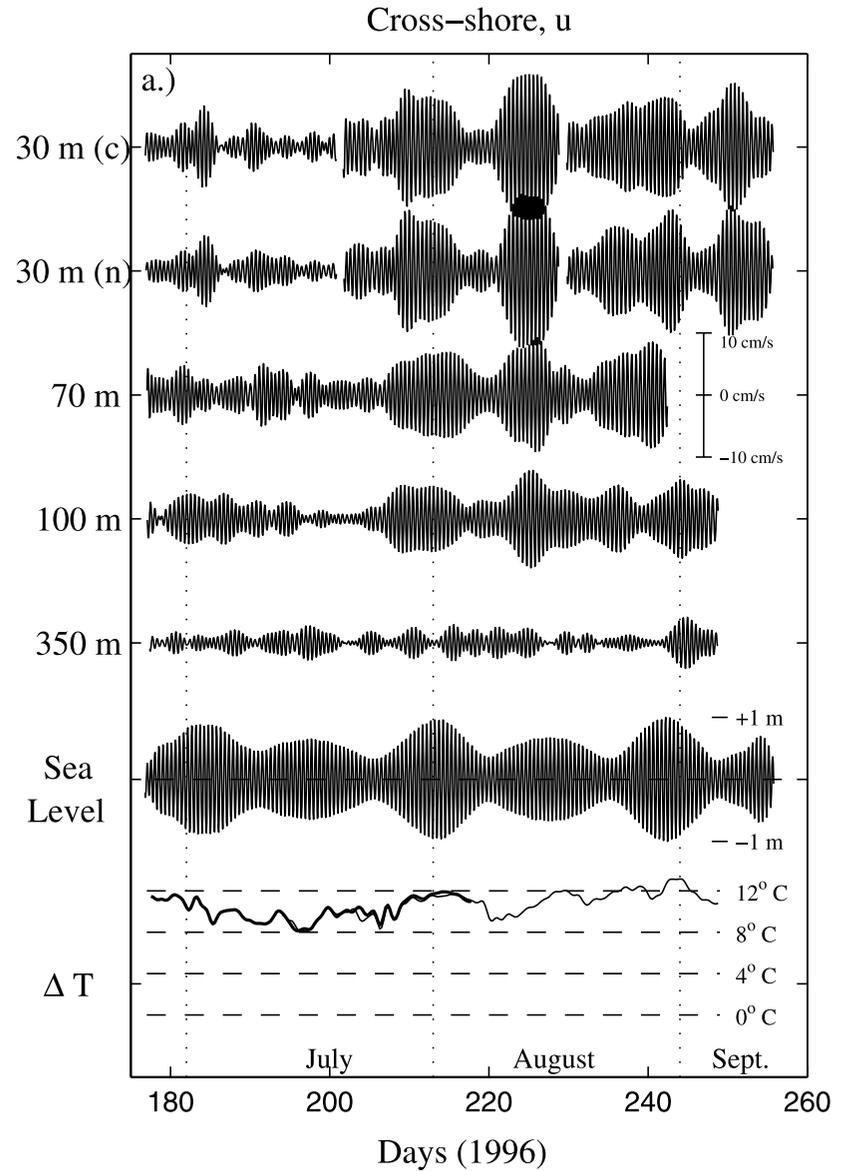
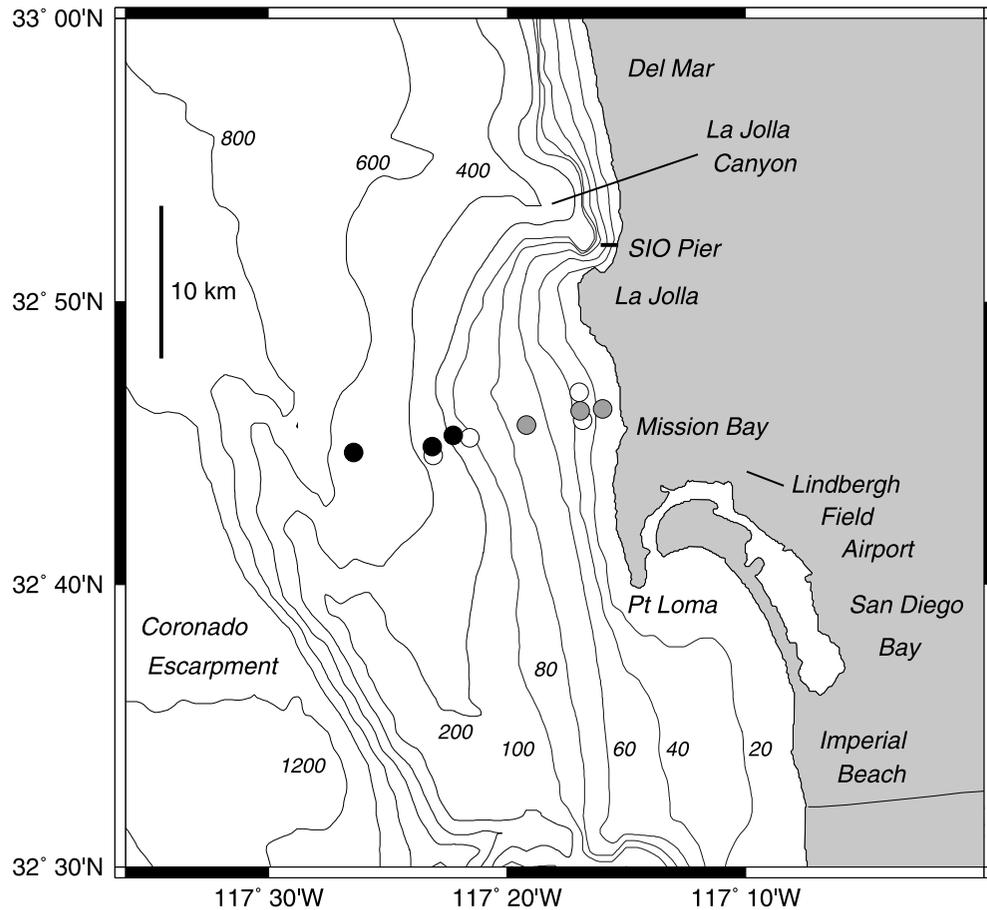
regular beating of semi-diurnal (12 hour) and diurnal (24 hour) signals

## Internal tide: a mess!

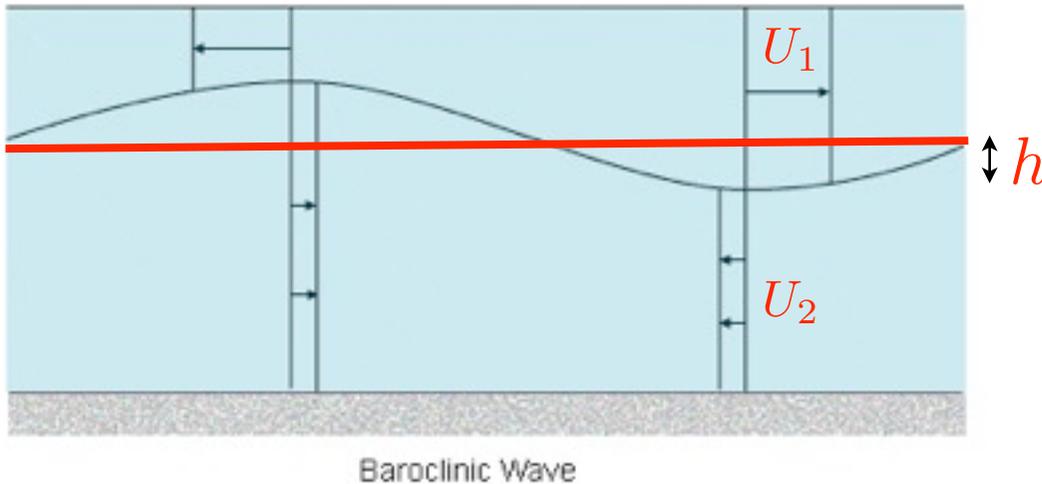
Changing stratification, mesoscale currents, eddies, ...

# More local internal tides

Lerczak, Winant and Hendershott, 2003



# Complicating factors: nonlinearity



## Linear waves

$$\frac{\partial h}{\partial t} + c_0 \frac{\partial h}{\partial x} = 0$$

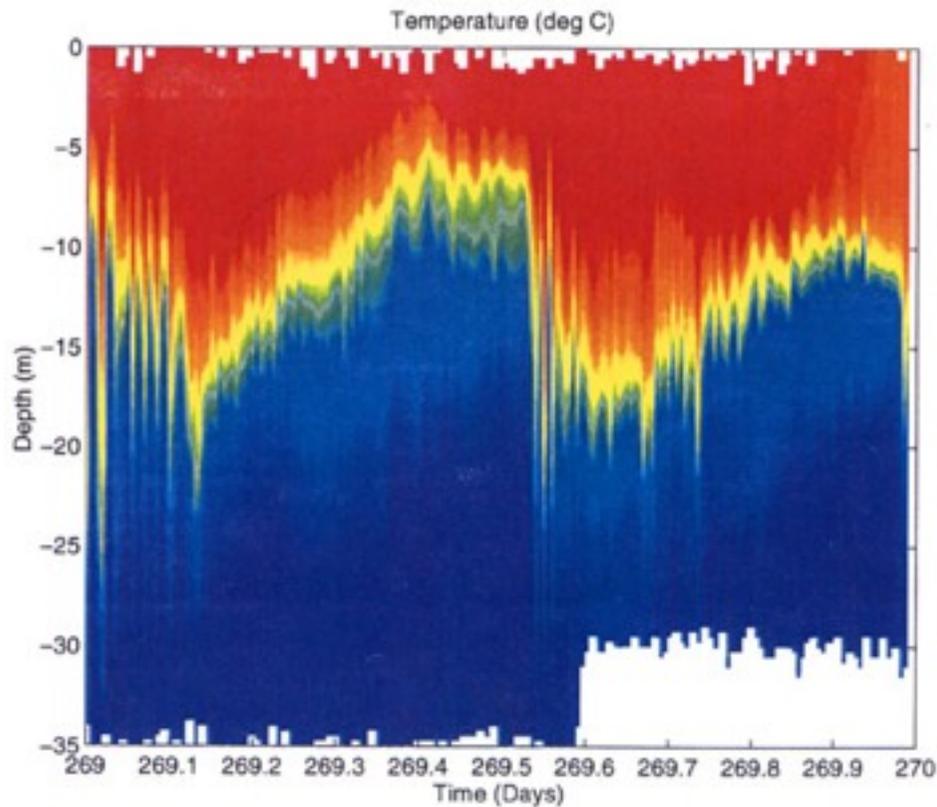
$$h(x, t) = \cos(x - c_0 t)$$

## Non-linear waves

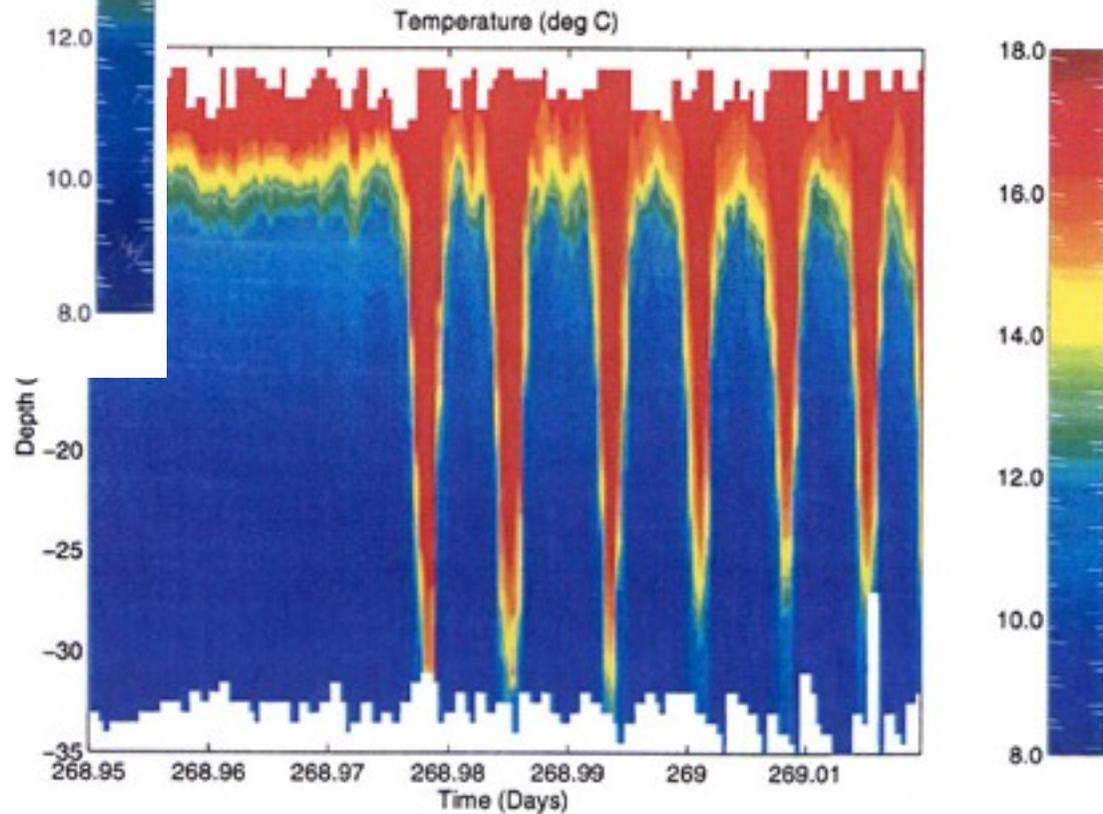
$$\frac{\partial h}{\partial t} + (c_0 + h) \frac{\partial h}{\partial x} = 0$$

When wave amplitude gets 'large' (shallow water), crest of wave moves faster, so wave starts to steepen. This can take several forms...

# Solitons: internal waves of unusual size



nonlinear steepening  
balanced by dispersion

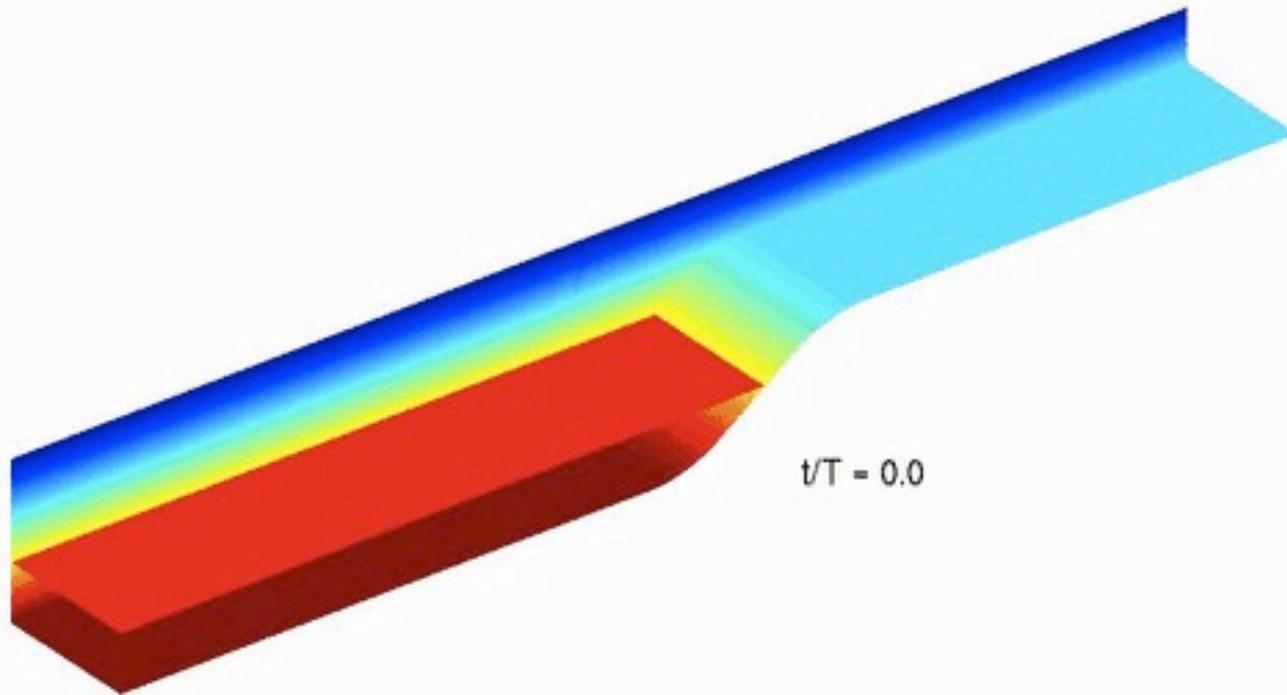


← 24 hours →

← 100 minutes →

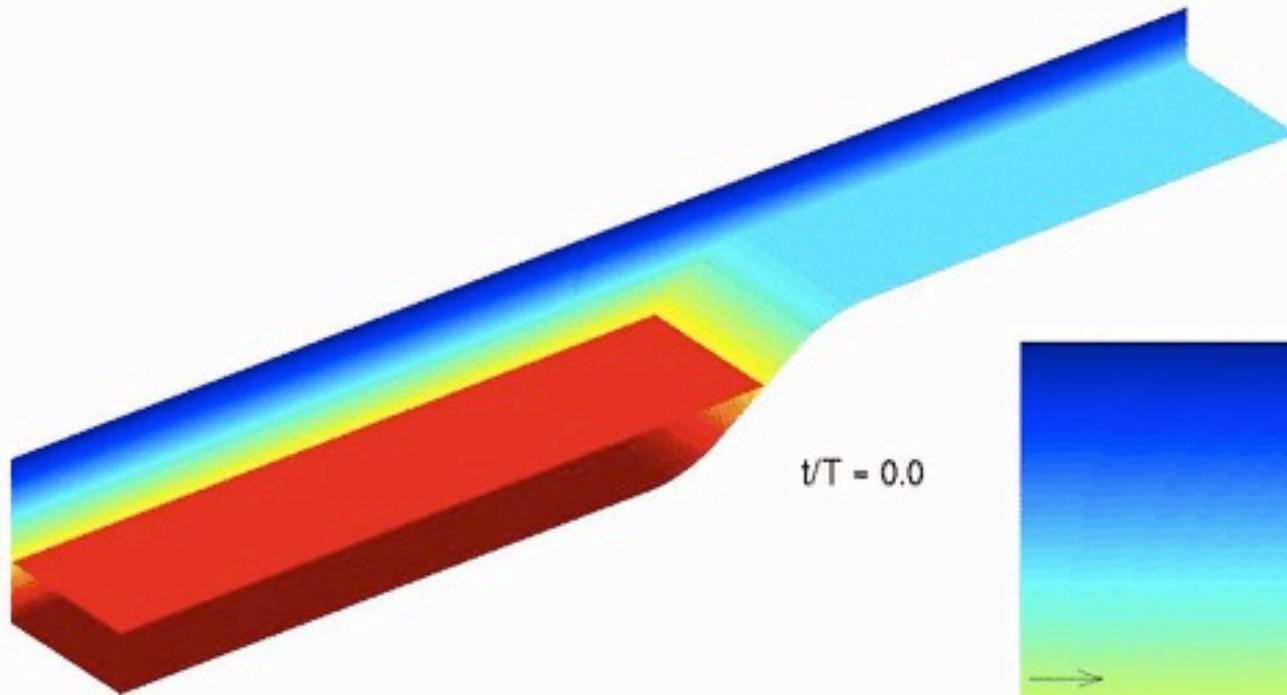
Stanton and Ostrovsky  
GRL 24(14) 1998

# Nonlinear internal tides: bores

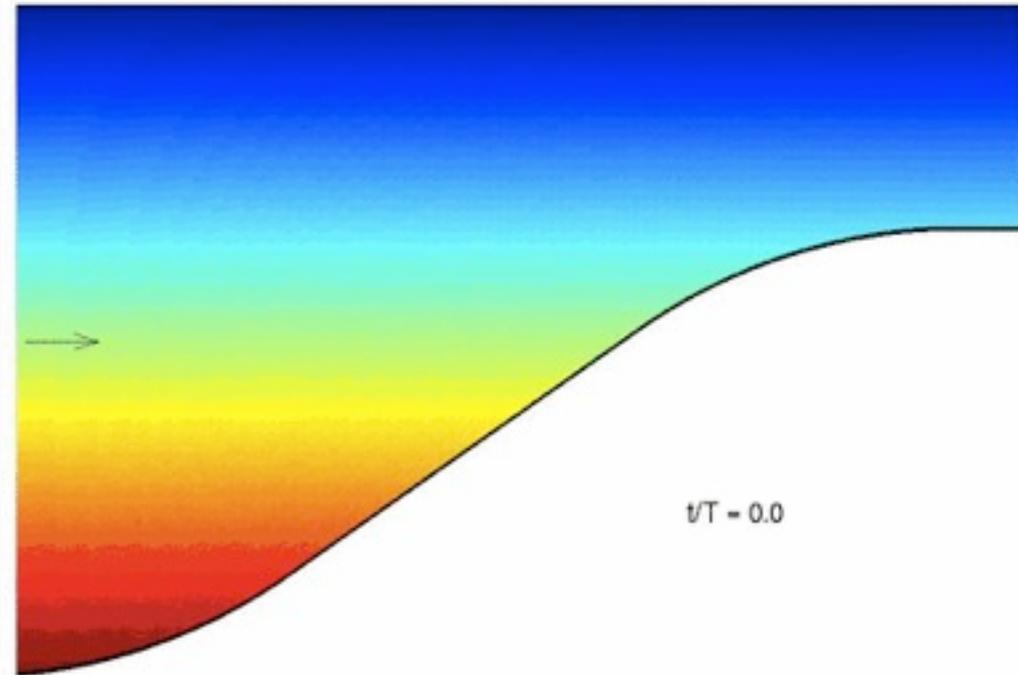


courtesy of S. K. Venayagamoorthy  
and O. Fringer, Stanford

# Nonlinear internal tides: bores



courtesy of S. K. Venayagamoorthy  
and O. Fringer, Stanford

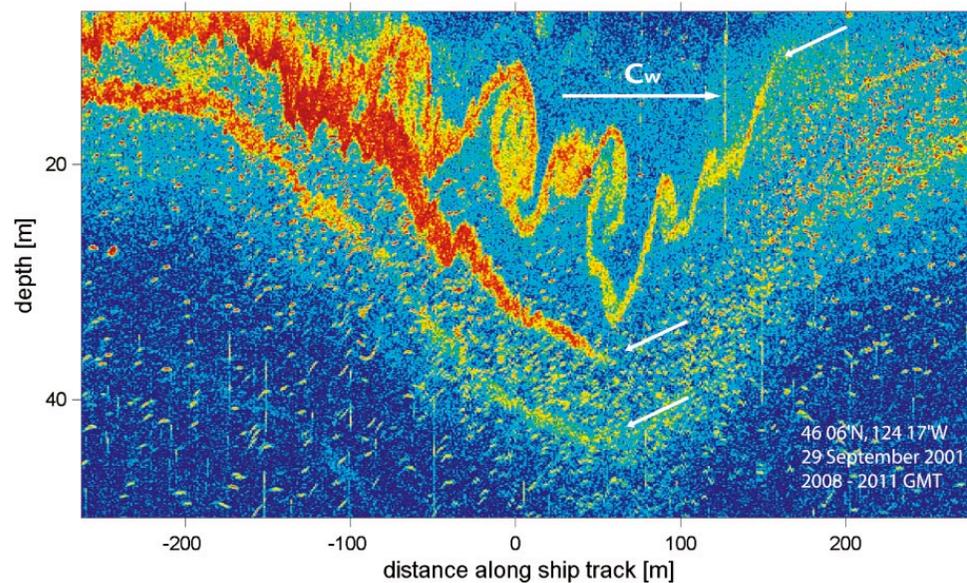


# Why you should care

- Internal-wave fluctuations often dominate any signal you measure. Up/down CTD casts. Moorings.
- Internal-wave shear produces turbulence and mixing. Most mixing at interface / thermocline, can bring nutrients up into the euphotic zone. (next week)
- May create net on or offshore transport of mass / nutrients / larvae / ???

# Consequences of Internal Waves

Wave breaking mixes the ocean (next week).

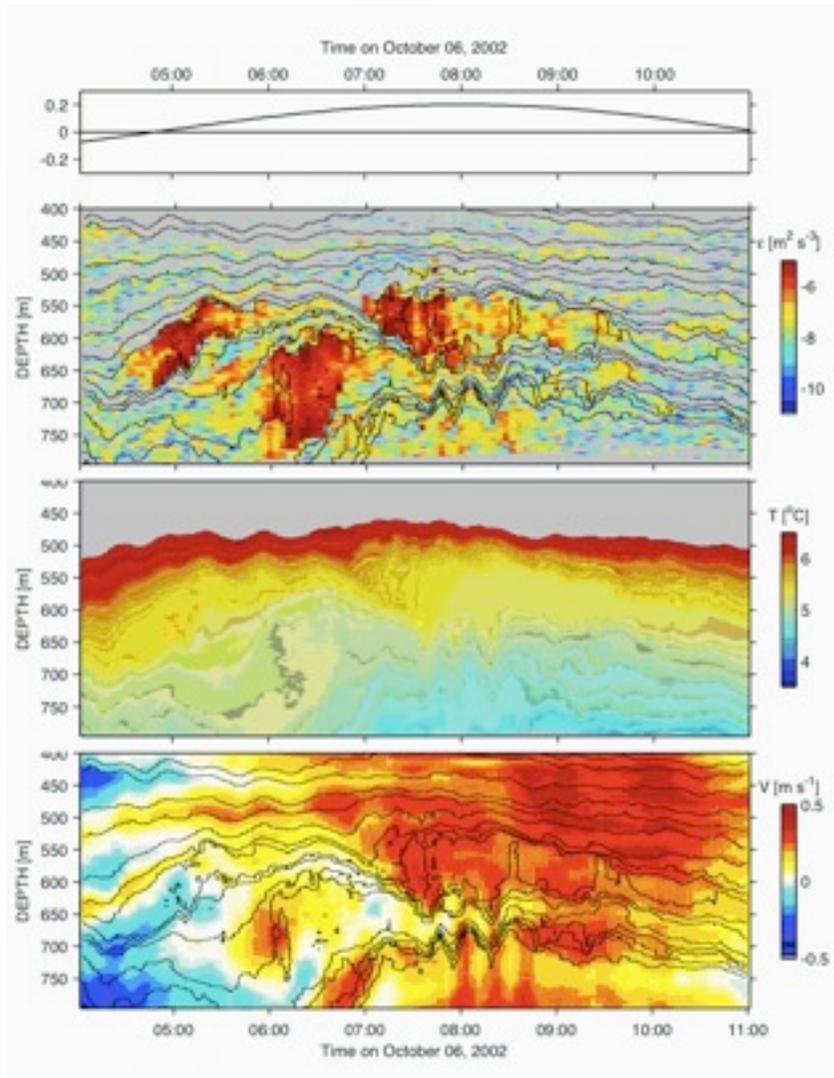
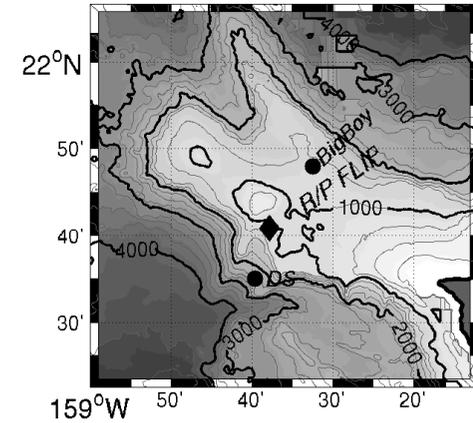


Moum et al 03

# Hawaiian Ocean Mixing Experiment (HOME)

Huge overturns as internal tide sloshes up and down a steep slope

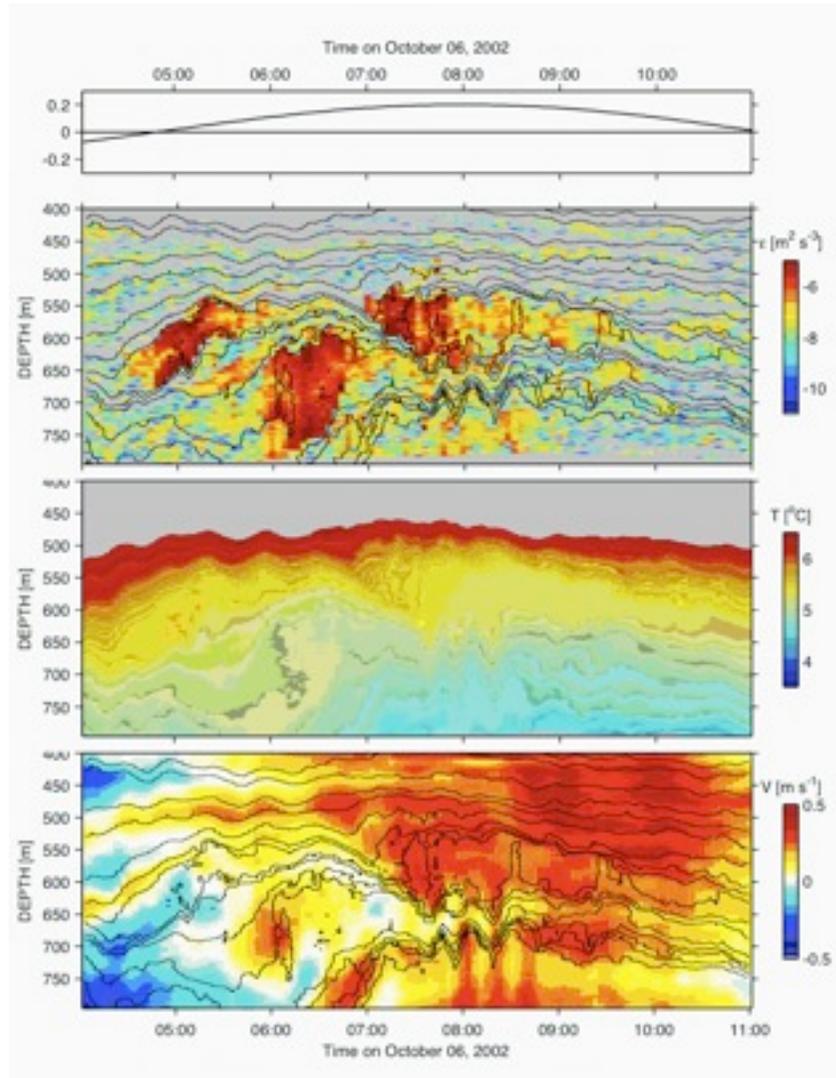
Klymak et al 07  
Levine and Boyd 06  
Aucan et al 05



Klymak et al 07

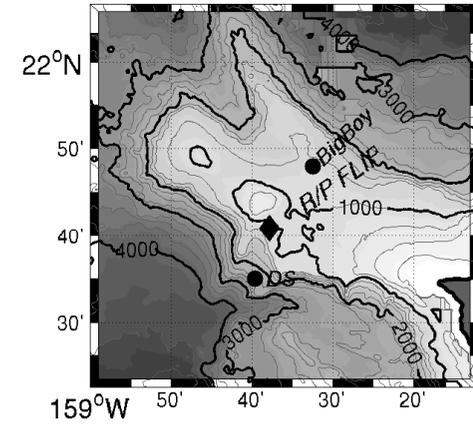
# Hawaiian Ocean Mixing Experiment (HOME)

Huge overturns as internal tide sloshes up and down a steep slope



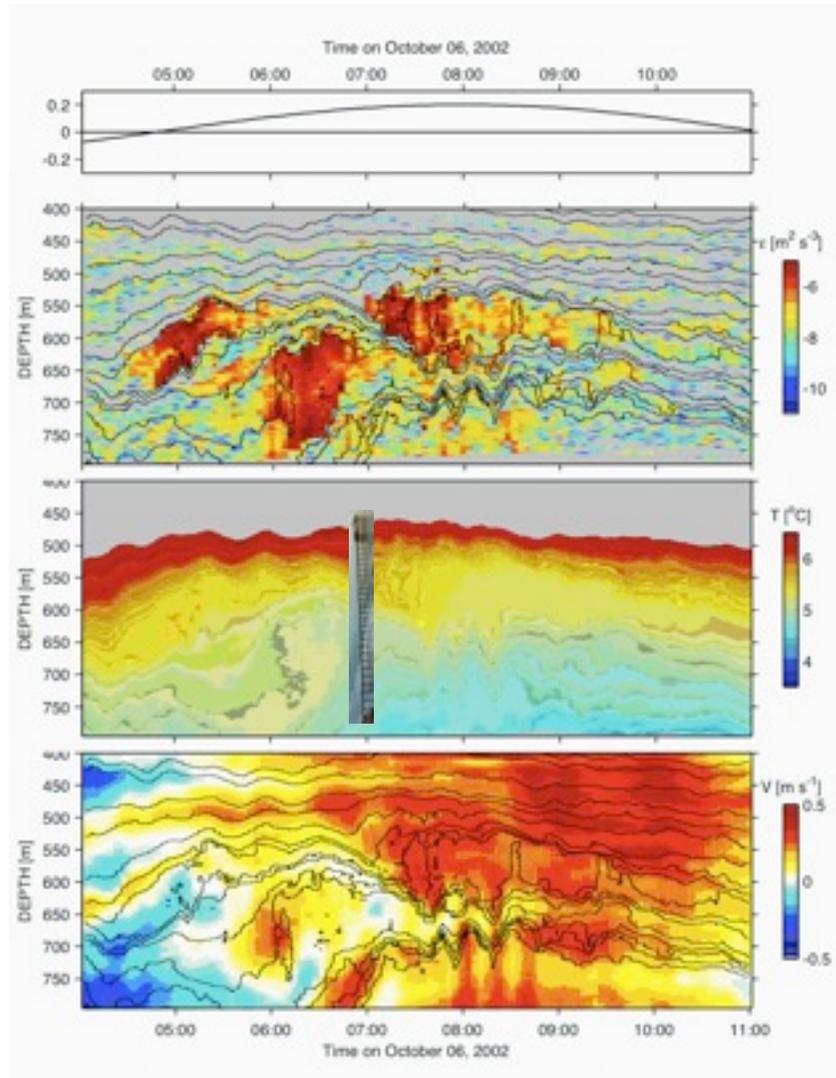
Klymak et al 07

Klymak et al 07  
Levine and Boyd 06  
Aucan et al 05



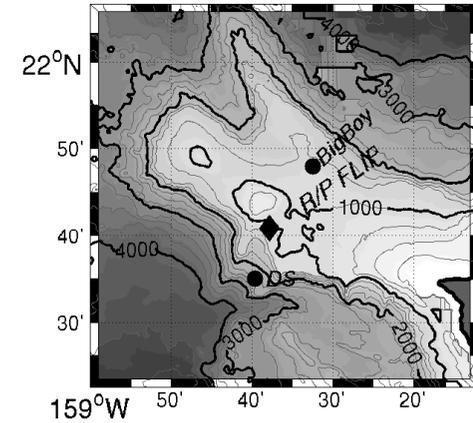
# Hawaiian Ocean Mixing Experiment (HOME)

Huge overturns as internal tide sloshes up and down a steep slope

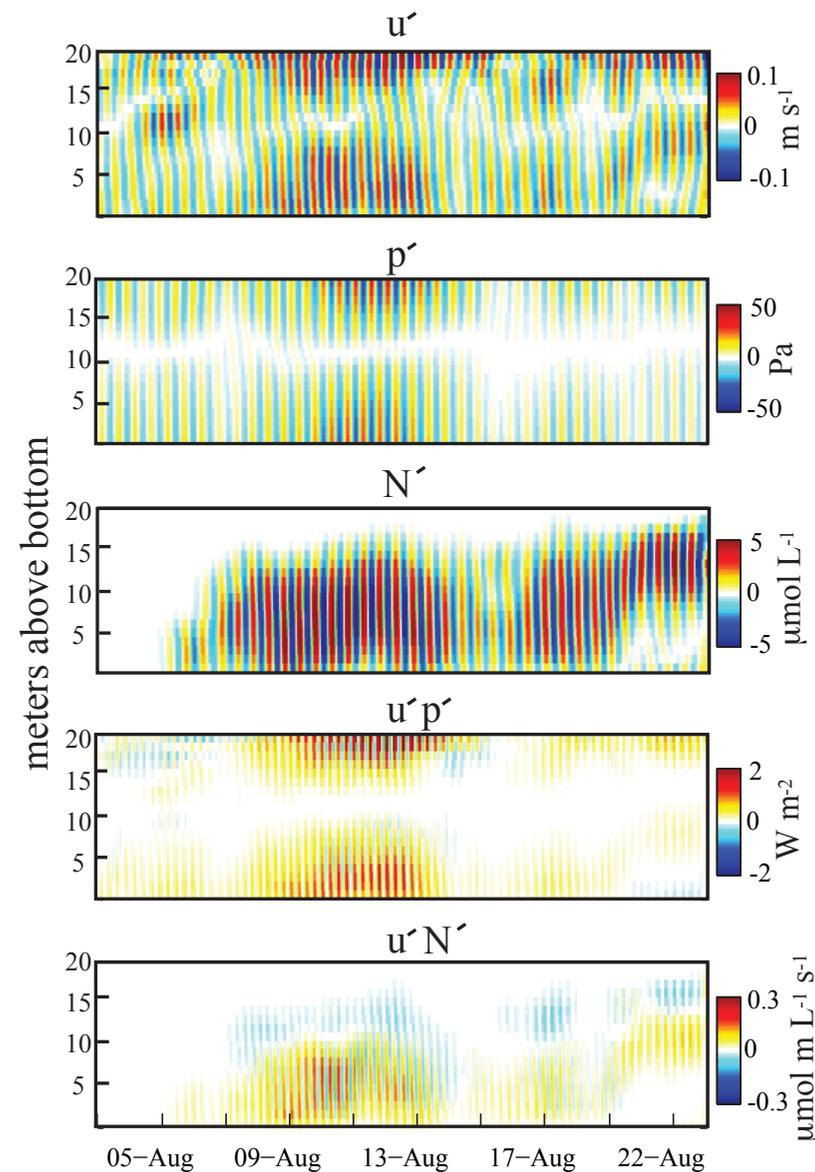
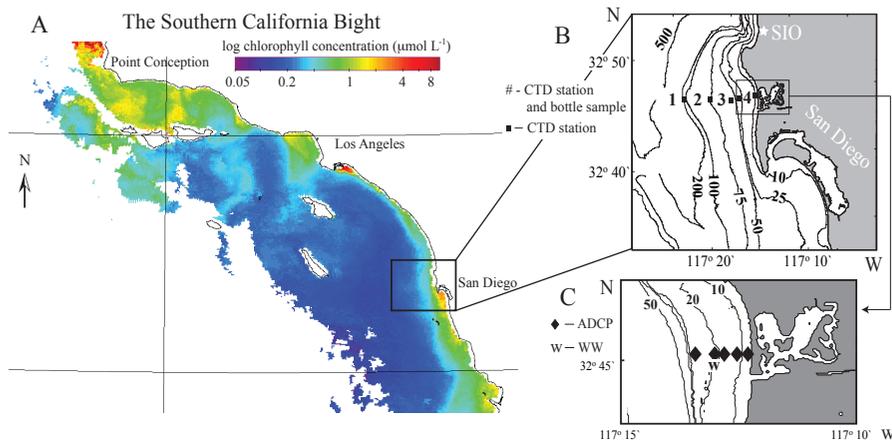


Klymak et al 07

Klymak et al 07  
Levine and Boyd 06  
Aucan et al 05

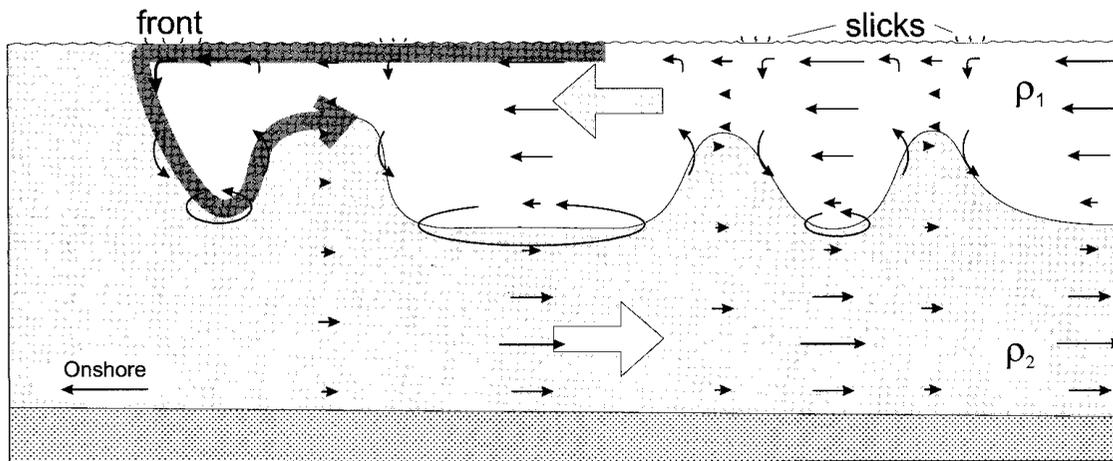


# IW transport larvae/nutrients



Drew  
Lucas, SIO

# Larvae transport onshore



Convergence at the front of a wave train

Only strong upward swimmers can stay in the front

