SIO 210 - Solution - week 1

1. If the highway is congested, we can take the density, ρ in each lane to be constant. If the flow is steady the flux of cars across the two sections (one with a single lane, the other with two lanes) has to be the same. Since the flux is equal to the product of the density times the velocity, the velocity must be twice as fast on the single lane section. Try it: you will notice that you actually speed up right after the two lanes have merged. If the traffic is not congested, you have no way of evaluating the ratio of the densities in the two sections, and so you cant tell whether the cars speed up or slow down (could be either way).

2. Since the cross-sectional area decreases with distance from the faucet, and the density remains the same, the velocity must increase.

3. Use thee hydrostatic equation in the form $p = -\rho g z$, the pressure at the bottom $(z = -h)$ is $p = \rho g h$. So the force is $F_{bottom} = \rho g h A$, where A is the surface of the bottom of the glass, equal to the weight of water.

4. For the Erlenmeyer flask, the pressure on the bottom is the same as in the problem above, and the force on the bottom is the same as well, However the water exerts a force on the sides, perpendicular to the sides. We decompose this force into two components. The horizontal components cancel each other, but there is a net resultant vertical force F_E, such that the weight of the fluid in the flask is equal to $F_{bottom} - F_E$.

5. Consider two sections perpendicular to the stream: one at location 1, the other, a distance h beneath the first, at location 2. Because the pressure is constant (equal to atmospheric pressure, we can write Bernouilli's equations as: $V_2^2 = V_1^2 + 2gh$, and the mass conservation equation for constant density tells us that $Q = V_1 A_1 = V_2 A_2$ is constant. These expressions can be manipulated into an expression for g:

   $$g = \frac{Q^2}{2\rho h}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

   As you all realized, this is a terrible way to measure g, since it depends on difference between squares of a measurement.