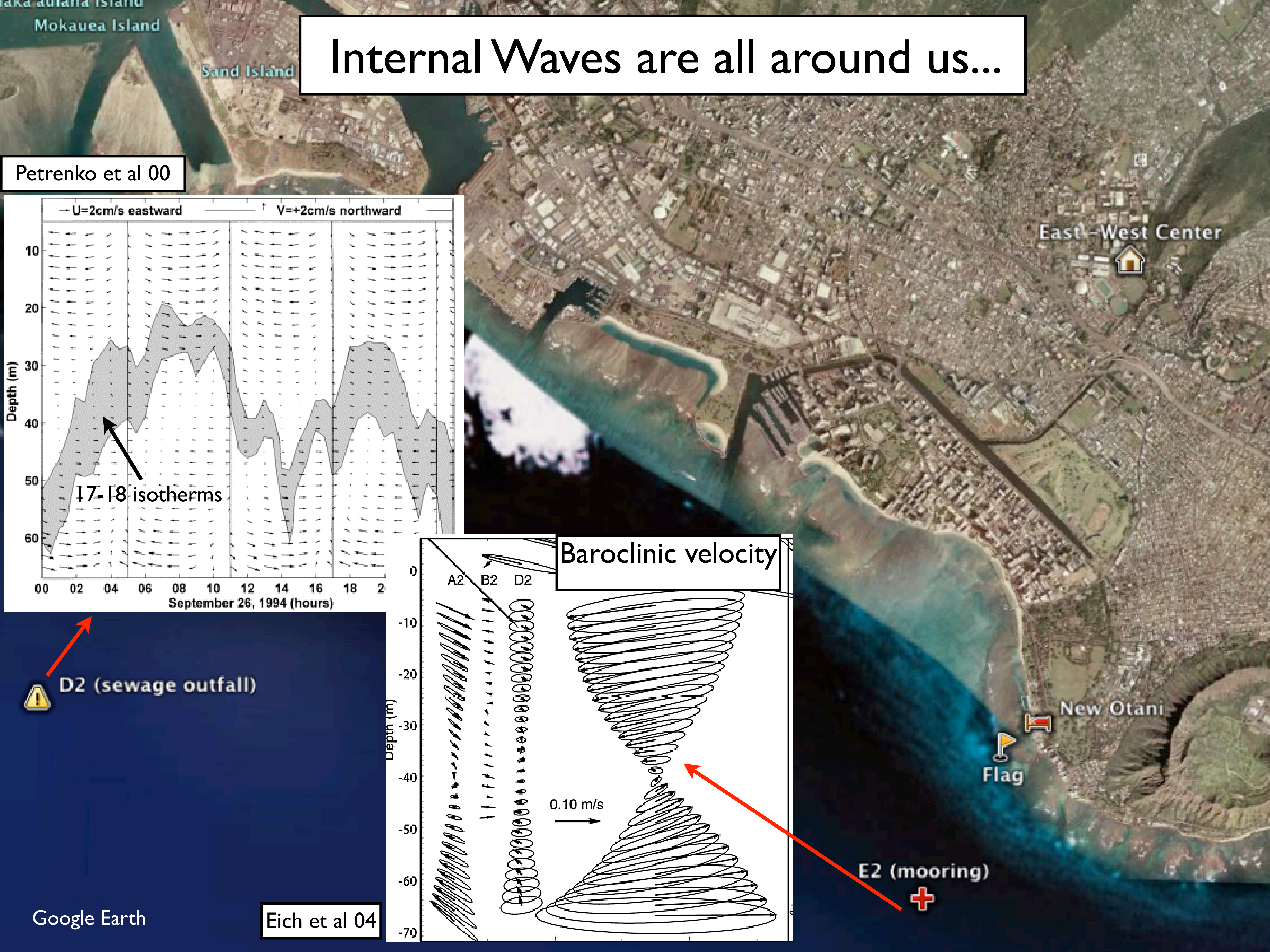
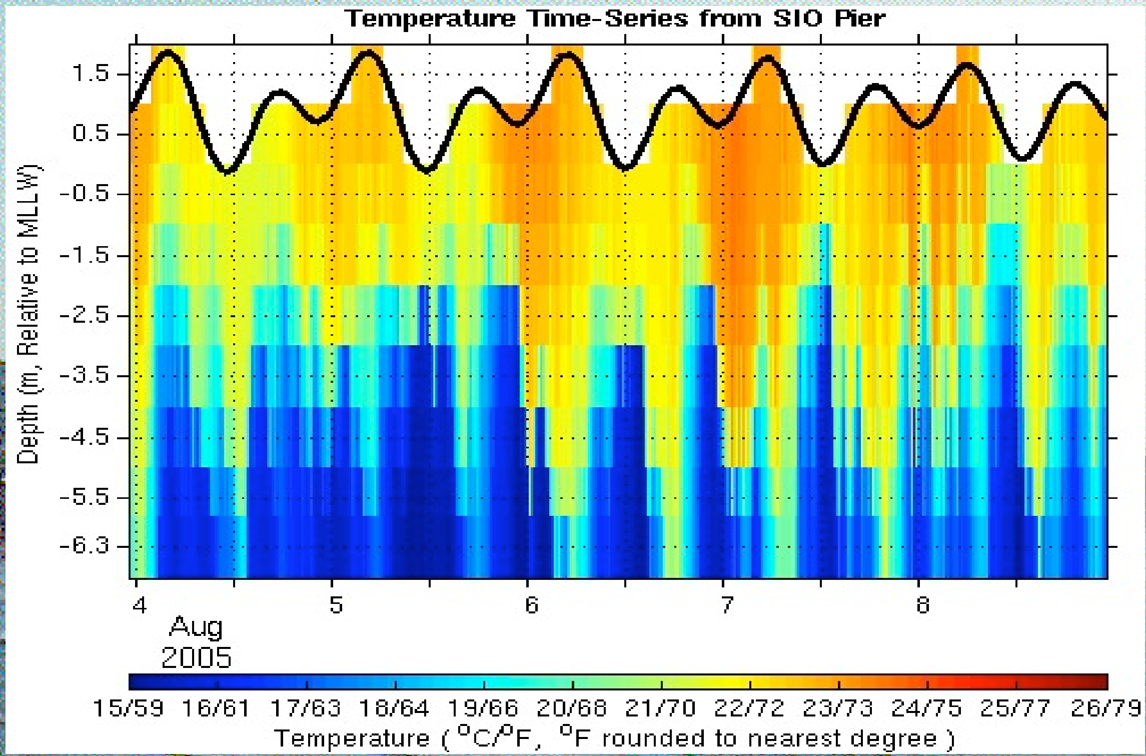
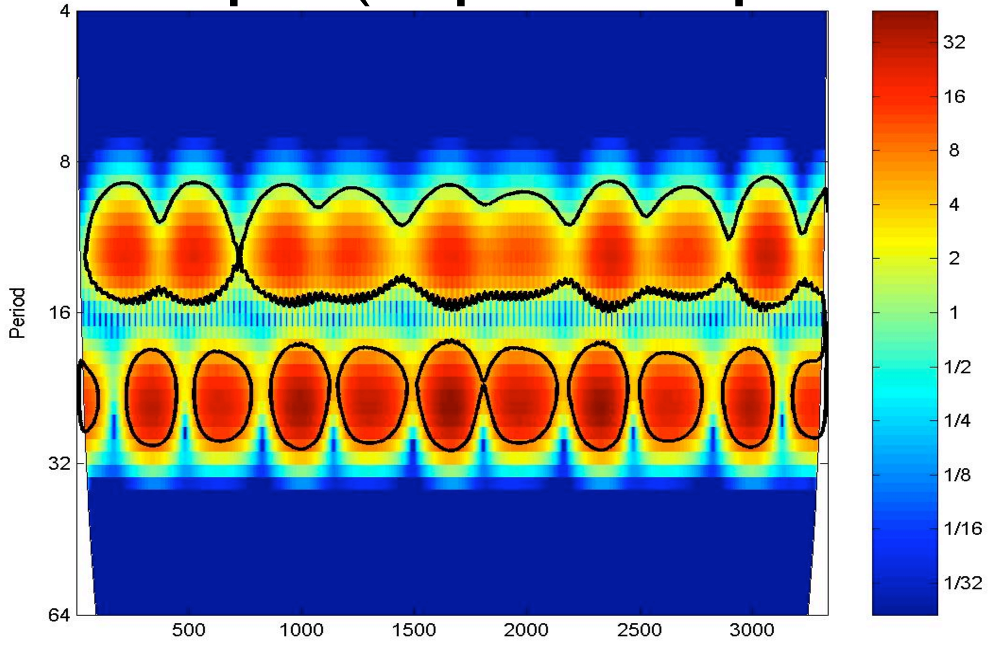

Internal Waves are all around us...
Mokauea Island
Sand Island
Petrenko et al 00
U=2cm/s eastward
V=+2cm/s northward
Depth (m)
17-18 isotherms
September 26, 1994 (hours)
East-West Center
D2 (sewage outfall)
Baroclinic velocity
A2
B2
D2
Depth (m)
0.10 m/s
New Otani
Flag
E2 (mooring)
Google Earth
Eich et al 04

# Erratic internal waves at SIO Pier

data and wavelet analysis courtesy of E. Terrill, SIO



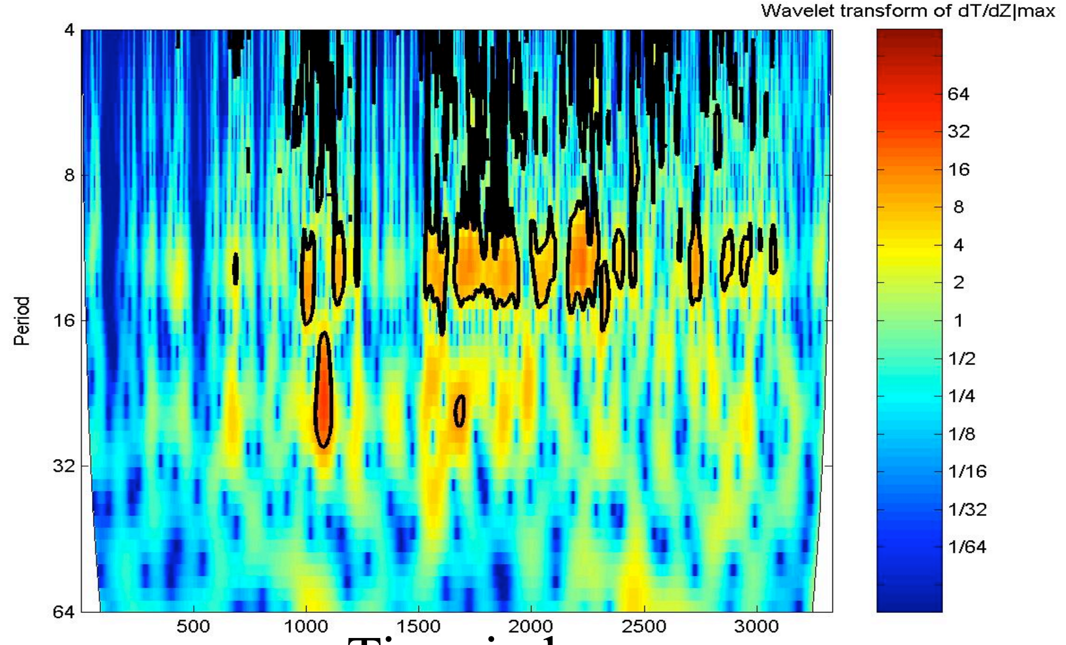

## Barotropic (depth-independent)



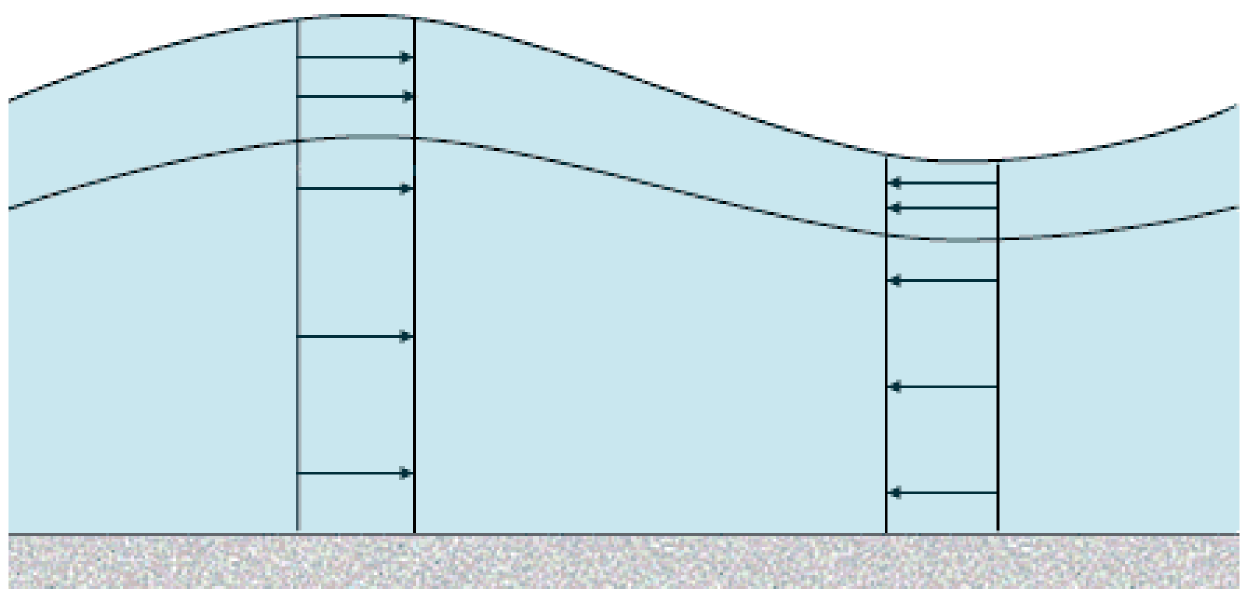

semi-diurnal

diurnal

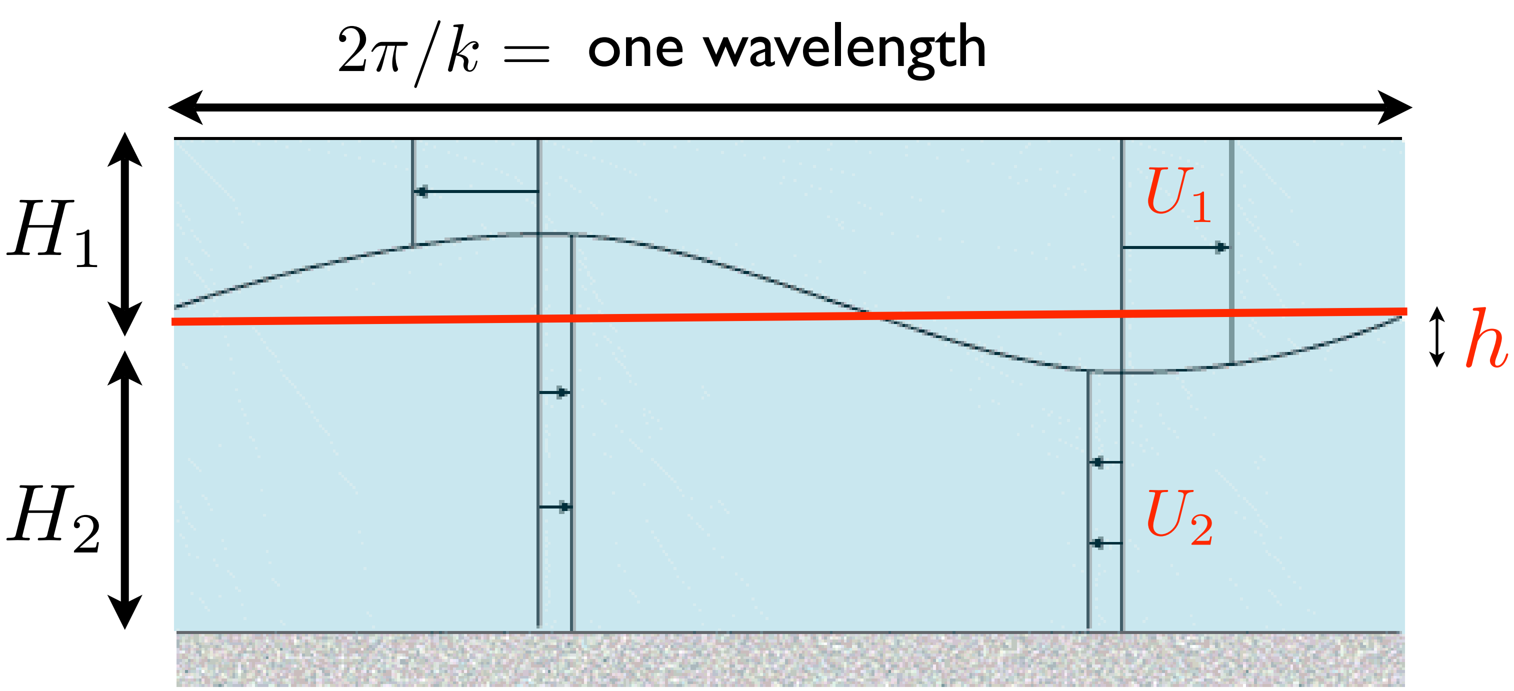
## Baroclinic (depth-dependent)



# Simple interfacial internal wave



Barotropic Wave




Baroclinic Wave

$$\mathbf{h = -h_0 \cos(kx - \omega t)}$$

$$\mathbf{U_1 = \frac{\omega h_0}{H_1 k} \cos(kx - \omega t)}$$

$$\mathbf{U_2 = -\frac{\omega h_0}{H_2 k} \cos(kx - \omega t)}$$

after Gill,
*Atmosphere-Ocean Dynamics*

# Internal wave equations

Linearize equations of motion

$$
\begin{aligned}
\frac{\partial u}{\partial \mathrm{t}} &= \cancel{-\vec{u}\cdot\nabla u} + fv - \frac{1}{\rho}\frac{\partial p}{\partial \mathrm{x}} + \cancel{\nu\nabla^2 u} \\
\frac{\partial v}{\partial \mathrm{t}} &= \cancel{-\vec{u}\cdot\nabla v} - fu - \frac{1}{\rho}\frac{\partial p}{\partial \mathrm{y}} + \cancel{\nu\nabla^2 v} \\
\frac{\partial w}{\partial \mathrm{t}} &= \cancel{-\vec{u}\cdot\nabla w} - \frac{1}{\rho}\frac{\partial p}{\partial \mathrm{z}} + \cancel{\nu\nabla^2 w} - g \\
\frac{\partial \rho}{\partial \mathrm{t}} &= -\vec{u}\cdot\nabla\rho + \cancel{\kappa\nabla^2\rho} \\
\nabla\cdot\vec{u} &= 0
\end{aligned}
$$

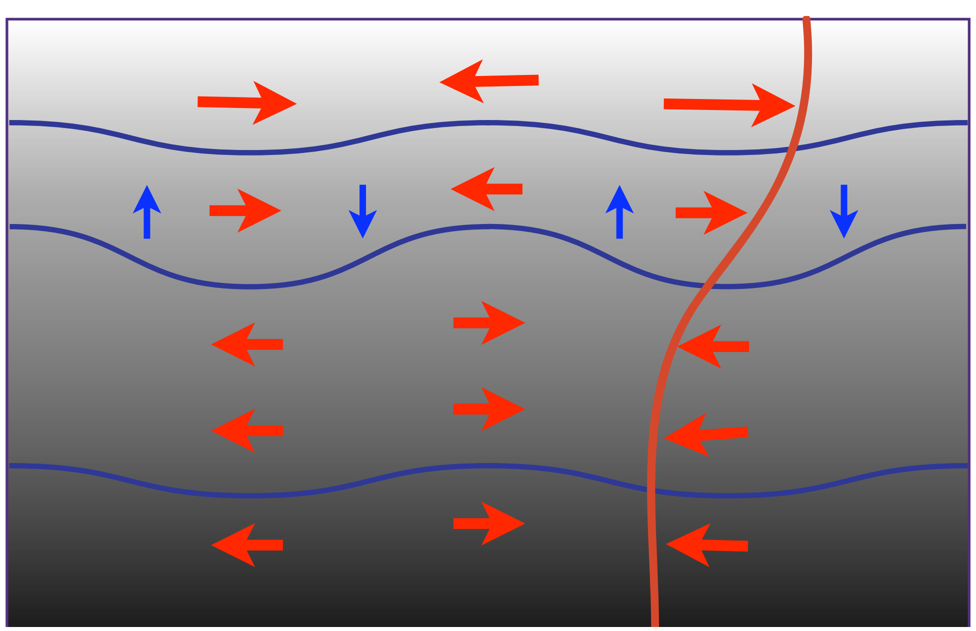
Try a solution of the form

$$u(x,y,z,t) = \hat{u}e^{-i[kx+ly+mz-\omega t]}$$

Get polarization and dispersion relationships

$$\omega^2 = \frac{(k^2+l^2)*N^2+m^2*f^2}{k^2+l^2+m^2}$$

Low-mode versus high-mode

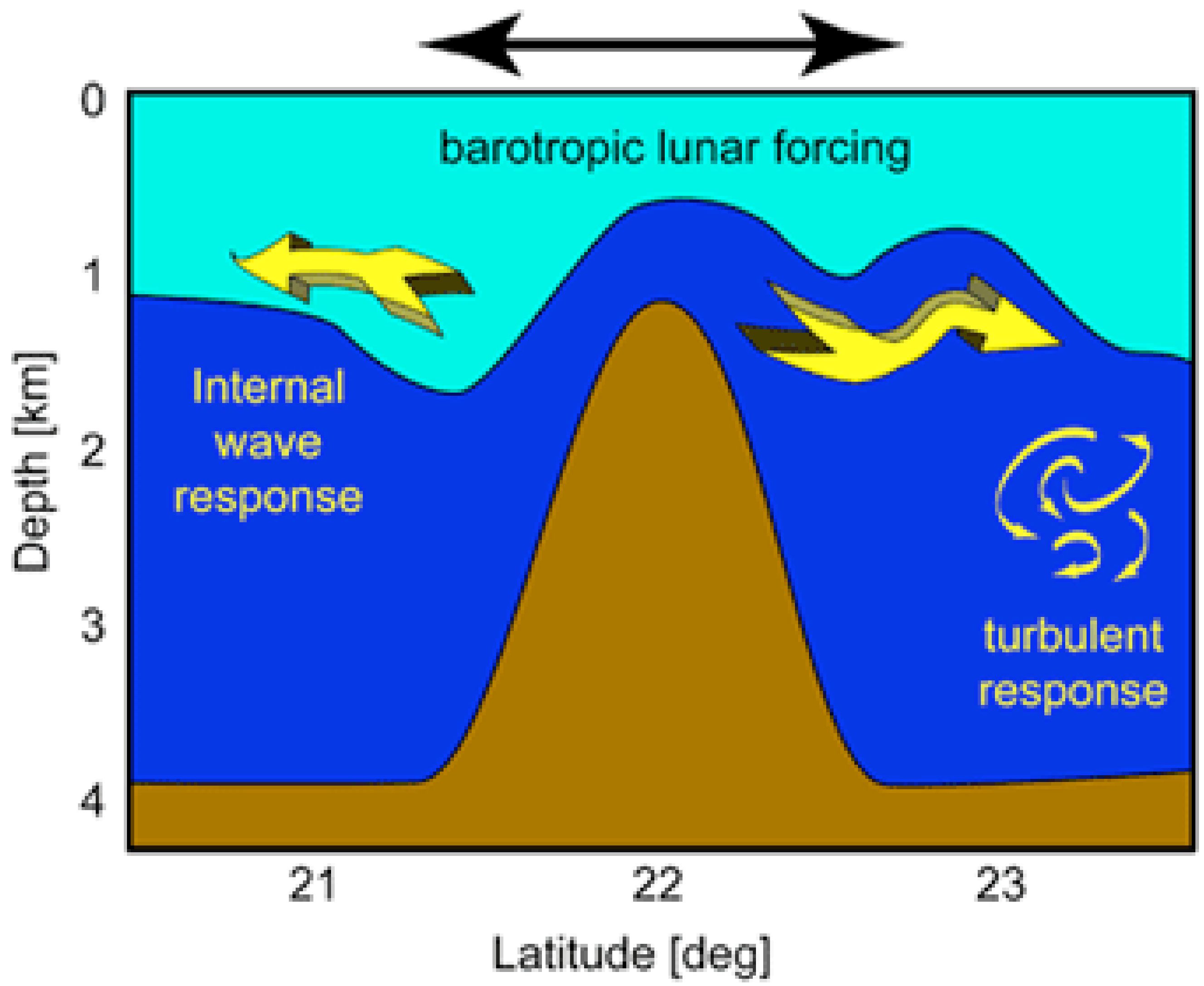


$$\mathbf{U = \Psi(z)\cos(kx - \omega t)}$$

(Glenn Flierl)

# What generates internal waves?

1) Barotropic tide sloshing over topography



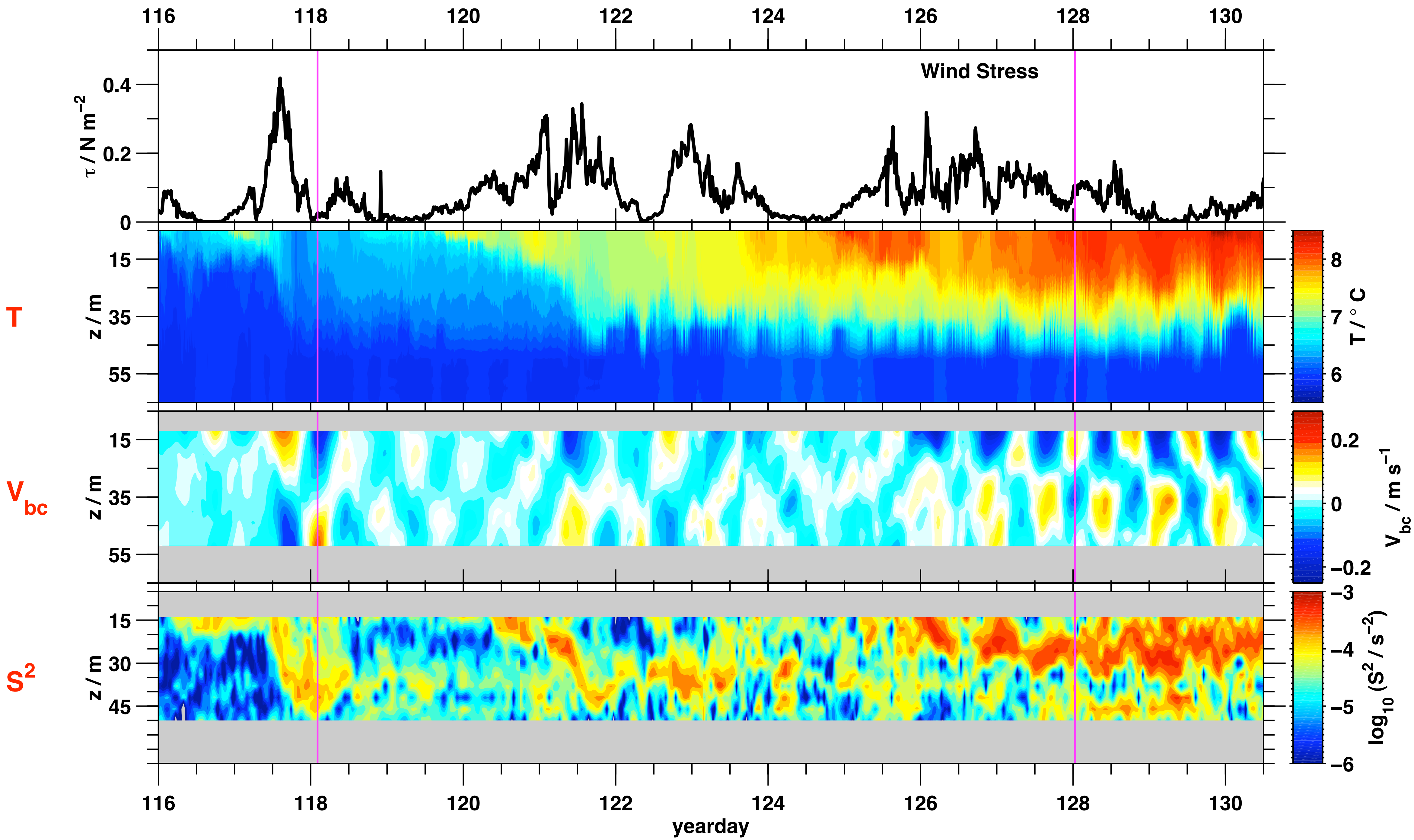

(J. Nash)

**Internal Tide:** An internal wave with a tidal frequency, usually once in 12.4 hours = M2

Often generated at the continental shelf break, with waves propagating both on and off shore.

# What generates internal waves?

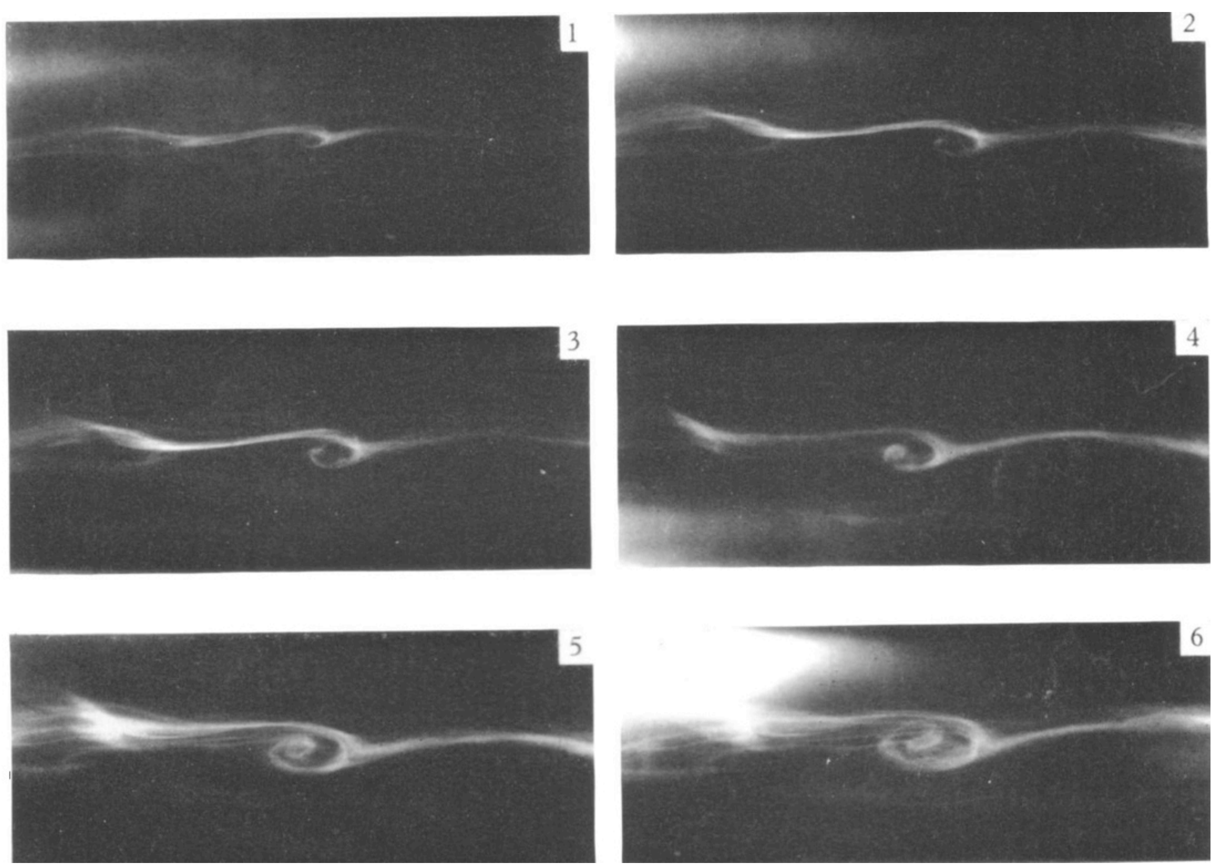
## 2) Wind makes near-inertial internal waves



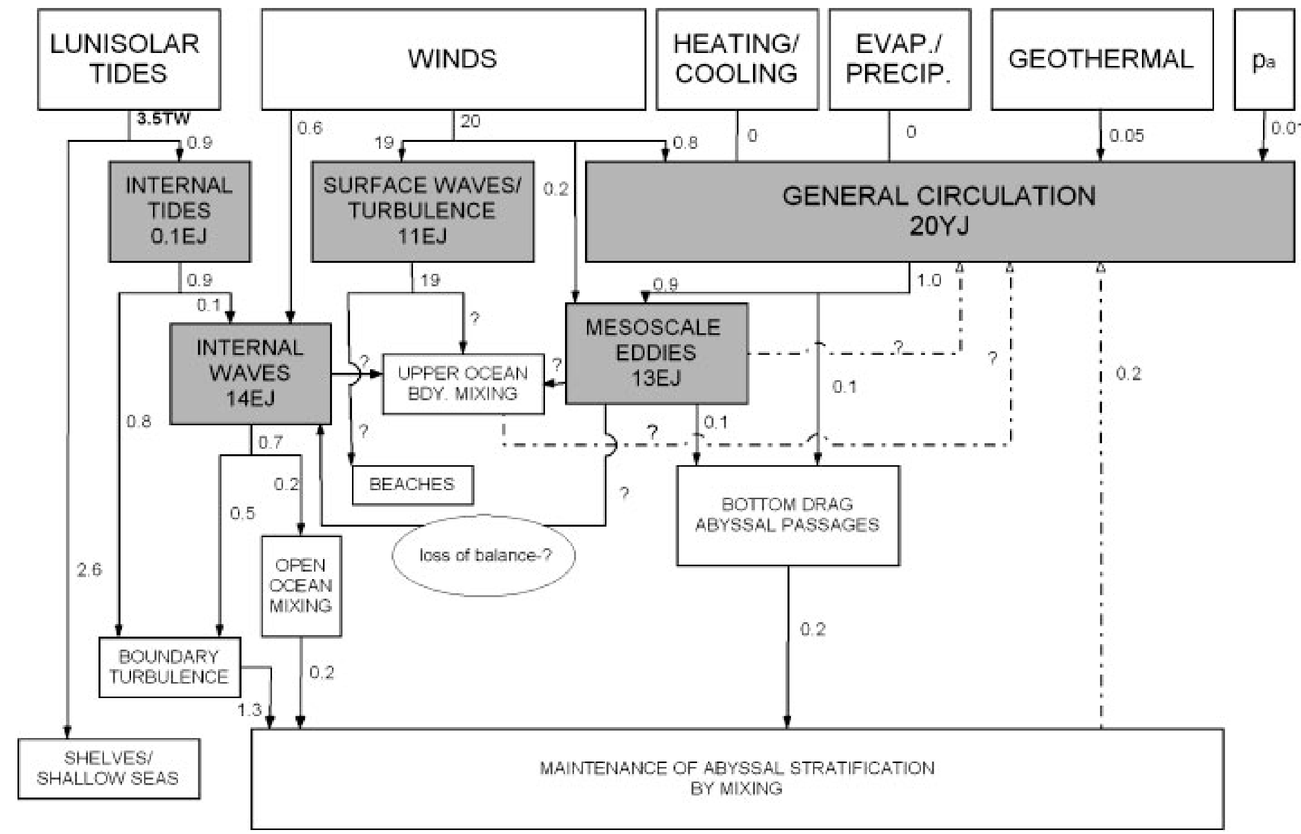

(MacKinnon and Gregg, JPO, Dec 05)

# Internal wave breaking mixes the ocean

Waves break by shear instability
or convective overturning.




Woods 68



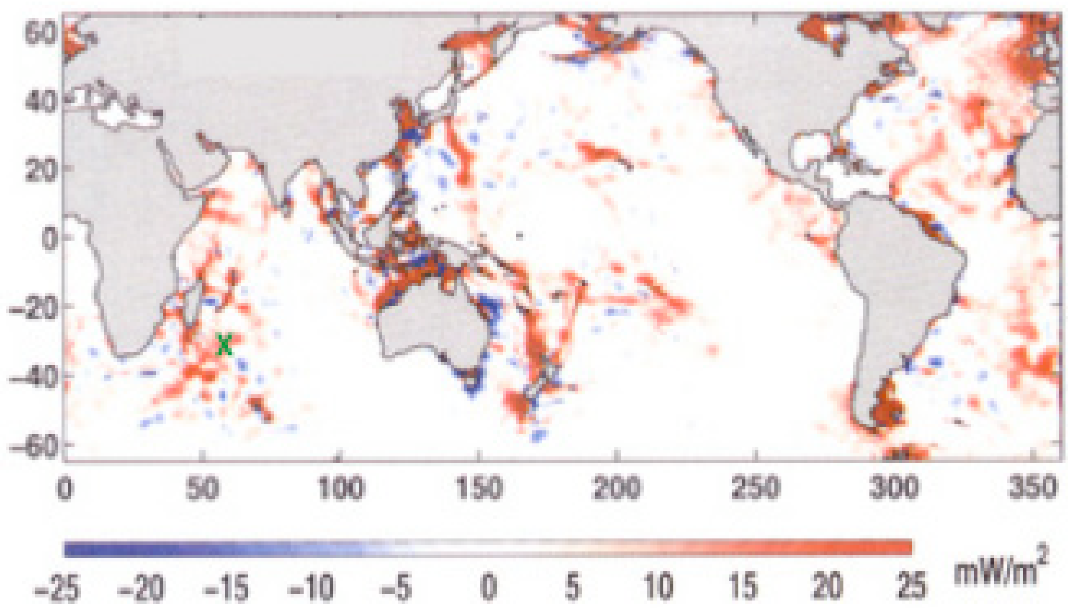

Wunsch and Ferrari 04

May provide enough total power to drive the global meridional overturning cirulation

Cant’ explicitly resolve internal waves in climate models.
3 steps to parameterize their role:

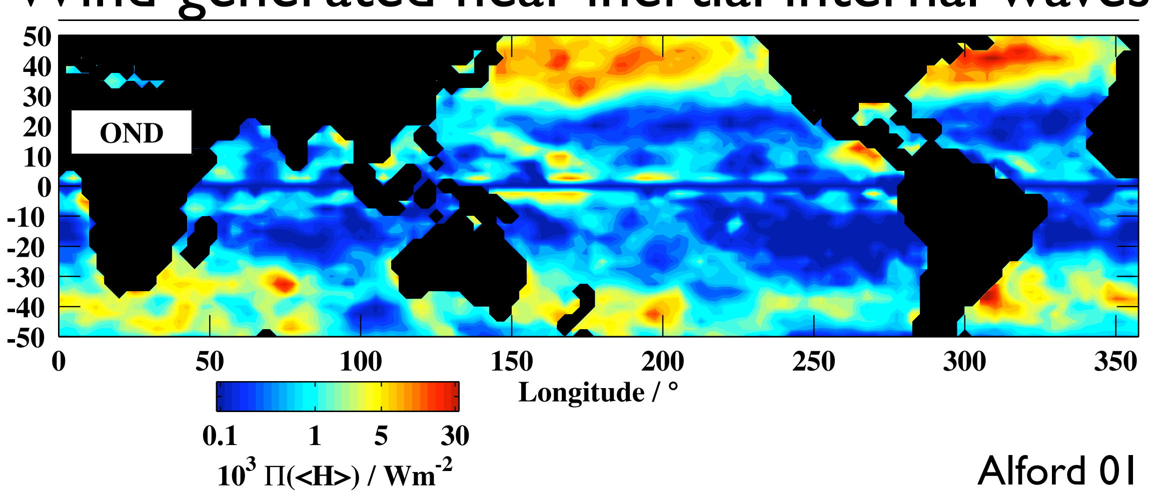
# 1) Wave generation

Internal-Tide Generation




Egbert and Ray 01

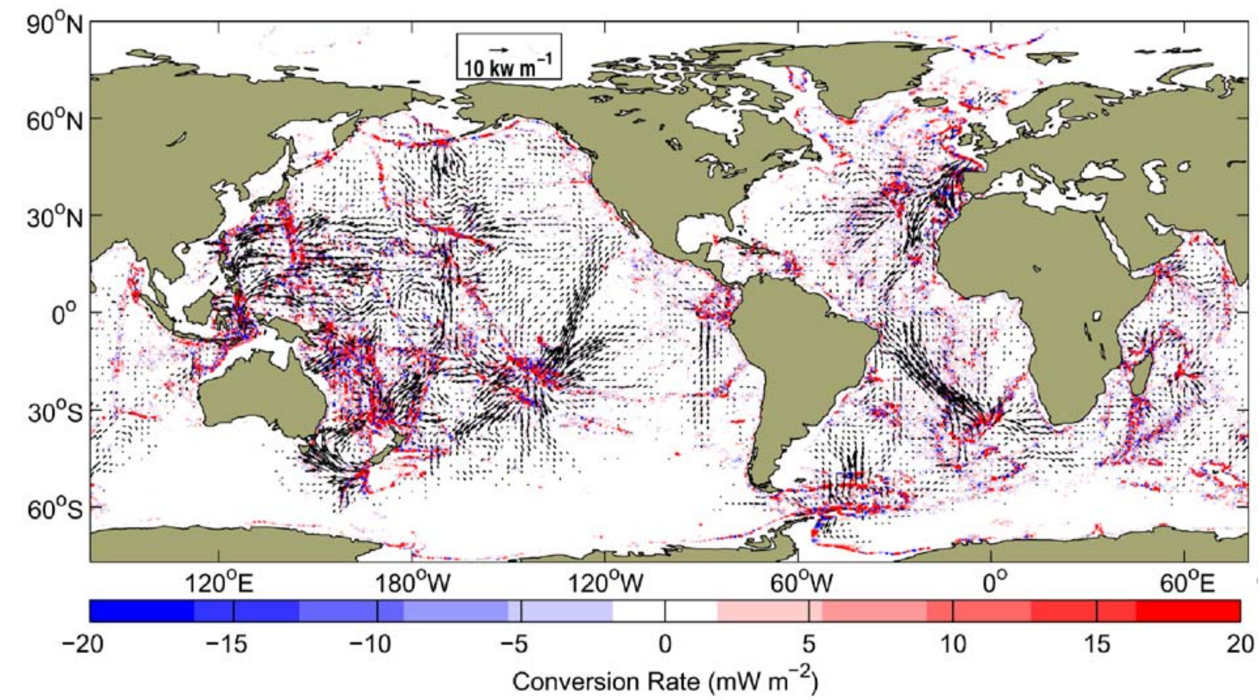
Wind-generated near-inertial internal waves




Alford 01

# 2) Wave propagation

Internal-Tide Propagation




# 3) Wave breaking ...

# Statistical models of wave breaking

"With three parameters I can fit an elephant"
-Lord Kelvin

Generated waves generally have large (vertical) scales and low frequencies

Resolution of GCMs somewhere in between

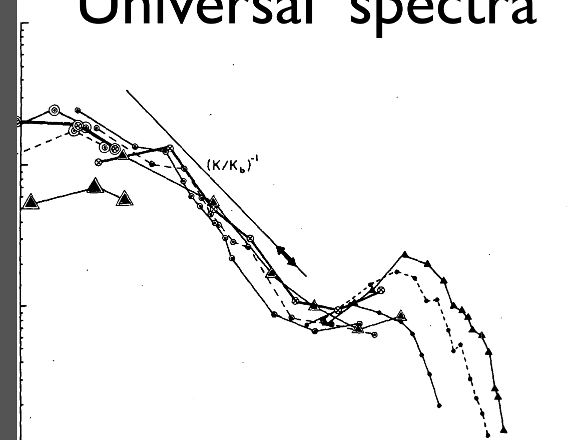
Breaking waves generally small (vertical) scale. Small-scale waves have the most shear.

$E_{\text{low-mode}}$

Ambient Conditions:
-stratification
-latitude
-mesoscale features

Black Box

'Universal' spectra



Gargett et al 81

(9)

(1)

(4)

Dissipation rate ($\epsilon$)

## Models of down-scale energy transfer

- Triad theory of weakly nonlinear wave-wave interactions (e.g.McComas and Muller)

$$\frac{\partial U}{\partial t} = \ldots - \mathbf{U}\cdot\nabla U$$
$$U = U_1 e^{i(\mathbf{k_1}\cdot\mathbf{x}-\omega_1 t)} + U_2 e^{i(\mathbf{k_2}\cdot\mathbf{x}-\omega_2 t)} + U_3 e^{i(\mathbf{k_3}\cdot\mathbf{x}-\omega_3 t)} + \ldots$$

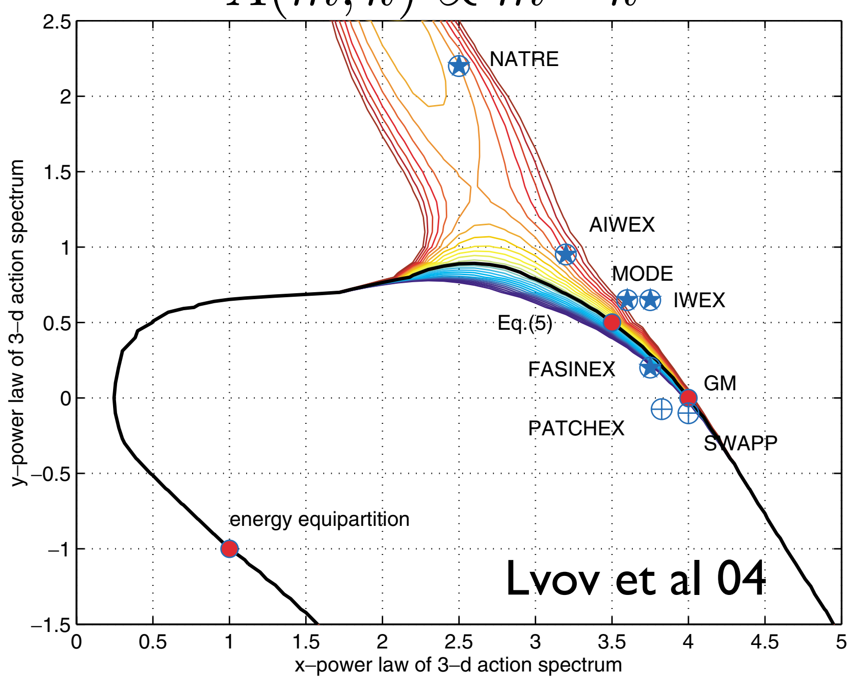
- Eikonal calculations of small-scale waves propagating through spectrum of larger waves (Henyey et al)

Steady flux to smaller vertical scales

$$F \propto \hat{E}^2\ N^2$$

- Theoretically predicted 'family' of steady-state spectra ...hamiltonian....

$$A(m,k) \propto m^{-y}k^{-x}$$



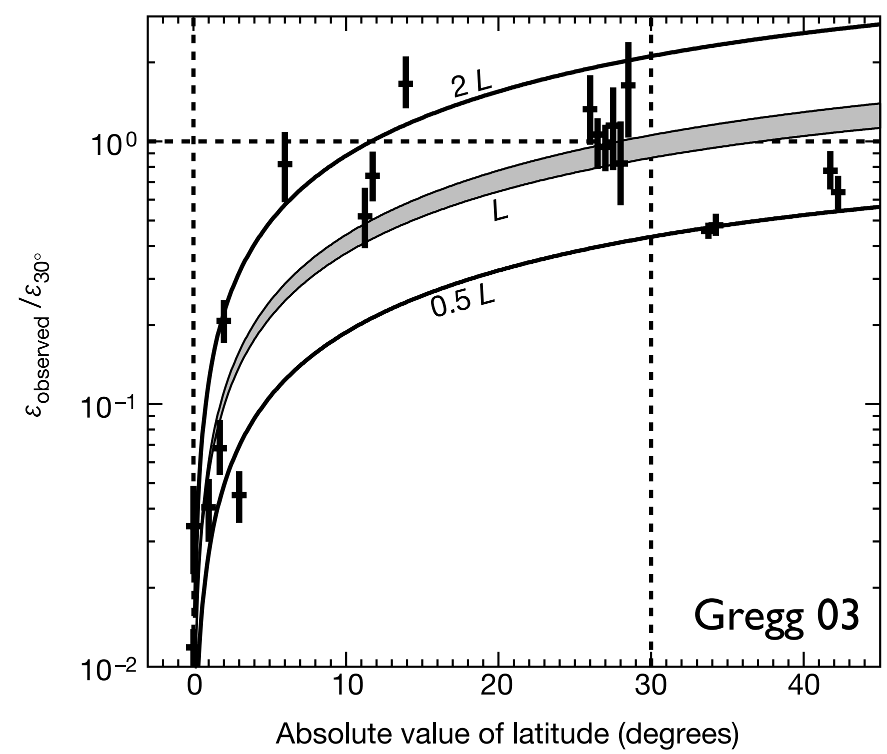

## Experimental verification

- Garrett-Munk empirical spectrum

$$E(m,\omega) \propto m^{-2}\omega^{-2}$$

- Observed dissipation rates agree with predictions, based on measurements of shear and strain. (Gregg 89, Polzin et al 95)

$$\varepsilon = \varepsilon_{30^\circ}\big(N, \Phi_{\text{shear}}(m), \Phi_{\text{strain}}(m)\big) \times L(\theta, N)$$




- Numerical simulations of internal-wave interactions. Confirm GM spectrum as steady state and predicted rates of down-scale energy flow. ( Hibiya et al, Winters and D'Asaro, etc)

Wave propagation through resolved mesoscale

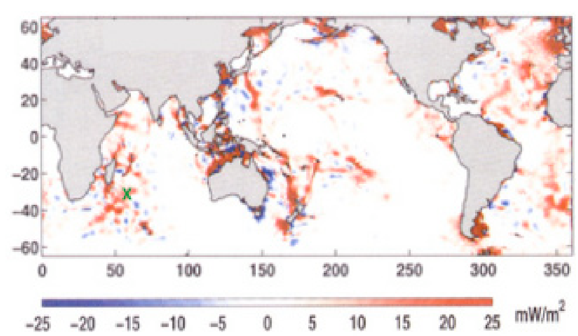
Spectral flux (down-scale)

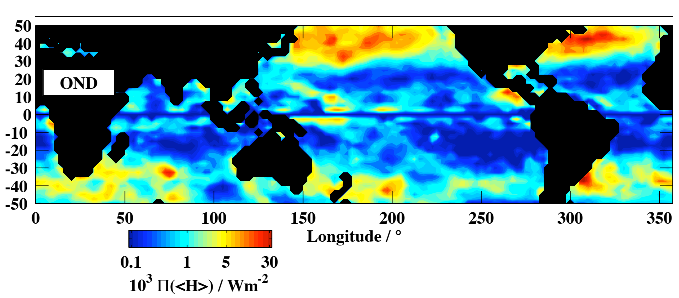
Wave breaking (small scales)

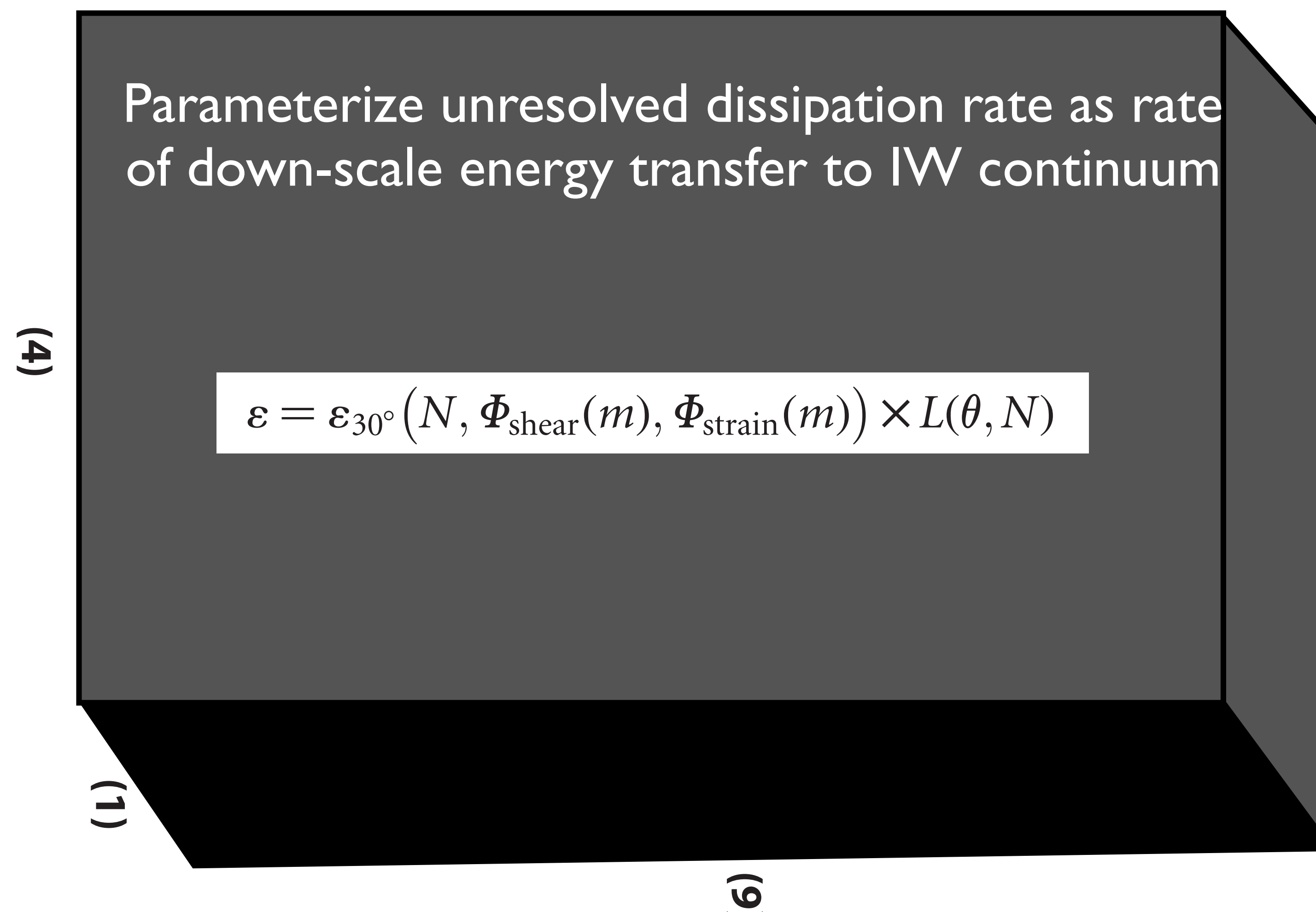
$$E_t(\vec{k},\vec{x},t) = S_o(k) - \nabla_x \cdot (C_g\ E) - \nabla_k \cdot F(k) - S_i(k)$$

## Sources

- low mode
- estimate from observations
- explicitly resolve in GCM (?)



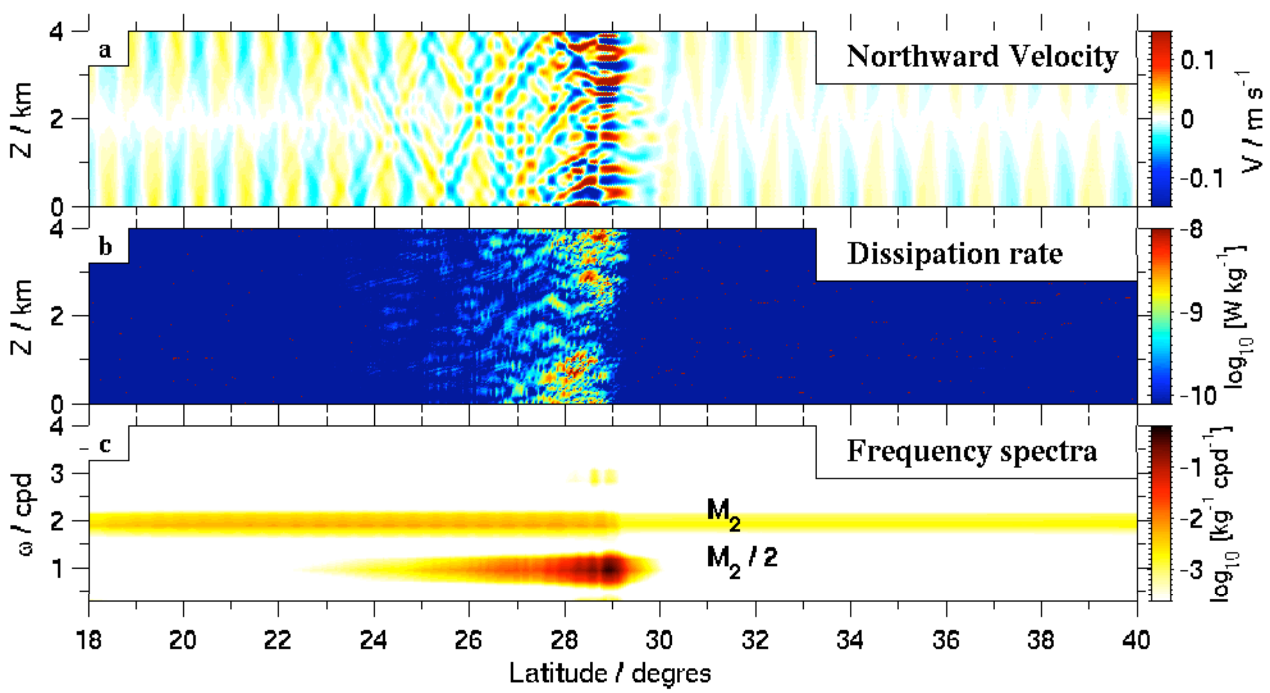




**But the problem is that there are some exceptions where wave energy doesn't cascade in this nice way, and these exceptions, though localized, might dominate mixing in key places. Best to illustrate through examples...**

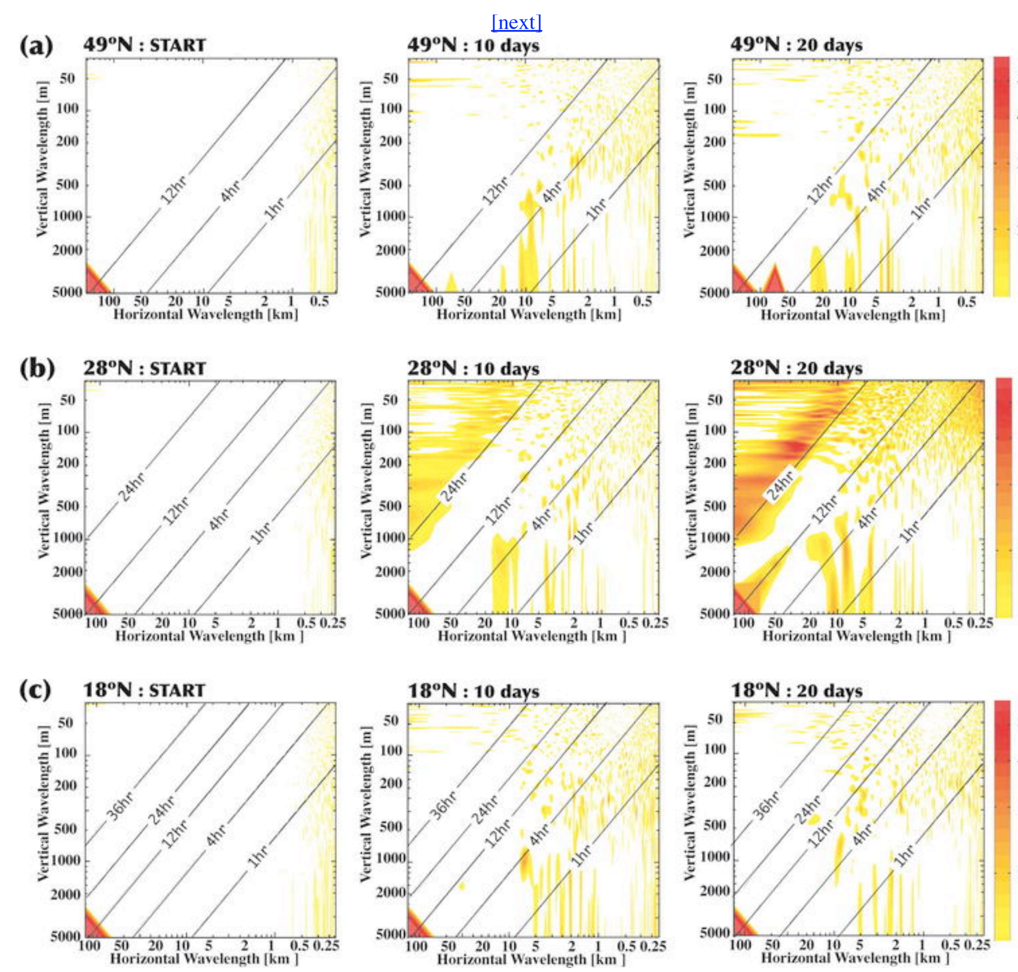
# Problem I: Fast wave interactions

Parametric Subharmonic Instability (PSI): Decay of a initial wave (e.g. low-mode internal tide), into two smaller-scale waves of half the frequency. Seems to be particularly efficient/resonant when the subharmonic frequency is equal to the local intertial frequency (28.9 N/S).

## Numerical evidence for fast PSI near 29



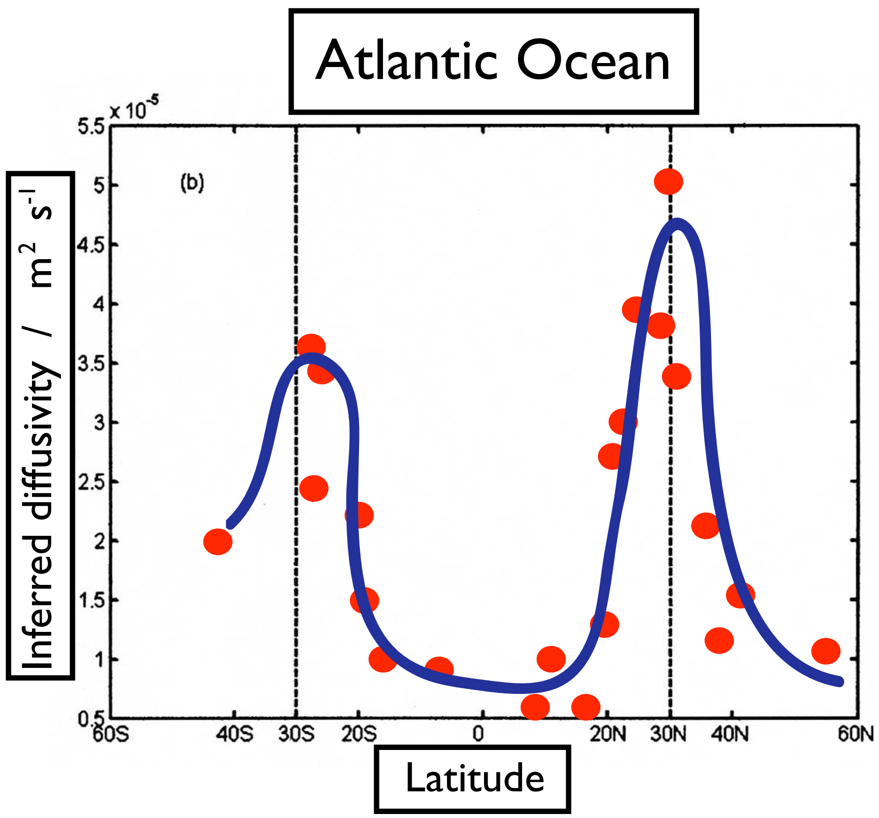

MacKinnon and Winters 05

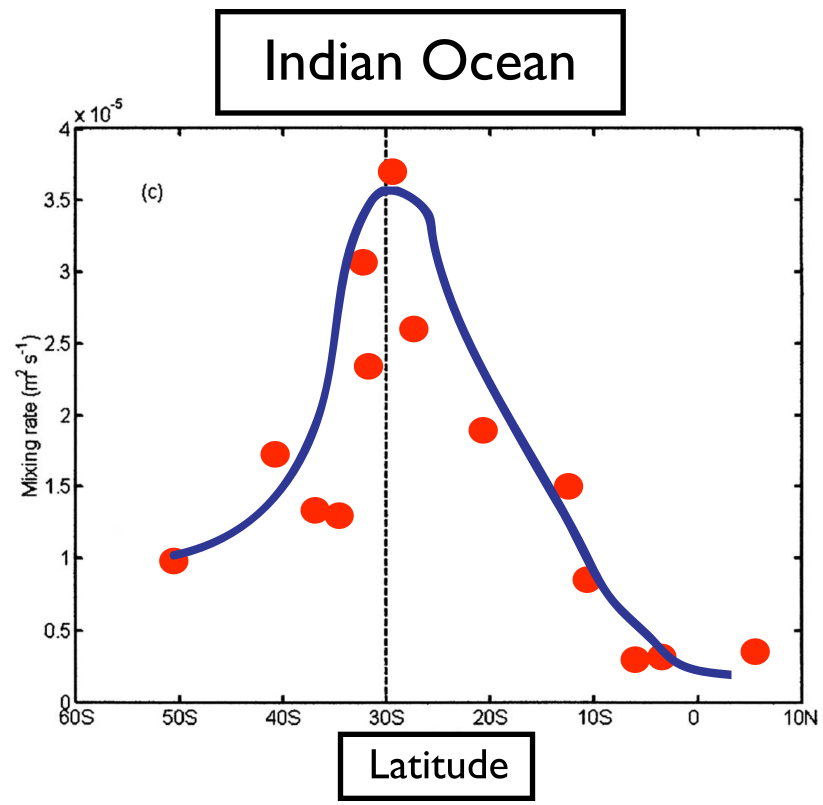


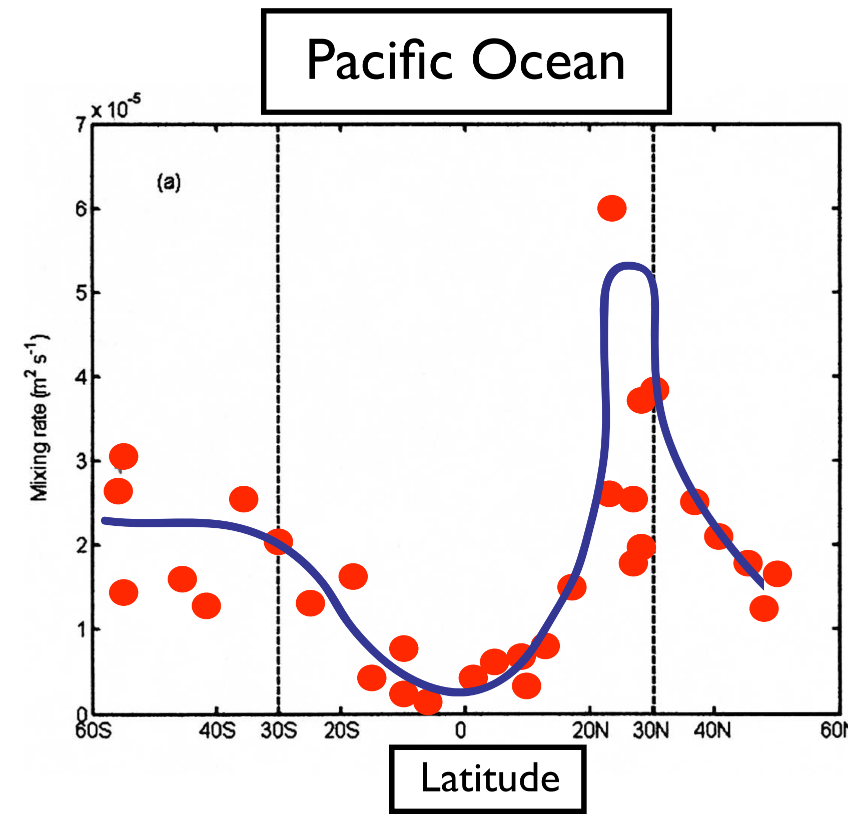

Furuichi et al 05

# Observational evidence for something special near 29

'Dissipation rate' of internal tide, as inferred from convergences in internal tide energy fluxes measured with satellite altimetry.



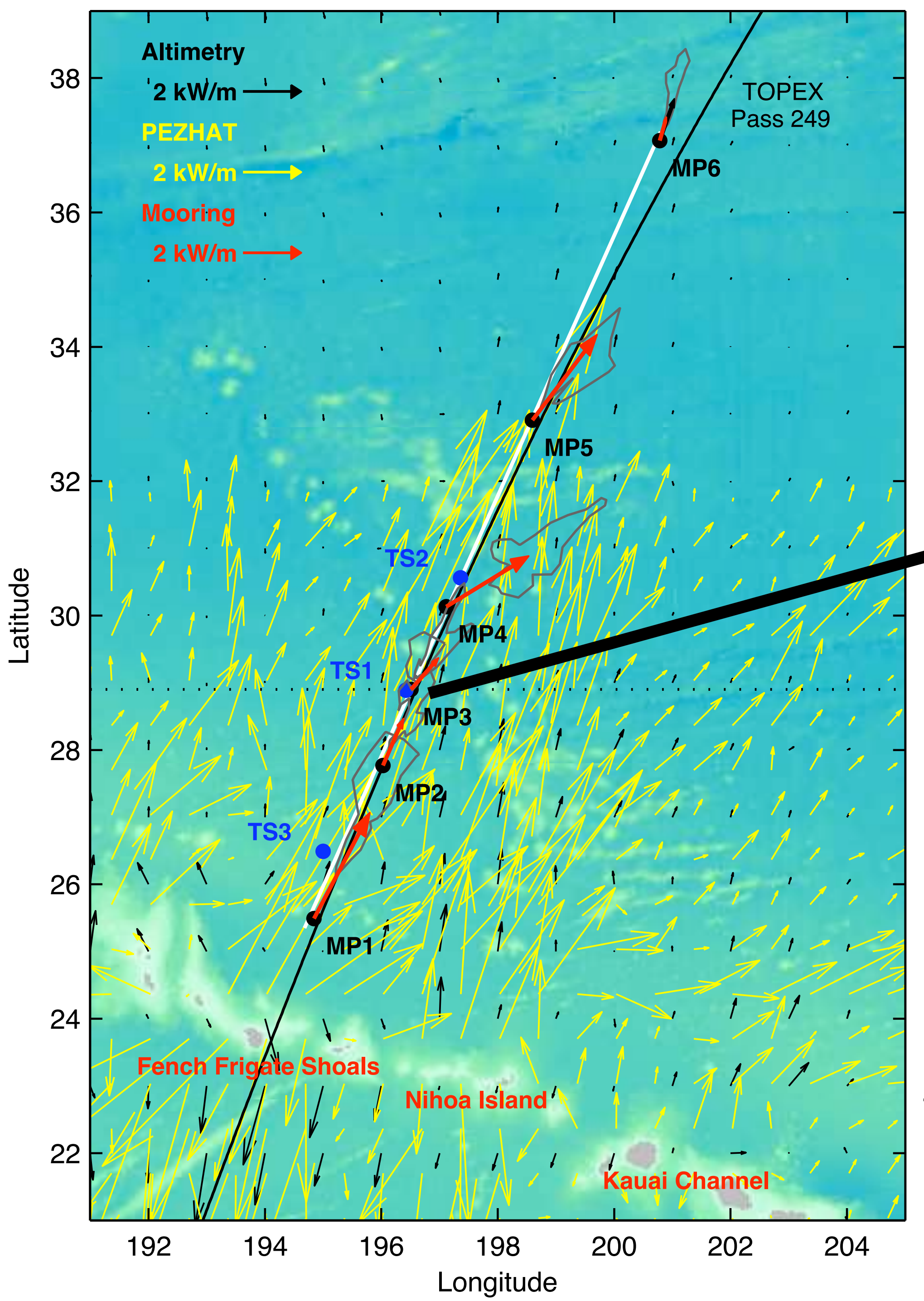

Tian et al 06:

(suggestive blue lines added by me)

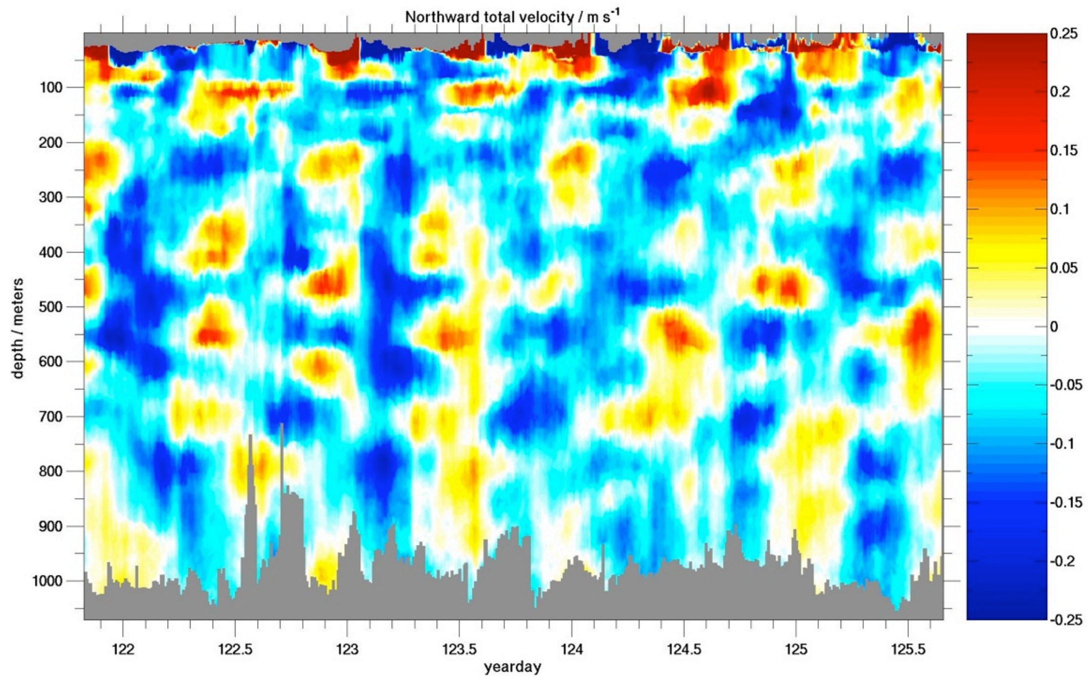
# Observational evidence for fast PSI near 29

## Internal Waves Across the Pacific (IWAP)

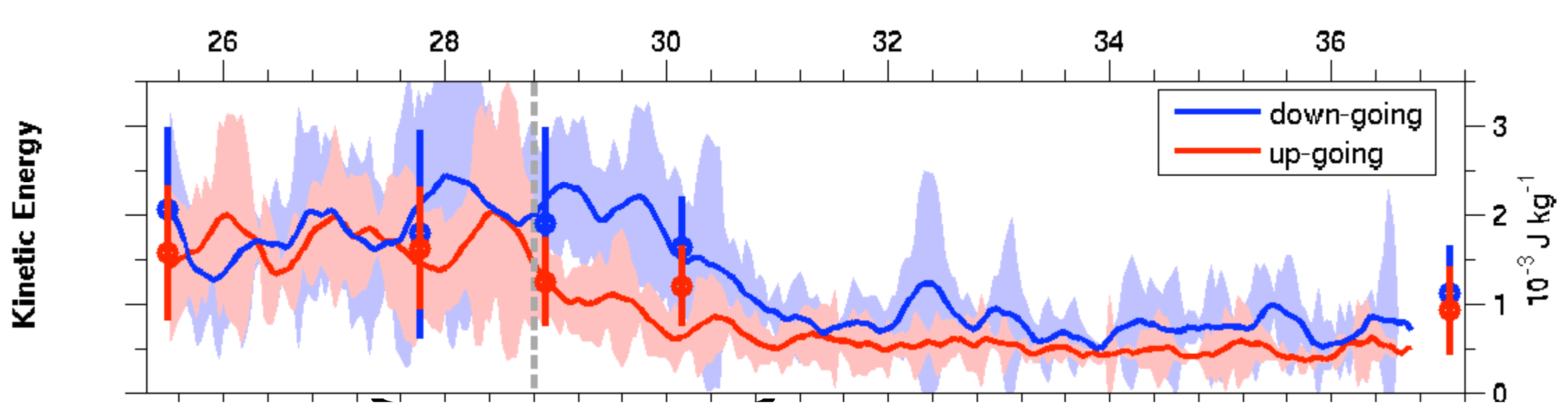
Alford, Klymak, MacKinnon, Munk, Pinkel



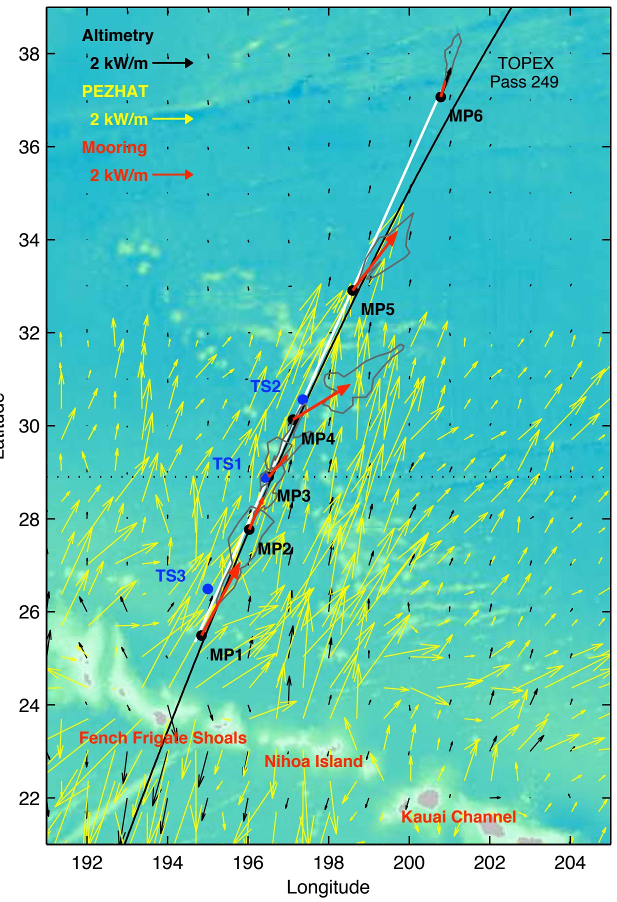

### Velocity at the 'critical' latitude

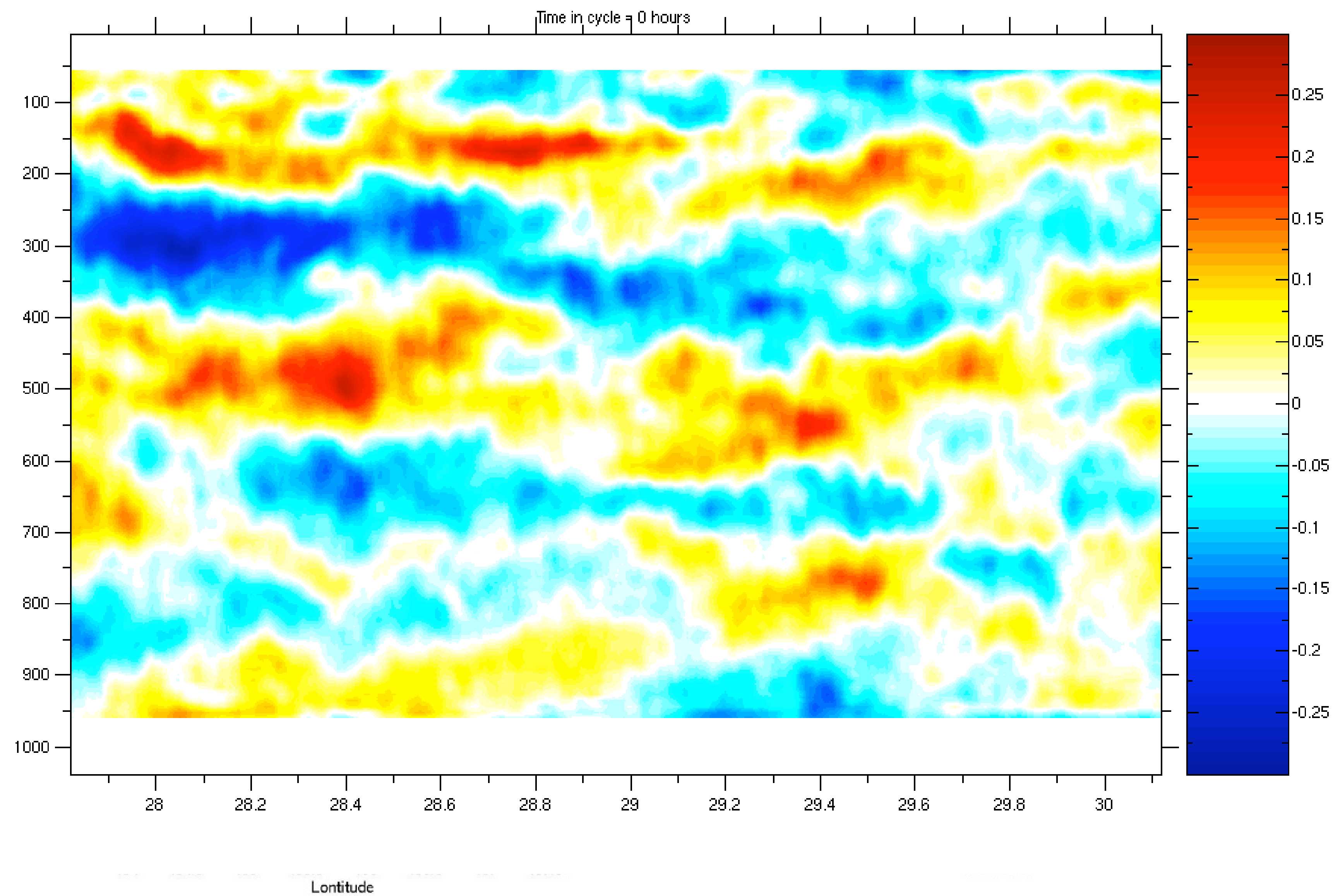


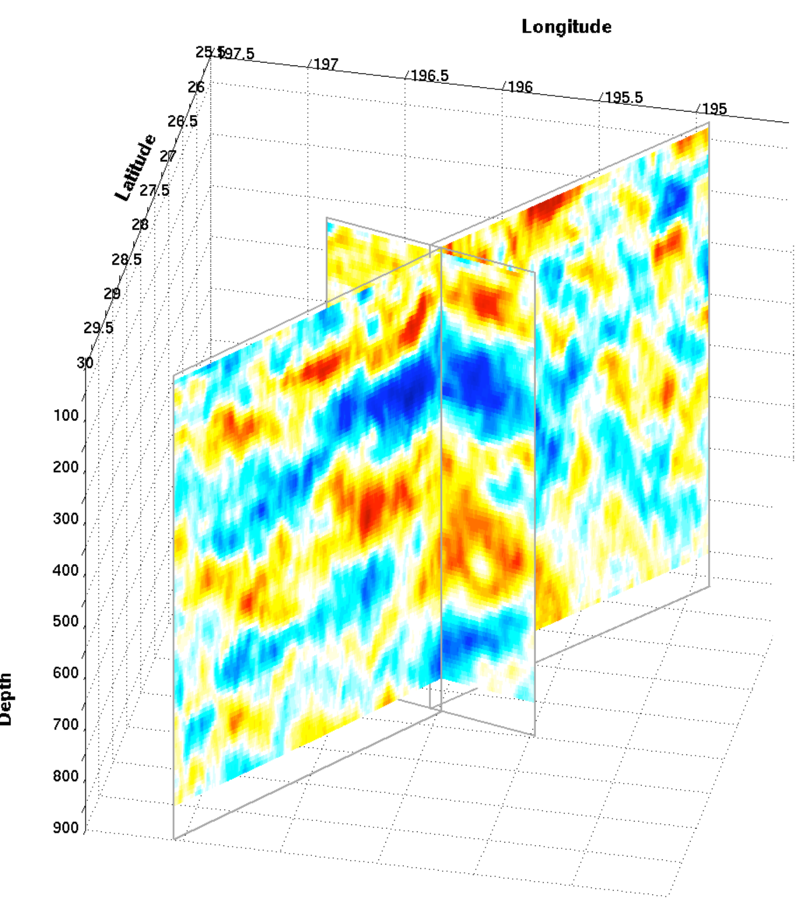

### Average near-inertial kinetic energy vs. latitude



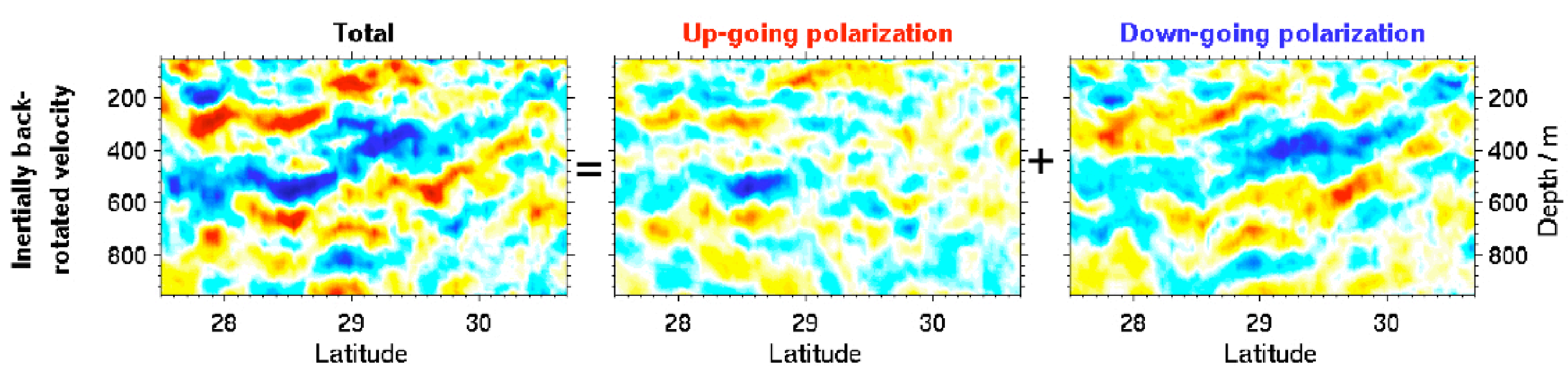
# Problem II: coherent waves



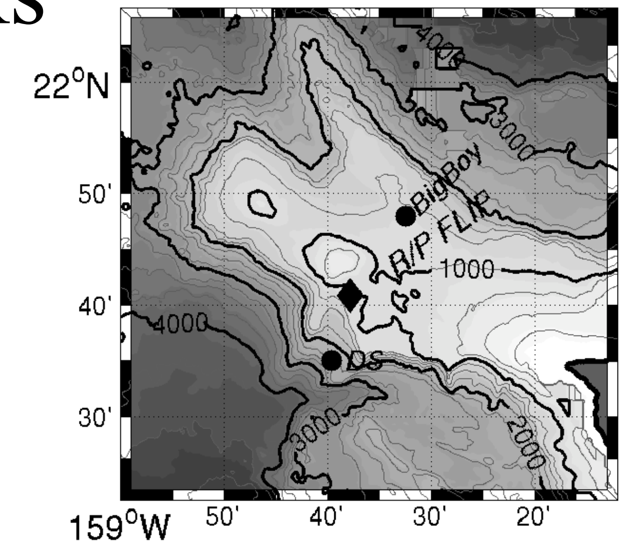

## North-south section of back-rotated velocity



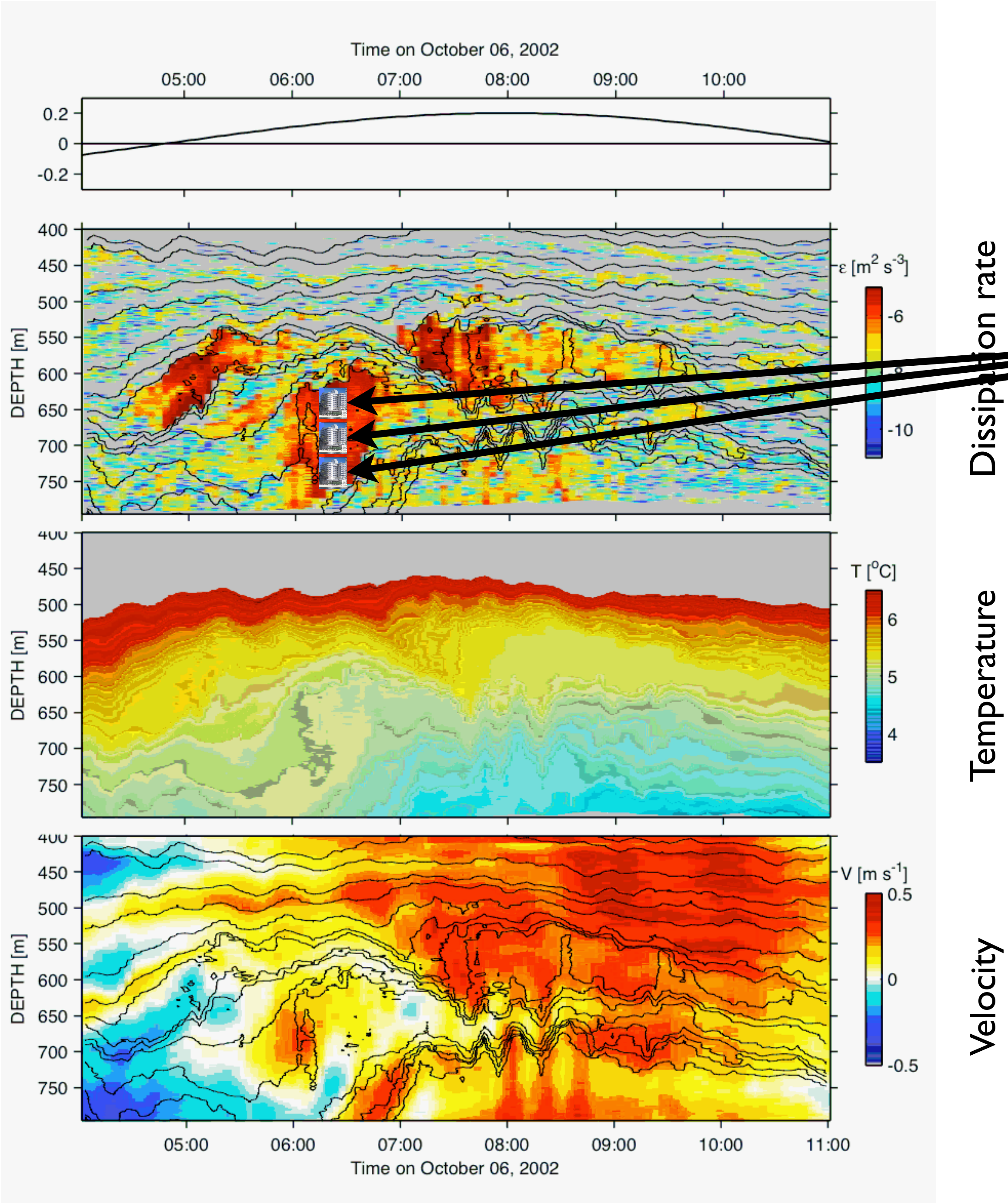
# Problem III - internal tide ‘directly breaks’

## Hawaiian Ocean Mixing Experiment (HOME)

Levine and Boyd 06
Aucan et al 05




Huge overturns as internal tide sloshes up and down a steep slope

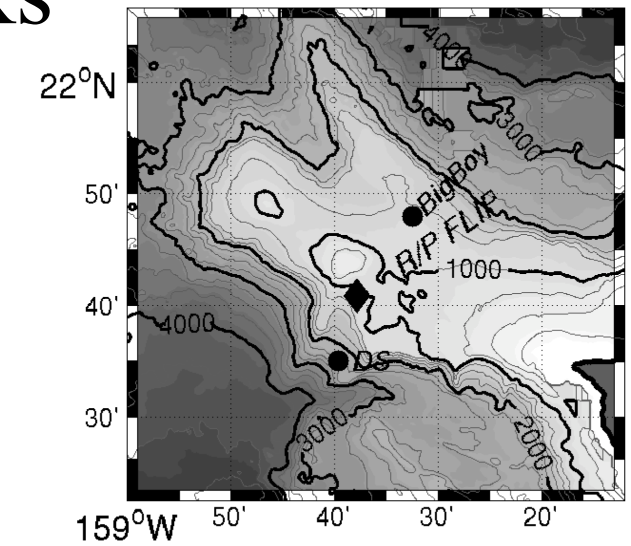



Klymak et al 07

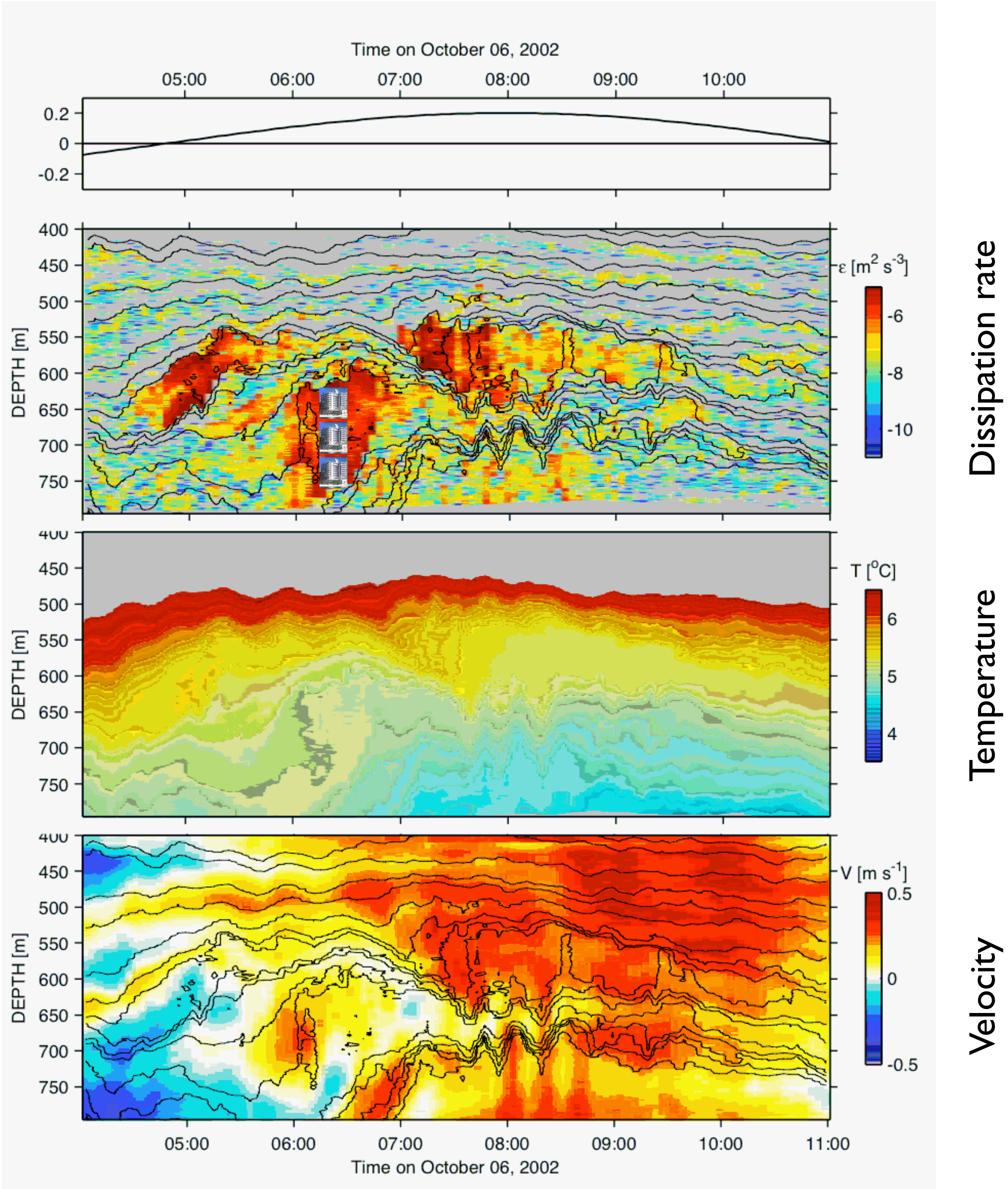
# Problem III - internal tide ‘directly breaks’

## Hawaiian Ocean Mixing Experiment (HOME)

Levine and Boyd 06
Aucan et al 05

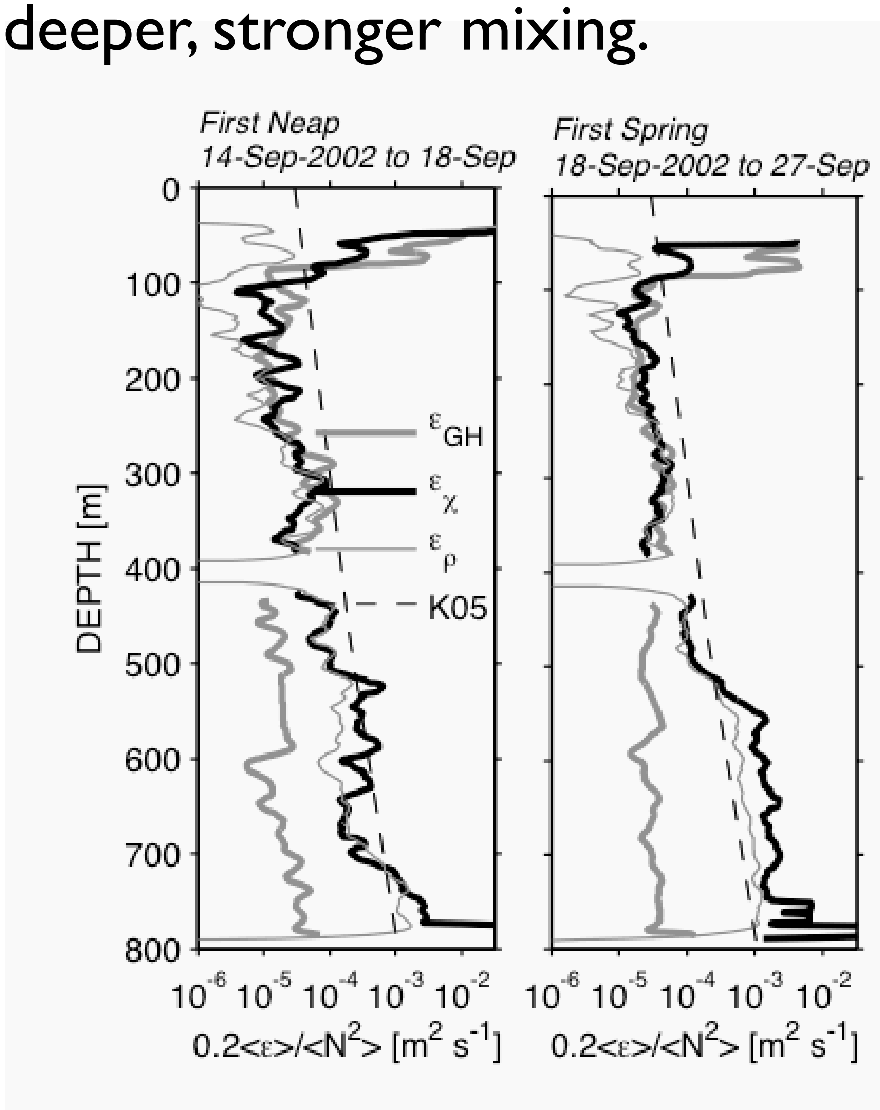



Huge overturns as internal tide sloshes up and down a steep slope



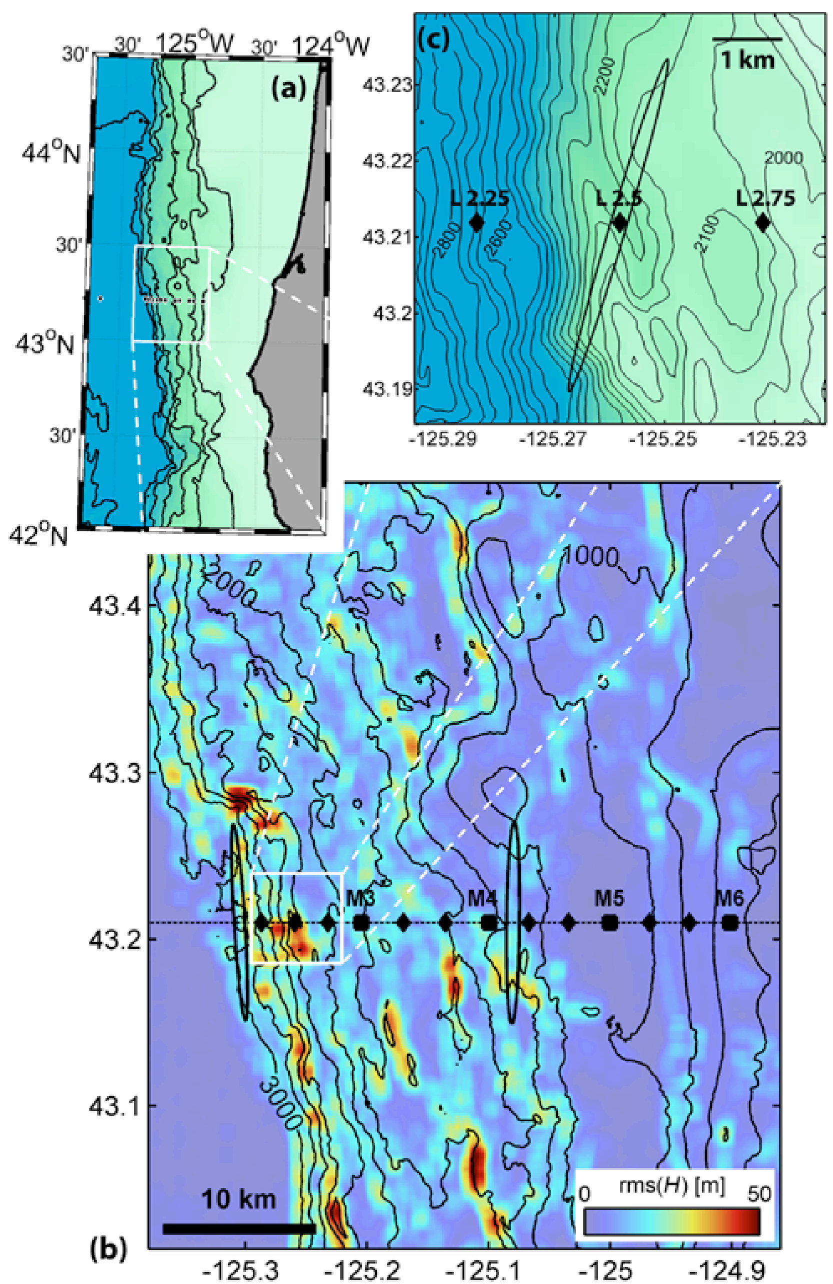

Klymak et al 07

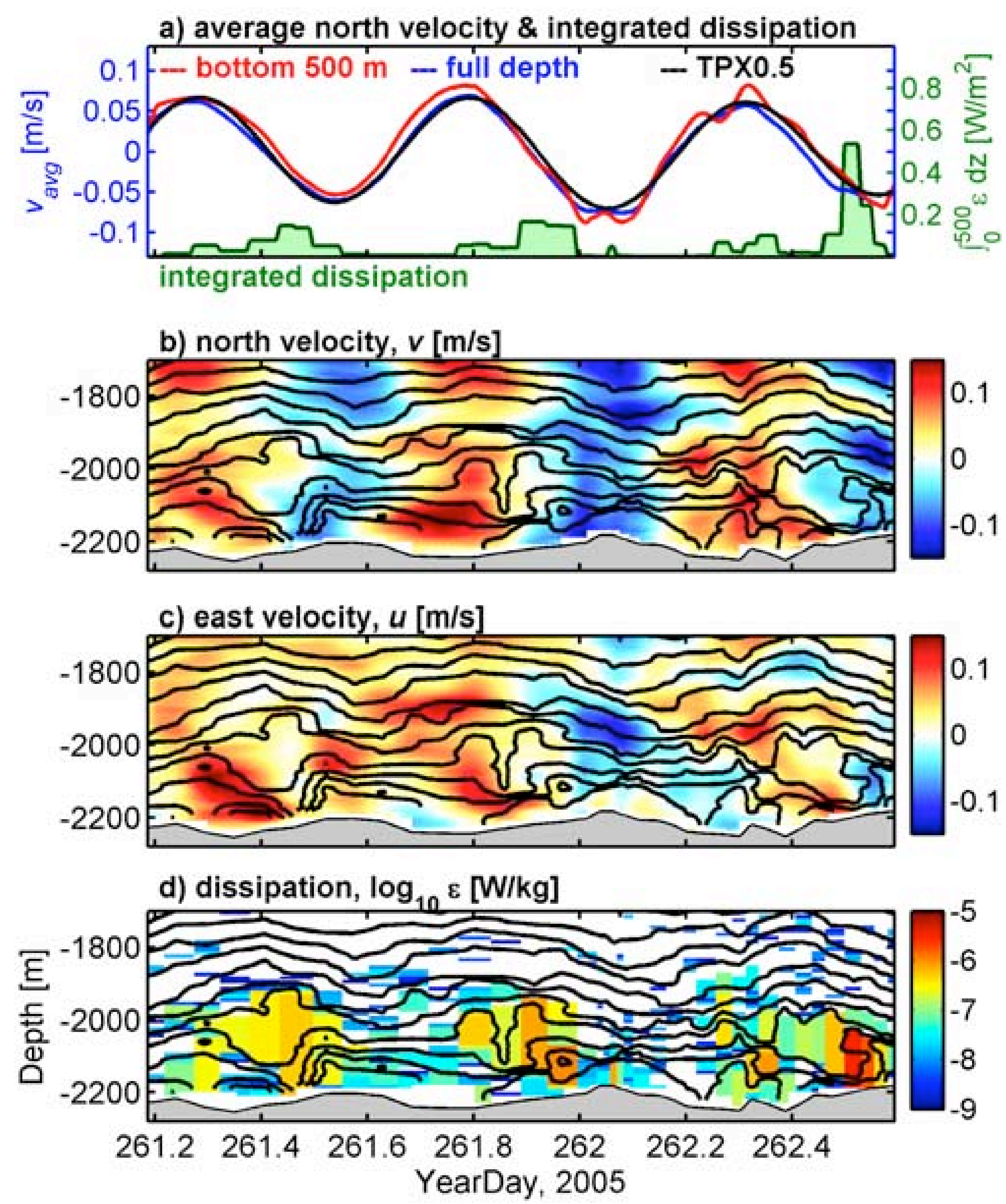
Gregg-Henyey parameterization works in upper water column, but not for the deeper, stronger mixing.



# Directly breaking internal tide on Oregon slope

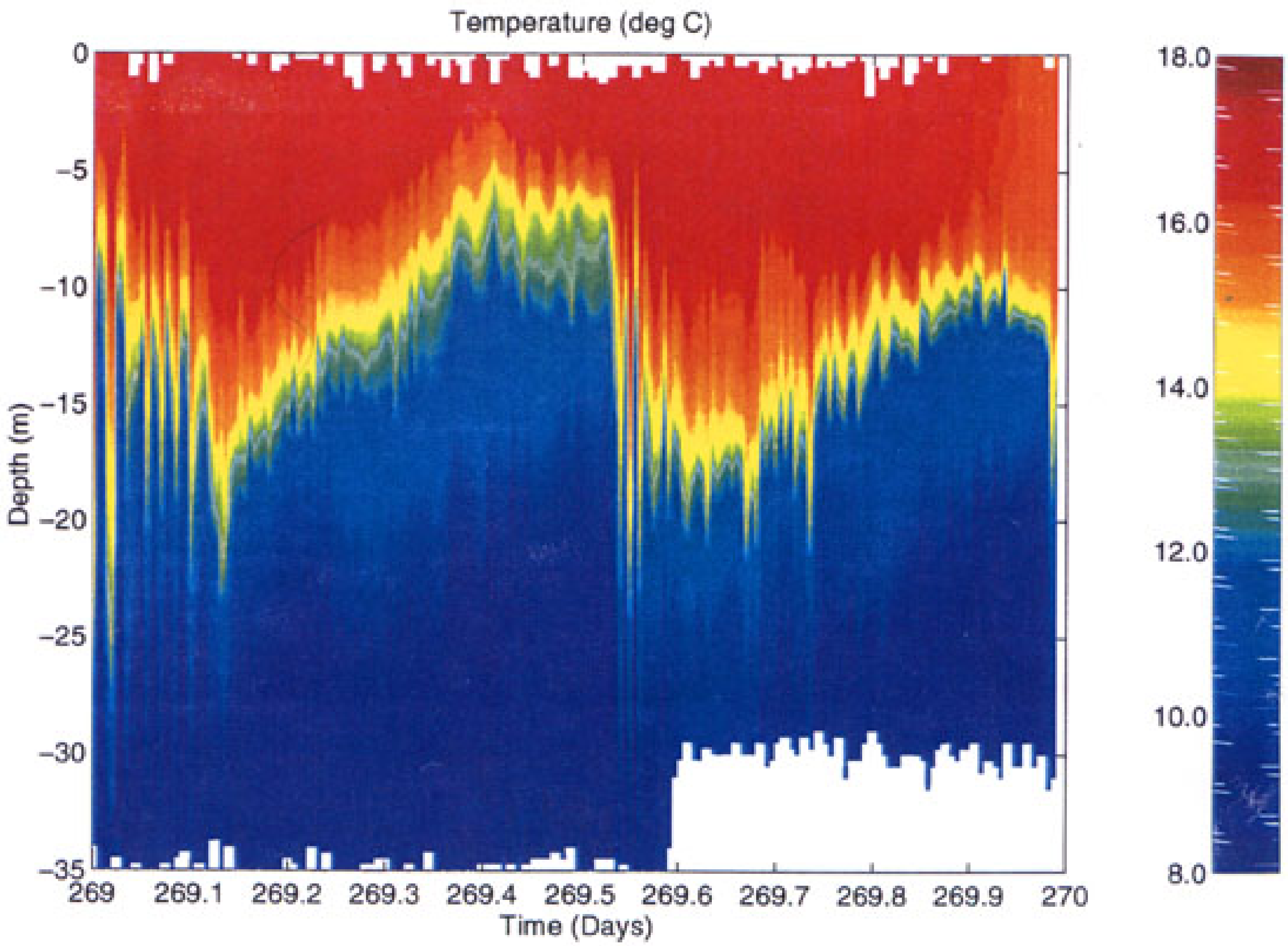
Similarly huge overturns, but in a place not predicted by global internal tide generation maps.



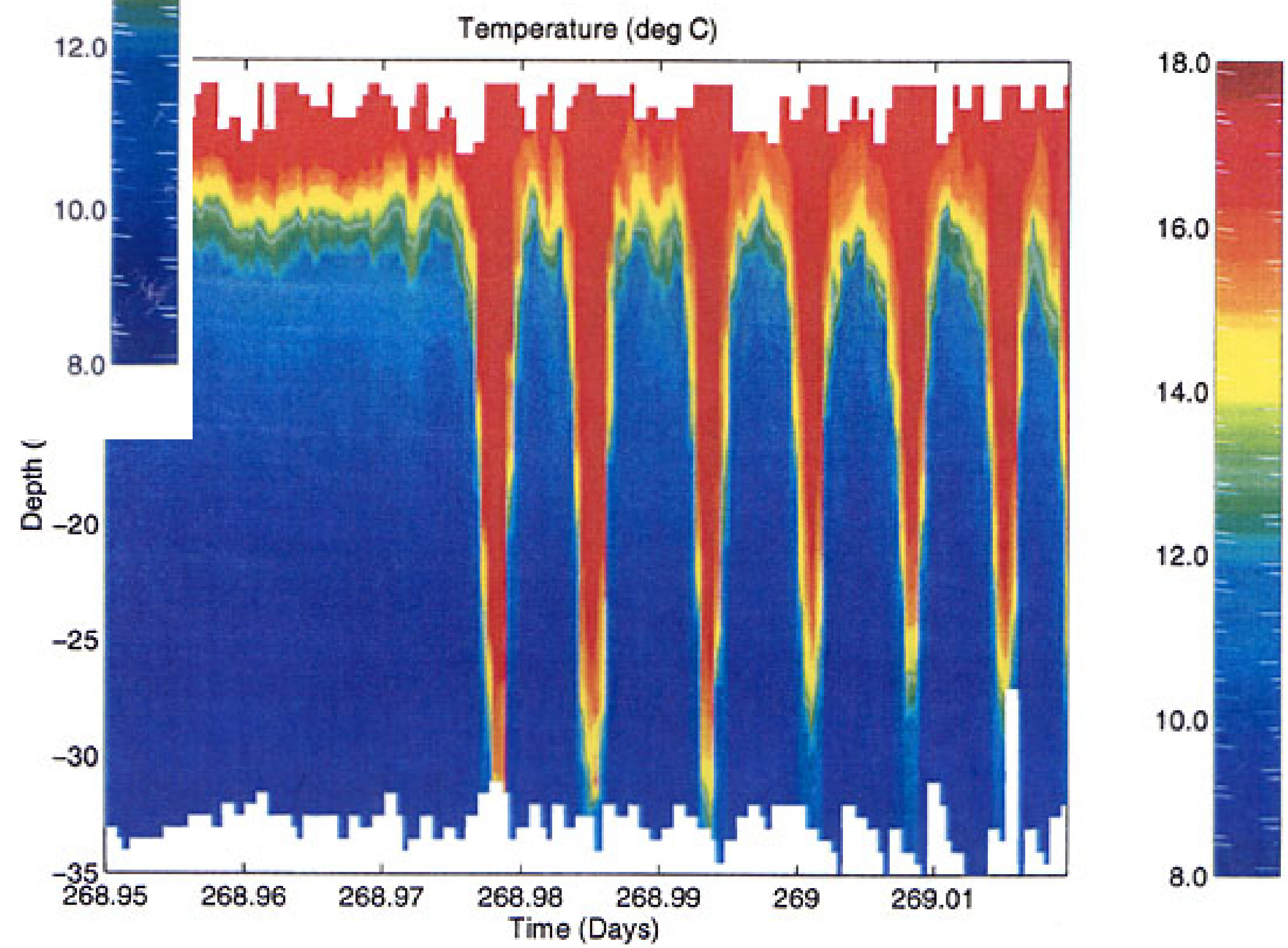

Nash et al 07

# Problem IV: Solitons (internal waves of unusual size)




24 hours

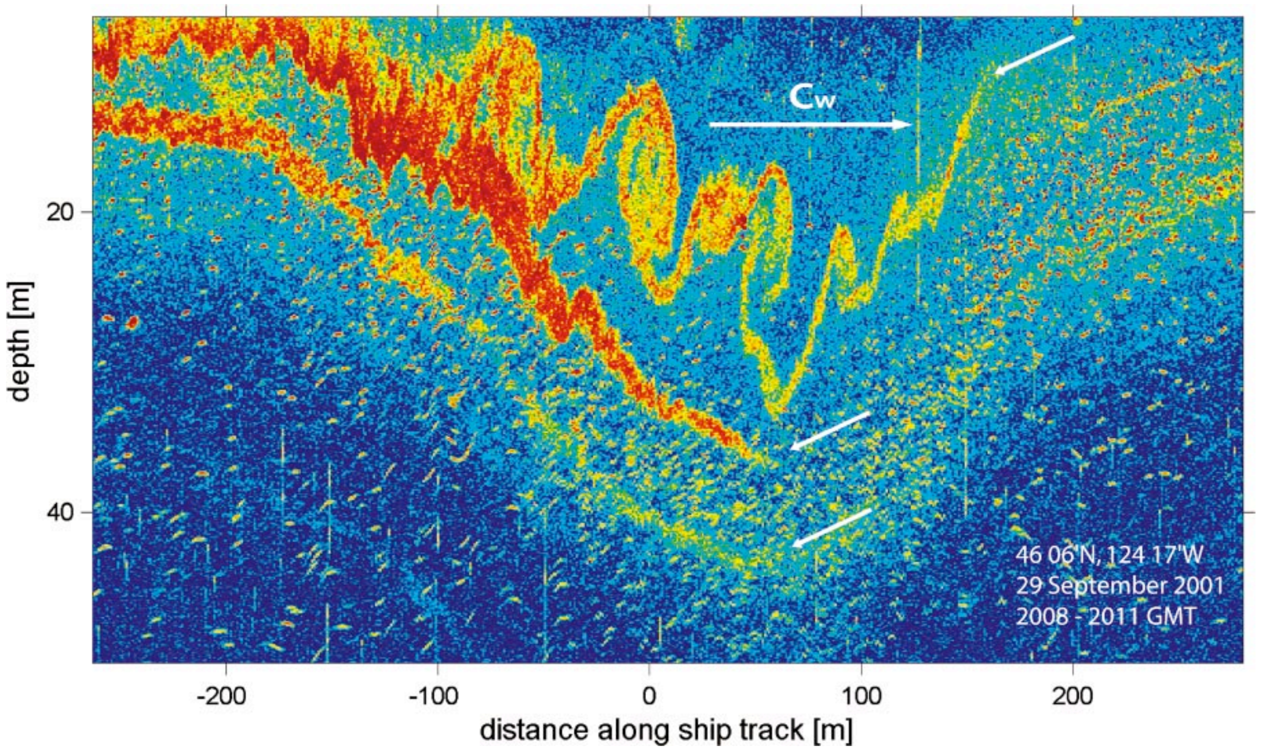
nonlinear steepening
balanced by disperson



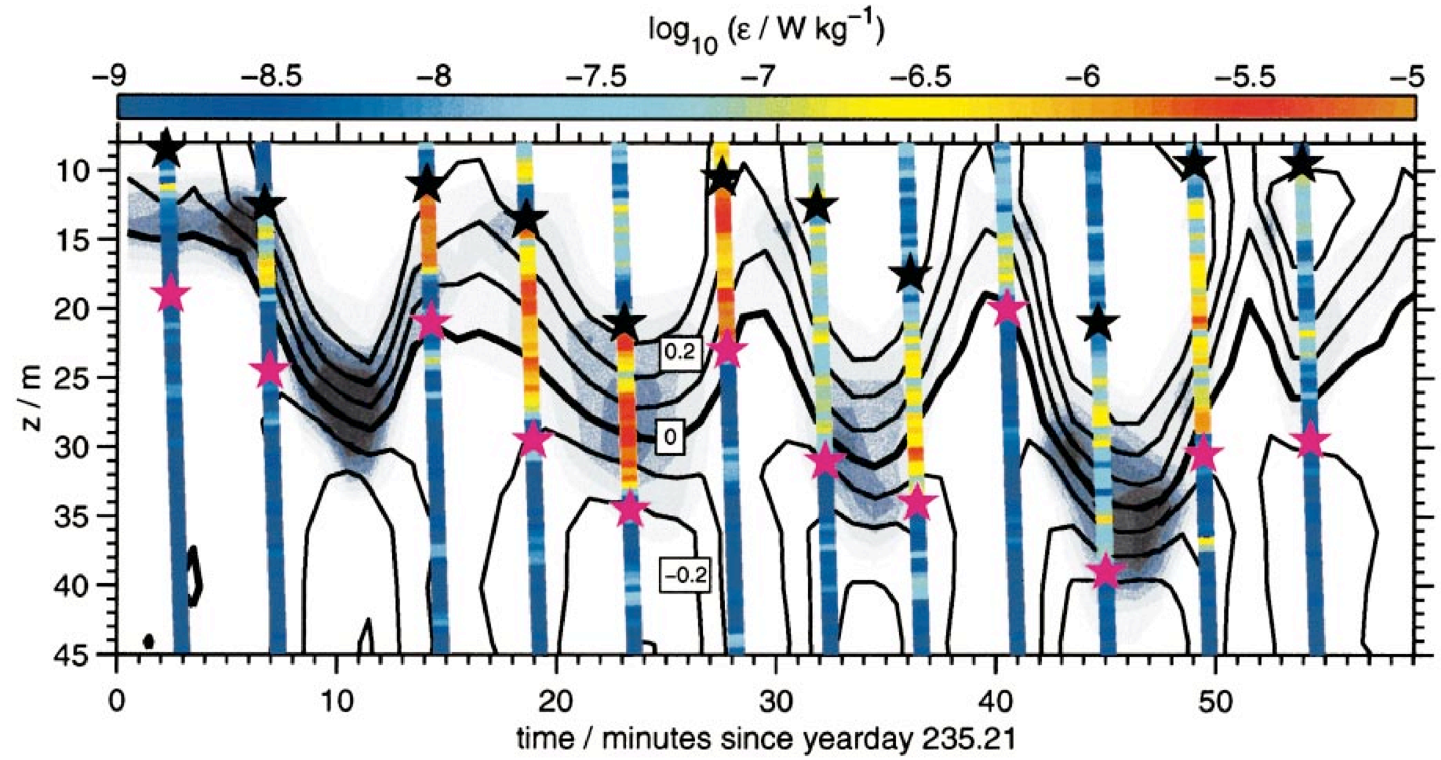

Stanton and Ostrovsky
GRL 24(14) 1998

100 minutes

# Extreme soliton breaking



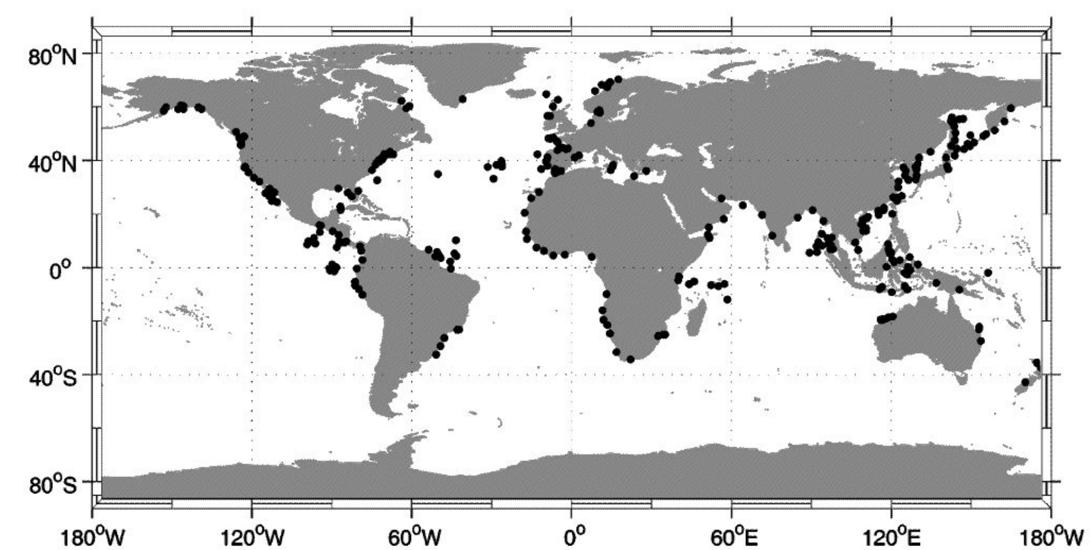

Moum et al 03




MacKinnon and Gregg 03

50-75 % of total average daily mixing occurs in these few minutes

## Solitons around the world




Apel et al 06

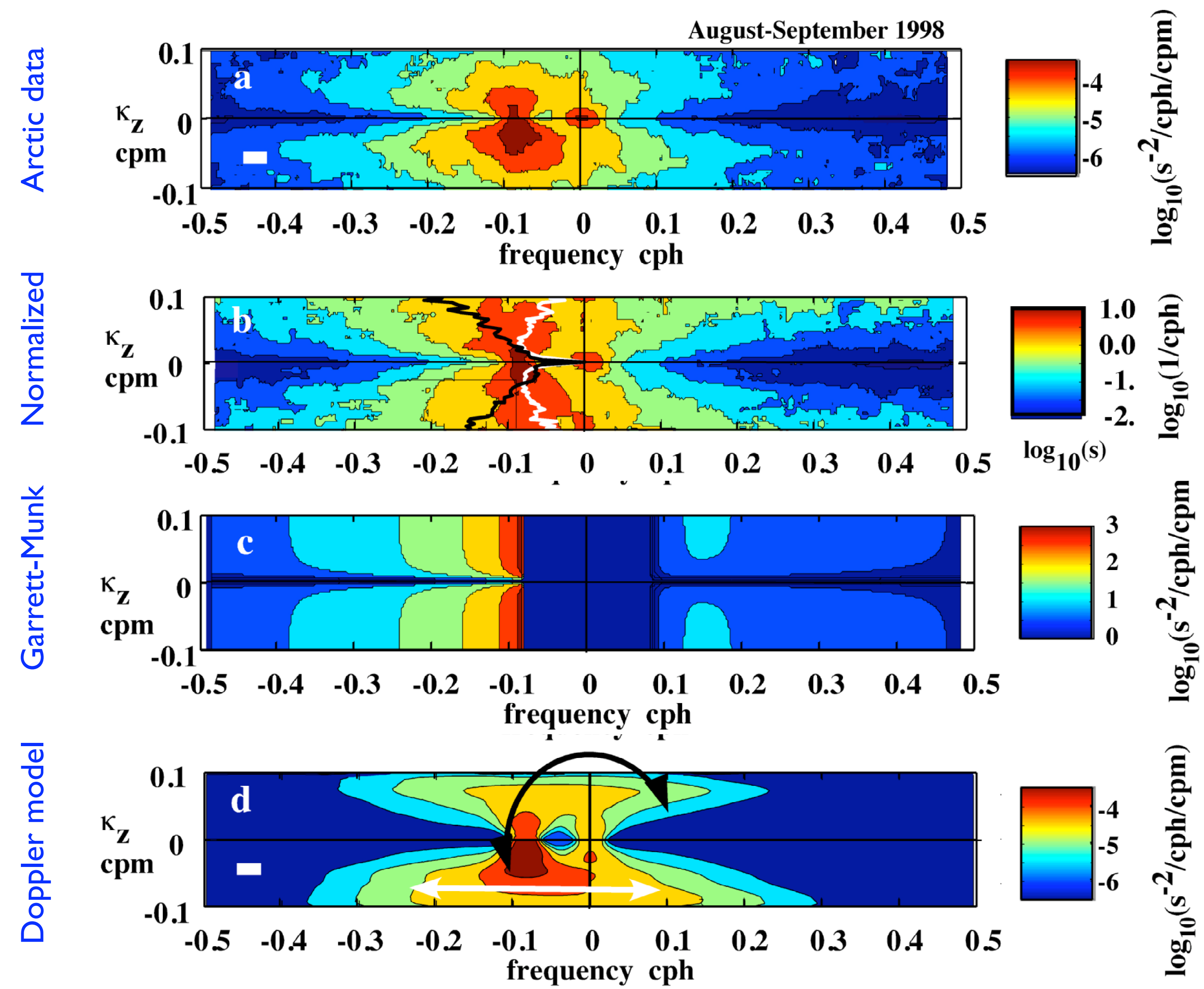
# Is the spectrum a myth? (the personalist approach)

Doppler shifting makes for broadened frequency band at higher wavenumbers.

$$s_{Eul} = s_0 \exp(i\,\underline{k}*\underline{x} + \underline{k}*(\underline{V}\,t) - \omega\,t)$$

Intrinsic shear only at inertial and vortical (f=0) frequencies, advected by horizontal currents

Perhaps the internal-wave continuum is in the eye of the beholder, at least the high-wavenumber part where all the shear lives.



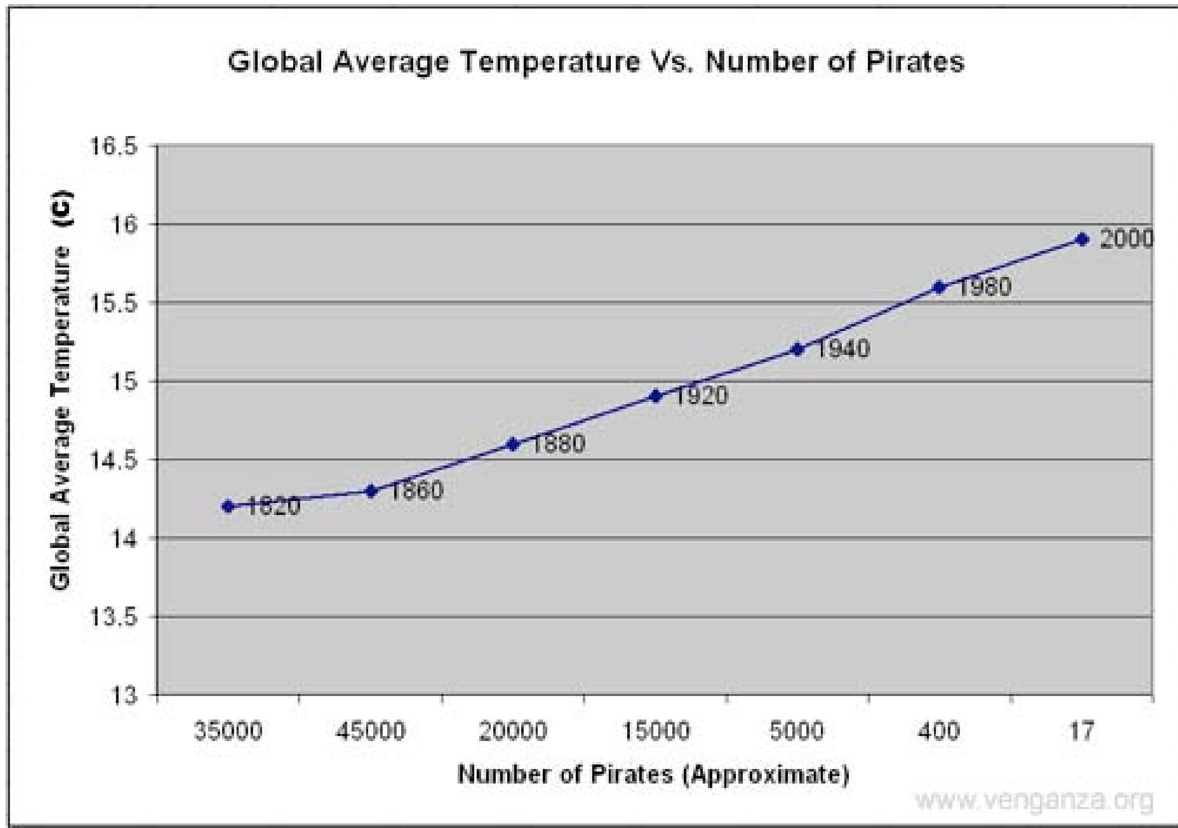

Pinkel 07

# Conclusions?

- Statistical models of internal-wave breaking are based on rates of down-scale energy transfer through steady, gaussian fields of incoherent waves. Such models appear to work some places, but many of these are ‘boring’ places.
- In other places, mixing is dominated by extreme events, mostly because ‘special physics’ is taking place.
- How much do extreme events matter?

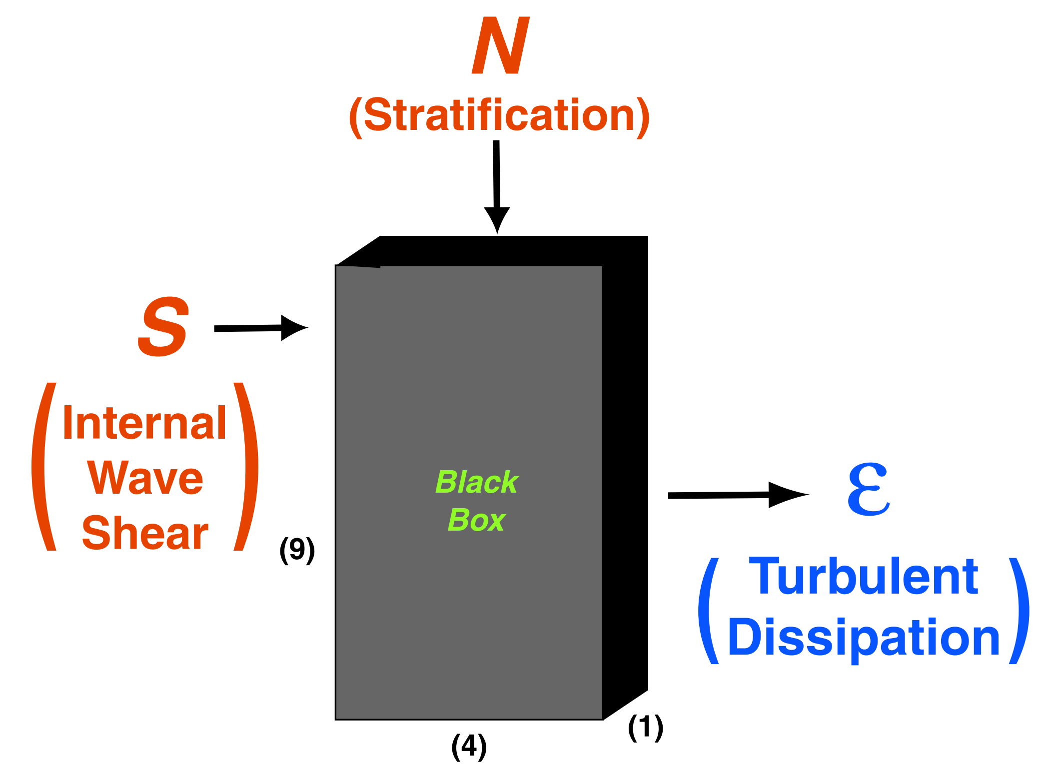
Coastal ocean - probably quite a bit, in shallow water most internal wave breaking is abnormal.

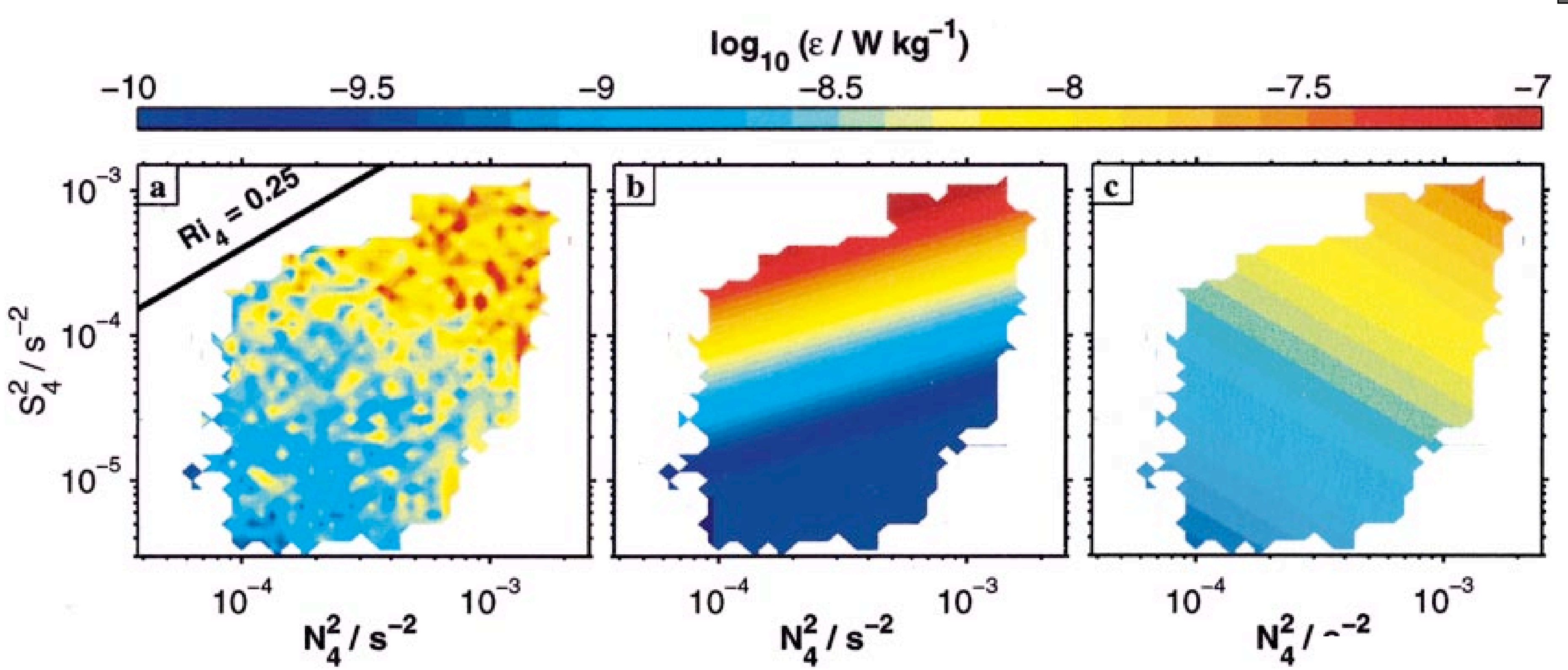
Open ocean - hard to say.  Factor of 2 near hawaii, averages out to not much basin-wide.  But perhaps extreme breaking events are undersampled.



# Breaking internal waves

Even if you can't directly see turbulence, can parameterize turbulent mixing in terms of observed (low-resolution) internal-wave shear.



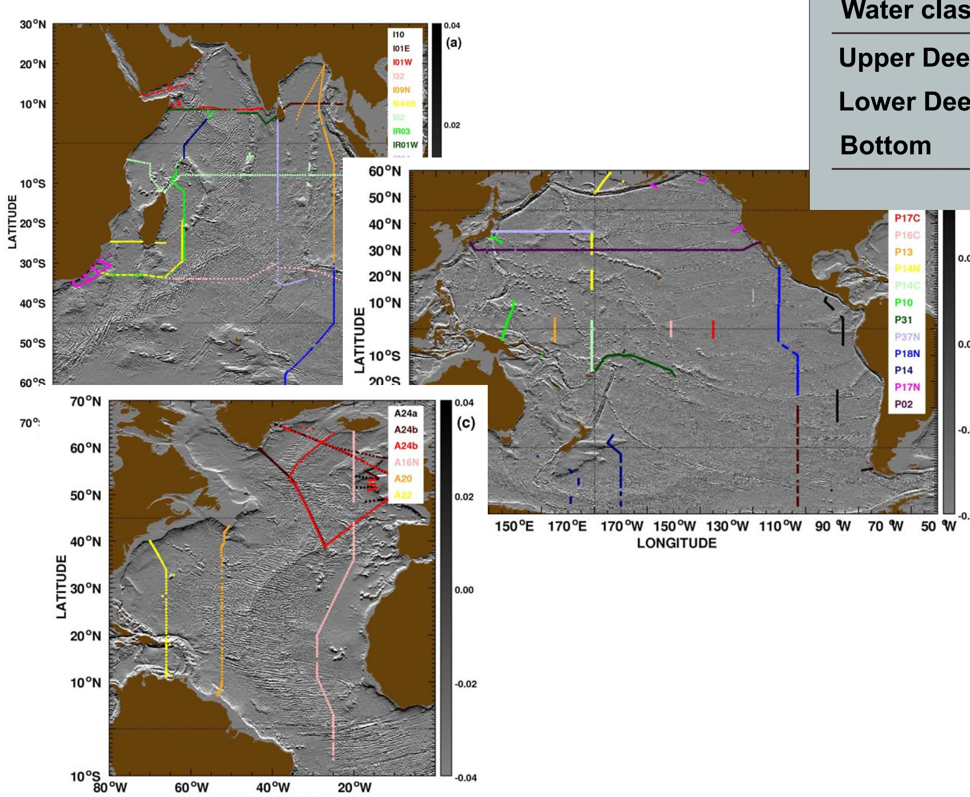

$$\epsilon_{\mathrm{MG}} = \epsilon_0 \left(\frac{N}{N_0}\right)\left(\frac{S_{lf}}{S_0}\right) \quad (\mathrm{W\ kg^{-1}})$$

# are we missing mixing still?

| Program/ Experiment | Diffusivities O(×10$^{-4}$) m$^2$/s | Notes |
|---|---|---|
| C-SALT/ SFTRE | 0.1 (above/below stairs)<br>1 (in staircase) | Double-diffusive staircase |
| PATCHEX | 0.1 (thermocline) | Thermocline processes |
| Tropic Heat 1/ Tropic Heat 2 | 0.1 (in thermocline)<br>1 (in undercurrent) | Equatorial processes |
| NATRE | 0.1 (thermocline)<br>1 (in NADW) | First open-ocean tracer release |
| BBTRE | 0.1 (in thermocline)<br>1 (in NADW)<br>>10 (in AABW) | Basin-scale survey |
| HOME | 0.1 (away from ridge)<br>1 (near the ridge) | Energy budget study |

**TABLE 1** Spatial-average across-isopycnal diffusviities, as estimated by various budget methods. In the first six rows, the bottom region lies between neutral surface 28.1 and the sea floor (see Figure 1), and from 27.96 to 28.07 for the deep layers. Generally, these lie between about 3800 m and the sea floor, and between about 2000 m and 3800 m, respectively, but with considerable spatial variations. The last six estimates are from restricted basins or channels, but all values are spatial averages over the interior and boundary layers

| Ocean/Depth | $\langle K \rangle$ (10$^{-4}$ m/s) | Reference |
|---|---|---|
| Atlantic, bottom | 9 ± 4 | Ganachaud & Wunsch (2000) |
| Indian, bottom | 12 ± 7 | " |
| Pacific, bottom | 9 ± 2 | " |
| Atlantic, deep | 3 ± 1.5 | " |
| Indian, deep | 4 ± 2 | " |
| Pacific, deep | 4 ± 1 | " |
| Scotia Sea | 30 ± 10 | Heywood et al. (2002) |
| Brazil Basin | 3–4 | Hogg et al. (1982) |
| Samoan Passage | 500 | Roemmich et al. (1996) |
| Amirante Trench | 10.6 ± 2.7 | Barton & Hill (1989) |
| Discovery Gap | 1.5–4 | Saunders (1987) |
| Romanche Fracture Zone | 100 | Ferron et al. (1998) |




## Indirect estimates

**Inverse methods have been applied on global scales to estimate mixing rates**

**Ganachaud and Wunsch 2000**

| Water class | density $\gamma_n$ | $k_v$ cm$^2$ s$^{-1}$ |
|---|---|---|
| Atlantic deep | 27.96 < $\gamma_n$ < 28.1 | 3 ± 1.5 |
| Indian deep | " | 4 ± 2 |
| Pacific deep | " | 4 ± 1 |
| Atlantic bottom | $\gamma_n$ > | 9 ± 4 |
| Indian bottom | 28.1 | 12 ± 7 |
| Pacific bottom | " | 9 ± 2 |

**Lumpkin & Speer 2006**

| Water class | density $\gamma_n$ | $k_v$ cm$^2$ s$^{-1}$ |
|---|---|---|
| Upper Deep | 27.0 < $\gamma_n$ < 27.8 | 0.3 ± 0.5 |
| Lower Deep | 27.8 < $\gamma_n$ < 28.1 | 1 ± 0.5 |
| Bottom | $\gamma_n$ > 28.1 | 3 ± 1 |