Pseudo-spectral models of internal-wave interaction

Jennifer MacKinnon
Kraig Winters

with generous help from
Jody Klymak
Rob Pinkel
Matthew Alford
Walter Munk

Scripps Institution of Oceanography
University of California, San Diego

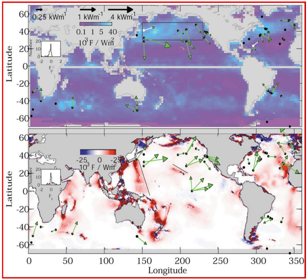
http://cfd.ucsd.edu

13 October 2004

What sets global mixing patterns?

"You **must** believe something other than PSI matters" - Lou

Energy input into low-frequency waves



Alford 03, Nature

Mesoscale interactions
(eddies, fronts, ACC)

Wunsch
Ferrari
Heywood & Naveira-Garabato
Rainville

$\vec{c}_g E_{\omega_0}(\vec{x})$ $\xrightarrow{\text{1}}$ IW Continuum $\Psi(\vec{k}, \omega, \vec{x})$ $\xrightarrow{\text{2}}$ $\varepsilon(\vec{x})$

Topographic effects
(scattering, reflection, hydraulics)

Legg
Thurnherr
St. Laurent and Garrett
HOME
TWIST

1 "The first jump": Energy loss from low-frequency (tidal or near-inertial) waves (Hibiya et al)

$$\tau = \tau\,(E?, k?, f?, ??)$$

2 Statistical cascade downscale to turbulence (random phase approximation)
e.g. Kunze (yesterday), Lvov (this morning), Gregg 03, Polzin, McComas and Mullter, Henyey et al, etc

Which step is the rate-controlling process?
Which step sets global patterns of mixing?

Questions:

1. **How do nonlinear interactions control the rate of energy transfer out of a coherent internal tide?**
2. **What are the implications for the spatial distribution of tidal mixing?**

Nearfield

Inspired by Brazil Basin experiment

Upward propagating tide (f-plane)

Mode 10

Farfield

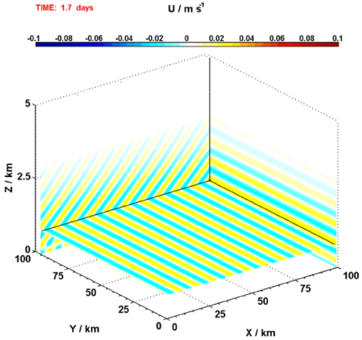
Inspired by HOME

Northward propagating tide (β-plane)

Mode 1

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u}\cdot\nabla\mathbf{u} - 2\Omega\times\mathbf{u} - \frac{1}{\rho_0}\nabla p - \frac{g\rho'}{\rho_0} + \mathbf{F_u}(z) + \nu\nabla^p\mathbf{u}$$

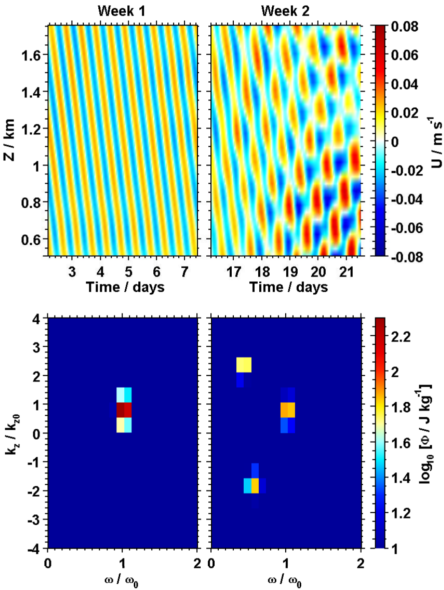
- Boussinesq, non-hydrostatic
- Pseudospectral
- Horizontally periodic
- Free-slip top/bottom boundaries
- 3-Dimensional
- Sponge layer aloft
- Constant stratification
- Infinitesimal white noise



$F_u(z)$: Near bottom tidal forcing

Rate of energy removal at smallest scales

$$\varepsilon = \nu_j\left[\frac{\partial}{\partial x_j}^{p/2} u_{kk}\right]^2$$



A sum of waves

$$
\begin{aligned}
u &= u_0(T)e^{i[\mathbf{k_0}\cdot x - \omega_0 t + \phi_0(T)]} \\
&+ u_1(T)e^{i[\mathbf{k_1}\cdot x - \omega_1 t + \phi_1(T)]} \\
&+ u_2(T)e^{i[\mathbf{k_2}\cdot x - \omega_2 t + \phi_2(T)]} + \ldots
\end{aligned}
$$

interact on a slow timescale (T)

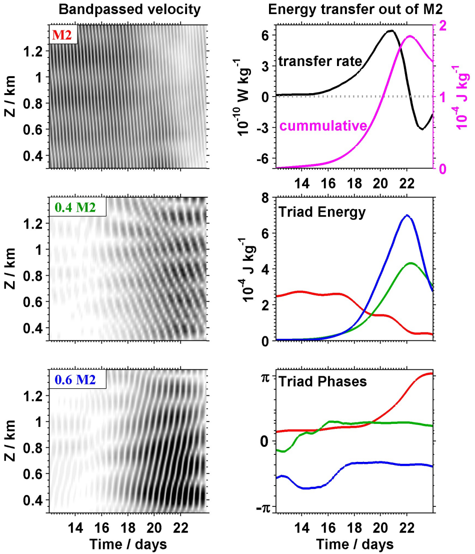
$$\frac{d}{dT}u_0 = -u_1 \cdot \nabla u_2 - u_2 \cdot \nabla u_2 + \ldots$$

Resonant energy transfer when

$$
\begin{array}{ccccc}
\omega_1 & = & \omega_2 & + & \omega_3 \\
(\omega_0) & & (0.43\omega_0) & & (0.57\omega_0)
\end{array}
$$

$$
\begin{array}{ccccc}
\mathbf{k_{z1}} & = & \mathbf{k_{z2}} & + & \mathbf{k_{z3}} \\
(k_{z0}) & & (2.4k_{z0}) & & (-1.5k_{z0})
\end{array}
$$

Nearfield III: Energy transfer in finite events



A sum of interacting waves

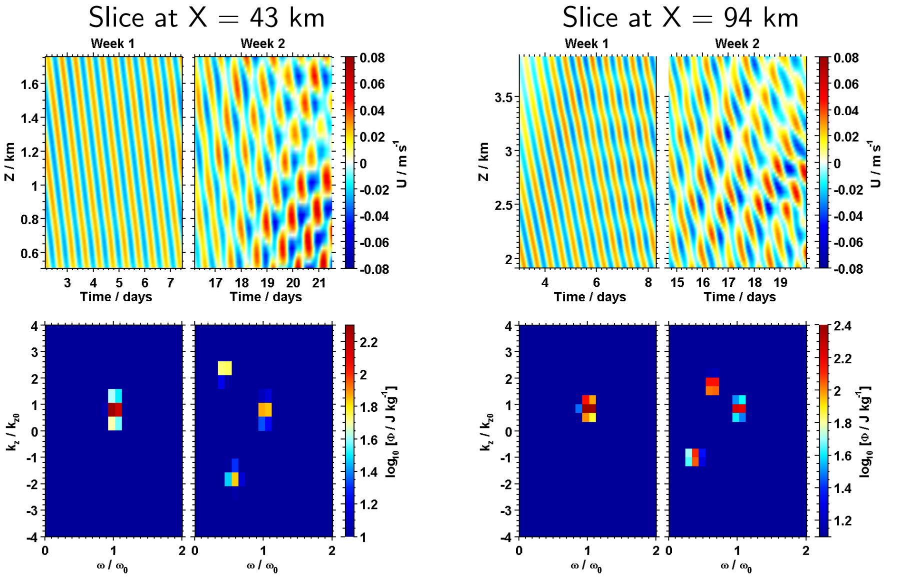
$$
\begin{aligned}
u &= u_0(T)e^{i[\mathbf{k_0}\cdot x-\omega_0 t+\phi_0(T)]} \\
&+ u_1(T)e^{i[\mathbf{k_1}\cdot x-\omega_1 t+\phi_1(T)]} \\
&+ u_2(T)e^{i[\mathbf{k_2}\cdot x-\omega_2 t+\phi_2(T)]} + \ldots
\end{aligned}
$$

exchange both energy ...

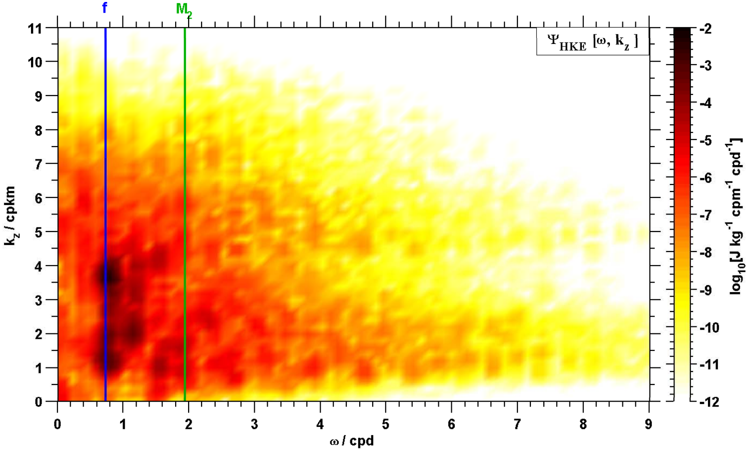
$$\frac{d}{dT}u_0 \approx -(k_1+k_2)\, u_1 u_2 \cos(\phi_1+\phi_2-\phi_0)$$

AND phase

$$\frac{d}{dT}\phi_0 \approx (k_1+k_2)\, \frac{u_1 u_2}{u_0} \sin(\phi_1+\phi_2-\phi_0)$$

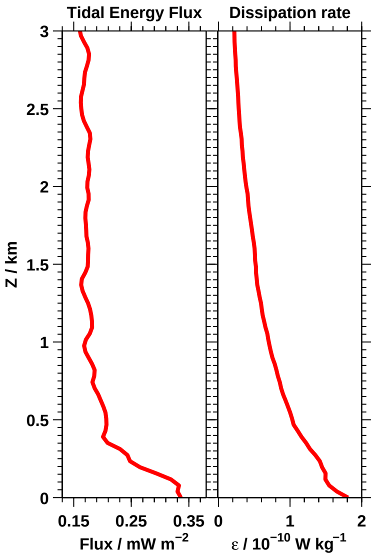

Slice at X = 43 km
Week 1
Week 2
Slice at X = 94 km
Week 1
Week 2
Z / km
Time / days
U / m s^{-1}
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
k_z / k_{z0}
ω / ω_0
$\log_{10} [\Phi$ / J kg$^{-1}]$

Nearfield V: Steady-state spectra



Broad inertial peak - without wind generated waves

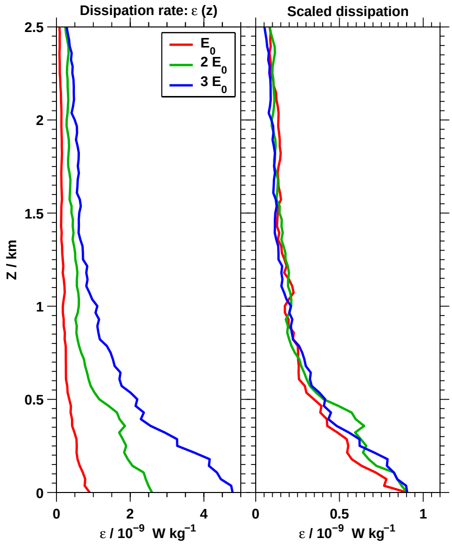
Nearfield VI: Steady-state vertical structure



- Energy transfer out of the tide
  $\tau \approx 1$ day
- Order of magnitude faster than timescale predicted for an incoherent tide (*Olbers and Pomphrey 81*)
- Recipient (near-inertial) waves move slowly and dissipate relatively locally
- Vertical extent of 'mixing hotspot':

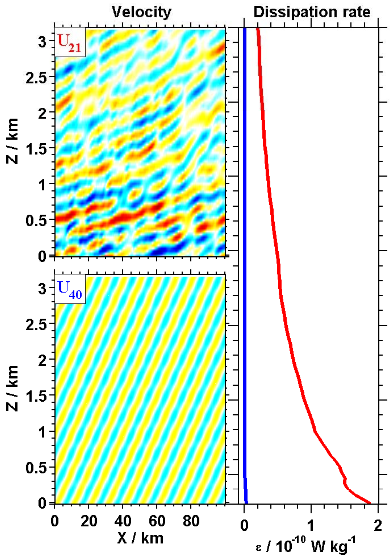
$$
\begin{aligned}
z_0 &\sim c_{gz} \times \tau \\
&\sim .0125\mathrm{m\ s^{-1}} \times 1\mathrm{day} \\
&\sim 1\mathrm{km}
\end{aligned}
$$

Nearfield VII: Changing tidal strength



- Increasing energy increases dissipation
- Yet **vertical shape** unchanged.... why?
- Stronger nonlinearity mean both faster energy exchange AND faster de-phasing $\Rightarrow$ same 'event-averaged' net energy transfer.

Nearfield VIII: Changing latitude



- Triad requires daughter waves to be within the internal wave band

$$\begin{aligned} \omega_1 + \omega_2 &= \omega_0 = M_2 \\ \omega_1, \omega_2 &\geq f \end{aligned}$$

- Not possible at latitudes poleward of 28.9°(where f $= M_2/2$)
- Hence, **no mixing hotspot!**

Nearfield conclusions and expectations

1. **Basic physics**
   - Fast ($\tau = $ 1-2 days) energy loss from a **coherent** internal tide
   - Transfer in a series of determinstic events - total energy loss limited by progressive **de-phasing** of component waves $\Rightarrow$ net transfer rate doesn't depend on tidal amplitude.
   - Daughter waves both near subharmonic ($\omega_1 + \omega_2 = \omega_0$)
   - But dynamics dominated by $U \cdot \nabla U$, NOT stretching/straining
2. **Practical implications: mixing hotspots near rough topography**
   - Decay scale of hotspots set by rate of energy loss from tide

$$L \sim c_g(M_2) \times \tau$$

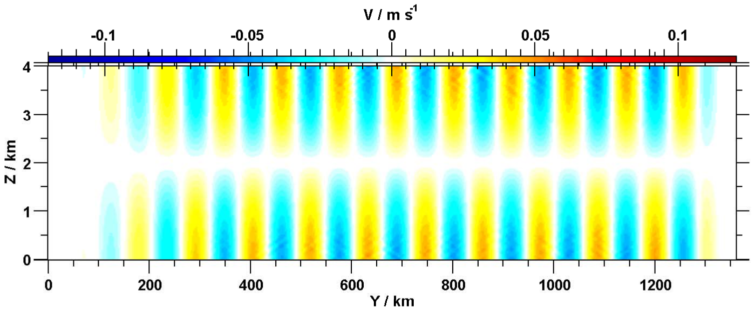
   - On/off latitude dependence: must be poleward of 28.9 (Hibiya)

What about the waves that get away?

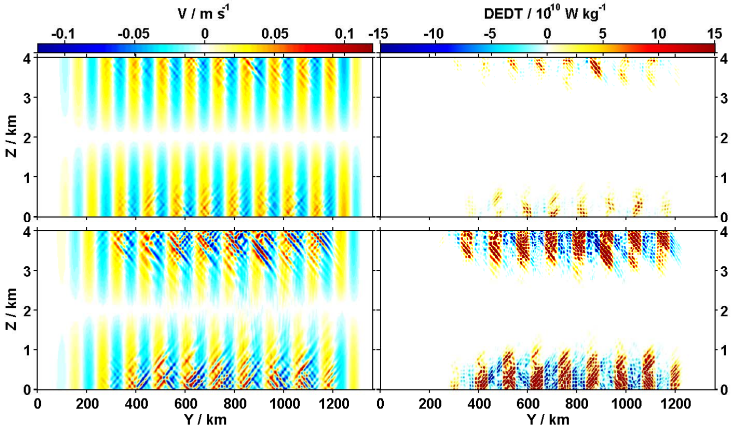
Farfield I: Mode-one tide on an f plane

- Mode-one M_2 tidal body force at left edge
- Realistic (2-10 kW/m) northward energy flux
- Sponge layer removes energy at right end of box
- Infinitesimal background noise seeds wave interactions

- f-plane
- Three-dimensional
- Free-slip top and bottom boundaries
- Constant stratification

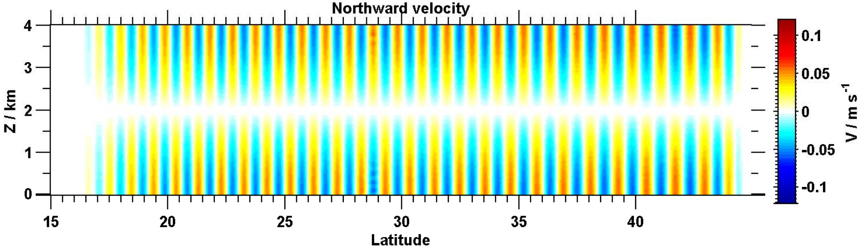


Farfield I: Mode-one tide on an f plane

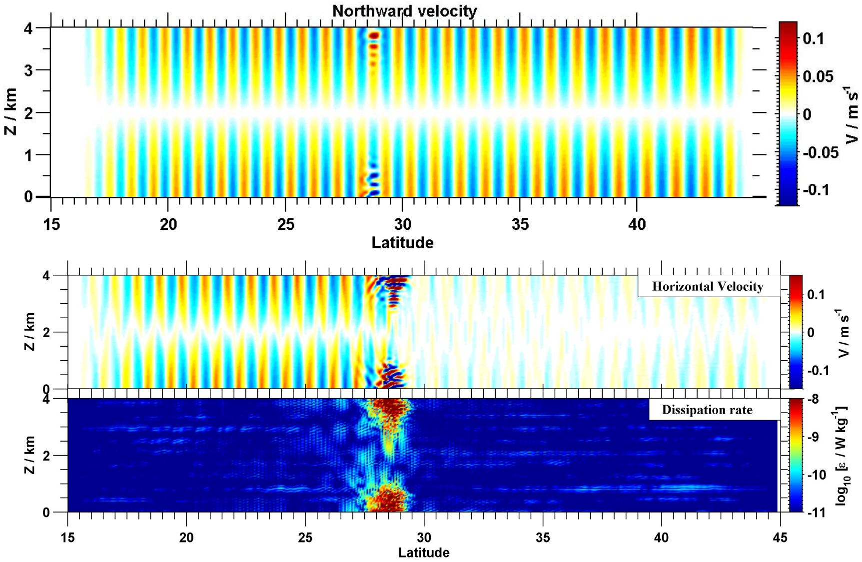


- Scale-separated, classic PSI-like triad.
- Fast energy transfer limited by de-phasing due to **group speed** propagation of subharmonic waves (hence faster de-phasing at low latitudes...)

Farfield II: Mode-one tide on a beta plane



Farfield III: Subtropical Catastrophe!



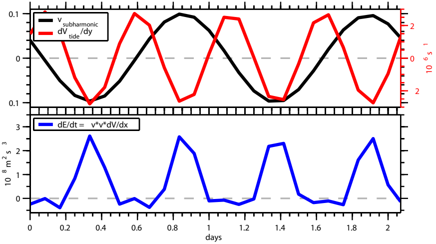
Farfield IV: Catastrophe Dynamics

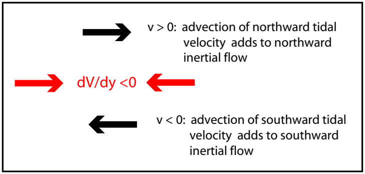
- Consider interaction between an internal (tidal) wave and near-inertial (subharmonic) motions

$$v = v_{\text{subharmonic}} + {\color{red}V_{\text{tide}}}$$

- Growth of subharmonic instability

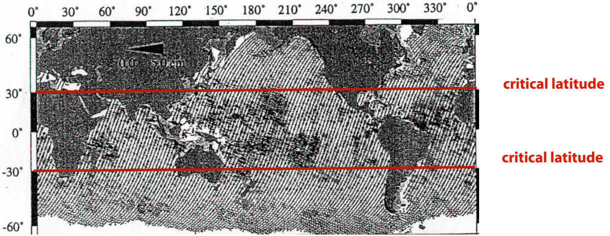
$$\frac{dv}{dt} = -v \cdot \nabla {\color{red}V} \ldots$$





Observational evidence of rapid PSI

- Satellite altimetry detects phase-locked baroclinic tide



Baroclinic M_2 tidal amplitude [Kantha and Tierney 97]

- Subharmonic signal observed in Hawaiian Ocean Mixing Experiment, locked to spring-neap cycle [Rainville and Pinkel, in prep]
- Latitude dependent turbulent diffusivities [Hibiya 04]
- **However,** non-phase locked signal observed poleward of critical latitude [Alford, in prep].

Conclusions

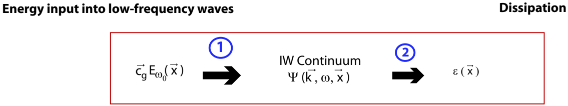
Where does tidal energy mix?

First order guess : Tide propagates, internal-wave continuum does not.

1. Prognostic: 'First jump' of energy out of propagating tide controlled by an interesting zoo of deterministic wave dynamics.
   - Nearfield: Decay scale of mixing hotspots near topography

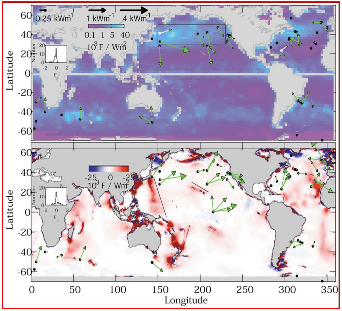
     $$L \sim c_g(M_2) \times \tau$$

   - Farfield: Scale separated energy transfer limited by group speed of subharmonic waves $\Rightarrow$ catastrophic energy loss at $28.9^\circ (c_g(M_2/2) = 0)$
2. Diagnostic: continuum locally grows to a level and shape consistent with rate of energy transfer from propagating tide



Where does tidal energy mix?

Energy input into low-frequency waves



Alford 03, Nature

Mesoscale interactions
(eddies, fronts, ACC)

Wunsch
Ferrari
Heywood & Naveira-Garabato
Rainville

$\vec{c}_g E_{\omega_0}(\vec{x})$ $\xrightarrow{1}$ IW Continuum $\Psi(\vec{k}, \omega, \vec{x})$ $\xrightarrow{2}$ $\varepsilon(\vec{x})$

Topographic effects
(scattering, reflection, hydraulics)

Legg
Thurnherr
St. Laurent and Garrett
HOME
TWIST