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Radiating instability and small-scale stochastic wind forcing

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Unlike their atmospheric counterparts, swift oceanic currents, except over equatorial regions, are not strictly zonal largely because the oceans are bounded by meridional continental boundaries. These currents are strongly shaped by these boundaries and the associated bottom topographies. Away from these swift boundary currents, the open ocean is full of small scale eddies, hidden behind which are the recently discovered quasi-zonal banded structures, the “latent jets”, so-called because of their small amplitude (we refer to them as “quasi-zonal jets in the ocean” hereafter). The origin of these latent zonal structures and their formation mechanism has been the subject of an ongoing discussion. It is not clear if we can directly apply turbulence theories based on a zonal reentrant channel model, as discussed extensively in other chapters of this book, to the generation of the quasi-zonal jets in the ocean.

One interesting phenomenon is that these jets often extend all the way to eastern boundaries, e.g. Figure 1b in Galperin (2004), Figure 4 in Nakano and Hasumi (2005), Figure 5 in Centurioni et al. (2008), Figure 2 in Maximenko et al. (2008). Several ocean-specific mechanisms related to oceanic eastern boundaries have been proposed. For example, zonal jets in the open ocean can be generated by beta plumes originating in a baroclinic meander at an eastern boundary (Afanasyev et al., 2011) or the radiating instabilities emitted from an unstable eastern boundary current (Hristova et al., 2008; Wang et al., 2012).

Here we focus on the generation mechanism of zonal jets in the open ocean by radiating instability of an eastern boundary current. This newly proposed mechanism is still in an early stage of development. This review is mostly based on Talley (1983a); Fantini and Tung (1987); Kamenkovich and Pedlosky (1996); Pedlosky (2002); Hristova et al. (2008); Wang et al. (2012, 2013). We first review the history of radiating instability on a beta plane, then the linear stability problem of a meridional flow, and finally the nonlinear radiating instability of an eastern boundary current. The nonlinear problem is more relevant to the generation of zonal jets in the ocean. Finally we discuss several wave-based theories (O’Reilly et al., 2012; Qiu et al., 2013).

Radiating instability on a beta plane

Radiating instability refers to an instability that can couple to waves that can radiate away from the source region. The radiation can be of various forms such as gravity, acoustic or Rossby waves. Here we focus on the radiating instability through Rossby waves on a beta plane, which has been shown to be relevant to the generation of quasi-zonal jets in the ocean. Before discussing the generation mechanism, let us first review the historical development of the theory.

Early studies of radiating instabilities are mainly motivated by the need to address the origin of eddies in the open ocean away from swift boundary currents. By early 1970s, eddies in the ocean had been known to be part of the ocean circulation, but had just started to draw much attention after a general recognition of their importance to the ocean circulation. Comprehensive field experiments such as the Soviet POLYGON (Brekhovskikh et al., 1971) and the international MODE-I initiated by Henry Stommel highlighted the turbulent nature of the oceans. Although it is relatively intuitive to understand the eddies of the Gulf Stream, it as less clear for eddies in the open ocean because of a lack of an obvious energy source. Subsequent theoretical developments suggested two main mechanisms regarding the origin of the observed eddies in the open ocean. One direct mechanism is baroclinic instability that can aid the release of the vast potential energy of the mid-ocean circulation to generate energetic eddies (Gill et al., 1974; Robinson and McWilliams, 1974; Pedlosky, 1975; Spall, 2000). An alternative mechanism is the radiation of mechanical energy originating from the instabilities of vigorous boundary currents such as the Gulf Stream (Wyrtki et al., 1976; Pedlosky, 1977).

The initial theoretical development focused on the linear response of a resting ocean to a steady localized forcing (Flierl et al., 1975; Pedlosky, 1977; Harrison and Robinson, 1979). For example, Pedlosky (1977) idealized a meandering current like the Gulf Stream as a spatially periodic disturbance traveling in the zonal direction with speed c and wave number k, and studied the conditions under which the disturbances can radiate energy southward. It is found that the extent to which disturbances can radiate through a resting ocean depends on local criticality in terms of baroclinic instability, so that the local potential vorticity structure of the open ocean flow controls both the local production of baroclinic eddies and the transmission of remotely generated wave energy. A series studies expanded the theory to include nonlinearity (Malanotte-Rizzoli, 1984) and transient forcing (Malanotte-Rizzoli et al., 1987), which are not discussed further here since they are not our focus. Motivated readers are encouraged to read the cited literature and the references therein.

The radiation mechanism in the above mentioned studies is put into the context of radiating instabilities by Talley (1983b,a) who studied the radiating capability of the intrinsic instabilities of barotropic and baroclinic zonal jets. She found that whether or not instability can radiate depends strongly on whether the instabilities satisfy the Rossby wave dispersion relation in the ambient far field. Instabilities usually propagate in the same direction as the mean flow. As a result, westward flow more easily supports instability radiation than eastward flow since Rossby waves always have westward phase speed.

The orientation of the mean current makes a huge difference to the instability of the current and its ability to radiate energy to the far field. Mean currents, which are tilted with respect to the zonal direction, are more unstable and their instabilities are more able to radiate (Fantini and Tung, 1987; Kamenkovich and Pedlosky, 1996, 1998a,b; Walker and Pedlosky, 2002; Pedlosky, 2002). A meridional current, representing ocean boundary currents, can be regarded as an extreme case of the non-zonality. Fantini and Tung (1987) noticed that a meridional current can generate radiating instability more easily than the zonal flows discussed in Talley (1983b,a). They showed that unstable waves are able to propagate energy eastward even in the presence of realistic dissipation. One would expect more radiating instability to occur for these meridional boundary currents.

Recently Hristova et al. (2008) studied and compared the linear radiating instability of two meridional boundary currents, one along western boundary and the other along eastern boundary both in a barotropic and in a baroclinic framework. In the barotropic setup, Kelvin-Helmholtz type of instability due to the horizontal velocity shear supports radiating instabilities both from a western boundary current and from an eastern boundary current. In the baroclinic setup, instability is provided by Kelvin-Helmholtz instability and baroclinic instability. Notably, the energy source from baroclinic instability significantly contributes to the energy balance for almost all unstable modes for both eastern and western boundary currents. One of the unstable baroclinic modes resembles the one found by Walker and Pedlosky (2002), suggesting the linkage between the instability of a baroclinic boundary current and the instability of a meridional channel flow. Hristova et al. (2008) found that the structure of the radiating mode from an eastern boundary has a long zonal tail, and proposed that those radiating modes can potentially generate quasi-zonal jets in the ocean.

Although both barotropic and baroclinic radiating modes exhibit long zonal tails and the baroclinic energy conversion can be important both by supporting unstable baroclinic radiating mode and by coupling to a barotropic radiating mode, the barotropic component itself seems to be sufficient to generate long-tailed quasi-zonal jets. The zonal wavelength and envelope decay scale of a barotropic radiating mode can be much longer than its baroclinic counterpart (Hristova et al., 2008). Based on a barotropic model, Wang (2011); Wang et al. (2012, 2013) used both linear stability analysis and nonlinear numerical simulations to demonstrate that radiating instabilities from a barotropic eastern boundary with realistic parameters can generate zonal jets in the ocean interior with observed properties.

Linear inviscid stability equation for a meridional flow

A completely realistic model needs to consider both baroclinic and barotropic energy sources, but our discussion here concentrates on the barotropic problem as a simplified example of how radiating instabilities can produce zonal jets. The linear stability equation based on the barotropic quasi-geostrophic vorticity equation for its simplicity,

$$\partial_t q + J(\psi, q) = \mathcal{F} \tag{1.1}$$

$$q = \nabla^2\psi + \beta y \tag{1.2}$$

where ψ is the streamfunction $(-\psi_y, \psi_x) = (u, v)$, q the potential vorticity, $\mathcal{F}$ the external forcing and dissipation, J the Jacobin operator, and β the meridional gradient of the Coriolis parameter. Consider a basic steady solution $\overline{q}$ (hereafter we use overbar to represent the time mean basic state), which satisfies

$$J(\overline{\psi}, \overline{q}) = \overline{\mathcal{F}}. \tag{1.3}$$

For a parallel zonal flow, $J(\overline{\psi}, \overline{q}) = \overline{\mathcal{F}} = 0$ meaning that no external forcing is needed to maintain a zonal parallel flow. But external forcing is needed for a meridional parallel flow to balance the divergence of planetary vorticity advection $\beta\overline{v} = \overline{\mathcal{F}}$.

The linear stability problem examines the evolution of a small perturbation to the basic state. Considering small perturbations denoted by primed quantities, Eq. (1.1) with Eq. (1.3) is linearized to

$$\partial_t q' + J(\overline{\psi}, q') + J(\psi', \overline{q}) = 0 \tag{1.4}$$

in which the friction and quadratic terms of the perturbation are neglected.

For a velocity jet with a typical width scale L_b and velocity scale V, the linearized equation Eq. 1.4 is nondimensionalized as

$$(\partial_t + \overline{u}\partial_x + \overline{v}\partial_y)\nabla^2\psi' + (\beta^* - \overline{u}_{yy} + \overline{v}_{xy})\psi'_x - (\overline{v}_{xx} - \overline{u}_{xy})\psi'_y = 0 \tag{1.5}$$

where $\beta^* = \beta L_b^2/V$ and the subscripts denote partial derivatives.

Equation 1.5 is further simplified to (after dropping the primes)

$$(\partial_t + \overline{u}\partial_x)\nabla^2\psi + (\beta^* - \overline{u}_{yy})\psi_x = 0 \tag{1.6}$$

for zonal flows, and to

$$(\partial_t + \overline{v}\partial_y)\nabla^2\psi + \beta^*\psi_x - \overline{v}_{xx}\psi_y = 0 \tag{1.7}$$

for meridional flows. The vorticity gradient induced by a basic zonal shear flow is parallel to the gradient of planetary vorticity, which simplifies the derivation of integral theorems, e.g. the Rayleigh-Kuo theorem (Rayleigh, 1880; Kuo, 1949). However, the first order term appearing in Eq. 1.7

introduces new modes of instability, which makes it difficult to extend the integral theorems to flows with a meridional component (Fantini and Tung, 1987; Kamenkovich and Pedlosky, 1996). Nonzonal flows can be expected to be more unstable, and even a small meridional tilt can destabilize an otherwise stable zonal current (Kamenkovich and Pedlosky, 1996). It also introduces a zonal asymmetry in the instability properties.

We here consider a meridional parallel flow. Since the coefficients in Eq. 1.7 are only a function of x, the perturbation solution consists of an eigenfunction in x and trigonometric wave-like structure in y and t

$$\psi = \Re\left(A\phi(x)e^{il(y-ct)}\right), \tag{1.8}$$

where ϕ is an eigenfunction. Substituting Eq. 1.8 into Eq. 1.7 gives the stability equation of a meridional flow

$$\phi_{xx} + \frac{\beta^*}{il(\overline{v}-c)}\phi_x - \left(l^2 + \frac{\overline{v}_{xx}}{\overline{v}-c}\right)\phi = 0. \tag{1.9}$$

Boundary conditions

Consider a basic sheared flow $\overline{v}(x)$ with constant velocity in the far field ($x \to \pm\infty$), which is set to zero in the following without loss of generality ($\overline{v}|_{x\to\pm\infty} = 0$). Then in the far field, Eq. 1.9 becomes the barotropic Rossby wave equation since $\overline{v} = \overline{v}_{xx} = 0$, resulting in far field wave-form solutions with zonal wavenumber k satisfying the Rossby wave dispersion relation

$$\psi = \Re\left(Ae^{ikx}e^{il(y-ct)}\right) \tag{1.10}$$

$$k^2 + \frac{\beta^*}{lc}k + l^2 = 0, \tag{1.11}$$

where k and c are allowed to be complex but l is set to be real. Note that the roots of k are always a pair of complex conjugates for all eigenmodes since the product of the two roots is l^2, which is real. Explicitly considering c and k being complex, the far field solution is written as

$$\psi = \Re\left(Ae^{ik_r}e^{il(y-c_rt)}\right)e^{-k_ix}e^{lc_it}. \tag{1.12}$$

The imaginary part of the phase speed c represents the growth or decay rate of the initial small perturbations. The imaginary part of k represents the zonal structure of the envelope of eigenfunction amplitude, which can be used to determine whether or not an eigenmode is radiating.

Another boundary condition for a mode which radiates in continuously differentiable velocity profile can be applied outside the jet region at some convenient finite value of x, namely,

$$\phi_x + ik\phi = 0, \tag{1.13}$$

in which k is the proper solution of Eq. 1.10 which yields outgoing radiation. This condition is applied at an arbitrary point x in the region where there is no mean flow. It ensures the continuity of the wave function and its derivative with the radiating Rossby wave. It is also conveniently turns the semi-infinite region into a finite region for numerical calculation. We need to determine how to apply the two roots for k satisfy the two boundary conditions. As the perturbation energy in the far field is either zero for non-radiating modes or finite for unstable radiating modes, eigenfunctions in both cases are expected to decay away from the energy source region, which in our case is the basic meridional flow. For a meridional flow in an infinite domain, the k with negative imaginary part is used for the western boundary condition and the one with positive imaginary part for the eastern boundary condition. We will discuss this in more dynamical detail in the next section.

Identifying a radiating mode

We aim to study the instability of a meridional jet and identify the modes that can radiate. It is straightforward to understand that a neutral mode in a plane wave form is radiative. However, an unstable mode, regardless of whether it is radiating or not, has a spatially decaying structure. We need a more subtle criterion in order to separate the unstable trapped and radiating mode as they both appear to decay in space.

One criterion is the phase speed condition. Given a meridional wavenumber l and fixed parameter β, we can find the associated eigenvalues c by solving the stability equation 1.9 with proper boundary conditions (Eq. 1.13). The distinction between a radiating mode and a trapped mode becomes evident in the small growth rate limit $c_i \to 0$. The phase speed c can be approximated by a Taylor expansion in terms of a small k_i at $k = k_r$

$$c_r + ic_i = c(k_i = 0) + ik_i\frac{\partial c}{\partial k}(k_i = 0) + \mathcal{O}(k_i^2). \tag{1.14}$$

Since l is real, k_i must be real if c is real for a radiating wave satisfying the Rossby dispersion relation. Matching the real parts of Eq. 1.14 gives

$$c_r = -\frac{\beta^* k_r}{l(k_r^2 + l^2)} \tag{1.15}$$

This is the so-called phase speed condition.

Matching the imaginary part of Eq. 1.14 requires that

$$c_i \approx k_i\frac{\partial c}{\partial k}(k_i = 0) = \frac{k_i}{l}c_g^x(k_i = 0) \tag{1.16}$$

where c_g^x represents the zonal group velocity. For an unstable mode $c_i > 0$, k_i and c_g always share the same sign, i.e. westward group velocity $c_g^x < 0$ corresponds to negative k_i, which is also used in the western boundary condition and vice verse. This is consistent with the physical mechanism that the decaying envelope in a radiating mode is produced by the propagating unstable wave packet generated earlier. By the time a wave package generated earlier reaches the far field, the amplitude of the unstable mode at the source region becomes even larger. So for an unstable radiating mode, its spatial decay scale $1/k_i$ is intimately linked with how fast it grows locally (c_i) and how fast it can propagate away (c_g^x). Following $c_i \to 0$ we would expect $k_i \to 0$ for unstable radiating mode, otherwise the mode is trapped.

One commonly used practical but not very strict criterion is that the radiating mode should “look wavy”, which is measured by k_r/k_i. The wavy structure indicates that a radiating mode is of a wave whose decay scale is long compared to its wavelength, a condition necessary to reveal the wave character of the radiation. For those with $k_r/k_i > 1$, their zonal decay scale $1/k_i$ is longer than the zonal oscillation scale $1/k_r$ so that its eigenfunction looks like a plane wave modulated by a slowly decay envelope. This criterion is practically convenient but obviously not a sufficient condition as rapid decay can be due to large growth rate.

To summarize, one can use $k_r/k_i > 1$ as the first check, and follow $c_i \to 0$ to see whether $k_i \to 0$, and finally the phase speed condition to identify radiating modes. The phase speed condition is a necessary condition. $c_i \to 0$ as $k_i \to 0$ is a sufficient condition. $k_r/k_i > 1$ is an empirical condition. One needs also check that the relation between spatial decay, group velocity and growth rate (15) is satisfied for a truly radiating mode.

Linear radiating instability

Asymmetry exists between westward and eastward radiating instabilities due to the first derivative of streamfunction associated with planetary beta. This asymmetry is easier understood intuitively in terms of the asymmetric Rossby wave propagation. The zonal group velocity is westward for long zonal wavelengths ($k < l$ for a fixed l) but eastward for short zonal wavelengths ($k > l$) although the phase speed of planetary Rossby wave is always westward. The meridional scale $2\pi/l$ is set by the most unstable mode in this barotropic case, which is about $6L_b$. For baroclinic modes, however, the deformation radius is a better measure of the transition in k from eastward to westward energy propagation. With the same baroclinicity, waves emitted from the eastern boundary are always of a longer zonal wavelength than those from the western boundary. Obviously, both energy sources need to be considered in a realistic model, but we only consider the barotropic problem as a simplified example of how radiating instabilities can produce zonal jets. Consider two boundary currents, one along a western boundary and the other along an eastern boundary. The boundary condition at the solid wall is non-permeable for both currents, but for the far field is $c_g^x > 0$ for the western boundary case and $c_g^x < 0$ for the eastern boundary case. If they exist, radiating modes of a western boundary current will be of short zonal wavelength, more affected by lateral friction, and will decay faster than those long waves radiated from an eastern boundary current.

Fantini and Tung (1987) explicitly considered a western boundary current represented by a piece-wise constant velocity profile with finite friction, and showed that a range of long meridional waves (compared to the width of the boundary current) are able to overcome local friction and radiate into a region far from the boundary current. They also showed that the radiating unstable modes are confined within a small wavenumber range over the long wave end. The growth rate is large for trapped modes but very small for radiating modes.

Kamenkovich and Pedlosky (1996) studied linear stability of a nonzonal (meridionally tilted) jet and considered both barotropic and baroclinic continuous velocity profiles. They demonstrated that the nonzonal orientation of the jet leads to the emergence of weakly unstable radiating modes, whereas all unstable modes of a purely zonal current are trapped. In the along-jet direction, these radiating modes are longer than the trapped ones; in the cross-jet direction, the radiating modes are neither symmetric nor antisymmetric. The radiating properties are not significantly different between the barotropic and baroclinic systems, although the baroclinicity modifies the linear modes of the barotropic problem and results in the emergence of a new type of a radiating mode. The analysis of the barotropic and baroclinic energy conversion terms demonstrated that one mode type exists mainly due to the barotropic mechanism, whereas the baroclinic mechanism is important for the remaining two mode types. The results suggest that radiating instabilities can exist as long as the mean stationary flow is not purely zonal.

Hristova et al. (2008) extended the study of Fantini and Tung (1987) by explicitly considering the asymmetry between a western boundary current and an eastern boundary current using the same piece-wise velocity profile. They confirmed the finding of Fantini and Tung (1987) for a western boundary current, but found that an eastern boundary current supports radiating instability over a wider meridional wavenumber range and with longer wavelength than does an equivalent western boundary current. In addition, Hristova et al. (2008) also studied a baroclinic case in which baroclinic instability provides an additional energy source for perturbations. They showed that baroclinic energy conversion is the dominant energy source for perturbations in a western boundary current, but only accounts for 50% energy source in the case with an eastern boundary current. Another 50% comes from the horizontal velocity shear through barotropic instability. While baroclinic instability can be important in supporting baroclinic mode, which can couple to barotropic modes and dramatically change the instability property, we here keep the problem as simple as possible and do not consider the baroclinic case.

Both Fantini and Tung (1987) and Hristova et al. (2008) used a broken line velocity profile for the sake of computational simplicity. This also reduces the continuous differential equations to a set of algebraic equations.

Wang et al. (2013) considered a continuous velocity profile represented by a bounded Bickley jet with a focus on the case with an eastern boundary current

$$\overline{v} = -V sech^2\left(\frac{x - x_0}{L_b}\right) \tag{1.17}$$

where x_0 denotes the location of the center of the Bickley jet, and L_b the cross-stream length scale of the boundary current.

Figure 1.1 shows an example of the linear results in Wang et al. (2013). The dashed and solid lines correspond to varicose and sinuous modes first found by Lipps (1962) but with modified structures. The varicose and sinuous modes are no longer symmetric and anti-symmetric because the two modes can project onto each other due to the beta effect associated with the first derivative in x. To illustrate the cross-projection, we perform a simple perturbation analysis assuming $\beta \ll \mathcal{O}(1)$. The eigensolution in terms of streamfunction can be expanded as

$$\psi = \psi^{(0)} + \beta^* \psi^{(1)} + \mathcal{O}(\beta^{*2}) \tag{1.18}$$

Substituting Eq. (1.18) into Eq. (1.7) and collecting terms with the same order of β^* gives

$$(\partial_t + \overline{v}\partial_y)\nabla^2\psi^{(0)} - \overline{v}_{xx}\psi_y^{(0)} \quad = \quad 0 \tag{1.19}$$

$$(\partial_t + \overline{v}\partial_y)\nabla^2\psi^{(1)} - \overline{v}_{xx}\psi_y^{(1)} \quad = \quad -\psi_x^{(0)} \tag{1.20}$$

The zeroth order equation 1.19 is the same stability equation for a jet on an f plane, and has the unstable sinuous mode and varicose mode for a basic Bickely jet. The first order correction $\psi^{(1)}$ slightly modifies the two basic modes to reflect the β influence. The first order equation shows that the two basic zero-order modes have a forcing effect on the order one term $\psi^{(1)}$. The x-derivative leads to a 90 degree phase shift, so that the sinuous (varicose) mode imposes an antisymmetric (symmetric) forcing structure. As a result, the zeroth order modes lose their original symmetry. Readers may refer to Wang (2011) for more detailed analysis of the cross-projection of the originally orthogonal unstable modes induced by beta effect.

Figure 1.1a shows the growth rates of the modified sinuous (solid line) and varicose modes (dashed line). While the most unstable mode is a modified sinuous mode, the modified varicose mode supports radiating instability over a wider wavenumber range as shown in Figure 1.1b (red color). The solid and dashed lines in Figure 1.1b represent the real part of the eigenvalues of the unstable modes, lc_r. The symbols represent the frequencies would be if the unstable modes satisfy Rossby wave dispersion relation, i.e., the symbols correspond to $\omega_r = -\beta k_r/(k_r^2 + l^2)$, where k_r is calculated according to Equation 1.10. The instabilities with a matching lc_r and $-\beta k_r/(k_r^2 + l^2)$ (marked by red symbols) can radiate. It is clear that the modified varicose mode has more potential to radiate. The two critical wavenumbers dividing radiating and trapped modes are $l = 0.46$ and $l = 0.74$ for the modified varicose and sinuous mode, respectively. Note that the approximately linear relationship between ω_r and l (lines in Figure 1.10b) should not be compared to the conventional Rossby wave dispersion relation, because k_r here is not a fixed quantify, but rather it changes as l varies. The relationship between ω_r and l is an intrinsic property of the instability of the boundary current. One would expect a different $\omega_r(l)$ function for a different current profile.

There are both long and short wave cutoffs in the growth rate, unlike for the piece-wise continuous velocity profile case where short wave cutoff is absent due to the infinite background shear at the velocity jump. The modified varicose mode is the only unstable mode over the long wave end with a long-wave cutoff $l = 0.125$. Radiating instability occurs over the meridional wavenumber range $0.125 < l < 0.46$ for the modified varicose mode and $0.706 < l < 0.74$ for the modified sinuous mode.

Historically, less attention has been paid to the varicose mode in barotropic instability studies of a zonal shear current because of their smaller growth rate compared with the sinuous mode. The varicose mode, however, becomes crucial here because most of the radiating modes are modified varicose modes.

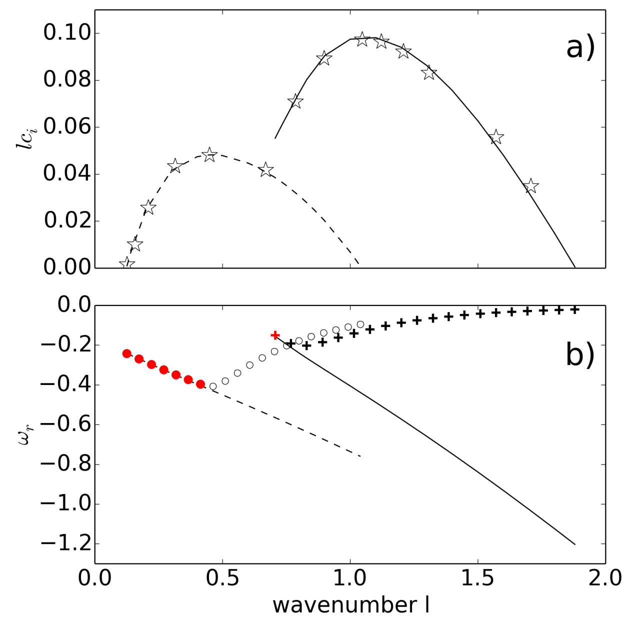

Figure 1.1 (a) shows the growth rates calculated by a shooting method (lines) and an initial value method (stars). (b) shows the real frequencies calculated from eigenvalues, $\omega_r = lc_r$ (lines) and $\omega_r = -\beta^* k_r/(k_r^2 + l^2)$ where $\beta^* = 0.41$ (symbols). In (a) (b), the dashed lines represent modified varicose modes, and the solid lines represent modified sinuous modes. The red symbols in (b) marks the unstable modes that satisfy the matching condition.

Figure 1 shows an example of the eigenfunction (real part) associated with a radiating mode (dashed line) and a trapped mode (solid line). The "wavy structure" is clearly shown in the radiating mode, but not in the trapped mode. In this example, $k_r/k_i = 190 > 1$ for the radiating mode and $k_r/k_i = 0.43 < 1$ for the trapped mode. Assume the boundary current width $L_b = 100km$, then the instability has meridional wavelength of less than $400km$ but a zonal wavelength of more than $4000km$. In the nonlinear studies shown next we can see that the most unstable mode has wavelength about 400 km. The strong anisotropic property of the radiating instability from an eastern boundary current may play a role in the generation of quasi-zonal jets in the ocean observed in, for example, Maximenko et al. (2005). In contrast, radiating instability from a western boundary current is more isotropic, i.e., $k \sim l$, and less likely exhibits asymmetric zonal-jet structures (Hristova et al., 2008).

Nonlinear radiating instability

Uncertainties exist in the linear theory. It is not clear whether radiating instabilities extend to finite amplitudes and how energetically important the radiating instabilities are since they have small growth rates. The theory is extended to nonlinear regime in Wang (2011); Wang et al. (2013) and its relevance to the generation of quasi-zonal jets in the ocean is demonstrated in Wang et al. (2012).

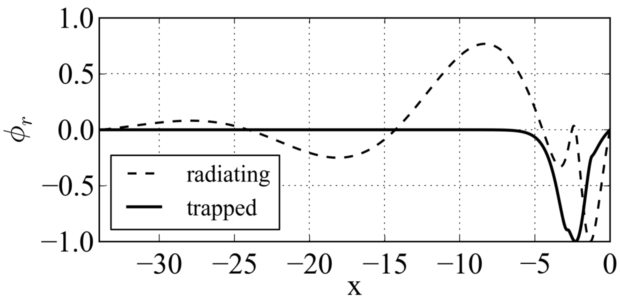

Figure 1.2 The real part of the eigenfunctions of the radiating (dashed) and trapped (solid) modes as a function of x (x is normalized by L_b). The radiating mode has $l = 0.4$, $k = 0.21 - i0.0011$, and the trapped mode has $l = 0.82$, $k = 0.32 - i0.74$.

Consider weakly nonlinear radiating instability using a numerical solution of Eq. (1.1). With an eastern boundary current $\overline{v}$ fixed in time and a Laplacian friction applied to perturbations, Eq. (1.1) becomes

$$(\partial_t + \overline{v}\partial_y)\nabla^2\psi + \beta\psi_x - \overline{v}_{xx}\psi_y - \nabla \cdot A_H \nabla\psi = 0. \tag{1.21}$$

A_H is strongly increased at the western boundary to remove energy and enstrophy

$$A_H = A_H^w - (A_H^w - A_H^e)\exp(\frac{x}{\alpha L_x}), \tag{1.22}$$

where α controls the decay scale. This function changes from approximately A_H^w at the western boundary to A_H^e at the eastern boundary. The boundary current is represented by Eq. (1.17). The numerical model domain is L_x in the zonal direction and L_y in meridional direction discretized by N_x and N_y grids. The nonlinear radiating instability is investigated by studying the behavior of a small perturbation ψ to the basic state $\overline{v}$. The parameters used in Wang et al. (2013) are listed in Table (1.1). Note that the small L_y is deliberately chosen to decrease the model's spectral resolution so that behaviors of the first several discrete modes can be studied individually. For L_y=700 km, the only resolved unstable wavelength is 350km, since the unstable wavenlength range is between approximately 250 and 550 km (Figure 1.3). Then the question is whether the frictionally suppressed radiating mode can contribute to the energy radiation from the boundary current to the interior.

We aim to study the fundamental element of nonlinear dynamics, the triad-interaction, in which three waves satisfy the condition for resonance, described below, between them. Here by reducing the model L_y, we can single out one unstable mode, denoted as the primary mode, with a wavenumber l_2 and frequency ω_2. The goal is to test whether we can find the other two modes that satisfy resonance condition with the primary mode: $l_1 + l_2 + l_3 = 0$, $\omega_1 + \omega_2 + \omega_3 = 0$, with a hope that one of the two modes is a radiating mode.

The evolution of the first four modes, i.e. modes with wavelengths L_y/i , $i = [1, 2, 3, 4]$, is studied in Wang et al. (2013) under two scenarios. In the first scenario, the single unstable mode with wavelength 350km resonates with the frictionally suppressed radiating mode with wavelength 700km through subharmonic instability. Note that the resonance condition for a triad containing the linearly unstable mode is possible only if the growth rate of that mode is small compared to the real frequencies of the components of the triad. No resonance can occur if the unstable mode has an order one growth rate since the sum of the frequencies will not be zero and the resonance denominator, which is proportional to the sum of the complex frequencies, will not be small enough. If the growth rate is order one, the development time for the instability will lead to the growth of the unstable wave to finite amplitude before the sharing of energy between members of the triad can be made manifest. We keep the growth rate of the most unstable mode as small as possible by reducing the forcing amplitude by trial and error. The mechanism of energy transfer in this limit is most clear and rigorous but we believe it illuminates a general process of energy transfer in the spectrum.

The linearly stable radiating mode becomes non-linearly unstable under this scenario by tapping into the energy of the most unstable mode with half of its wavelength. The radiating mode can take away almost a quarter of the total perturbation energy from the boundary region into the interior. In the second scenario with a slightly different β, the most unstable mode and the linearly stable radiating mode become non-resonant, so that the radiating mode becomes energetically insignificant.

Figure (1.4) shows the first scenario. Three stages are clearly shown in the time evolution of the streamfunction at an arbitrary location within the boundary current (a). There is a clear separation in the time evolution because the specific parameter set in Table (1.1) are chosen to reduce the growth rate of the most unstable mode to as small a value as possible in order to isolate and identify the three stages of development. During stage I, the most unstable mode quickly stands out and from random noise initialization and grows exponentially, while other modes decay. The subharmonic instability between the most unstable mode and its superhamonic starts to occur during stage II, but the radiating mode is still in the developing stage with an amplitude several orders of magnitude smaller. As a result, only the structure of the most unstable mode is observable. During stage III, both the most unstable mode and the radiating mode come into play. The long tail of the radiating mode becomes evident. The linearly stable radiating mode becomes nonlinearly unstable.

One of the main results in Wang et al. (2013) is the demonstration of the significant energy radiation from an unstable eastern boundary current even though there is no unstable radiating mode. They intentionally studied several discrete waves with a focus on the fundamental element of nonlinear dynamics, the triad interaction. The mechanism is expected to be applicable to more general cases. For example, elongated eddies was also observed in a study with a meridionally tilted jet (Kamenkovich and Pedlosky, 1996).

Generation of quasi-zonal jets

The phenomenon of perturbations propagating westward into the ocean interior from an eastern boundary is not

Parameter	Value	Parameter	Value
L_y	700km	A_H^e	$10^2 m^2/s$
L_x	5000km	α	0.15
N_y	32	x_0	-100km
N_x	256	L_b	50km
A_H^w	$10^4 m^2/s$	V	$0.11 m/s$
β	$1.8 \times 10^{-11}/ms$		

Table 1.1. *Parameters used in the nonlinear study in Wang et al. (2013).*

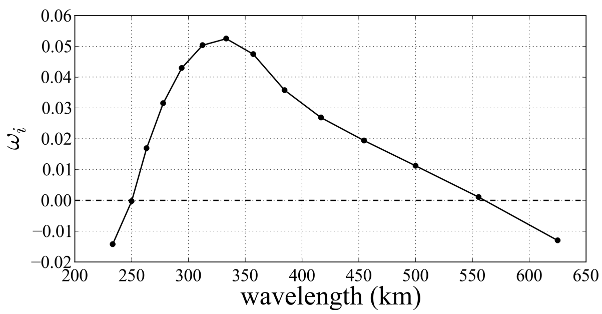

Figure 1.3 Linear growth rates normalized by V/L_b as a function of meridional wavelength. The growth rates are calculated by fitting the time series of the integrated perturbation kinetic energy to exponential curves, $EKE = Ce^{2\omega_i t}$, where $\omega_i = lc_i$ is growth rate.

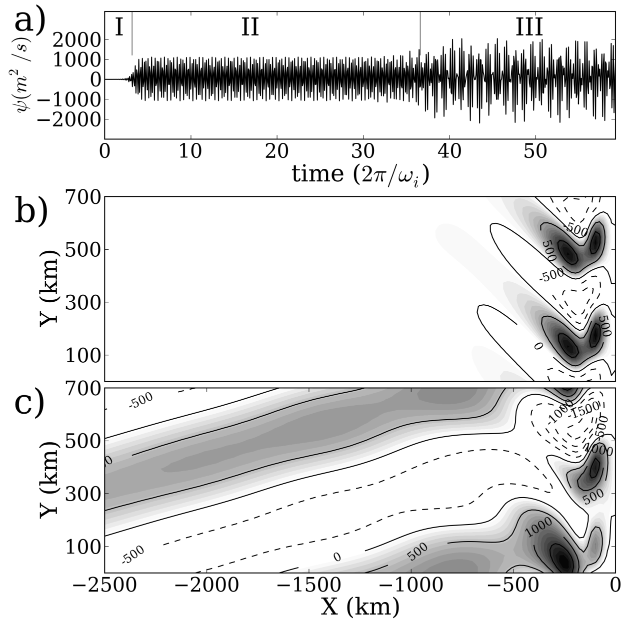

Figure 1.4 (a) shows the time series of the streamfunction at a fixed location within the boundary current. The time-axis is non-dimensionalized by $2\pi/\omega_i$ in which ω_i of the most unstable mode is chosen. The streamfunction snapshots at $t = 20$ and $t = 50$ are shown in panel (b) and (c), respectively. Letters I, II and III indicate different stages. Unit m^2/s is used in all panels.

a new idea. It is the specific horizontally elongated structure of the radiating instabilities that connects them to the generation of the quasi-zonal jets in the ocean that is newly of interest. As a natural extension of Wang et al. (2013), Wang et al. (2012) demonstrated the connection by examining whether the radiating instabilities of an eastern boundary current with realistic oceanic parameters can generate quasi-zonal jets in the ocean with observed properties. Wang et al. (2012) used the same model and the same set of parameters listed in Table (1.1) but with a mid-latitude β ($2\times10^{-11}/ms$) and the boundary current velocity $V = 0.2m/s$, which is consistent with the observed values (Brink and Cowles, 1991).

The simple model can reproduce realistic zonal structures comparable to observations. Figure (1.5) shows the model result along with a figure of high-pass filtered mean dynamic ocean topography from Maximenko et al. (2008). The snapshot of the surface height anomaly (Figure 1.5b) clearly shows elongated zonal structures. The instantaneous surface height anomaly reaches $9cm$ with velocity scale about $10cm/s$. The ten-year averaged surface height anomaly has an amplitude of $1.5cm$, which is comparable to results from observations averaged over a similar time period (Maximenko et al., 2008; Melnichenko et al., 2010).

Note that meridional tilt of the quasi-zonal structure exists in the snapshot but not in the time averaged field. The tilt in the snapshot is due to the simultaneous westward and northward propagation of radiating instabilities. The wave crests/troughs emitted at later time lay to the north of those emitted at earlier time. The angle of the tilt in the snapshot depends on the direction of the perturbation propagation. In a meridional reentrant channel model, this meridional tilting is averaged out over a long time period. The mean field only shows the residue with straight east-west banded structure, whose meridional wavelength is set by the radiating instability of the boundary current and in this case comparable to observed values.

Discussion

While the radiating instabilities from an eastern boundary generate instantaneous vacillating quasi-zonal jets in the ocean, the irregularity of the coastlines and topography, which is known to play an important role in anchoring coastal filaments and enhancing the growth of meanders and eddies (Kelly, 1985; Brink and Cowles, 1991), may additionally trigger stationary perturbations in the ocean interior. Davis et al. (2013) investigate the generation of zonal striations in the North Pacific using a primitive equation model and confirm the relevance of the radiating instability of an eastern boundary current in generating zonal striations in the ocean interior.

There exist other relevant *wave-based* theories. O'Reilly et al. (2012) showed that zonal jets emerge in a two-layer quasi-geostrophic ocean model forced by large-scale stochastic wind. The zonal jets are wave-like perturbations resulting from the secondary instability of baroclinic Rossby waves emanated from an eastern boundary with long meridional but short zonal scales. Qiu et al. (2013) used a $1\frac{1}{2}$-layer reduced gravity model and showed that zonal jets form as a result of the breakdown of primary waves that are forced

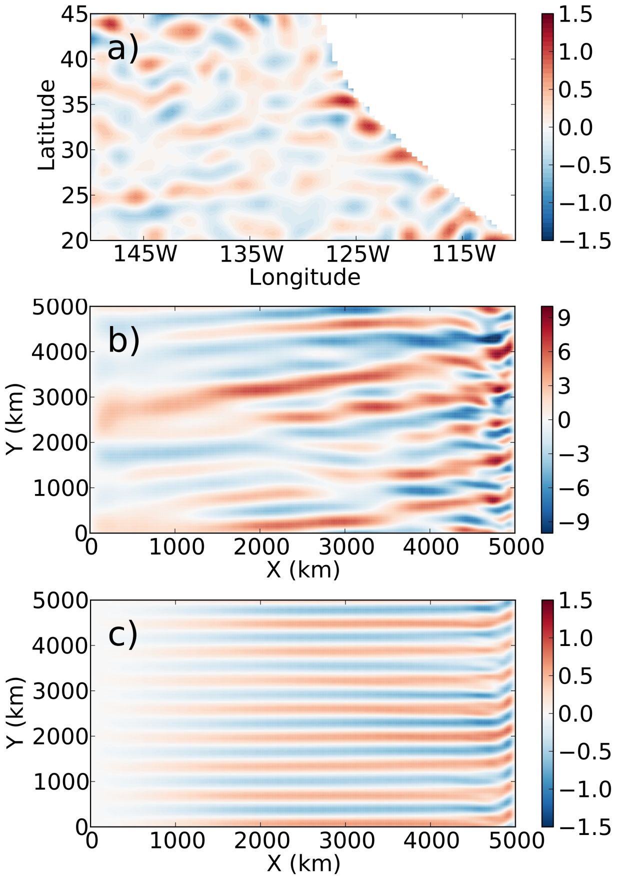

Figure 1.5 a) is a reproduction of the figure 2e in Maximenko et al. (2008) showing the high-pass filtered mean dynamic ocean topography (cm). The corresponding geostrophic zonal velocity has amplitude of $O(1cm/s)$ (Melnichenko et al., 2010). b) shows a snapshot of the surface height anomaly in the model (cm). The anomaly is the deviation of surface height from its meridional mean. c) is the 10-year time average of the model surface height anomaly (cm).

by annually oscillating winds and emanated from eastern boundary. As in O'Reilly et al. (2012), zonal jets in Qiu et al. (2013) emerge from instabilities of Rossby waves with long meridional and short zonal wavelengths. However, the instability mechanism in Qiu et al. (2013) is essentially nonlinear triad-interaction of three primary waves, unlike the intrinsic instability of the meridional primary Rossby waves in O'Reilly et al. (2012). A similar mechanism related to the secondary instability of primary meridionally-oriented instability is systematically studied in Berloff et al. (2009). They showed that the structure of the primary instability of an unstable baroclinic zonal flow resemble the shape of *noodles* oriented in the meridional direction. The secondary instability of those "noodles" results in zonal jets. These theories, together with the radiating instabilities of an eastern boundary current, all share similar fundamental physics, in a broad sense, that banded zonal jets can emerge from meridinally-sheared flows, either in a form of baroclinic Rossby waves, "noodle" instabilities or forced-stationary boundary currents.

The generation of quasi-zonal structures in the ocean interior by radiating instabilities from an eastern boundary current is clearly demonstrated in both linear and nonlinear studies. One of the main findings is that linearly stable, long radiating modes of an eastern boundary current can become nonlinearly unstable by resonating with short trapped unstable modes. This phenomenon is clearly demonstrated in the weakly nonlinear simulations. Results suggest that linearly stable longwave modes deserve more attention when the radiating instability of a meridional boundary current is considered. As remarked in Wang et al. (2012), since the simplest barotropic QG model can capture striations with wavelengths and amplitudes which resemble those observed in the satellite data, we anticipate similar features will occur in more complex systems. Although it is still unclear whether there is one universal mechanism that can explain striations observed throughout the world ocean, these results demonstrate that those close to the oceanic eastern boundary can be formed by radiating modes of the EBC which, in our case, overcome friction by nonlinear transfer of energy from the more unstable trapped modes. The dynamics shown here seems generic enough to point to the EBC as a major source.

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