1. The Pacific Ocean is approximately 10,000 km wide and approximately 5,000 m deep. Consider a west-to-east cross-section across the whole width of the Pacific, from Japan to California. Assume that there is a narrow western boundary current and a very broad interior flow across most of the section.

(a) If the water in that cross-section is moving southward at 1 cm/sec, calculate the total southward volume transport, in MKS units. (Ignore the western boundary current for this calculation.)

(b) If this same amount of water is returning northward in a western boundary current that is 100 km wide (and still 5 km deep), calculate the average northward velocity of the western boundary current.

(c) If the average oxygen content of the northward flow in the western boundary current is 150 µmol/kg, calculate the net northward flux of oxygen in the western boundary current, in units of µmol/sec. Use the information from (b) to calculate.

2. At the beginning of the course and in the first problem set, we introduced the non-dimensional parameters aspect ratio and Rossby number. Another important non-dimensional parameter for non-rotating, viscous flow is the Reynolds number.

(a) Use the steady-state equation (no time dependence, hence no acceleration), with no pressure gradient force:

\[ u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} \]

and assume the flow has characteristic velocity and length scales of U and L. The viscosity \( \nu \) is just a given property of the flow. The Reynolds number will be the ratio of the relative size of the advective term to the relative size of the diffusive term.

Write down the Reynolds number. (HINT: look at the unused slides at the end of the powerpoint for the non-rotating dynamics lecture.) (Second hint: the Reynolds number must have no dimensions, i.e. no units.)

(b) If the Reynolds number is high, is the flow very viscous or very inviscid?

(c) Turbulence occurs when a flow has very little viscosity. Does a highly turbulent flow have high or low Reynolds number?
3. Assume that the diffusivity $\kappa$ is given by its molecular value for water, of about 0.01 cm$^2$/sec. Just by looking at its units and this value, make a good guess at how long it would take a temperature signal to diffuse 10 meters in a very quiescent (non-moving) flow. (This could be, for instance, water-soaked dirt, for which the water is not moving at all, and we would ignore the effect of the dirt on the diffusion.)

4. The three momentum equations can be expressed in words as:

$$\text{acceleration} + \text{advection} + \text{Coriolis force} = \text{pressure gradient force} + \text{gravity} + \text{friction}$$

(a) Which term does not appear in the horizontal momentum equations?

(b) Which terms balance in inertial currents?

(c) Which terms balance in surface gravity waves?

(d) Which term contains the centrifugal force?

5. Consider a water parcel that moves from the sea surface far north in the North Atlantic (say 60°N) down into the ocean until it reaches the equatorial Atlantic, at a depth of about 2000 m. Suppose the parcel has a temperature of 3 °C and a salinity of 34.9, and that it doesn't mix with anything along its path.

(a) What will happen to its oxygen along this path? (You don't need to give values, just explain what happens to its oxygen content.)

(b) Likewise, what will happen to its chlorofluorocarbon content?

(c) What will happen to its tritium content?

(d) Suppose it is joined by a warmer, saltier water parcel that has the exact same density at the sea surface. Will they have the same in situ density when they reach the equator, at about 2000 m depth? If not, which will be denser?