SIO 210 Problem Set 2 Answer Key
October 16, 2018
Due October 25, 2018 (1 week)

Different # points each problem

10 points
1. If the sampling rate for a time series of, say, ocean velocity, is 1 day, and the length of the time series is 2 years, and you wish to do a time series analysis:
2 (a) What is the Nyquist frequency? $f = 1/(2\Delta t) = 1/(2 \text{ days})$
2 (b) What is the fundamental frequency? $N = 2*365 = 730$; $f = 1/(N\Delta t) = 1/(730 \text{ days})$
3 (c) If the region has semi-diurnal tides, will the tidal frequency be aliased?__yes_ Explain briefly. Semi-diurnal means peaks of tide occur two times per day, so frequency is higher than the Nyquist frequency. Therefore they will be aliased.
3 (d) If the phenomenon you are measuring is affected by El Nino, which has a time scale of 3 to 7 years, compare the length of time series you would need to resolve the El Nino cycle with this length of time series. Use the concept of 'degrees of freedom'. For a time scale of 3 (7) years, the fundamental frequency of the time series has to be at least 1/(3 years) [1/(7 years)], meaning the time series has to be at least 3 (7) years long for one realization of the phenomenon. With just one realization, there is about 1 degree of freedom, and so the phenomenon is not well sampled statistically. To sample El Nino well, need to sample at least 30 (70) years.
n.b. problem was too unspecific by saying 3 to 7 – students can answer with any length of time between 3 and 7, or provide a more nuanced answer including the range.

30 points
2. Consider heat transport and divergence associated with nearly horizontal gyre circulation. The Pacific Ocean is approximately 10,000 km wide. Its upper layer (wind-driven subtropical gyre – clockwise circulation) is approximately 1,000 m deep. Consider a west-to-east cross-section at about 30°N across the whole width of the Pacific, from Asia to North America, for this layer. Assume that there is a narrow, northward-flowing, western boundary current (Kuroshio) and a very broad, southward-flowing, interior flow across most of the section. For the following questions, assume that velocity does not vary with depth within this layer.

5 points each part
(a) If the water in that cross-section is moving southward at 1 cm/sec, calculate the total southward volume transport, in MKS units. (Ignore the Kuroshio for this calculation.)

Convert all distance units to meters. Do it before calculating, or do it within the calculation, either way.
$\Delta x = 10{,}000 \text{ km} = 10^7 \text{ m}$
$\Delta z = 1000 \text{ m} = 10^3 \text{ m}$ (doesn't need converting)
$v_{south} = 1\text{cm/sec} = 0.01 \text{ m/sec} = 10^{-2} \text{ m/sec}$
Volume transport V = velocity * cross-section area = $v_{south} * \Delta x * \Delta z = (10^{-2} \text{ m/sec})(10^7 \text{ m})(10^3 \text{ m}) = 10^8 \text{m}^3\text{/sec}$. (This is equal to 100 Sv, not necessary for the answer.)

(b) If this same amount of water is returning northward in a western boundary current that is 100 km wide (and still 1 km deep), calculate the average northward velocity of the western boundary current.

Calculate either as a simple proportion, or calculate from the volume transport of (a).

v_{wbc} = (10,000km)/(100km)*v_{south} = 100*v_{south} = 100 cm/sec = 1 m/sec

OR

$v_{wbc} = V/(\Delta x * \Delta z) = (10^8 m^3/sec)/(10^5\ m)(10^3\ m) = 1\ m/sec$

(c) If the average oxygen content of the northward flow in the western boundary current is 150 μmol/kg, calculate the net northward transport of oxygen in the western boundary current, in units of μmol/sec. Use the information from (b) to calculate.

Transport of a property, asked this simple way assuming oxygen and velocity are the same at every point in the cross-section, is density*concentration*volume transport

O_T = (1025 kg/m^3)*150 μmol/kg * $10^8 m^3$/sec = (1.54 x 10^5 μmol/m^3)* $10^8 m^3$/sec = 1.54 x10^{13} μmol/sec = 1.54 x10^7 mol/sec if you prefer those units.

(d) Suppose this circulation is transporting 1 PW of heat northward. If all of the northward flow is of one temperature and all of the southward flow is of another temperature, what is the temperature difference between the northward and southward flow? Use typical (uniform) values for density and specific heat, as given in class or in a textbook.

H_{net} = Volume transport x density x specific heat x (temperature difference)

Therefore temperature difference $\Delta T = H_{net}/(\rho\ c_p\ V)$

Recall that 1 PW = 1 x 10^{15} W

ΔT = 1 x 10^{15} W/[(1025 kg/m^3)*3850 J/(kg °C)* $10^8 m^3$/sec] = 2.5°C

(e) Explain why I asked you to calculate a temperature difference in (d), rather than the actual temperature. The net heat transport is proportional to the temperature difference between the northward and southward flows. The total heat transport at each grid point would have to be calculated in units of Kelvin. We don't usually refer to that calculation.

(f) Calculate the average air-sea heat flux between this section and the northern edge of the Pacific Ocean (Japan/Russia/U.S./Canada). Use very simplified assumptions about the width and length of this region (i.e. don't worry about calculating the exact dimensions, just approximate it). Ignore Bering Strait – assume there is no leakage out of this large "box".

Average air-sea flux = (heat transport)/(area)

Width: say 5000 km = 5 x 10^6m

Meridional length: say 30 degrees latitude = 30*100km = 3000km = 3 x 10^6 m

Area = width*length = 15 x $10^{12} m^2$

Average air-sea flux = 1x10^{15}W/(15 x $10^{12} m^2$) = 0.67 x 10^2W/m^2 = 67W/m^2

16 points total

3. Surface gravity waves: refer to upcoming Mackinnon lecture notes for Oct. 23 or material in DPO chapter 8.

2 points

(a) Write down the expression for phase speed c for surface gravity waves if their wavelength λ is much shorter than the water depth. $c = (g/k)^{1/2}$

2 points

(b) Are these 'deep water' waves or 'shallow water' waves? Explain briefly. These are 'deep water' waves, meaning that the orbital motion of particles in the waves does not feel the bottom.

2 points

(c) Write the expression for wavenumber k in terms of wavelength λ. $k = 2\pi/\lambda$

5 points

(d) Evaluate the phase speed for a wavelength of 10 meters________ and for a wavelength of 500m.____________

Use $g = 9.8\ m/sec^2$

$k = 2*3.1416/10m = 0.628/m$

$c = (g/k)^{1/2} = (9.8\ m\ sec^{-2}/0.628\ m^{-1})^{1/2} = 3.95\ m/sec$

$k = 2*3.1416/500m = 0.0126/m$

$c = (g/k)^{1/2} = (9.8\ m\ sec^{-2}/0.0126\ m^{-1})^{1/2} = 27.9 m/sec$

5 points

(e) Using these results, suppose the waves are generated by a storm 1000 km away.

Which wave from (d) will reach you first? ___The long waves will arrive first.

How long will it take for the 500 m wavelength waves to reach you?__

time = distance/speed

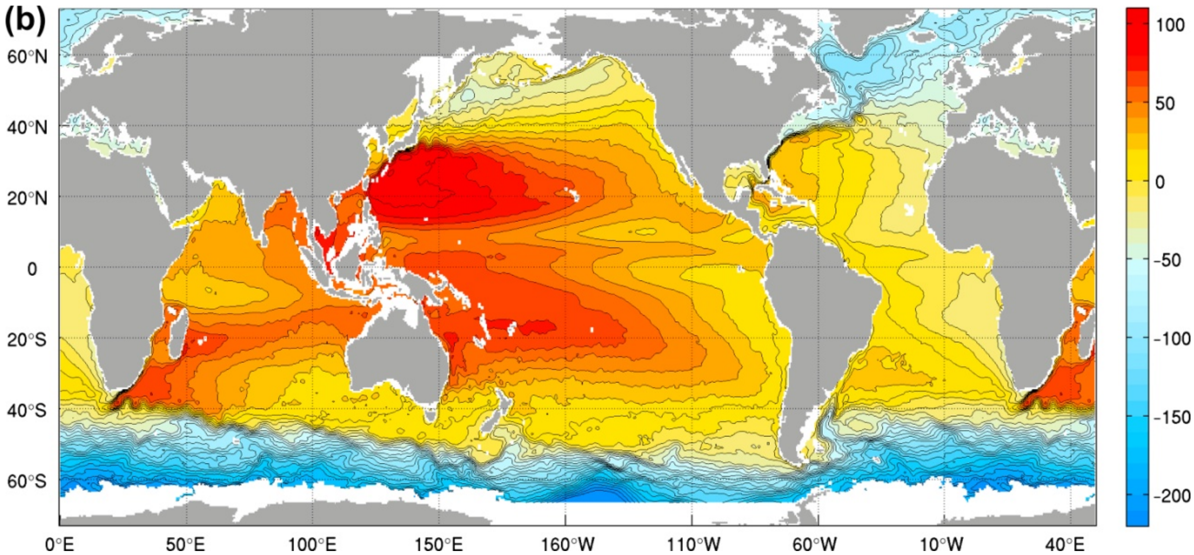
$T = 10^6 m/27.9\ m\ sec^{-1} = 3.58x10^4\ sec$

You can convert this to hours: 1 hour = 60*60 sec = 3600 sec

$T = 3.58x10^4\ sec/3.6x10^3\ sec/hr = 1\ hour$

24 points total

4. This is a map of sea surface height (not exactly, but close enough for this problem set). (DPO Fig. 14.2b). Contours are in centimeters.



4 points

(a) Look at the equator in the Pacific Ocean. Estimate the difference in sea surface height (SSH) from the western side to the eastern side of the Pacific, along the equator. _I needed to explain that the contour intervals are 10 cm (Figure 14.2 in DPO). Using this information or just guessing from the colorbar, the estimated height difference is Δz = 40 to 50 cm, or 0.4 to 0.5 m. Use this in part (b).

10 points (5 points for using the hydrostatic balance, and 5 points for using

(b) What is the pressure gradient force (pgf) associated with this SSH difference?

$pgf = -\partial p/\partial x$

Evaluate: $pgf = -\Delta p/\Delta x$

First use hydrostatic balance to calculate the pressure in the west and in the east. Since you need only the pressure difference between west and east, you only need to calculate the extra amount of pressure in the west, due to the 0.5 m of extra water compared with the east. Therefore no calculation is needed for the east.

Use hydrographic balance to calculate the extra pressure in the west:
Assume water density of 1025 kg/m^3, and gravity of 9.8 m/sec^2.

Δp: estimate this from hydrostatic balance, using Δz from part (a):
$0 = -\partial p/\partial z - \rho g$
Switch to Δ notation to calculate (rather than integrating the pde).
$\Delta p = -\rho g \Delta z = -(1025 \text{ kg/m}^3)(9.8 \text{ m/sec}^2)(0.5 \text{ m}) = 5022.5 \text{ kg m}^{-1} \text{ sec}^{-2}$

Δx: Estimate distance: longitudes are 150°E to 90°W at the equator.
There are 111 km per degree at the equator:
$\Delta x = 120° \times 111 \text{ km/°} = 13320 \text{ km} = 13.32 \times 10^6 \text{ m}$

pgf = $-\Delta p/\Delta x$ = 5022.5 kg m^{-1} sec^{-2}/(13.32 x 10^6 m) = 3.77 x 10^{-4} kg m^{-2} sec^{-2}

10 points (5 points each)

(c) If this pgf is associated only with acceleration, calculate the (eastward) acceleration.

Use the eastward force balance: acceleration and pgf terms only.
a = $\Delta u/\Delta t = -(1/\rho)\Delta p/\Delta x$ = -(1/1025 kg/m^3)(3.77 x 10^{-4} kg m^{-2} sec^{-2}) = 3.68 x10^{-7} m sec^{-2}

Calculate the (eastward) velocity along the equator after 1 month (assuming starting from rest).
Δt = 1 month = (30 days/month)(24 hours/day)(60 min/hour)(60 sec/min) = 2.59 x 10^6 sec
Δu = (a)* Δt = (3.68 x10^{-7} m sec^{-2})(2.59 x 10^6 sec) = 0.95 m/sec ~ 1 m/sec

20 points total

5. Water masses.

Look at the zonal (east-west) section from the N. Atlantic at 24°N (DPO Fig. 9.22, included here). For each of the following: circle and identify on the sections, and explain what property(ies) you used to identify the water masses.

4 points each of the 5 parts

(a) Identify the mixed layer. What are its salinities, temperatures, oxygen relative to the rest of the water column?
Very thin layer at the top of each section, very hard to identify. This is a summer data set and so it should be thin. The oxygen section is perhaps most useful since it shows a slight subsurface maximum, which cannot be in the mixed layer. So the slightly lower oxygen within the top 10s of meters is where the mixed layer is.
The mixed layer properties: very warm (> 25°C), salty (> 36.8 or more), slightly lower oxygen than slight subsurface maximum but overall high compared with layer down to 1000 to 1500 m depth. The deeper waters have high oxygen concentration, but that is because they are cold compared with the surface waters.

(b) Identify the Mediterranean Water.
This is the vertical salinity maximum in the eastern Atlantic between 1000-1500 m, and does encompass all of the noisy salinity fluctuations in this depth range in the central Atlantic.

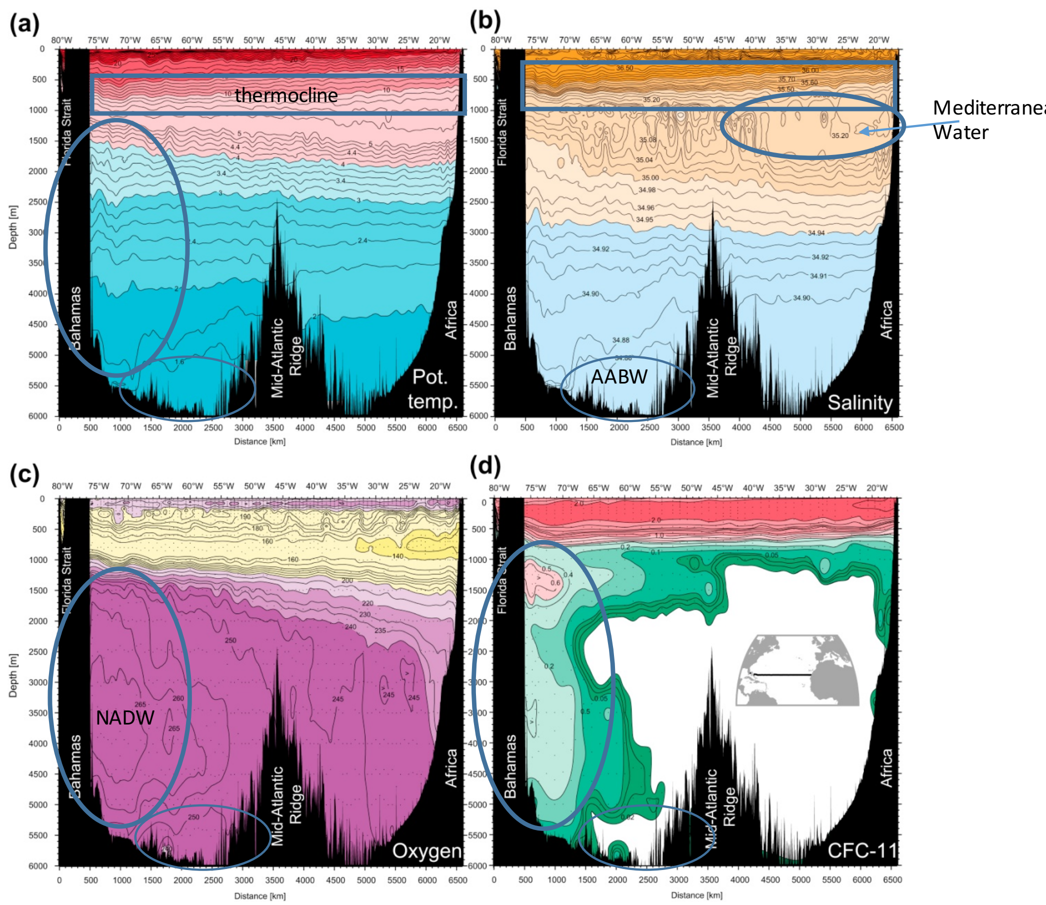
(c) Identify the main thermocline.
This is the layer between about 5°C and 18°C or warmer, in which temperature changes rapidly with depth. It is also a 'halocline' in which salinity changes rapidly with depth.

~~(d) Identify the Mediterranean Water~~

(e) Identify the North Atlantic Deep Water

This is the layer of high oxygen below about 1500 m. It is especially well developed in the west where oxygen is highest. The high CFC-11's at 1000-1500 m are the uppermost part of the NADW, originating in the Labrador Sea. Temperature ranges from about 1.6 to 5°C on this section.

(f) Identify the Antarctic Bottom Water
AABW is the coldest water west of the Mid-Atlantic Ridge, and includes no CFC-11 (marked by its absence) because this section is a long way from Antarctica. Oxygen is lower than in the NADW. Temperature range is < 1.6°C, salinity is < 34.86 (lowest salinity of the deep waters), oxygen is < 250 µmol/kg.



(a)
(b)
(c)
(d)
thermocline
Mediterranea
Water
AABW
NADW
Pot.
temp.
Salinity
Oxygen
CFC-11
Florida Strait
Bahamas
Mid-Atlantic
Ridge
Africa
Depth [m]
Distance [km]