1. Consider the concentration in a volume of the ocean of a conservative tracer (call it chlorofluorocarbon or CFC). The units of concentration are (moles tracer)/(kg seawater), that is, moles/kg.

(a) Explain what it means for a tracer to be conservative: (short answer)

(b) What does it mean for mass to be conserved? (short answer)

(c) Is it possible for the concentration of tracer within a volume of water to change while the mass in the volume does not change? YES or NO (circle one)

Explain.

d) Consider the volume in the figure. There is flow into the left side of the volume and flow out of the right side of the volume. The inflow has a tracer concentration $C_1$ and velocity $u_1$. The outflow has a tracer concentration $C_2$ and velocity $u_2$. The area of the inflow is $A_1$ and the area of the outflow is $A_2$.

\[
\begin{align*}
&u_1 = 0.1 \text{ m/sec} & u_2 = \text{???} \\
&A_1 = 10 \text{ kilometers} \times 100 \text{ meters} & A_2 = 5 \text{ kilometers} \times 100 \text{ m}, \\
&C_1 = 1 \text{ pmol/kg,} & C_2 = 0.8 \text{ pmol/kg}
\end{align*}
\]

To make calculations easy, use a density of $1000 \text{ kg/m}^3$. 
d.1) Given that mass is conserved, what can you say about $u_2$, if you are told the values of $u_1$, $A_1$ and $A_2$?

d.2) If $u_1 = 0.1 \text{ m/sec}$, $A_1 = 10 \text{ kilometers} \times 100 \text{ meters}$, $A_2 = 5 \text{ kilometers} \times 100 \text{ m}$, Calculate $u_2$:

d.3) What is the FLUX of tracer on the left side (where the values are $C_1$, $u_1$, $A_1$)?

d.4) What is the TRANSPORT of tracer on the left side ($C_1$, $u_1$, $A_1$)?

d.5) What is the TRANSPORT of tracer on the right side (at $C_2$, $u_2$, $A_2$)?

d.6) Do you expect the tracer concentration within the volume to (circle one)
INCREASE          DECREASE           REMAIN THE SAME

2. Heat budgets. Consider a rectangular tropical region that is 1000 km from north to south and 10,000 km from east to west. Suppose the annual mean surface heating over this region is 20 W/m$^2$.

(a) Suppose the ocean temperature in this region is in steady state due to ocean currents/diffusion that transport heat. Calculate the net transport of heat by ocean currents/diffusion out of the region.
(b) If on the other hand, there are no ocean currents and no diffusion, and if the heating uniformly warms just the upper 200 meters of the ocean, calculate the change in temperature of this ocean layer over 1 year.

3. The salinity of the inflow to the Mediterranean Sea at the Strait of Gibraltar is about 36.3. The salinity of the outflow from the Mediterranean is about 37.8. (Both values are "psu" if you want "units"). The exchange is about 1 Sv. Calculate the average evaporation rate of the Mediterranean Sea.

4. Of the following terms, state which pair forms the primary balance for each of the following approximations.

Terms: Pressure Gradient Force, Temporal change in velocity, Coriolis 'force', Gravity, Tidal Potential, Centrifugal force

A) Hydrostatic Balance: ___________________ and _____________________
B) Geostrophic Balance: ___________________ and _____________________
C) Inertial Oscillations: ___________________ and _____________________
D) Surface Waves: ___________________ and _____________________

5. The Reynolds number.
The momentum equation in the x-direction, ignoring variations in the y and z directions, is:
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial x^2}
\]

or, if you prefer to work in terms of differences:

\[
\frac{\Delta u}{\Delta t} + u \frac{\Delta u}{\Delta x} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + \nu \frac{\Delta}{\Delta x} \left( \frac{\Delta u}{\Delta x} \right)
\]

Compare the sizes of the advective and viscous terms:

(a) Assume that you can assign a scale \( U \) to the \( x \)-velocity and \( L \) to the \( x \)-length. These are scales that are intrinsic to the flow. That is, \( L \) would be order (meters) for waves or order (thousands of kilometers) for large-scale circulation, etc. The scales are therefore basically just orders of magnitude.

Write the order of magnitude of just the advection and viscous terms in terms of \( U \) and \( L \). (Note that the difference \( \Delta \) has no scale, and that the \( u \) that occurs outside the derivative will have the same scale as the \( u \) that occurs within the derivative. Also, ignore the time dependent and pressure terms.)

(b) What is the ratio of the sizes of the advection and viscous terms? This ratio will have no dimensions ("non-dimensional"). The ratio is called the Reynolds number.

(c) What does the Reynolds number measure or represent? If the Reynolds number is large, what is the relative importance of the advective and viscous terms in the equation of motion?

(d) Calculate a typical Reynolds number for the following. You will need to choose an appropriate velocity and length scale for each phenomenon.

(i) The Gulf Stream, with molecular viscosity.

(ii) The Gulf Stream, with eddy viscosity.

(iii) A surface wave with molecular viscosity.