SIO 210: Dynamics VI: Potential vorticity

• Variation of Coriolis with latitude: “β”
• Vorticity
• Potential vorticity
• Rossby waves

READING:
Review Section 7.2.3
Section 7.7.1 through 7.7.4 or Supplement S7.7
(figures are taken from supplementary chapter S7)
Section S7.8.1
Upper ocean circulation

Gyres, western boundary currents, Antarctic Circumpolar Current, equatorial circulations

Schmitz (1995) (DPO Fig. 14.1)
Absolute surface height, related to surface geostrophic velocity

Maximenko et al., GRL, 2003 (DPO Fig. 14.2)
Review: Coriolis parameter

\[ f = 2\Omega \sin \varphi \] is the “Coriolis parameter”

\[ 2\Omega = 1.458 \times 10^{-4}/\text{sec} \]

At equator (\( \varphi = 0, \sin \varphi = 0 \)):
\[ f = 0 \]

At 30° N (\( \varphi = 30°, \sin \varphi = 0.5 \)):
\[ f = 0.729 \times 10^{-4}/\text{sec} \]

At north pole (\( \varphi = 90°, \sin \varphi = 1 \)):
\[ f = 1.458 \times 10^{-4}/\text{sec} \]

At 30° S (\( \varphi = -30°, \sin \varphi = -0.5 \)):
\[ f = -0.729 \times 10^{-4}/\text{sec} \]

At south pole (\( \varphi = -90°, \sin \varphi = -1 \)):
\[ f = -1.458 \times 10^{-4}/\text{sec} \]
Coriolis parameter variation with latitude: $\beta$

Rotation around the local vertical axis is what matters for Coriolis.

Rotation around local vertical axis goes:
- from maximum counterclockwise at North Pole,
- to 0 at equator,
- to maximum clockwise at South Pole.

$$\beta = \frac{\Delta f}{\Delta y} = \frac{\Delta f}{(\text{Earth radius} \times \Delta \text{latitude})} = \frac{2 \Omega \cos \phi}{R_{\text{earth}}}$$

Maximum at the equator and 0 at the poles.

11/7/18

Talley SIO210 (2018)
Vorticity

Vorticity $\zeta > 0$ (positive)  
Vorticity $\zeta < 0$ (negative)

Use the Greek letter $\zeta$ for vorticity
Potential vorticity

CONSERVED, like angular momentum, but applied to a fluid instead of a solid

Potential vorticity \( Q = \frac{\text{planetary vorticity} + \text{relative}}{\text{(column height)}} \)

Potential vorticity \( Q = \frac{f + \zeta}{H} \) has three parts:

1. Vorticity (”relative vorticity” \( \zeta \)) due to fluid circulation
2. Vorticity (”planetary vorticity” \( f \)) due to earth rotation, depends on local latitude when considering local vertical column
3. Stretching \( 1/H \) due to fluid column stretching or shrinking

The two vorticities (#1 and #2) add together to make the total vorticity \( = f + \zeta \).

The stretching (height of water column) is in the denominator since making a column taller makes it spin faster and vice versa.
Potential vorticity balances: examples with each pair of two terms

Conservation of potential vorticity (relative and stretching)

\[ Q = \frac{(f + \zeta)}{H} \text{ is conserved} \]

Conservation of potential vorticity (relative plus planetary)

\[ Q = \frac{(f + \zeta)}{H} \text{ is conserved} \]
Potential vorticity balances: examples with each pair of two terms

Conservation of potential vorticity (planetary and stretching)

This is the potential vorticity balance for “Sverdrup balance” for the large-scale general circulation (gyres) (next set of slides)

\[ \beta v = f w_z \]

DPO Figs. S7.28
Rossby waves: westward propagating mesoscale disturbances

These waves result from the potential vorticity balances that include the β-effect (variation of f with latitude)

Surface-height anomalies at 24 degrees latitude in each ocean, from a satellite altimeter. This figure can also be found in the color insert. Source: From Fu and Chelton (2001).
Sea surface height animations: Rossby waves

https://www.youtube.com/watch?v=F8zYKb2GoR4
Rossby wave: potential vorticity with time dependence

**Westward phase propagation**

- Imagine column is pushed north.
- It stretches to conserve $f/H$.
- Produces downward slope to east, creating southward geostrophic flow that pushes any columns there back to south.
- Produces downward slope to west, creating northward geostrophic flow there that pushes columns on west side northward, thus moving the northward motion to the west of the initial displacement.
- This implies westward propagation.

For these long Rossby waves, balance is between $f$ and $H$ changes:

$$Q = \frac{f}{H}$$

For short Rossby waves (not shown), balance is between $f$ and relative vorticity $\zeta$:

$$Q = \frac{(f + \zeta)}{H_0}$$

where $H_0$ is constant.
SSH wavenumber spectra showing difference in westward and eastward propagating energy

(a) Frequency and (b) wavenumber spectra of SSH in the eastern subtropical North Pacific, using 15 years of satellite altimetry observations. The dashed line in (a) is the annual frequency. In the wavenumber panel, solid is westward propagating, and dashed is eastward propagating energy. Source: From Wunsch (2009).
Observed phase speeds from altimetry: “almost” Rossby waves

- Phase speeds from SSH (dots)
- Rossby wave phase speeds (curves)

- Similarity of the two suggests that the observed propagation is very close to Rossby waves.

- (Difference between the observed and theoretical has provided basis for many analyses/publications.)

DPO Figure 14.19

(a) Westward phase speeds (cm/sec) in the Pacific Ocean, calculated from the visually most-dominant SSH anomalies from satellite altimetry. The underlying curves are the fastest first-mode baroclinic Rossby waves speeds at each latitude. (b) The ratio of observed and theoretical phase speeds, showing that the observed phase speeds are generally faster than theorized. Source: From Chelton and Schlax (1996)