Nonlinear vorticity balance of the Antarctic Circumpolar Current

Chris W. Hughes
Proudman Oceanographic Laboratory, Liverpool, UK

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[1] Using a recently published global ocean mean surface dynamic topography, based predominantly on drifter data, the near-surface vorticity balance of the Antarctic Circumpolar Current (ACC) is calculated. With a little smoothing, the dynamic topography is found to produce good estimates of even such highly differentiated quantities as the relative vorticity advection. Two clear modes of flow are found in the ACC: meanders of wavelength 300–500 km, in which the nonlinear term is important, resulting in a balance between advection of relative and planetary vorticity, as in a stationary equivalent-barotropic Rossby wave (this implies a surface divergence which is at least partly balanced by an opposite divergence at depth); and a flow with divergence associated with topographic features. It is tentatively concluded that the divergence of this latter flow is a scaled measure of bottom pressure torque. The inferred bottom pressure torque shows large-scale topographic interactions, as well as a strong influence of some sharp topographic features such as the fracture zones and Macquarie ridge system south of the Tasman Sea, and the narrow ridges in Drake Passage.


1. Introduction

[2] The Antarctic Circumpolar Current (ACC) is the largest current in the world, transporting approximately 144 Sv (1 Sv = 10⁶ m³ s⁻¹) around the world between about 40°S and 65°S [Cunningham et al., 2003]. As a result of its high latitude, relatively weak stratification, and strong eastward flow, baroclinic Rossby waves cannot propagate to the west in the ACC, and it can be identified from satellite altimetry as a region in which transient disturbances (be they eddies, waves, or meanders) propagate to the east, in contrast to the predominant westward propagation in lower latitudes [Hughes et al., 1998].

[3] As we know from spin-up experiments [Anderson and Gill, 1975; Anderson and Killworth, 1977], the westward propagation of baroclinic Rossby waves is the mechanism by which the flow in a subtropical gyre is isolated from the effect of topography, permitting a Sverdrup balance to develop. It therefore appears that the strength of the ACC is also related to its strong interaction with topography.

[4] A current which flows much faster than the baroclinic Rossby wave speed approximates an equivalent-barotropic structure, in which the streamlines at depth are parallel to those at the surface [Hughes and Killworth, 1995]. This equivalent-barotropic structure has been noted in models of the ACC [Killworth, 1992; Killworth and Hughes, 2002], and has been used to great advantage in mapping the flow of the real ACC [Sun and Watts, 2001, 2002; Watts et al., 2001].

[5] The topographic interaction is essential in order to maintain a balance of angular momentum in the band of latitudes unblocked by continents. As Munk and Palmén [1951] first pointed out, the eastward wind stress would accelerate a much stronger current if it were balanced by friction or eddy momentum fluxes of any plausible magnitude, so it must be balanced by pressure differences across topography fluxing angular momentum to the solid earth via “form stress.” Numerous model studies have confirmed this balance [Ponte and Rosen, 1994; Stevens and Ivchenko, 1997; Gille, 1997; Hughes and de Cuevas, 2001].

[6] These pressure differences across topography also have the effect of upsetting Sverdrup balance by producing “bottom pressure torques” which permit meridional flows other than those driven by wind stress curl. As shown by Hughes and de Cuevas [2001], the balance between wind stress and form stress at each latitude ensures that these bottom pressure torques balance the wind stress curl in a zonal integral (when averaged over a few degrees of latitude to reduce the effect of nonlinear terms). This means that the meridional deflection of the ACC due to wind stress curl at some longitudes is balanced by that due to bottom pressure torques at other longitudes (in fact this also occurs in subtropical gyres, in which the equatorward transport driven by wind stress curl in the ocean interior is returned in a poleward western boundary current flowing over the continental slope, balanced by the bottom pressure torque).

[7] Where a meridional (depth-integrated) flow occurs, which is not driven by wind stress curl, it must therefore be
balanced by the bottom pressure torque. Equivalently, it must imply a vertical velocity at the ocean floor: bottom pressure torque requires a gradient of pressure along an isobath, producing a geostrophic flow toward (or away from) the topography, which must therefore imply an upslope (or downslope) flow. The linearized balance is

\[ \beta V = J(p_b, H) + \nabla \times \tau, \]  

where \( \beta = \partial f / \partial y \), the northward derivative of the coriolis parameter, \( V \) is the depth-integrated northward mass transport \( (\int \rho \, v \, dz) \), \( p_b \) is bottom pressure, \( H \) is ocean depth, \( \tau \) is surface wind stress, and the Jacobian can be expressed as \( J(p_b, H) = \mathbf{r} \cdot (\nabla p_b \times \nabla H) \) where \( \mathbf{r} \) is a unit vector in the local vertical. Using geostrophy, plus the kinematic boundary condition \( u \cdot \nabla H = -w_b \) where \( w_b \) is vertical velocity at the bottom (or just above a viscous boundary layer), this can also be written as \( J(p_b, H) = -p \partial \omega_b / \partial y \), where \( p \) is density.

[8] In principle then, the pattern of the depth-integrated flow should tell us the vertical velocity at the ocean floor. The equivalent-barotropic nature of the flow helps us in this, since the depth-integrated flow should be simply related to the surface flow. However, one major complication is the neglected nonlinear terms in the barotropic vorticity balance (1). As shown by Hughes and de Cuevas [2001], these are large at short length scales, where “short” in the ACC can be anything up to about 10 degrees of latitude or longitude. This is sufficient to mask much of the relationship between \( \beta V \) and bottom pressure torque in the model diagnostics. If we wish to see more clearly how the flow interacts with bottom topography, we must find a way to take account of these nonlinear terms.

[9] The purpose of this paper is to use a map of the mean surface flow in the ACC to investigate the nonlinear terms in the vorticity balance. The map is an update of the drifter-based global mean surface dynamic topography of Niiler et al. [2003]. The update, by Niiler et al., incorporates large-scale information from the difference between satellite altimetry and the GRACE (Gravity Recovery and Climate Experiment) geoid, but this does not have much effect at length scales shorter than 1000 km, with which we are concerned here (N. A. Maximenko, personal communication, 2005). Niiler et al. [2003] noted a possible systematic error due to Stokes drift in surface waves, which may be large in the Southern Ocean, but should be significantly improved by incorporation of the GRACE data. In addition, the simple method of accounting for wind slip uses a single global parameterization which does not account for regional differences in Ekman layer dynamics, or small-scale wind stress variations known to be associated with sea surface temperature gradients [Chelton et al., 2004]. Errors are certainly present, but will be very inhomogeneous and difficult to predict. The analysis presented here can be thought of as a way to investigate the errors by seeing whether the field makes dynamical sense on the length scales investigated.

[10] The dynamic topography is differentiated by first-order centered differencing. Each differentiation uses data from four grid points at the corners of a box to calculate the derivative at the center of the box. The second derivative then produces values at the original grid points. It is to be expected that errors will increase as the dynamic topography is differentiated, and this is indeed the case. However, a little experimentation showed that the worst of the errors (predominantly at the grid scale) could be damped out by a single pass of a \( 3 \times 3 \) grid point box-car smoother after each differentiation. The final fields were regridded for plotting by linear interpolation onto a one-third degree Mercator grid, and subjected to a final pass of the smoother (now on a finer scale as the grid scale on the Mercator grid is between about 33 and 15 km). This smoothing reduces both signal and noise at small length scales, in a manner which will be discussed in more detail later. The geostrophic flow speed shown in Figures 1a and 2a still exhibits some grid-scale noise in the regions of weaker flow, particularly south of the ACC, but the stronger currents appear to be very well represented. The broad character of the flow shows many instances of steering by topography, but there are also meanders which do not appear directly related to topography.

[11] Relative vorticity \( \zeta \) was calculated by taking the curl of the surface velocity. It is plotted in Figures 3a and 4a, and has typical values of order \( 10^{-6} \) s\(^{-1}\), much smaller than \( f \).
Figure 1. Contours of (a, b) mean dynamic height, with contour interval 15 cm, (c) absolute vorticity \((f + \zeta)\), with variable contour interval, (d) dynamic height remapped to account for relative vorticity advection (15 cm contour interval), shown on top of color plots of (Figure 1a) mean geostrophic flow speed and (Figures 1b, 1c, and 1d) ocean bathymetry.
Figure 2. As for Figure 1, but showing a different range of longitudes.
Figure 3. Contours of mean dynamic height, with contour interval 15 cm, shown on top of color plots of (a) relative vorticity $\zeta$, (b) planetary vorticity advection $\beta v$, (c) relative vorticity advection $u \cdot \nabla \zeta$, and (d) total vorticity advection $u \cdot \nabla (f + \zeta)$. 

Scales: Vorticity ($10^{-6} \text{s}^{-1}$), Advection ($10^{-12} \text{s}^{-2}$)
Figure 4. As for Figure 3, but showing a different range of longitudes.
which ranges from $0.62 \times 10^{-4}$ to $1.3 \times 10^{-4} \text{ s}^{-1}$ in the region plotted. Figures 1c and 2c show the absolute surface vorticity $f + \zeta$, which shows that the effect of relative vorticity is to produce small-scale meanders in the contours. A given relative vorticity produces a larger meander if it occurs farther south, as $\beta$ is smaller farther south (by more than a factor of 2 at $65^\circ S$ as compared to $30^\circ S$). Again, some grid-scale effects appear south of the ACC, but for the most part the absolute vorticity looks reasonable. Interestingly, the meanders in the absolute vorticity often appear to follow the meanders in the mean flow, especially in the Agulhas retroflection south of Africa, and in the flow over the Pacific Antarctic Rise at $120^\circ W - 150^\circ W$. This suggests a tendency of the flow to conserve absolute vorticity. 

In order to test this, the effect of relative vorticity is “taken out” of the dynamic topography by replotting the contours in Figures 1d and 2d. At each longitude along the dynamic height contours, $f + \zeta$ is computed. Then an “adapted” contour location is determined by finding the latitude at which $f$ has the same value as $f + \zeta$ at the actual latitude of the contour. The contours are effectively moved to the latitude at which $\zeta$ would be zero, if absolute vorticity $f + \zeta$ were conserved as the contour moved. Another way to think of this is that where Figures 1b and 2b show the dynamic height as a function of longitude and $f$; Figures 1d and 2d show dynamic height as a function of longitude and $f + \zeta$. In a small number of places, most noticeably immediately south of S. Africa, contours fold over, meaning that a single contour has three latitudes at a given longitude. A simple regridding is not possible here, as this would result in multiple contours appearing in the same place. In these regions, only the southernmost contour is plotted, resulting in a step in the regridded topography.

The main effect of the regridding is to smooth out the flow, broadening the jets and removing most of the meanders. The broadening of the jets is a result of the fact that relative vorticity is positive to the north, and negative to the south of the axis of an eastward jet; westward jets would become narrower when replotted this way. Removal of the meanders, however, is an indication of the degree to which absolute vorticity is conserved following the flow.

This smoothing clearly shows that there is information in the mean relative vorticity calculated from the dynamic topography. In fact, where prominent meanders remain (south of Tasmania and the Tasman Sea, and in Drake Passage) they appear to be associated with very sharp changes in bathymetry at fracture zones and the Macquarie Ridge system. In the Agulhas retroflection region, the meanders are completely smoothed out except for a small remnant of the first meander which is clearly associated with steering over a topographic feature.

The fact that relative vorticity signals seem to be resolved by the dynamic topography prompted further investigation, and a more quantitative assessment of the effect of relative vorticity advection in the ACC.

3. Surface Vorticity Balance

As we have seen, relative vorticity is much smaller than planetary vorticity, which means that a quasigeostrophic interpretation is appropriate. From section 12.8 of Gill [1982], we can write the quasigeostrophic vorticity equation as

$$\frac{\partial}{\partial t} \zeta + \mathbf{u} \cdot \nabla \zeta = f w,$$

(2)

where $\zeta$ is a reference density, $p'$ is perturbation pressure, $D/Dt = \mathbf{u} \cdot \nabla$ with $\mathbf{u} = (u, v)$ the horizontal geostrophic velocity (for the steady case) and subscripts represent differentiation. As the dynamic topography is calculated from drifter data with a correction made for Ekman layer dynamics, we should interpret the velocities as those outside the Ekman layer, with negligible viscous force acting. Substituting $\rho_0 f u = -p'_x$, $\rho_0 f v = p'_y$, and $\zeta = v_x - u_y$ gives

$$\mathbf{u} \cdot \nabla (f + \zeta) = fw.$$

(3)

Equation (3) tells us that the total horizontal vorticity advection must be balanced by a vertical stretching term. When integrated over depth, this stretching term relates to the difference between vertical velocities at the surface and at the bottom of the ocean. As discussed in section 1, these vertical velocities are given by Ekman pumping at the surface, and bottom pressure torque (divided by $-\rho_0 f$) at the bottom. We cannot integrate over depth in this case, as we only have surface data, but the equivalent-barotropic nature of the flow should mean that the surface values of horizontal velocities are closely related to those at depth.

The terms on the left-hand side of (3) can be calculated from the dynamic heights. The planetary vorticity advection $\mathbf{u} \cdot \nabla f = \beta v$ is straightforward, and is shown in Figures 3b and 4b. The resulting pattern clearly shows topographic steering in places, but there is a lot of additional detail at zonal wavelengths of about $5 – 6^\circ$, typically equivalent to about 300–500 km.

The relative vorticity advection, $\mathbf{u} \cdot \nabla \zeta$, involves differentiating the dynamic height three times (with a pass of the smoother after each differentiation) to get $\nabla \zeta$, followed by a product with $\mathbf{u}$, after which a further pass of the smoother is applied. The result (shown in Figures 3c and 4c) is dominated by the 300–500 km wavelengths noted in $\beta v$, and is clearly anticorrelated with $\beta v$. This dominant range of scales is confirmed by both direct measurement of significant features, and by calculating the peak in the power spectrum at each latitude by Fourier analysis. This has been repeated both with and without smoothing, and appears to be a robust length scale (although it is possible that it is limited at the short wavelength end by limitations of the data).

The anticorrelation with $\beta v$ is highlighted by the change in character of the picture when plotting the sum of planetary and relative vorticity advection in Figures 3d and 4d. As in Figures 1d and 2d, most of the features related to meanders are gone, and the plot is much more clearly related to features in the bottom topography. The effects of sharp topographic gradients are again prominent, south of Tasmania and the Tasman Sea, and in Drake Passage.

The anticorrelation is more than just a visual impression. Taking the region between 35 and $65^\circ S$, the correlation between relative and planetary vorticity advection on the mercator grid is $-0.65$, and the regression coefficient between relative and planetary vorticity advection is very
close to \(-1\) (actually \(-1.00076\)), resulting in essentially zero correlation \((-0.0005)\) between relative and total vorticity advection. This conclusion, however, is dependent on the degree of smoothing used. The correlation is \(-0.54\) with no smoothing, \(-0.61\) with smoothing after only the first differentiation, and \(-0.63\) with smoothing after only the first and second differentiation. The corresponding regression coefficients are \(-0.39, -0.61,\) and \(-0.79,\) suggesting that \(|\beta v|\) is smaller than \(|u \cdot \nabla \zeta|\) at the surface.

[24] A regression coefficient smaller in magnitude than \(-1\) could result from two effects. The actual relative vorticity advection could match \(\beta v\) with a regression coefficient of \(-1\), in which case small scale noise would be responsible for a smaller regression coefficient when there is no smoothing. Alternatively, the actual regression coefficient could be smaller in magnitude than \(-1\), and the smoothing could be reducing the amplitude of the signal to make it appear to be \(-1\). This latter possibility has been investigated by performing the same analysis on several artificially generated dynamic topographies consisting of a sum of a sine wave in latitude, with amplitude 1 m and wavelength 20 degrees, plus a sine wave in longitude, with amplitude 10 cm and wavelength 4, 6, 8, or 10 degrees. For comparison, 6 degrees of longitude corresponds to 510 km at 40°S, and to 333 km at 60°S, matching the observed length scales well. It is found that the smoothing reduces the amplitude of signals at 4, 6, 8, and 10 degree wavelengths by factors of 0.51, 0.75, 0.85, and 0.9, respectively. This suggests that the actual regression coefficient should be close to 0.75. From this we can see that the regression coefficient with no smoothing, and with smoothing after the first differentiation only, are probably artificially small (in magnitude) because of the effect of small-scale noise, but that a regression coefficient of \(-1\) is probably artificially large in magnitude because of the effect of smoothing.

[25] Nonetheless, it is the smoothed data which account for the largest fraction of the variance of \(\beta v\), even when using the optimum regression coefficient for each version of the relative vorticity advection. Adding relative vorticity advection to \(\beta v\) reduces the variance of vorticity advection by a little over 42%.

3.1. Meander Dynamics

[26] The clear dynamical balance seen from this calculation confirms the remarkably high quality of the mean flow from the Niiler et al. [2003] data. It also gives us useful information about the dynamics of the meanders. For example, if we view the meanders as stationary Rossby waves (waves with a westward phase speed relative to an eastward flow, such that the phase speed and flow speed cancel), we would in general expect a three-way balance in which planetary vorticity advection is balanced by a combination of relative vorticity advection, and divergence. This can be seen from the linear, time-dependent quasigeostrophic relation for horizontal divergence (linearizing but retaining time dependence in equation (2), and using mass continuity in the Boussinesq approximation \(u_x + v_y + w_z = 0\)),

\[-\rho g \beta (u_x + v_y) = \beta p_x' + (p_{xx} + p_{yy})',\]

where the two terms on the right-hand side are related to \(\beta v\) and to \(\zeta\). Divergence is related to pressure via mass continuity in a manner which depends on the vertical wave mode considered, for example in a barotropic ocean of constant depth \(H\) we have

\[H(u_x + v_y) = -\eta,\]

where \(\eta\) is sea surface height displacement and is related to pressure anomaly \(p'\) via \(p' = \rho_0 g \eta\). Substituting these relations into 4 gives

\[\eta = \beta \eta + (\eta_x + \eta_y),\]

where \(R_0 = c/f\) is the Rossby radius, with \(c = (gH)^{1/2}\) the barotropic gravity wave speed. In an ocean of depth 4000 m at mid-latitude, \(c\) is typically 200 m s\(^{-1}\) and the Rossby radius is 2000 km. For baroclinic modes, a larger divergence is necessary to produce the same pressure perturbation, resulting in correspondingly slower gravity waves and a shorter Rossby radius. According to Chelton et al. [1998], typical values for first baroclinic mode of the ACC are \(c = 1.2-2.4\) m s\(^{-1}\), with the slower speeds farther north, and \(R_0 = 10-25\) km, with the larger values farther north.

[27] Substituting a wavelike form \(\eta = A \exp i(kx + ly - \omega t)\) gives

\[-\beta k = \omega \left(\frac{1}{R_0} + k^2 + l^2\right),\]

where \(-\beta k\) relates to \(\beta v\), \(\omega/R_0\) relates to divergence, and \(\omega(k^2 + l^2)\) relates to \(\zeta\). From this we see that the two terms on the right-hand side have the same sign, and their sum balances \(\beta v\). For wavelengths much longer than \(2\pi R_0\), the second term on the right is negligible and \(\beta v\) balances the divergence, leading to the long Rossby wave dispersion relation \(\omega = -R_0/\beta k\). For wavelengths much shorter than \(2\pi R_0\), the divergence term is negligible, leading to the short Rossby wave dispersion relation \(\omega = -\beta k(k^2 + l^2)\). From this we see that the dominant balance in long Rossby waves is between \(\beta v\) and divergence (or vortex stretching), and that in short Rossby waves is between \(\beta v\) and \(\zeta\).

[28] In our case, as we are considering the possibility of a stationary Rossby wave propagating against a mean flow, \(\zeta\) should be replaced by \(u \cdot \nabla \zeta\). The regression coefficient of \(-0.7\) and \(-1\) suggests that \(u \cdot \nabla \zeta\) either balances or more-than-balances \(\beta v\) in the meanders showing that they have the character of short Rossby waves. The wavelength of 300-500 km then tells us that they must be more like barotropic Rossby waves (short compared to \(2\pi \times 2000\) km) than baroclinic Rossby waves (short compared to \(2\pi \times 25\) km).

[29] Of course, the description in terms of linear wave modes is a gross simplification of the actual system which is not being advected by a constant, barotropic mean flow and independent of latitude, but by a mean flow which depends on both depth and latitude, and flows over sloping topography. Nonetheless, the result is consistent with the fact that the flow is supercritical with respect to baroclinic Rossby waves, which would imply that any standing meanders must have more the character of barotropic Rossby waves. The small divergence also gives hope that the meanders may be
susceptible to a two-dimensional analysis in terms of flow in an equivalent-barotropic jet although, as we have seen, topography may be important. The meridional shear of the mean flow, however, cannot be ignored. For a zonal jet, the northward gradient of relative vorticity is $-u_{\eta y}$, which can exceed the planetary vorticity gradient $\beta$. For example, the central regions of several of the observed ACC jets can be well fitted by sinusoidal curves of the form $u = a + b \sin(2\pi y/\lambda + d)$, with amplitude $b$ of about 0.1 m s$^{-1}$ and wavelength $\lambda$ of 5°. This leads to $-u_{\eta y} = 1.28 \times 10^{-11}$ m s$^{-1}$ at the jet center, equivalent to the value of $\beta$ at 56°S. Thus shear in the mean flow can double the effective value of $\beta$ in jet centers (and reduce it to near zero in the wings of jets).

[30] A simple consistency check for nondivergent waves can be made using the dispersion relation (7), with $R_0$ set to infinity. For this case, stationary waves ($-u = \omega/k$) require that $u = \beta/(k^2 + \beta^2)$. Taking $\beta = 1.47 \times 10^{-11}$ m s$^{-1}$ appropriate to 50°S, and assuming that $l = k\alpha$, where $\alpha$ is typically 0.5 or less (Figures 3c and 4c show that most features are at least twice as long meridionally as zonally), for a wavelength of 500 km this leads to

$$u = \frac{\beta}{k^2(1 + \alpha^2)} \left( \frac{9.3 \text{cms}^{-1}}{1 + \alpha^2} \right).$$

Taking $\alpha = 0.5$ decreases the speed by 25% from the $\alpha = 0$ case, and changing the wavelength to 300 km reduces the speed from 9.3 to 3.3 cm s$^{-1}$. Speeds like these are certainly seen in the ACC, but surface flow speeds actually reach considerably higher values (more than 20 cm s$^{-1}$ in Figure 1a). The enhancement of effective $\beta$ in these strong jets helps, but an additional problem is that it is not clear at what depth the value of $u$ is to be taken, since this simple analysis assumes a barotropic mean flow.

[31] This problem can be overcome, and topographic effects included, if we make the assumption that both mean flow and meanders are equivalent-barotropic. Consider a quasigeostrophic flow $u = -g\eta/f$, $v = g\eta/f$, where $\eta$ is given by

$$\eta = A(z) \left[ -u_{s}fz/g + Ce^{\pm jz} \right].$$

This produces a surface flow consisting of a constant eastward flow, on which are superimposed meridional flows with zonal wavenumber $k$. The vertical structure function $A(z)$ is defined to be 1 at the surface, and would typically decay with depth. Killworth and Hughes [2002] found a good fit in the case of the Ocean Circulation and Climate Advanced Modelling project model (OCCAM) for $A = [1 + \tanh(z + 300)/2100000][1 + \tanh(300)/210000]$ (z measured in meters from the ocean surface, positive upward). This profile decays almost exponentially away from the surface to a value of 0.1 at 3106 m, and 0.02 at 4840 m.

[32] If we integrate equation (3) from surface to sea floor, and assume $w = 0$ at the surface, then we obtain

$$h_0\beta_0 + h_1u_s\zeta_{ss} = -fw_0,$$

where $h_0 = \int_0^{h_s} A(z) \, dz$, $h_1 = \int_0^{h_s} A^2(z) \, dz$, the subscript $s$ refers to quantities at the surface, and $b$ to those at the sea floor. Given the OCCAM form of $A(z)$, $h_0$ is close to 1400 m, and $h_1$ is a little over 800 m for all depths over 3000 m.

[33] The kinematic boundary condition at the sea floor gives $w_b = \nu H = \alpha v H$, where $\alpha = A(-H)$. Substituting for $w_b$, and rewriting $\zeta_s$ and $v$ in terms of $\eta$ from geostrophy, then equations (9) and (10) lead to

$$h_0\beta - u_s^2 h_1 = \nu A H,$$

giving a dispersion relation

$$u_s = \frac{\beta h_0 - \nu A H}{k^2 h_1}.$$ 

For the flat-bottomed case, this is the same as equation (8), but multiplied by $h_0/h_1$ (for $\alpha = 0$). As $A$ is smaller than 1 beneath the surface, $\beta_0 > h_1$, so this results in an increase in the estimated flow speed, while unambiguously identifying that speed as the surface flow. The ratio $h_0/h_1$ depends on the form of $A(z)$ and on the ocean depth, but for the OCCAM form discussed above the ratio varies only from 1.5 in 2000 m, to 1.67 in 3000 m, to 1.78 at infinite depth, so this predicts a mean surface flow speed of about 16 cm s$^{-1}$, quite consistent with the values associated with the largest-scale meanders, especially if the enhancement of effective beta by lateral shear is accounted for.

[34] There appears to be no need of topographic interactions to explain the observed wavelengths of the meanders, but such interactions certainly cannot be ruled out, as a simple scaling suggests that they may be significant. From equation (12), topography can enhance $\beta$ by a factor of $(1 - \nu A H/\beta h_0)$. Taking $a = 0.05$ and $h_0 = 1411$ m (appropriate to a depth of 3860 m with the OCCAM form of $A(z)$), and a depth change of 1000 m over 500 km typical of a mid-ocean ridge, this factor is 1.54 at 50°S.

[35] It is interesting to note that even without topographic interactions (so that the depth-integrated divergence is zero and $w_b = 0$), this equivalent-barotropic form for the meanders implies a divergence at the surface. This is because, while $\beta v$ is proportional to $A(z)$, $u \cdot \nabla \zeta$ is proportional to $A^2$, and therefore decays more rapidly away from the surface. If the two are to balance in a depth integral, this means that $u \cdot \nabla \zeta$ must more-than-balance $\beta v$ at great depth. With the $A(z)$ structure used here, this results in a balance between the two at about 950 m, and in $u \cdot \nabla \zeta = -h_0/h_1 \beta v$ at the surface. With the ratio $h_0/h_1 = 1.7$, this would imply a regression coefficient of $-0.59$: somewhat smaller than our estimated value, but not inconsistently so, given the inherent uncertainty in the effect of smoothing.

[36] Clearly a more complete analysis of the meandering jets is required to understand their dynamics completely, but it seems clear that an equivalent-barotropic stationary wave gives a good first-order approximation to the dynamics of these stationary meanders, with some influence of topography likely.

3.2. Topographic Interactions

[37] The "clean" nature of Figures 3d and 4d strongly suggests that there are two quite separate physical processes represented in the data. Relative vorticity advection highlights wave-like dynamics in meanders. Subtracting out the
effect of the meanders then leaves a cleaner map, which equation (3) suggests is a measure of the large-scale divergence term $fw_E$. It is tempting to conclude that Figures 3d and 4d are plots of a quantity proportional to vertical velocity at the ocean floor, or to bottom pressure torque. That is indeed my tentative conclusion, but it is important to recognize that there are significant factors which cloud this interpretation.

[41] Wind stress curl thus appears to be a minor influence, leaving us free to interpret the large-amplitude features in Figures 3d and 4d as scaled maps of bottom pressure torque. From this point of view, they show many similar features to the results from OCCAM of Hughes and de Cuenas [2001]. Perhaps the most notable difference is the improved resolution of the measurements over the model results, due to the fact that it was necessary to smooth the model data in order to average out the effect of the model representation of topography as a series of plateaus bounded by cliffs. This higher resolution highlights the importance of very steep topographic jumps, such as the ridges in Drake Passage, the Macquarie ridge system, and the fracture zones south of the Tasman Sea. It is interesting that these latter regions appear to stimulate a series of downward steps in the bottom flow (predominantly negative $w_b$) in contrast to the Drake Passage region which produces a series of upward steps. In many other places, the patterns of inferred bottom pressure torque are surprisingly smooth, occurring at the dominant length scale of the topography itself.

[42] A final caveat in the interpretation as bottom pressure torque concerns the effect of eddy vorticity advection. In considering the steady vorticity balance, we have ignored a part of the nonlinear term $\mathbf{u} \cdot \nabla \zeta = \mathbf{u} \cdot \nabla \zeta + \mathbf{u}^' \cdot \nabla \zeta^'$, where an overline represents a time average, and prime represents deviation from the time average. We have looked at the first term on the right-hand side of this equation, but not the second. Preliminary calculations based on satellite altimetry suggest that it can be significant, and is also strongly correlated with topography, but tends to occur at shorter length scales than can be clearly resolved by the mean flow data. This is a subject of continued investigation.

4. Conclusions

[43] Using a drifter-based mean dynamic topography [Niler et al., 2003], the surface vorticity balance (excluding the Ekman layer) has been calculated for the Southern Ocean. This clearly shows two modes of behavior for the flow: meanders with wavelength 300–500 km in which relative and planetary vorticity advection tend to cancel, and a remnant balance in which total vorticity advection appears to be related to topographic steering.

[44] The vorticity balance of the meanders is consistent with an equivalent-barotropic model in which $\mathbf{u} \cdot \nabla \zeta$ more than balances $\beta v$ at the surface, balances $\beta v$ at some intermediate depth (suggested to be about 950 m), and less than balances $\beta v$ below that, the differences being taken up by divergence. It is not clear whether the residual depth-integrated divergence in the meander mode is significant, but scaling based on the OCCAM vertical structure function suggests it may be.

[45] If we tentatively make the assumption that depth-integrated divergence is unimportant to the meander dynamics at depth, then the equivalent-barotropic nature of the remaining component of the flow allows us to interpret the total vorticity advection as a measure of bottom pressure torque minus wind stress curl. Scaling shows that the contribution of wind stress curl is small.

[46] Interpreting the total vorticity balance as a measure of bottom pressure torque, we see many features similar to those predicted by the OCCAM model. In many places, the
bottom pressure torque is surprisingly smooth and appears to act at the dominant (large) length scale of the topography. Perhaps the most striking feature which differs from OCCAM is the strong effect of sharp topographic features such as the fracture zones and Macquarie ridge system south of the Tasman Sea, and the ridges in Drake Passage. This may be indicative of strong bottom pressure torques in these regions, or may be a result of incomplete cancellation of the meander mode at short length scales, resulting from the smoothing necessary in data processing.

[47] These results clearly demonstrate the high quality of the drifter-based data (in this case slightly enhanced by inclusion of satellite gravity from the GRACE satellite gravity mission and satellite altimetry). Because of the strong mean flow in the ACC, the nonlinear terms in the vorticity balance occur at length scales (related to the Rhines scale) which can be resolved by the data, in weaker flows this length scale will be shorter. Even in the ACC the resolution of the data set may still be limiting, however, and is particularly so in regions of low drifter coverage (such as south of the ACC). This can be expected to improve as more drifter data become available, and as a higher-resolution satellite geoid is produced by the GOCE (Gravity and Ocean Circulation Explorer) satellite scheduled for launch in 2006. In addition, the GOCE geoid will come with clear and spatially almost uniform error covariance estimates, making it possible to quantify errors in this kind of analysis.

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References


C. W. Hughes, Proudman Oceanographic Laboratory, Joseph Proudman Building, 6 Brownlow Street, Liverpool L3 5DA, UK. (cwh@pol.ac.uk)