the barotropic governor

effects of horizontal shear on baroclinic instability
\[ \bar{u}(y, t) \]
A curious observation

James and Gray 1986: atmospheric flow on a hemisphere with surface drag

INCREASE surface drag

INCREASE baroclinic conversion
model energetics
five values for drag

available potential energy (AZ)
also increased as the drag was reduced. However, KE and other measures of eddy activity such as the poleward eddy temperature flux declined as the drag was reduced.

The poleward temperature gradients for the longest values of $z$, were close to those of the equilibrium state. This result is confirmed by Fig. 7(b) which shows the mean values of the conversions from eddy available potential energy $AE$ to eddy kinetic energy $CE$, and from KE to $KZ$ $(CK)$ as a function of $zD$.

$CE$ (which is proportional to $[o^*P]$ and may be considered as a measure of baroclinic instability) decreased as $Z$, increased. At the same time, $CK$ remained nearly constant for $zD$ up to 10 days and then fell, albeit more slowly than $CE$, as the eddy activity was quenched. Thus, as the dissipation acting upon the eddies was reduced, the net input of energy into KE declined sharply resulting in a reduction of all measures of eddy activity.

For $zD = 250$ days, the values of $CE$ and $CK$ were nearly balanced. The drag was so slight that there was scarcely any direct dissipation of KE; rather, it was converted to some other form of energy in order to maintain a steady state.

3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0.4
0.8
1.2
1.6
2.0
2.4

Figure 7. Variation of the time-averaged $AZ$, $KZ$ and KE with the drag timescale $\tau_D$.

Figure 8. Variation of the time-averaged conversions $CE$ (eddy available potential energy to eddy kinetic energy) and $CK$ (eddy to zonal kinetic energy) with the drag timescale.

James and Gray, 1986

decreasing drag

I. N. James and L. J. Gray

potential and kinetic energy conversion

more drag

less drag

James and Gray, 1986
Think about this

1. shear deforms the growing unstable mode

2. if \( U_y > \sigma \), normal modes change significantly

3. NO shear \( \Rightarrow \) instability optimally extracts energy

4. so with shear, energy conversion must decrease

Eady might say \([u]_y/[u]_z \geq 0.31f/N\)
Or think about energy

\[
\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'} U_y + \left( \frac{Rk_R^2}{f_0} \right) \overline{v'T'} \overline{T_y} \, dx
\]

eddy energy

eddy KE to barotropic KE conversion

\[
\overline{u'v'} U_y > 0 \implies \text{perturbation energy given up to mean flow!}
\]

baroclinic conversion
Figure 14. Schematic diagram of the 'barotropic governor'. Energy conversion into KE and eventually KZ is balanced by drag. However, the vigour of baroclinic conversions are reduced as KZ increases due to strong horizontal shears inhibiting the baroclinic instability process.
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“... the normal modes contain the seeds of their own destruction... ” – I.N. James
it’s time to get Quantitative
The two layer model

linearize equations around

always assuming

\[ U = U(y) \hat{x} \]

\[ \psi \sim e^{ikx} \]
The "Young model" consists of assuming that and the perturbation stream functions are given by

With

2.3 The Young model (periodic shear)

2.2 The James model (shear in a channel)

We can rewrite this in the telling form where

The "Young model" consists of assuming that and the perturbation stream functions are given by and the perturbation stream functions are given by

Plugging these into (23) and (24) yields

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Linearized equations

Potential vorticity equations

"Diagnostic" equations

"Diagnostic" equations
Without shear

Use \[ U_1 = \Delta U, \quad U_2 = 0 \]
Without shear

Use $U_1 = \Delta U$, $U_2 = 0$

$$q_{1t} + \Delta U q_{1x} + k_R^2 \Delta U \psi_{1x} = 0$$
$$q_{2t} - k_R^2 \Delta U \psi_{2x} = 0$$
Without shear

Use \( U_1 = \Delta U \), \( U_2 = 0 \)

\[
q_{1t} + \Delta U q_{1x} + k_R^2 \Delta U \psi_{1x} = 0 \\
q_{2t} - k_R^2 \Delta U \psi_{2x} = 0
\]

with \( \psi_n = \phi_n e^{i(kx + \ell y) + \sigma t} \)

\[
\text{diagnostic eqns} \\
q_1 = \nabla^2 \psi_1 - k_R^2 (\psi_1 - \psi_2) \\
q_2 = \nabla^2 \psi_2 + k_R^2 (\psi_1 - \psi_2) \\
\text{eigenvalues} \\
\sigma = \sigma(k, \ell)
\]
Without shear

Use \( U_1 = \Delta U , \quad U_2 = 0 \)

\[
\psi \sim e^{i(kx + \ell y) + \sigma t}
\]

\[
\sigma = k \Delta U \left( \frac{2k_R^2 - k^2 - l^2}{2k_R^2 + k^2 + l^2} \right)^{1/2}
\]
Without shear

Use \( U_1 = \Delta U \), \( U_2 = 0 \)

\[ k_R = 500 \text{ km}, \quad \Delta U = 20 \text{ m/s}, \quad \ell = 0 \]

\[ \psi_j \sim e^{i(kx + \ell y) + \sigma t} \]

\[ \sigma = k \Delta U \left( \frac{2k_R^2 - k^2 - \ell^2}{2k_R^2 + k^2 + \ell^2} \right)^{1/2} \]
Can add beta

October 12, 2023

Figure 2: Growth rate $s = k c_i$ as function wavenumber, $(k, l)$, for four values of drag $\mu$ (indicated in top-left corner of each panel). Other parameter values are $\alpha_1 = \frac{8}{9}$ and $U = 5 \beta / (\alpha_2 m^2 R)$. The maximum growth rate is indicated at the bottom of each panel and there are six evenly spaced contours between zero and $\max(s)$. In the top left panel, with $\mu = 0$, there is both a high and low wavenumber cutoff. In the other panels, with $\mu \neq 0$, the low wavenumber cutoff is unchanged at $m = m^\ast$, but there is no high wavenumber cutoff.
Add shear, and stir…

growth rate [day$^{-1}$]

Increasing shear

theory $(A = 0)$

$A = 10^{-6}$ l/s

$A = 5 \times 10^{-6}$ l/s

$A = 10^{-5}$ l/s

zonal wave number
Add shear, and stir...

bifurcation in modal structure
Add shear, and stir…

\[ A = 10^{-5} \text{ sec}^{-1} \]

\[ |\psi_2|^2 \times e^{\sigma T} \]
Fastest growing modes

\[ |\psi_2|^2 \times e^{\sigma T} \]

- \( A = 10^{-6} \) l/s
- \( A = 5 \times 10^{-6} \) l/s
- \( A = 10^{-5} \) l/s

Theory (A = 0)

\[ A = 10^{-5} \text{ l/s} \]
\[ A = 5 \times 10^{-6} \text{ l/s} \]
\[ A = 10^{-6} \text{ l/s} \]
Energetics

\[
\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'} \overline{U_y} + \left( \frac{Rk^2}{f_0} \right) \overline{v'T'} \overline{T_y} \, dx
\]

eddy energy

eddy KE to barotropic KE conversion

baroclinic conversion

\[\overline{u'v'} \overline{U_y} > 0 \implies \text{perturbation energy given up to mean flow!}\]
Energetics

\[
\frac{\partial E}{\partial t} = -g^{-1} \int \bar{u}' v' U_y + \left( \frac{R_k R}{f_0} \right) \bar{v}' T' \bar{T}_y \, dx
\]
\[
\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'} U_y + \left( \frac{Rk_z^2}{f_0} \right) \overline{v'T'} \bar{T}_y \, dx
\]
Energetics

\[ \frac{\partial E}{\partial t} = - \int_{-L/2}^{L/2} \int_0^{2\Delta p} \left\{ u'v' \bar{U}_y + \left( \frac{RkR}{f_0} \right) v'T' \bar{T}_y \right\} \frac{dpdy}{g}. \]

baroclinic conversion

barotropic conversion

increasing shear

increasing shear
What we learned

• the presence of barotropic shear decreases baroclinic growth rates

• … but does NOT change conditions for stability

• eddy extracts potential energy, but sacrifices kinetic energy
More questions

- what do the modes look like when there is no channel?
- how do we go from
- other work?
thanks
What if I don’t like the channel?

try periodic boundary conditions

with \( U(y) = \left( \frac{S}{\ell} \right) \cos \ell y \)
What if I don’t like the channel?

\[ U(y) = \left(\frac{S}{\ell}\right) \cos \ell y \]
\[ \ell = \frac{4\pi}{L} \]