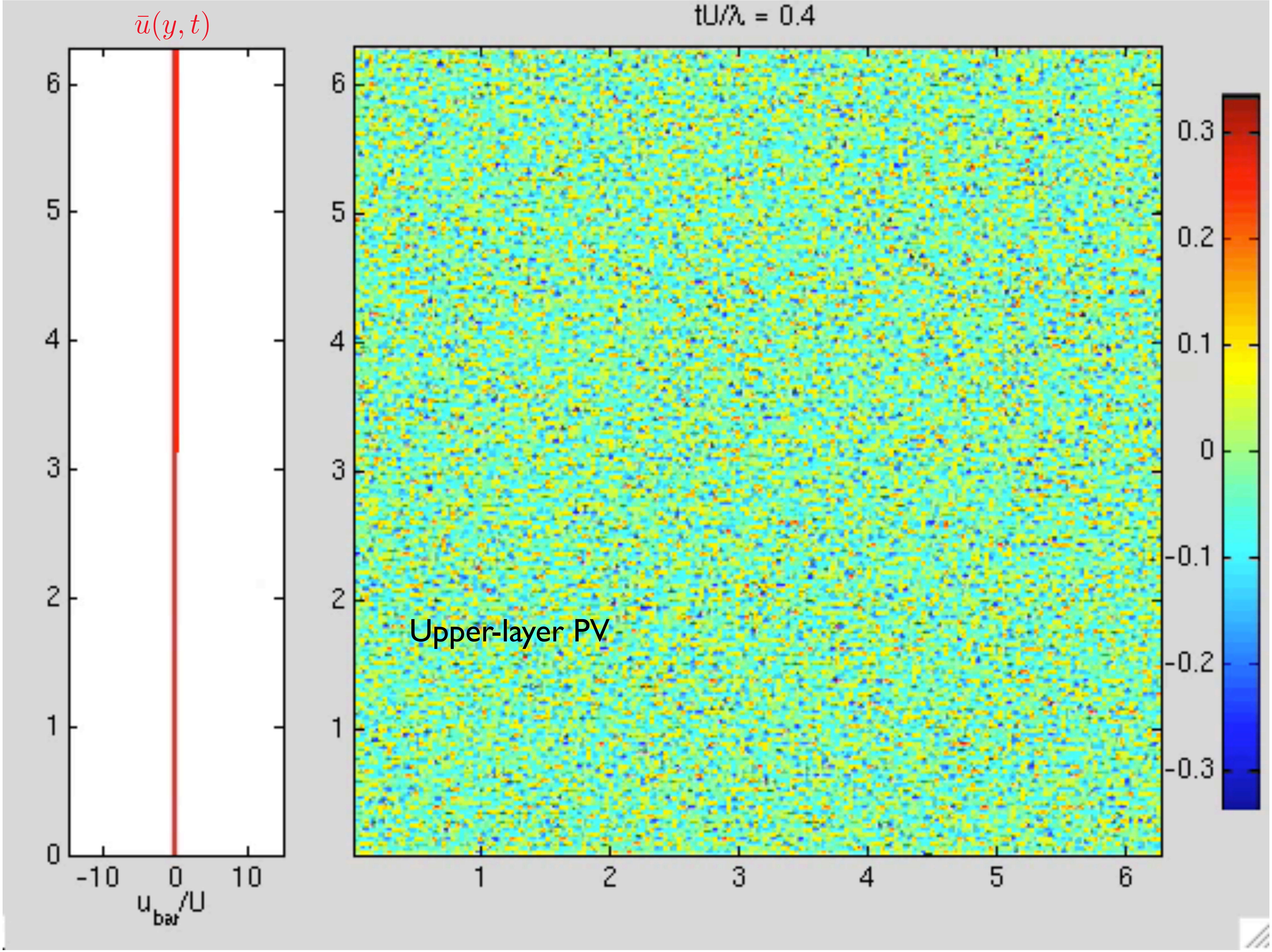
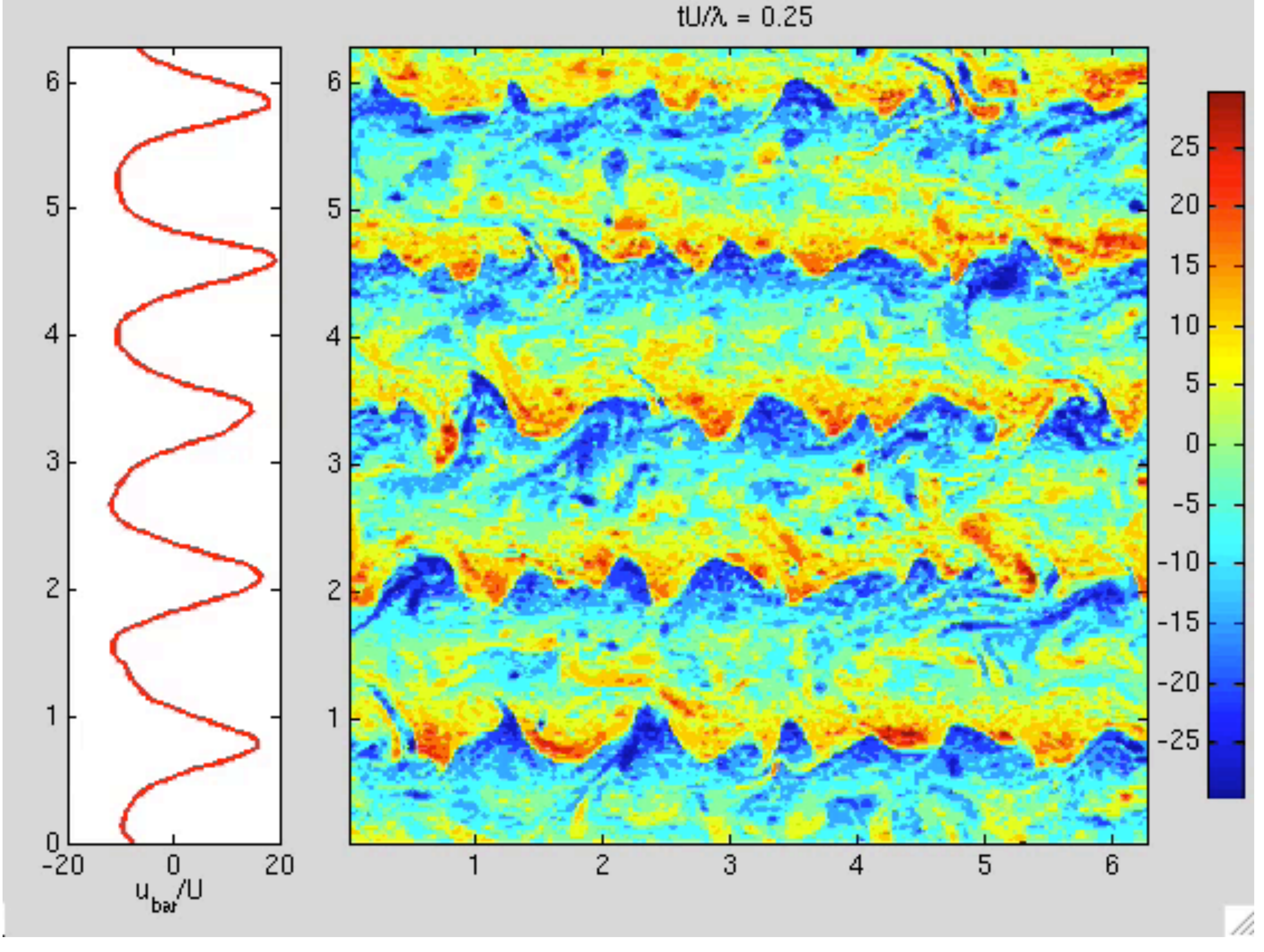
the barotropic governor

effects of horizontal shear

on baroclinic instability



$\bar{u}(y,t)$
u_{bar}/U
tU/λ = 0.4
Upper-layer PV



tU/λ = 0.25
u_bar/U
-20
0
20
0
1
2
3
4
5
6
25
20
15
10
5
0
-5
-10
-15
-20
-25

A curious observation

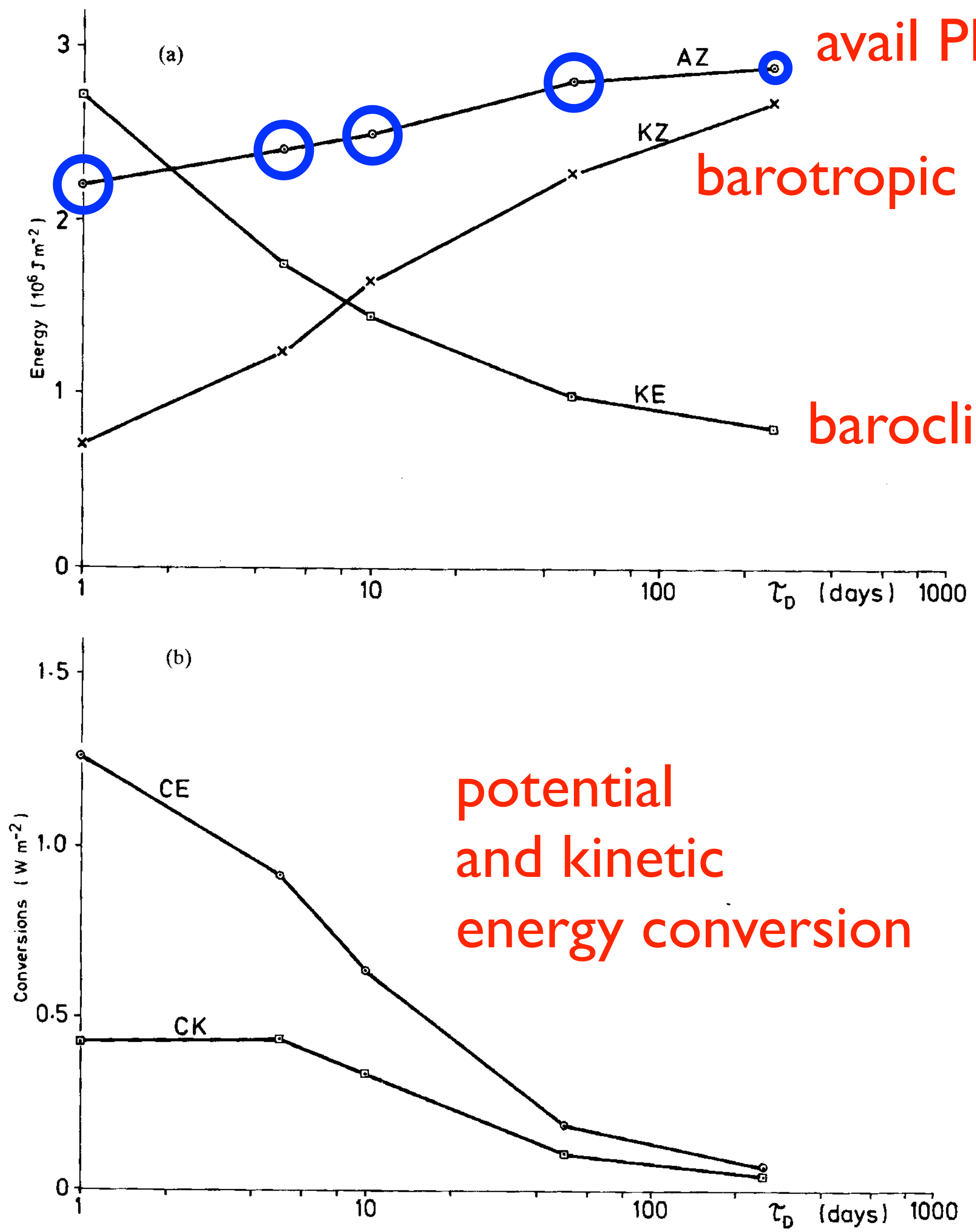
James and Gray 1986: atmospheric flow on a hemisphere with surface drag

INCREASE surface drag

INCREASE baroclinic conversion

model energetics

five values for drag



avail PE

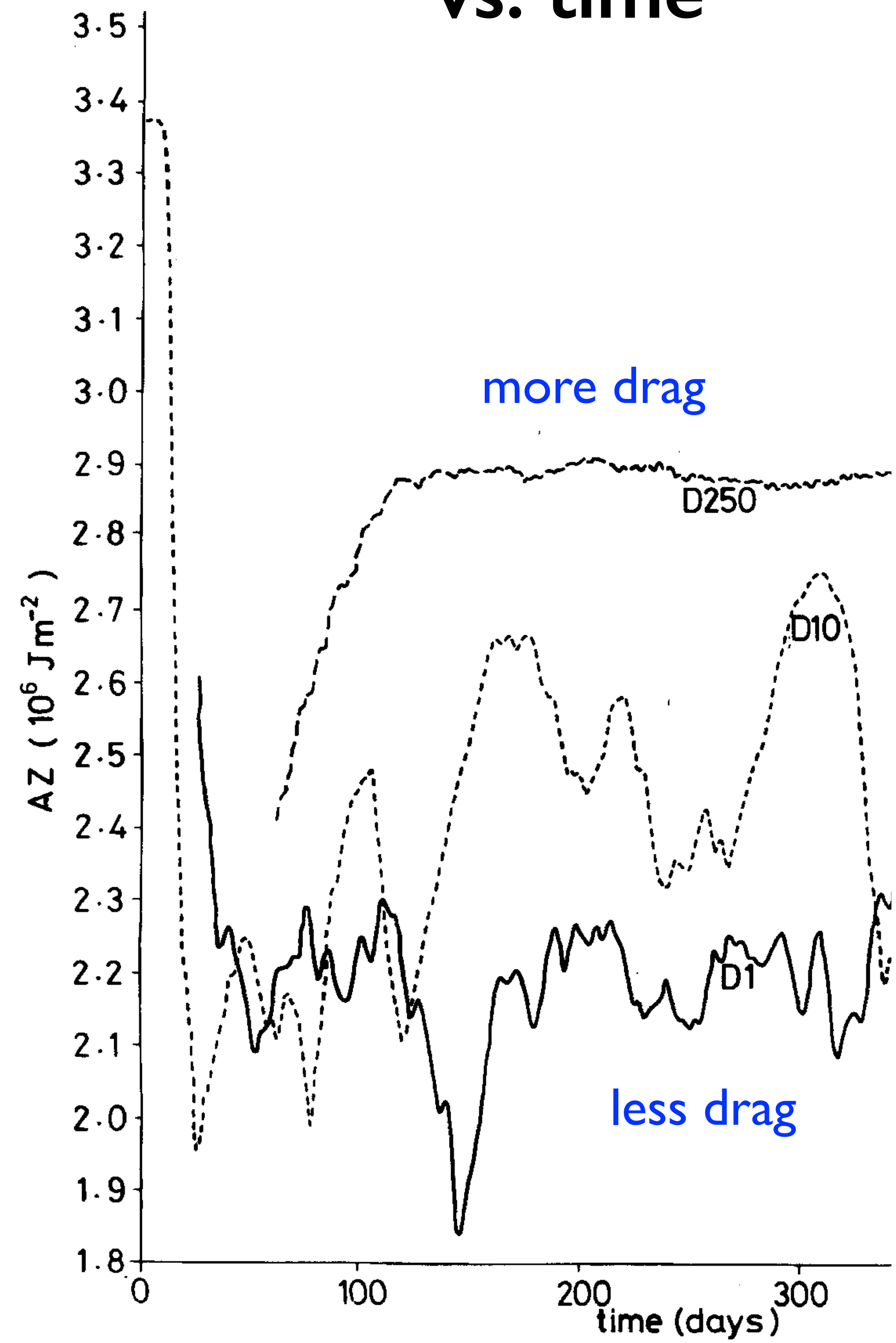
barotropic KE

baroclinic KE

potential
and kinetic
energy conversion

decreasing drag

pot. energy vs. time



more drag

less drag

James and Gray, 1986

Think about this

(U_y) $\psi \sim e^{\sigma t}$

1. shear deforms the growing unstable mode
2. if $U_y > \sigma$, normal modes change significantly
3. **NO** shear $\Longrightarrow$ instability optimally extracts energy
4. so **with** shear, energy conversion must decrease

Eady might say $[u]_y/[u]_z \gtrsim 0{\cdot}31 f/N$

Or think about energy

$$\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'}\, U_y + \left(\frac{R k_R^2}{f_0}\right) \overline{v'T'}\, \bar{T}_y \, \boldsymbol{dx}$$

eddy energy

eddy KE to barotropic KE conversion

baroclinic conversion

$$\overline{u'v'} U_y > 0 \implies$$

perturbation energy given up to mean flow!

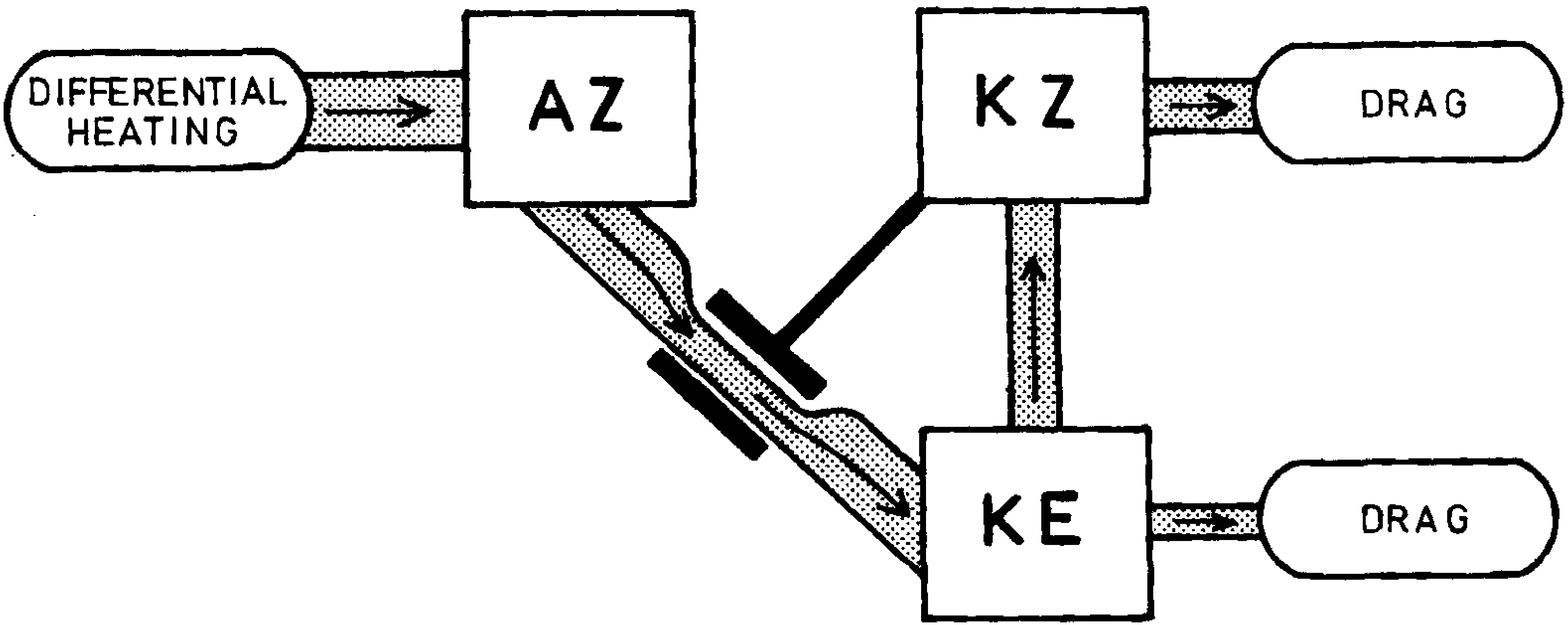
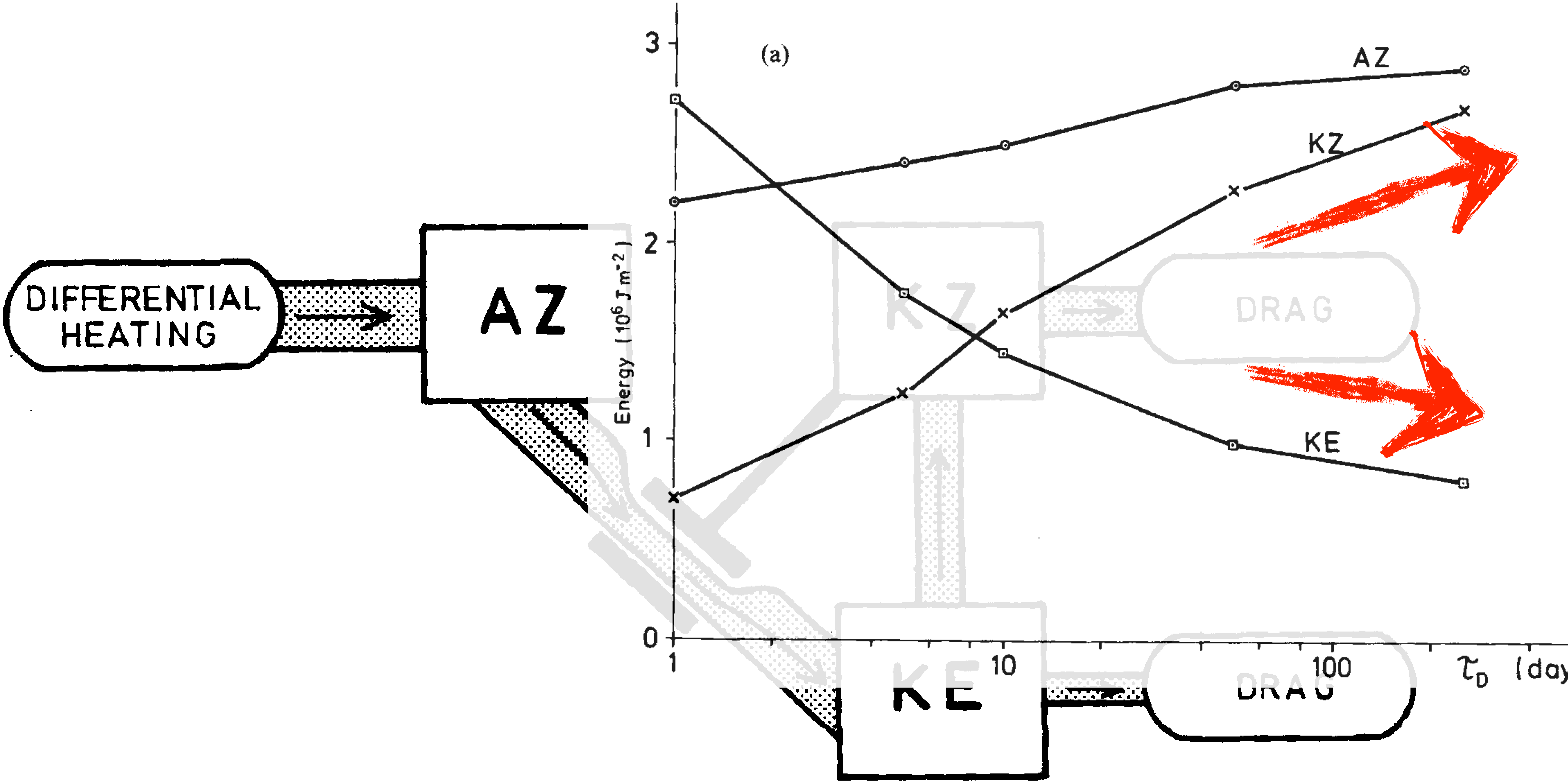


Figure 14. Schematic diagram of the 'barotropic governor'. Energy conversion into KE and eventually KZ is balanced by drag. However, the vigour of baroclinic conversions are reduced as KZ increases due to strong horizontal shears inhibiting the baroclinic instability process.



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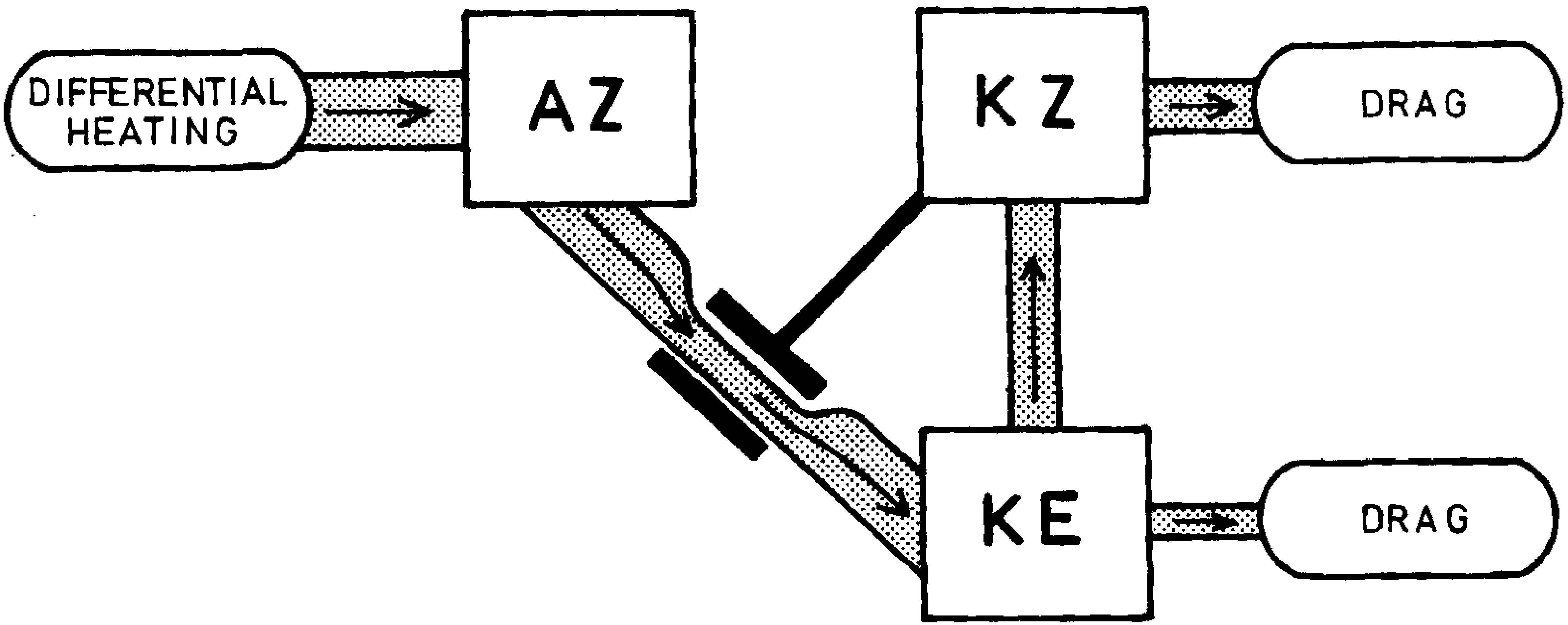

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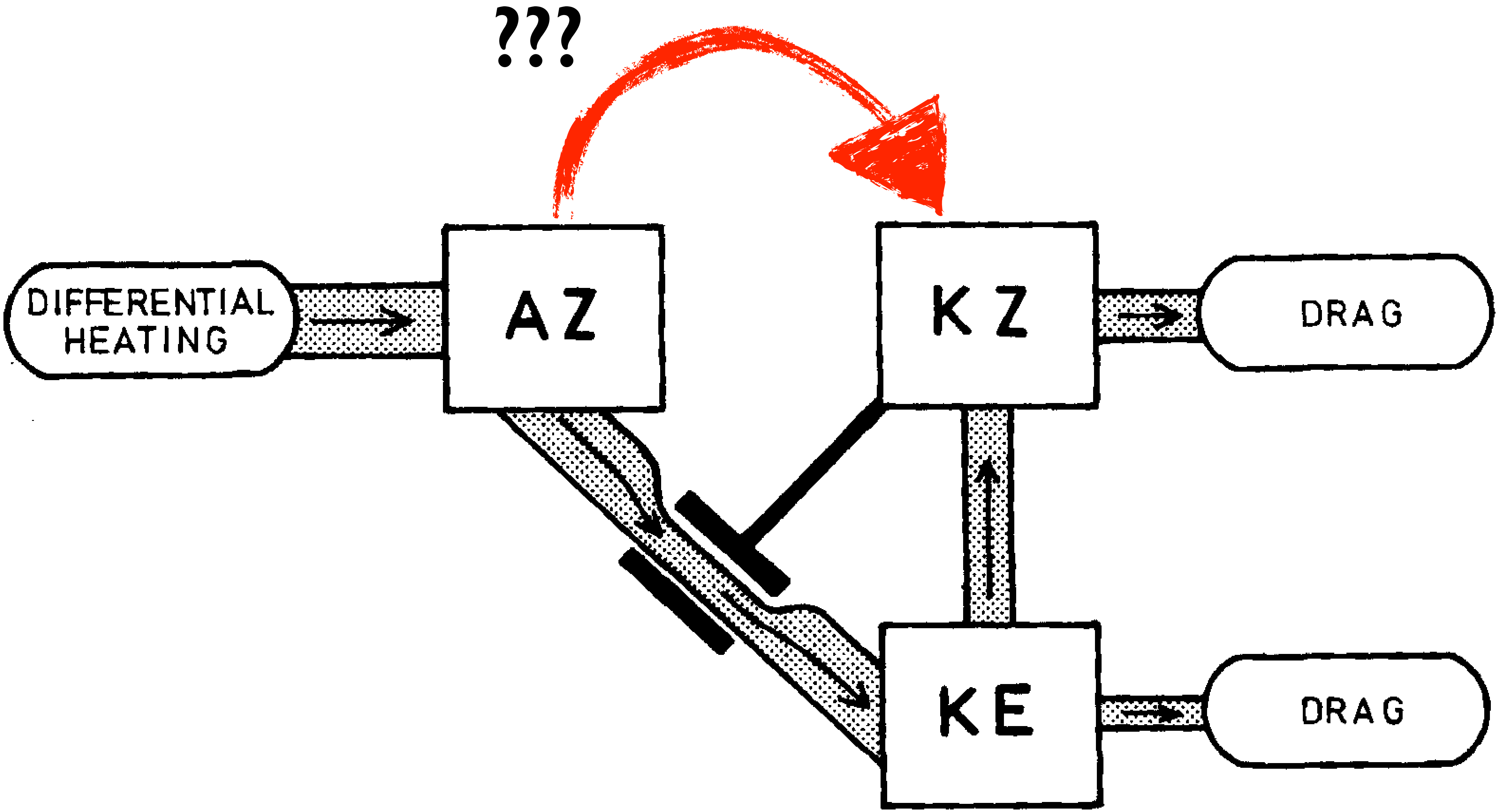

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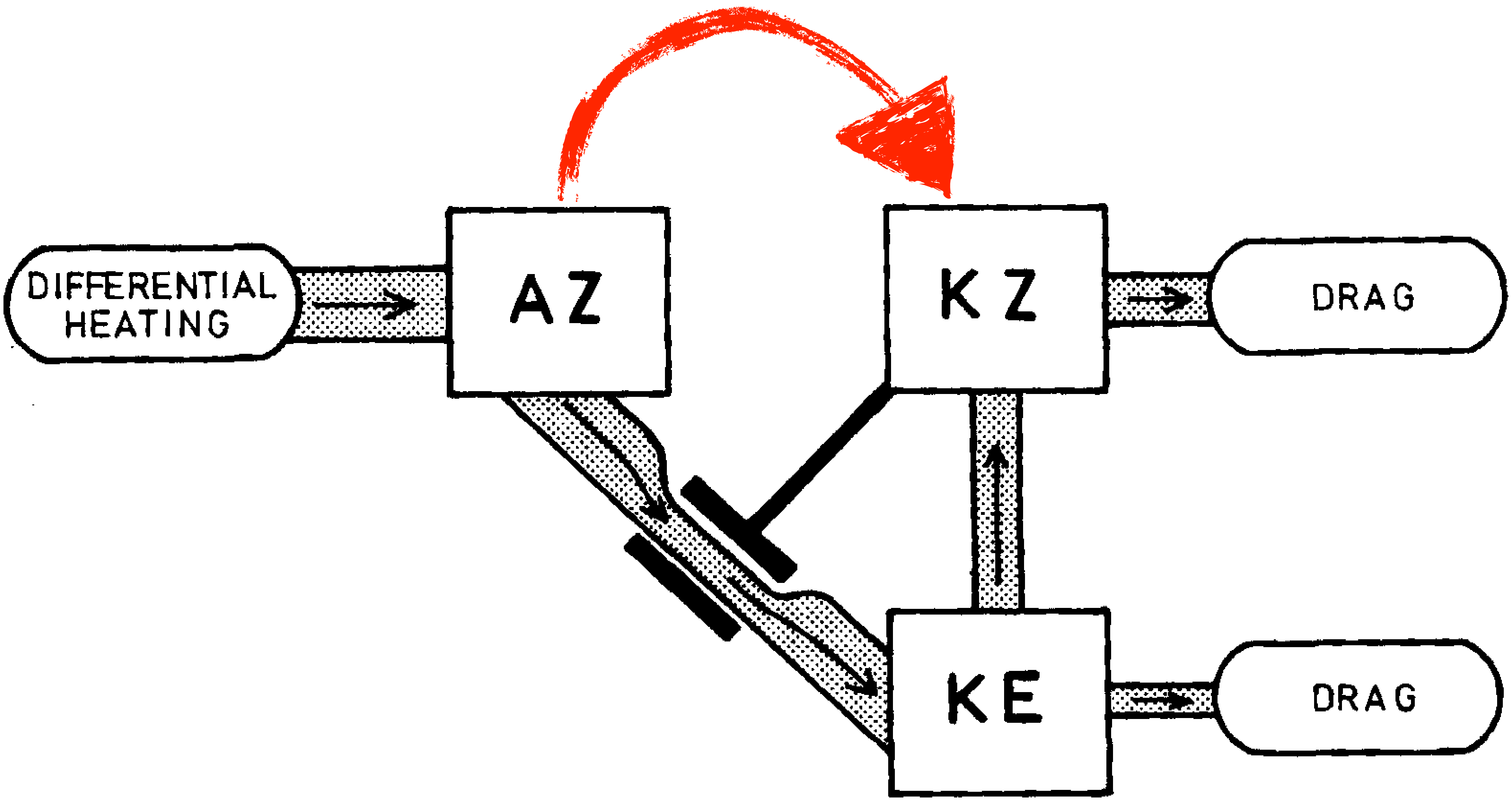
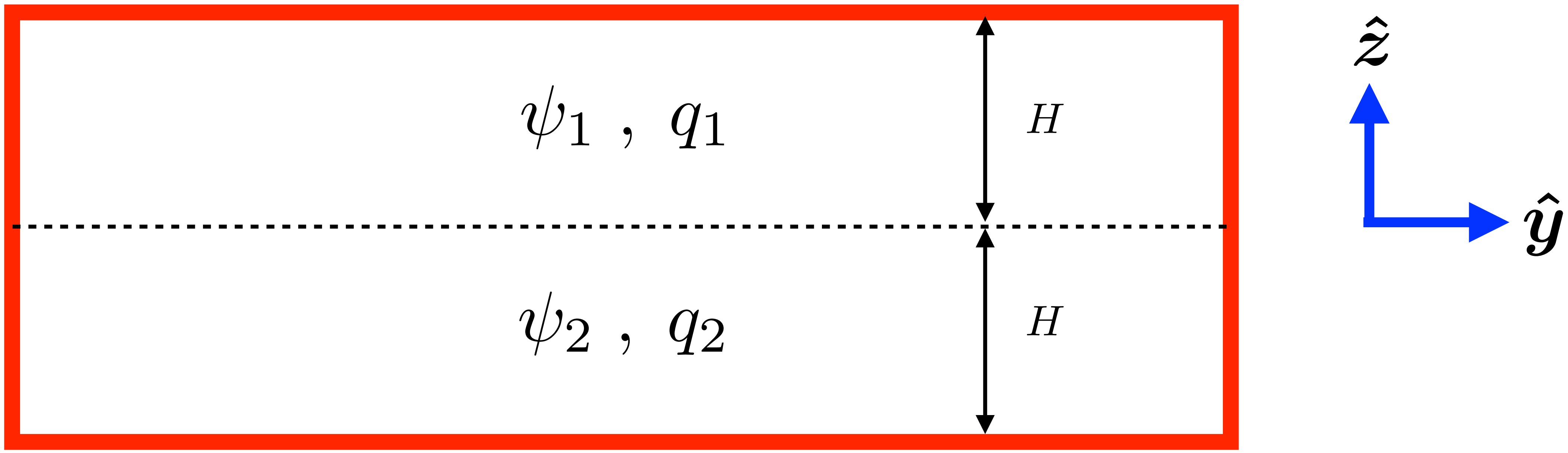


Figure 14. Schematic diagram of the 'barotropic governor'. Energy conversion into KE and eventually KZ is balanced by drag. However, the vigour of baroclinic conversions are reduced as KZ increases due to strong horizontal shears inhibiting the baroclinic instability process.

"… the normal modes contain the seeds of their own destruction… " – I.N. James it's time to get

QUANTITATIVE

The two layer model



linearize equations around $\boldsymbol{U} = U(y)\hat{\boldsymbol{x}}$

always assuming $\psi \sim \mathrm{e}^{\mathrm{i}kx}$

Linearized equations

Potential vorticity equations

$$q_{1t} + U_1 q_{1x} + \Big(\beta - U_{1yy}\Big)\psi_{1x} + k_R^2\Big(U_1 - U_2\Big)\psi_{1x} = 0$$

$$q_{2t} + U_2 q_{2x} + \Big(\beta - U_{2yy}\Big)\psi_{2x} - k_R^2\Big(U_1 - U_2\Big)\psi_{2x} = 0$$

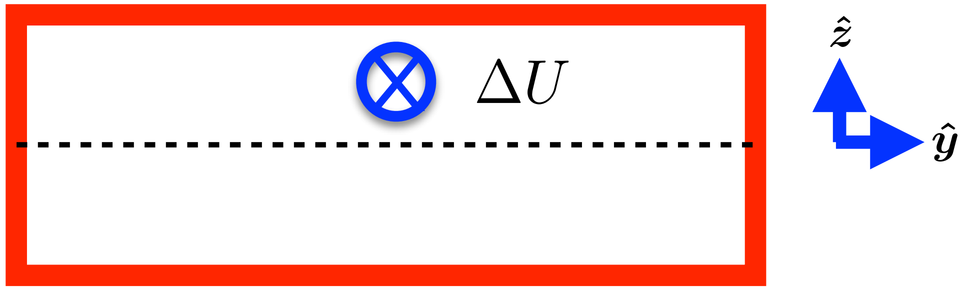
"Diagnostic" equations

$$q_1 = \nabla^2\psi_1 - k_R^2\Big(\psi_1 - \psi_2\Big)$$

$$q_2 = \nabla^2\psi_2 + k_R^2\Big(\psi_1 - \psi_2\Big)$$

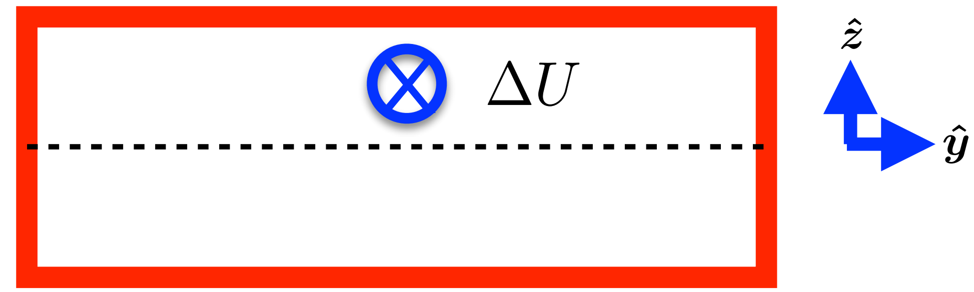
Without shear

Use $U_1 = \Delta U \,, \quad U_2 = 0$



Without shear

Use $U_1 = \Delta U \,, \quad U_2 = 0$

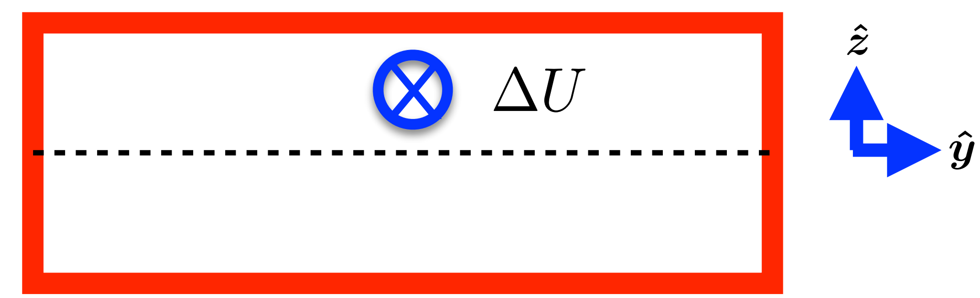


$$q_{1t} \; + \; \Delta U \; q_{1x} \; + \; k_R^2 \Delta U \; \psi_{1x} = 0$$

$$q_{2t} \; - \; k_R^2 \Delta U \; \psi_{2x} = 0$$

Without shear

Use $U_1 = \Delta U\,, \quad U_2 = 0$



$$q_{1t} \;+\; \Delta U\; q_{1x} \;+\; k_R^2 \Delta U\; \psi_{1x} = 0$$

$$q_{2t} \;-\; k_R^2 \Delta U\; \psi_{2x} = 0$$

with $\psi_n = \phi_n \mathrm{e}^{\mathrm{i}(kx+\ell y)+\sigma t}$ **+**

diagnostic eqns

$$q_1 = \nabla^2 \psi_1 - k_R^2 \big(\psi_1 - \psi_2\big)$$

$$q_2 = \nabla^2 \psi_2 + k_R^2 \big(\psi_1 - \psi_2\big)$$

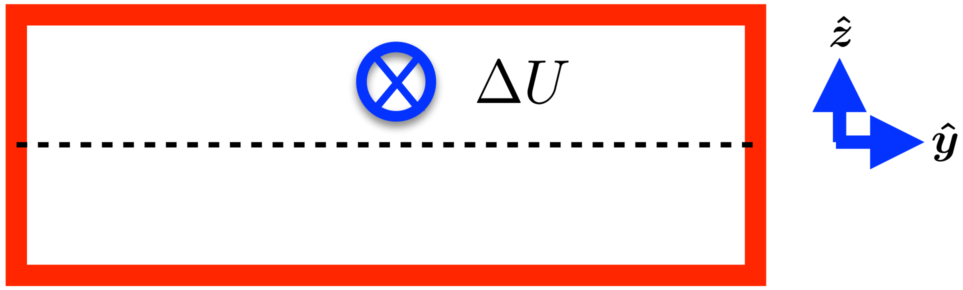
=

eigenvalues

$$\sigma = \sigma(k, \ell)$$

Without shear

Use $U_1 = \Delta U\,, \quad U_2 = 0$

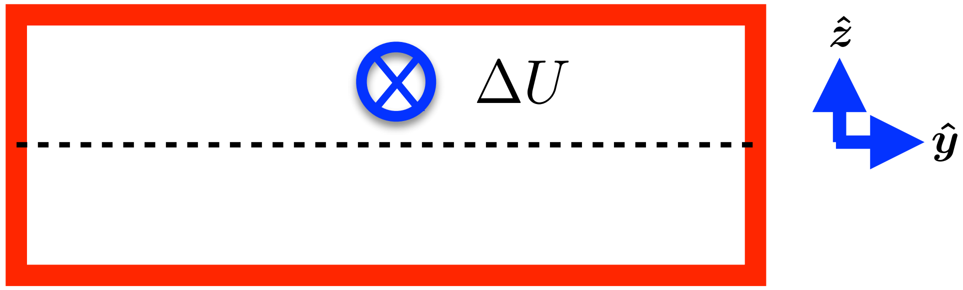


$$\psi \sim \mathrm{e}^{\mathrm{i}(kx+\ell y)+\sigma t}$$

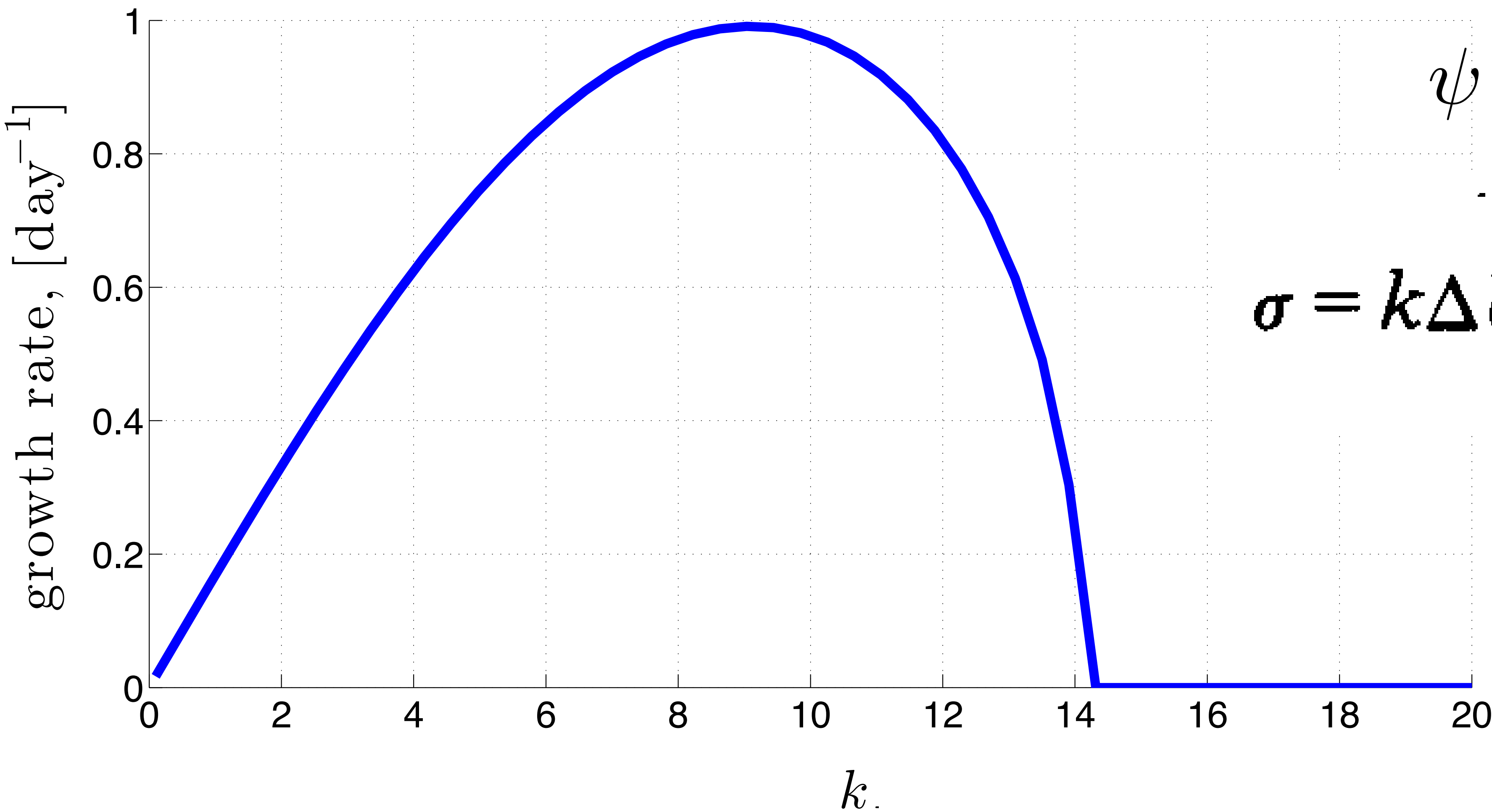
$$\sigma = k\Delta U\left(\frac{2k_R^{\,2} - k^2 - l^2}{2k_R^{\,2} + k^2 + l^2}\right)^{1/2}$$

Without shear

Use $U_1 = \Delta U \,, \quad U_2 = 0$



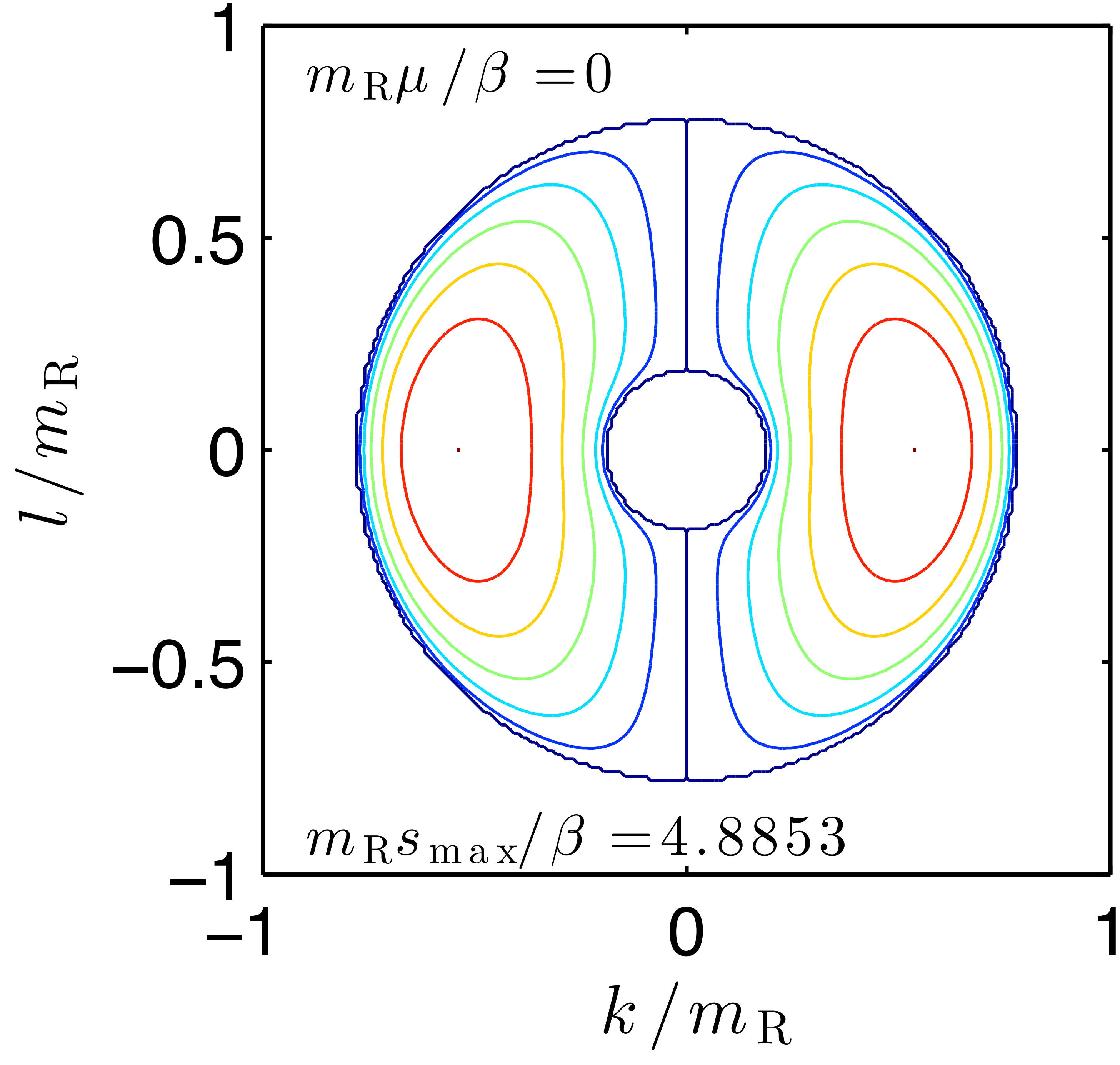
$k_R = 500$ km, $\quad \Delta U = 20$ m/s, $\quad \ell = 0$



$$\psi \sim e^{i(kx+\ell y)+\sigma t}$$

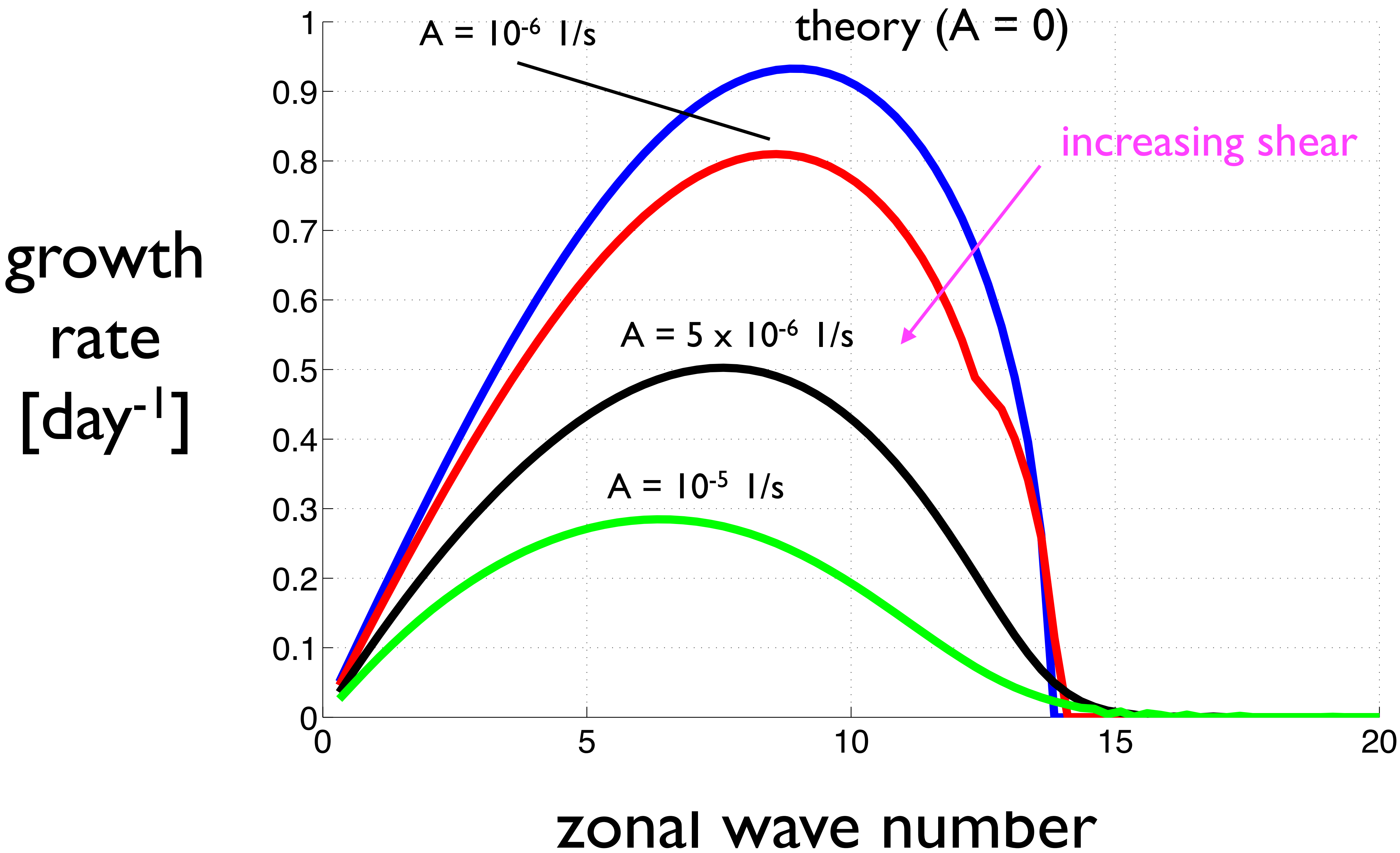
$$\sigma = k\Delta U \left(\frac{2k_R^2 - k^2 - l^2}{2k_R^2 + k^2 + l^2} \right)^{1/2}$$

Can add beta

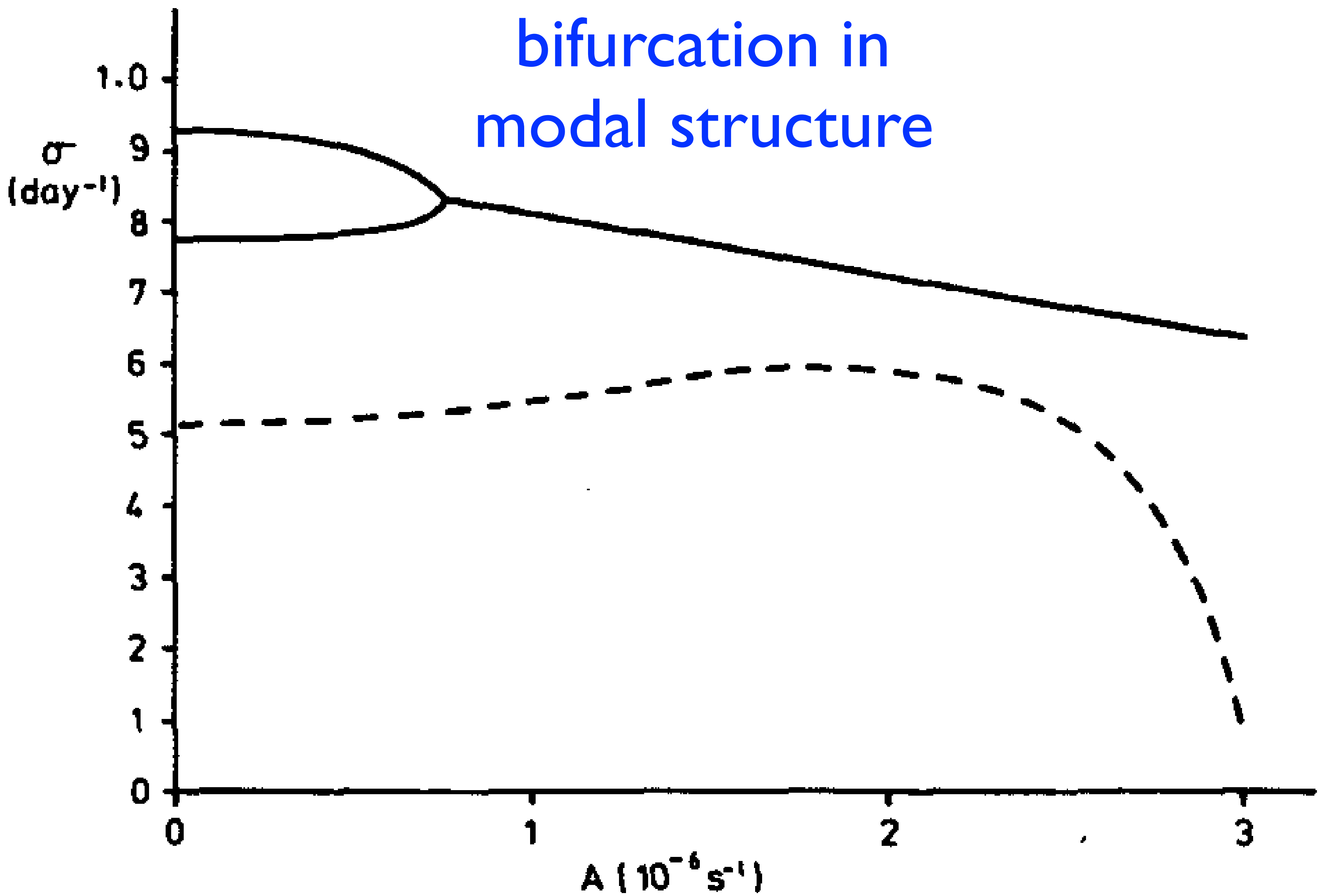


$m_\mathrm{R}\mu\,/\,\beta\;=0$
$m_\mathrm{R}s_\mathrm{max}/\,\beta\;=4.8853$
1
0.5
0
−0.5
−1
−1
0
1
$l\,/\,m_\mathrm{R}$
$k\,/\,m_\mathrm{R}$

Add shear, and stir…

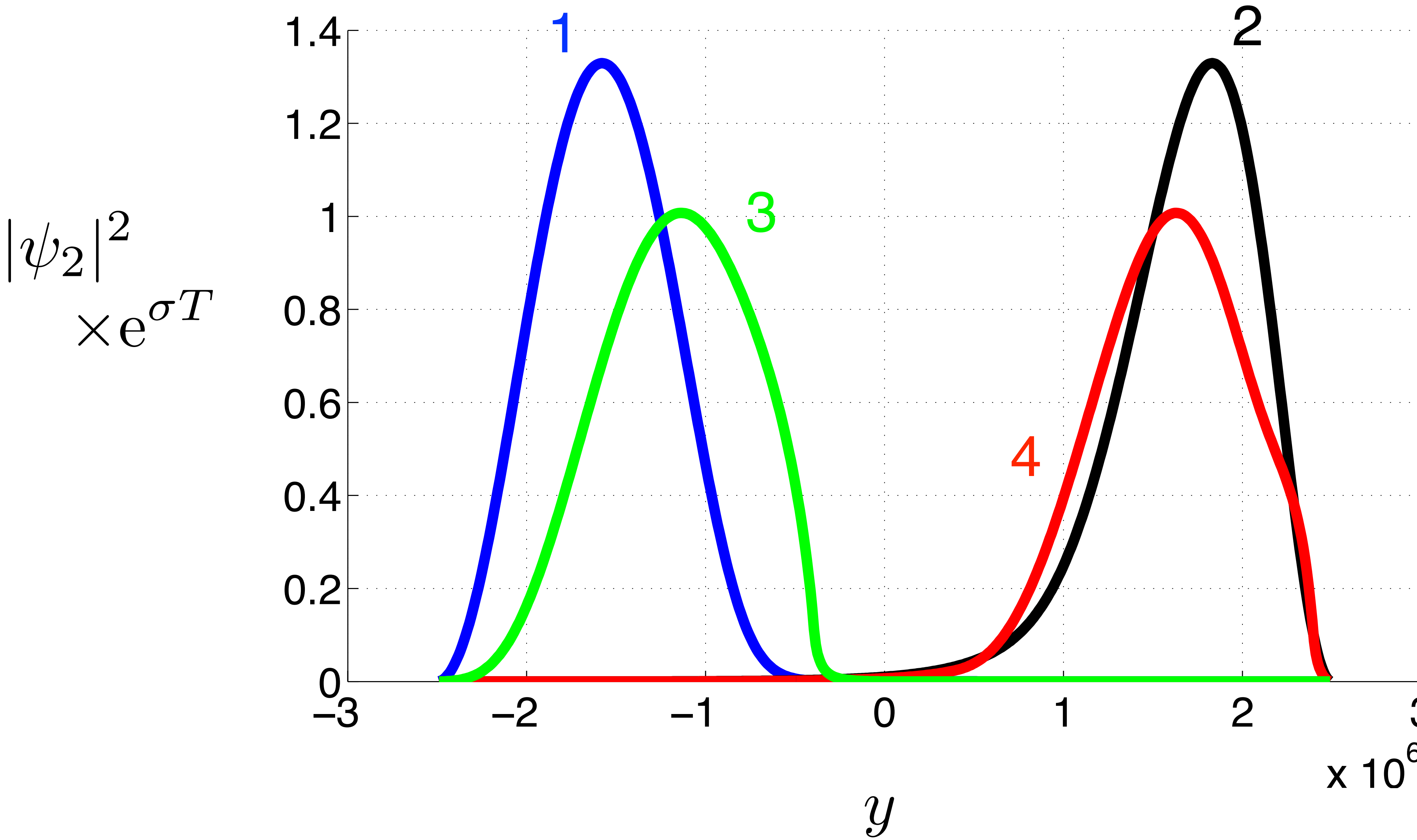


Add shear, and stir…

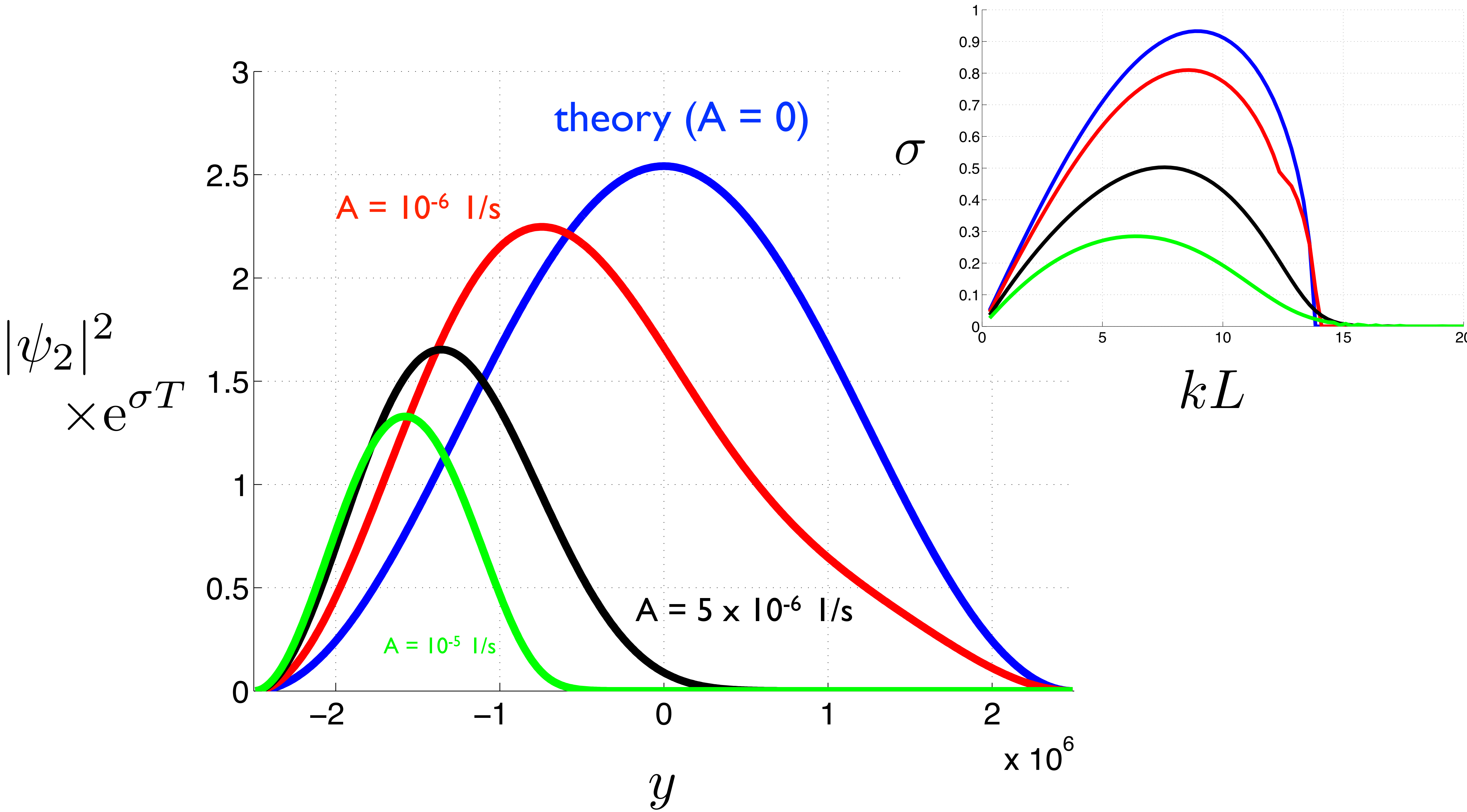


Add shear, and stir…

$A = 10^{-5}\ \mathrm{sec}^{-1}$



Fastest growing modes



Energetics

$$\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'}\, U_y + \left(\frac{R k_R^2}{f_0}\right) \overline{v'T'}\, \bar{T}_y \, \boldsymbol{dx}$$

eddy energy

eddy KE to barotropic KE conversion

baroclinic conversion

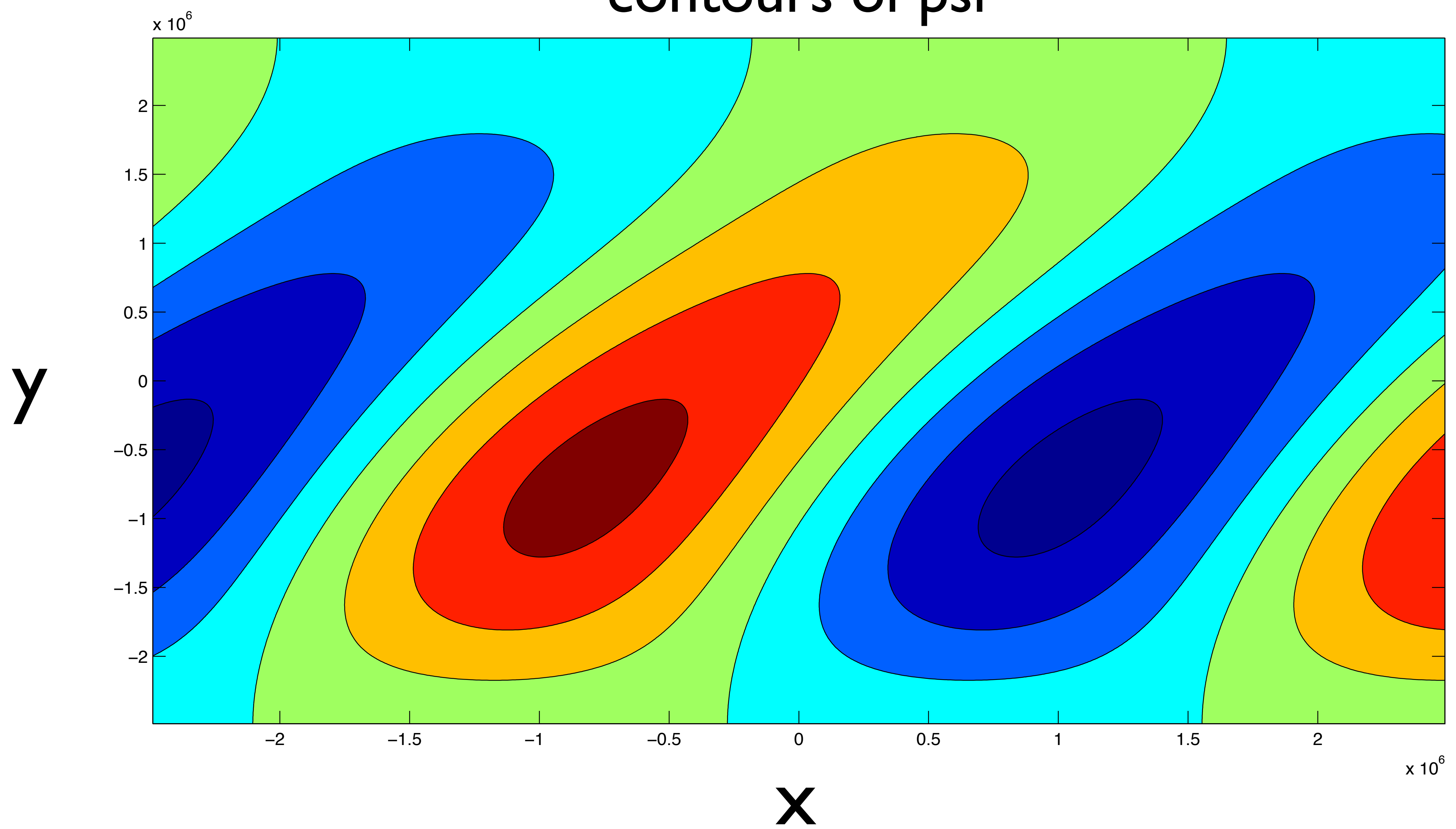
$$\overline{u'v'} U_y > 0 \implies$$

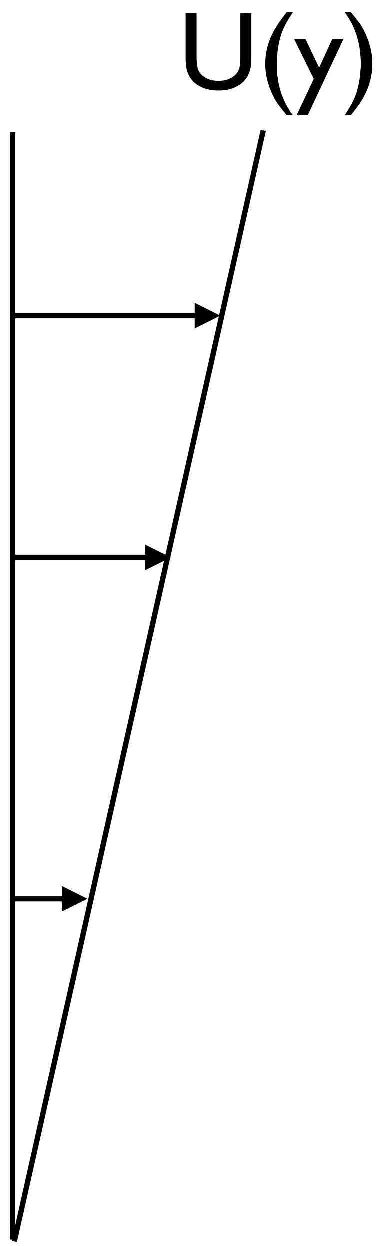
perturbation energy given up to mean flow!

Energetics

$$\underbrace{\frac{\partial E}{\partial t}}_{} = -g^{-1} \int \underbrace{\overline{u'v'}\, U_y}_{} + \left(\frac{Rk_R^2}{f_0}\right) \underbrace{\overline{v'T'}\, \bar{T}_y}_{}\, \boldsymbol{dx}$$

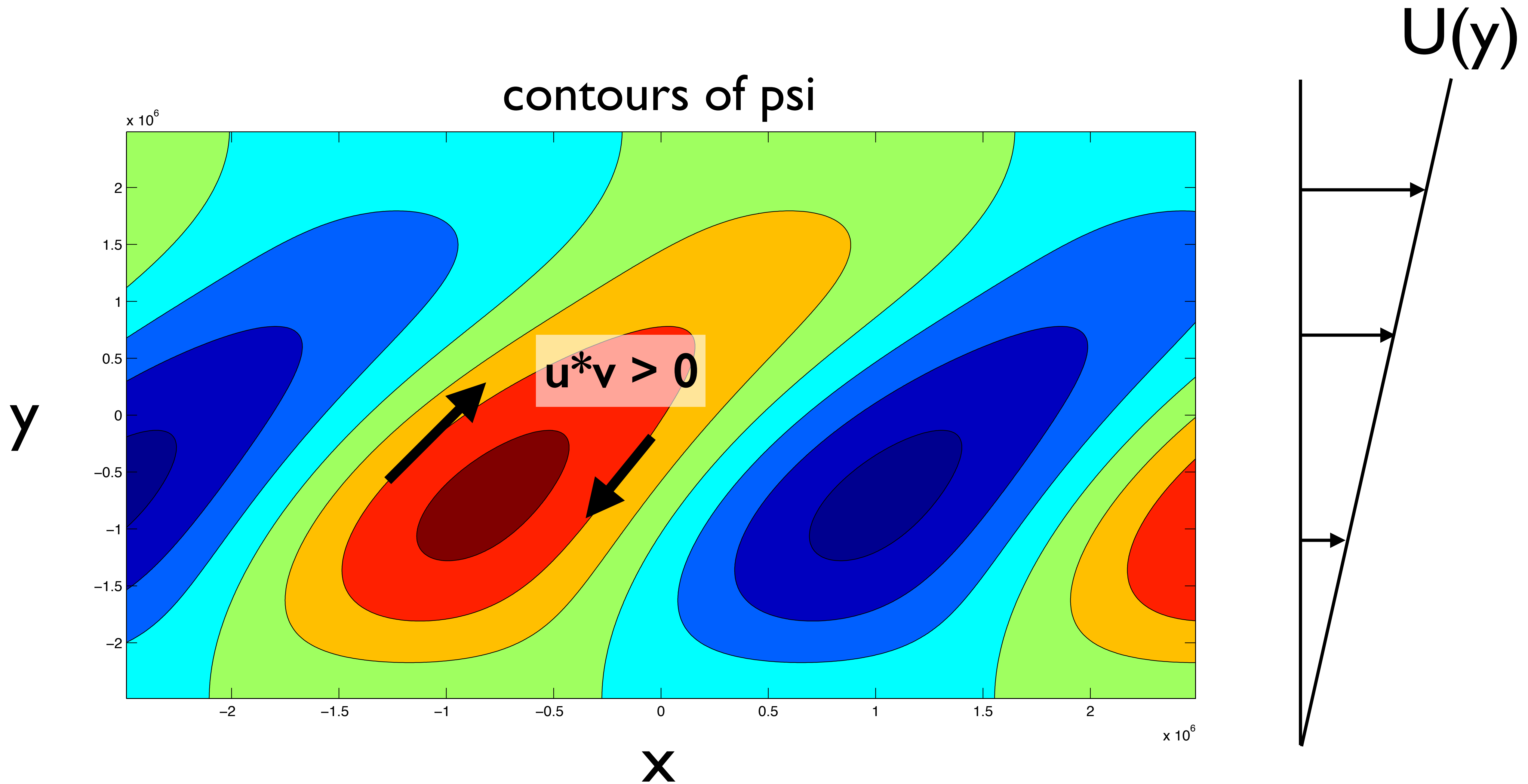
contours of psi





Energetics

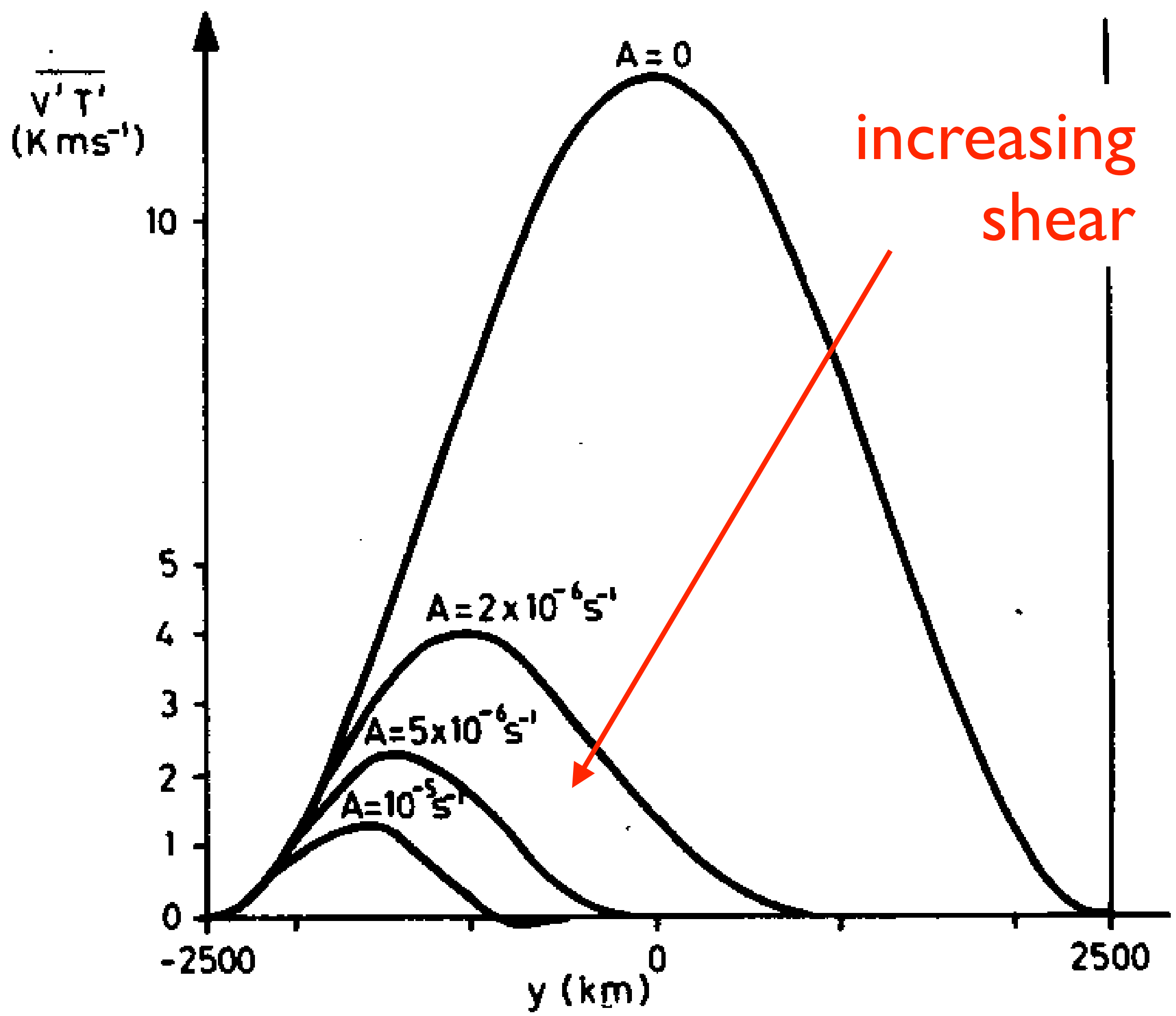
$$\frac{\partial E}{\partial t} = -g^{-1} \int \overline{u'v'}\, U_y + \left(\frac{Rk_R^2}{f_0}\right) \overline{v'T'}\, \bar{T}_y\, \boldsymbol{dx}$$



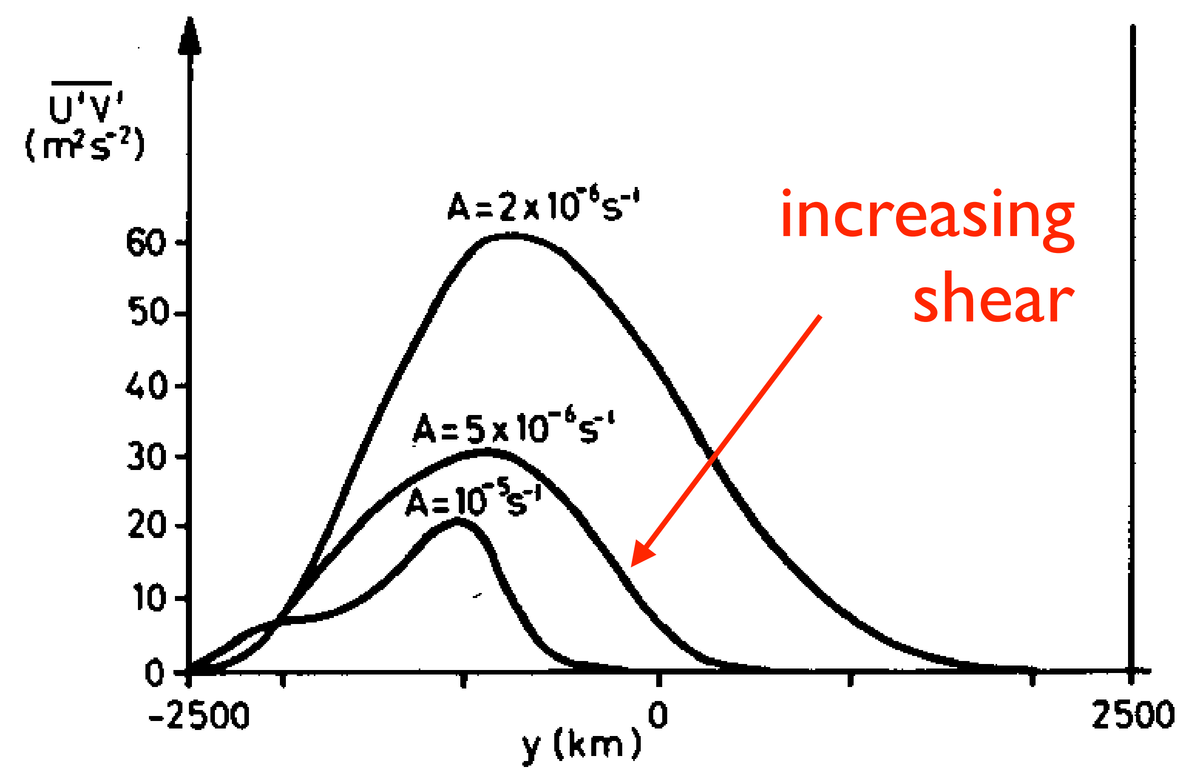
Energetics

$$\frac{\partial E}{\partial t} = -\int_{-L/2}^{L/2}\int_{0}^{2\Delta p}\left\{\overline{u'v'}\,\bar{U}_y + \left(\frac{Rk_R{}^2}{f_0}\right)\overline{v'T'}\,\bar{T}_y\right\}\frac{dpdy}{g}.$$

baroclinic conversion



barotropic conversion

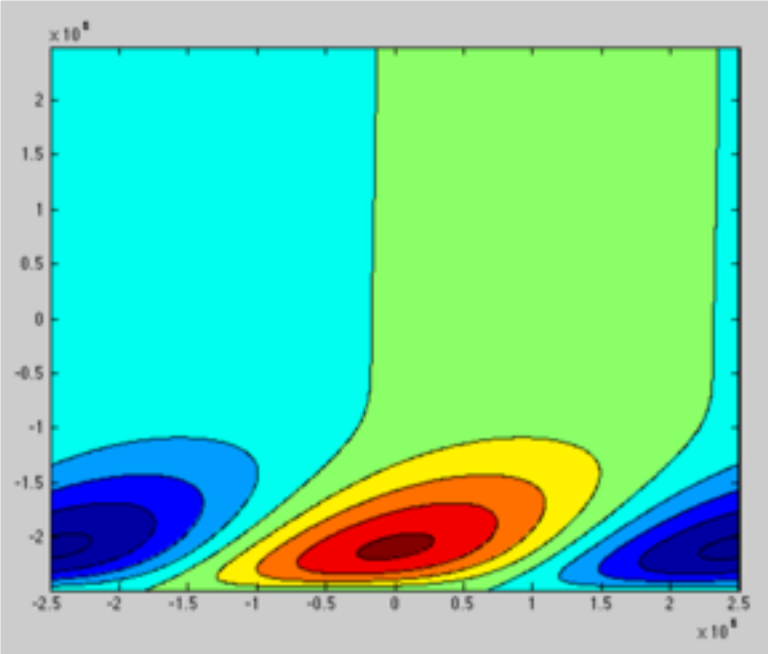


What we learned

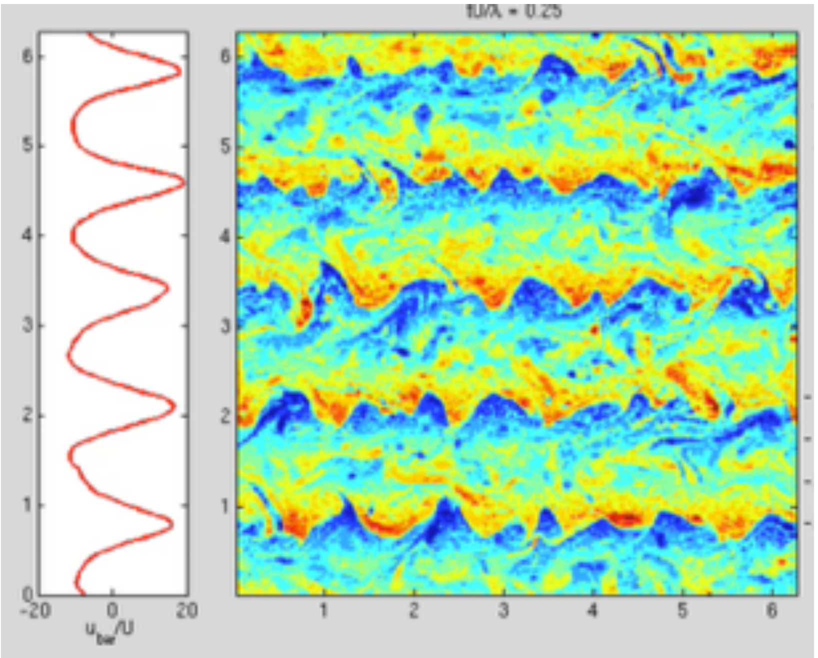
- the presence of barotropic shear *decreases* baroclinic growth rates
- … but does **NOT** change conditions for stability
- eddy extracts potential energy, but sacrifices kinetic energy

More questions

- what do the modes look like when there is no channel?
- how do we go from



to



- other work?


thanks

What if I don't like the channel?

try periodic boundary conditions

with $U(y) = (S/\ell)\cos \ell y$

What if I don't like the channel?

$$U(y) = (S/\ell)\cos \ell y$$

$$\ell = 4\pi/L$$