Atmospheric turbulence spectrum: competing theories

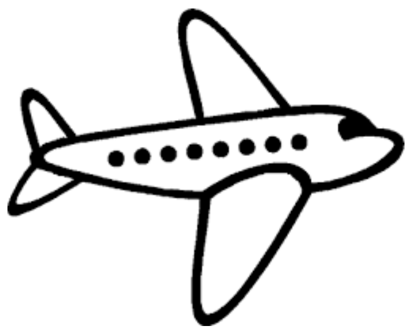
Explanations for Nastrom & Gage, JAS, **42**, 1985 by

Tung & Orlando, JAS, **60**, 2003
Tulloch & Smith, PNAS, **103**, 2006

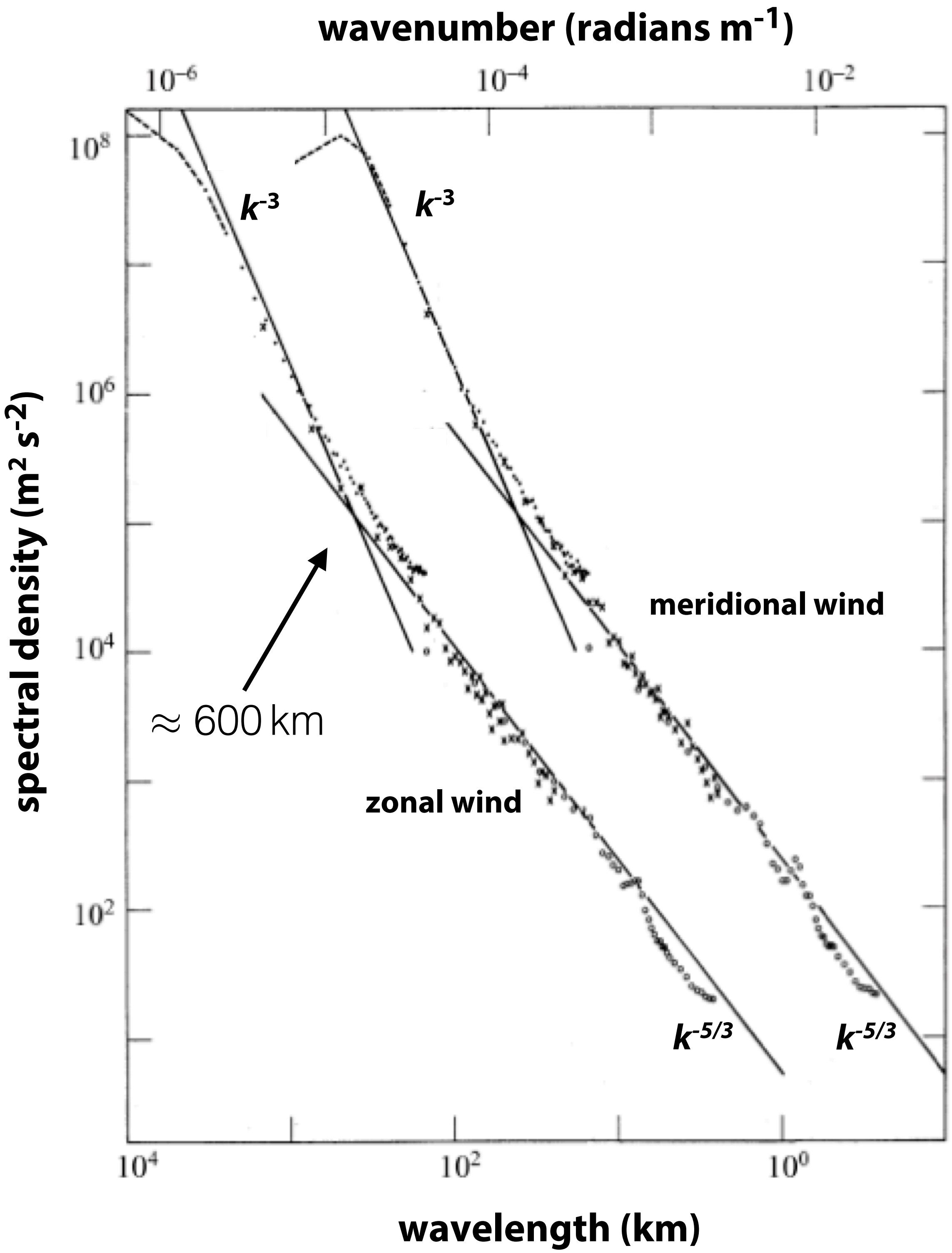
(k^p where $p = -3, -5/3, \ldots$)

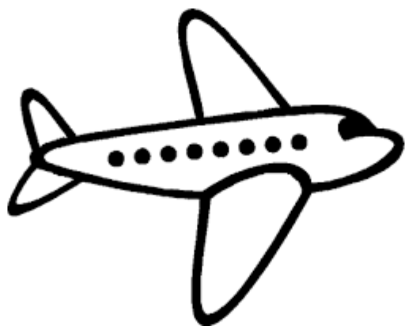
as told by Navid C

CASPO theory seminar, 6 Nov. 2015

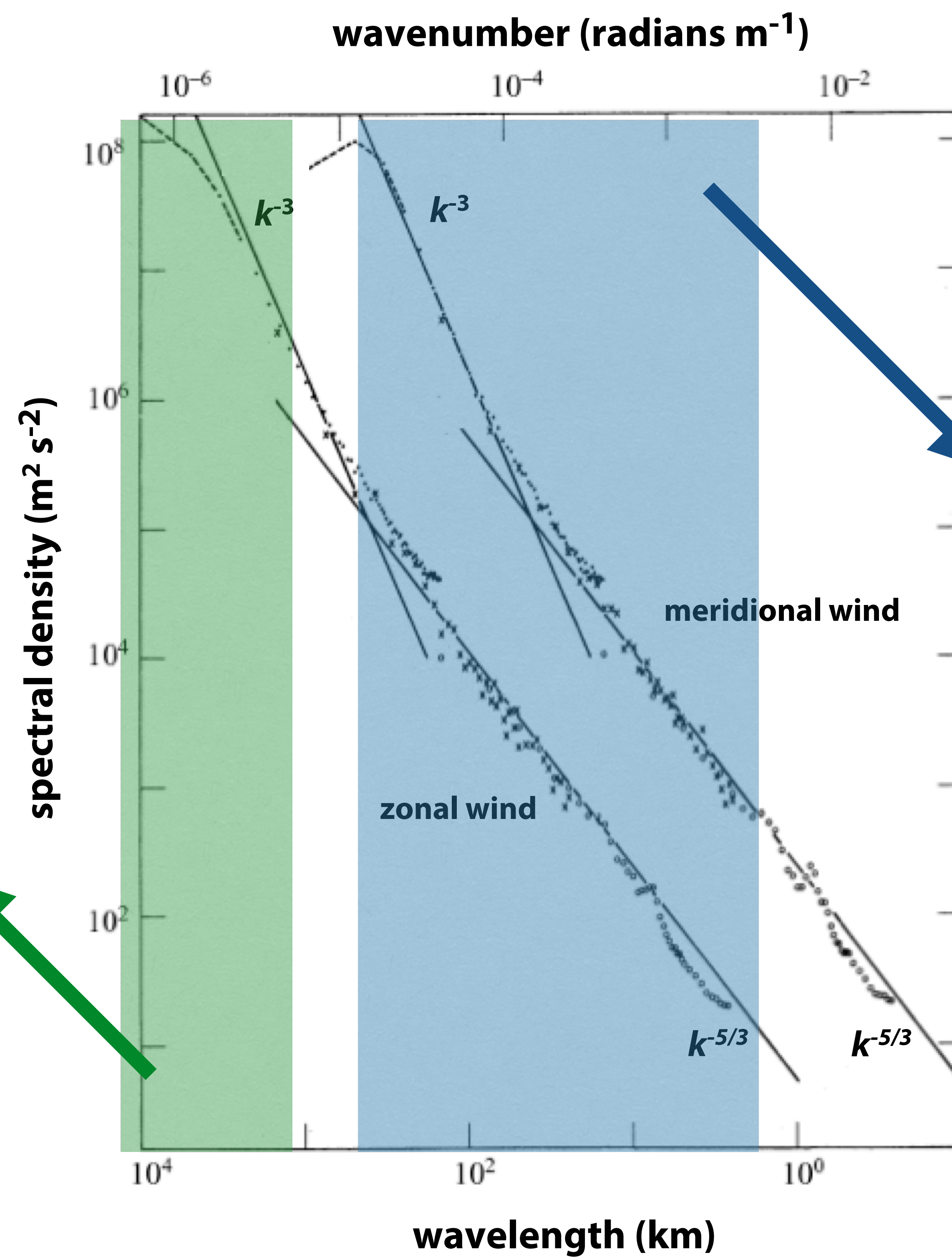


Atmospheric energy spectrum by Nastrom & Gage 1985





Atmospheric energy spectrum by Nastrom & Gage 1985



mesoscales

2 km < … < 600 km

synoptic scales

1000 km < … < 5200 km

Some possible (early) explanations

$k^{-5/3}$

- forward cascade predicted by 3D isotropic homogeneous arguments

  tropopause 10-12 km —>
  —> 3D effects only for scales <2 km

- internal gravity waves

  ruled out by Gage & Nastrom 1986

k^{-3}

- forward enstrophy inertial range (as in KBL theory)

  synoptic scales are definitely not inertial
  (baroclinic instability energetic scales)

- Charney 1971

  (several problems…)

Tung & Orlando 2003
reproduced an energy spectrum similar to
Gage & Nastrom with a 2-layer QG system

Tulloch & Smith 2006
(argued that Tung & Orlando are wrong and)
reproduced an energy spectrum similar to
Gage & Nastrom with a finite height SQG system

Tung & Orlando 2003

two-layer QG system

$$\partial_t q_1 + \mathrm{J}(\psi_1 - Uy, q_1) = 0$$
$$\partial_t q_2 + \mathrm{J}(\psi_2 + Uy, q_2) = -\nu_E(\partial_x^2 + \partial_y^2)\psi_2$$
$$q_1 = (\partial_x^2 + \partial_y^2)\psi_1 - \frac{k_R^2}{2}(\psi_1 - \psi_2) + \beta y$$
$$q_2 = (\partial_x^2 + \partial_y^2)\psi_2 + \frac{k_R^2}{2}(\psi_1 - \psi_2) + \beta y$$

$k_R, U, \beta, \nu_E, \nu_s,$ domain size   fixed parameters of the problem

Tung & Orlando 2003

hyperdiffusion
for numerical stability

two-layer
QG system

$$\partial_t q_1 + \mathrm{J}(\psi_1 - Uy, q_1) = -\nu_s(\partial_x^2 + \partial_y^2)^{10}\psi_1$$

$$\partial_t q_2 + \mathrm{J}(\psi_2 + Uy, q_2) = -\nu_E(\partial_x^2 + \partial_y^2)\psi_2 - \nu_s(\partial_x^2 + \partial_y^2)^{10}\psi_2$$

$$q_1 = (\partial_x^2 + \partial_y^2)\psi_1 - \frac{k_R^2}{2}(\psi_1 - \psi_2) + \beta y$$

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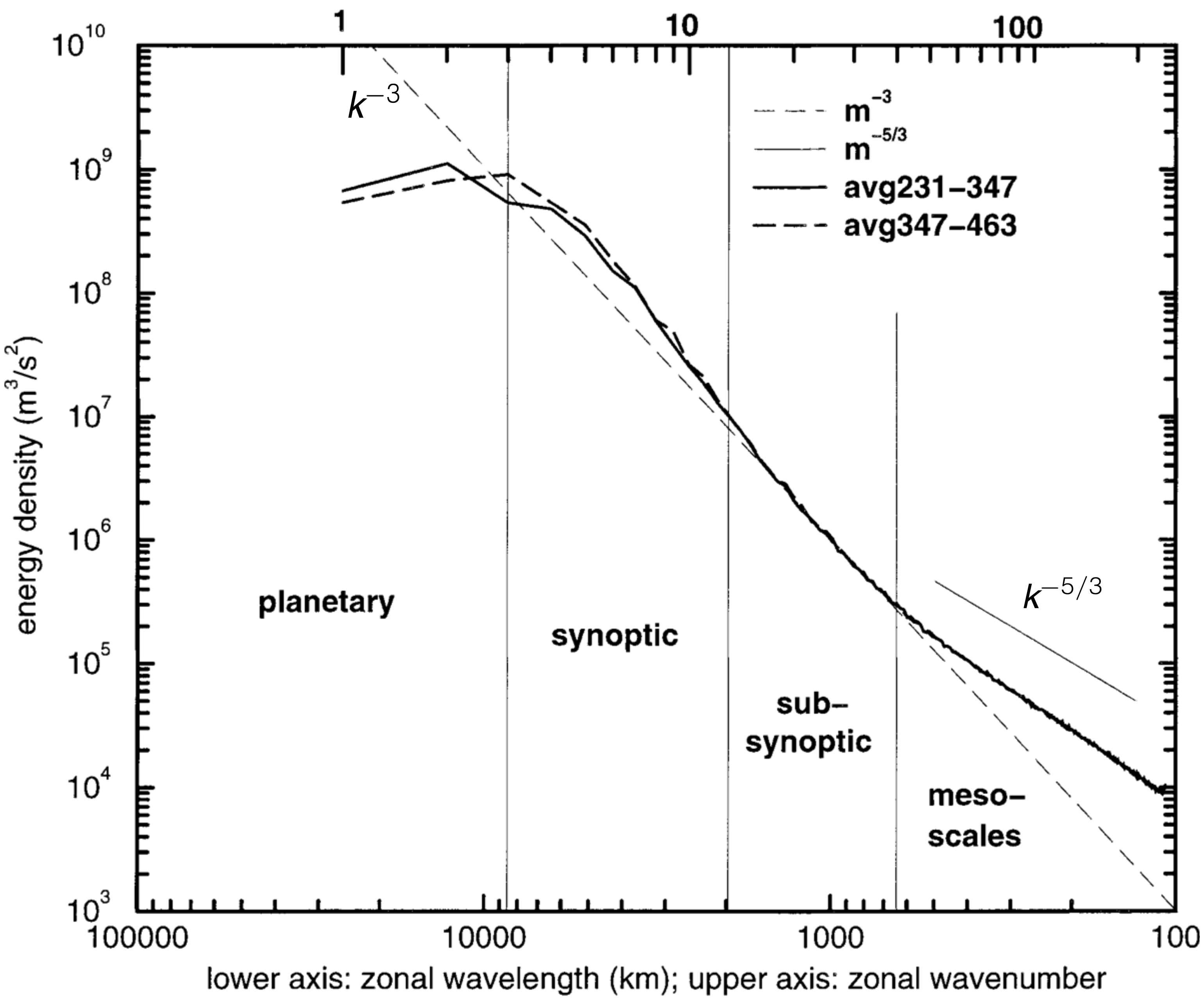
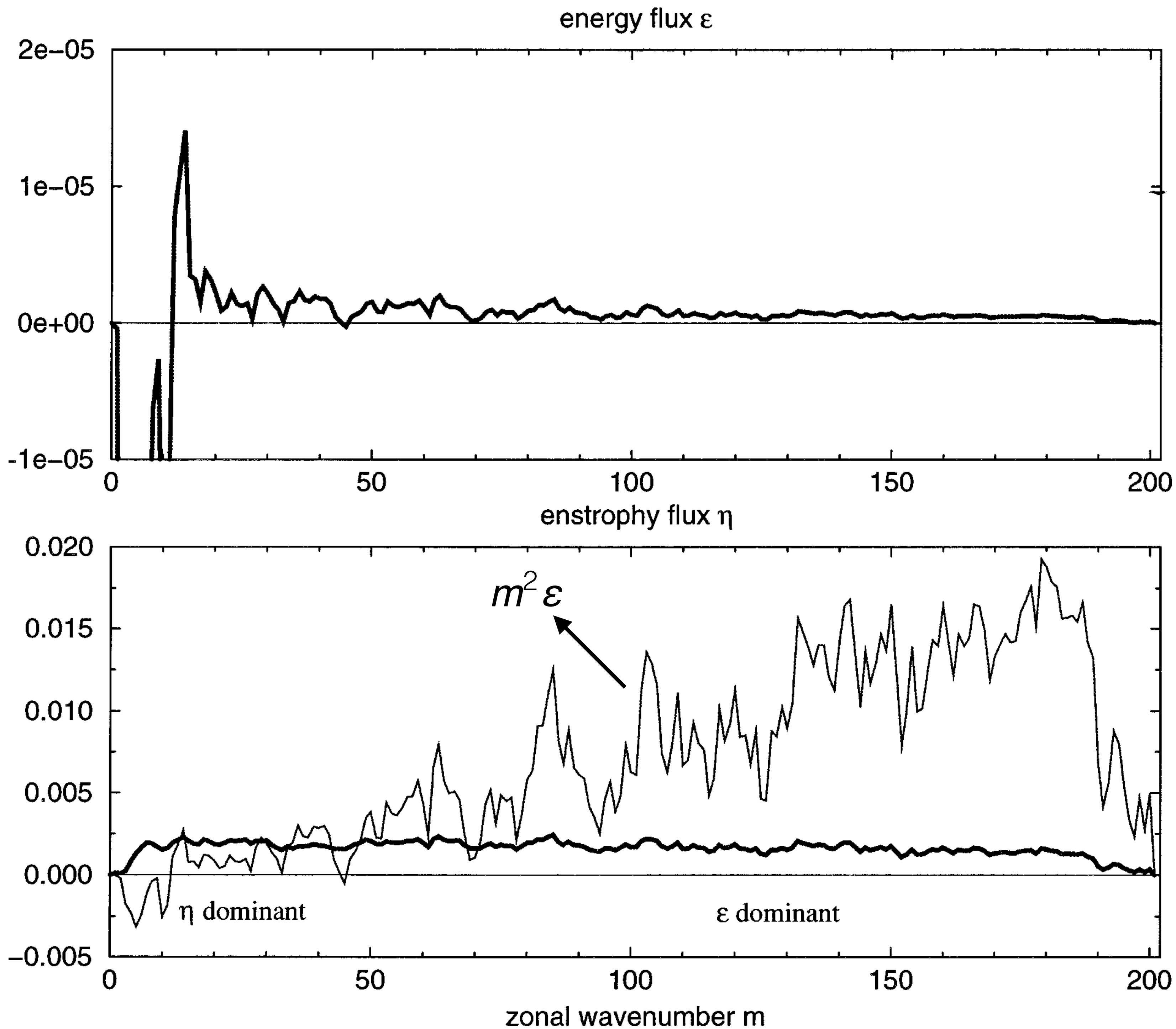
$k_R, U, \beta, \nu_E, \nu_s,$ domain size   fixed parameters of the problem

Tung & Orlando 2003

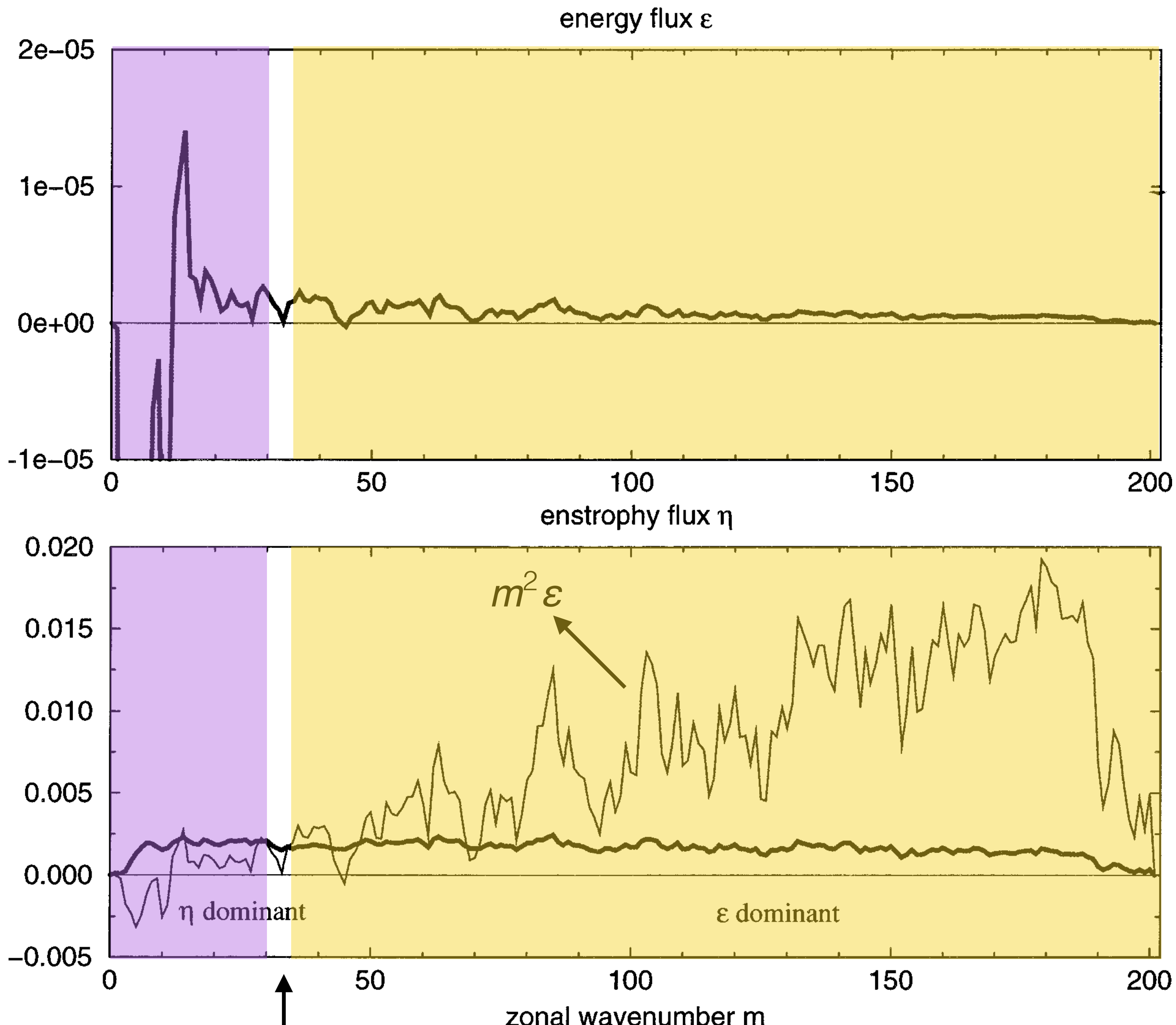
adjusted parameters to resemble
midlatitude tropopause



Tung & Orlando 2003



Tung & Orlando 2003



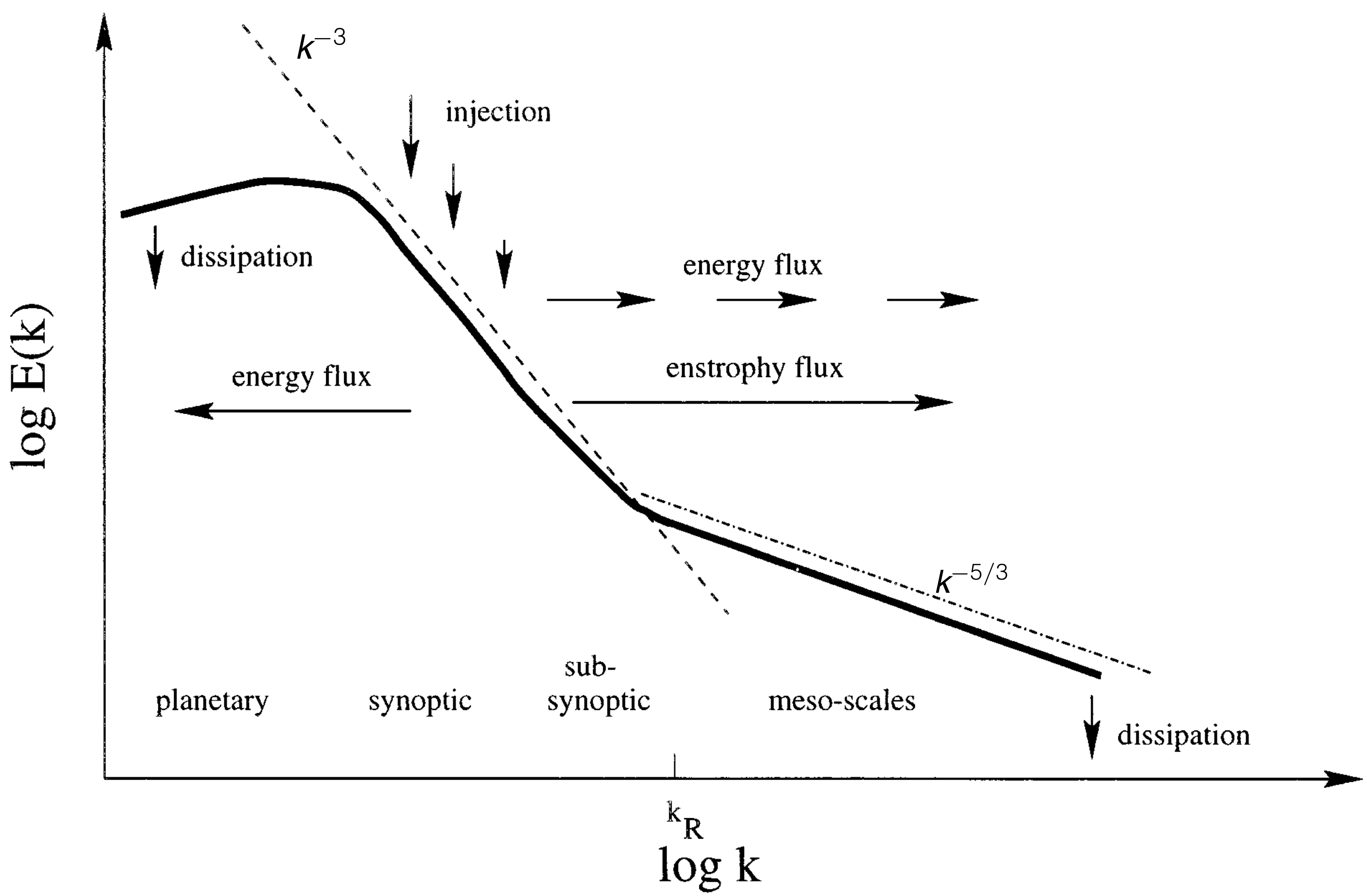
k^{-3}

$k^{-5/3}$

$k_t = \sqrt{\eta/\varepsilon} \approx 600\,\mathrm{km}$

Tung & Orlando 2003

enstrophy/energy fluxes based on their model



Tulloch & Smith 2006

SQG system

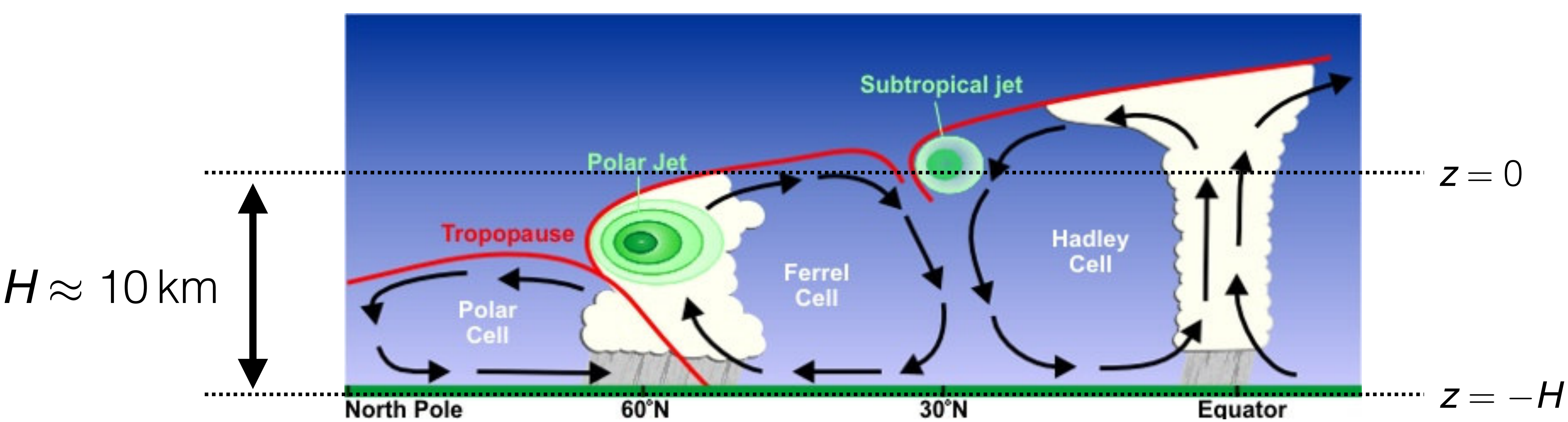
$$\partial_t\theta + \mathrm{J}(\psi,\theta) = 0\,, \quad z = 0$$

$$\theta = \partial_z\psi$$

$$q = (\partial_x^2 + \partial_y^2 + \sigma^{-2}\partial_z^2)\psi = 0\,, \quad z < 0$$

$$\partial_z\psi = 0\,,\ z = -\infty$$

$$\sigma = N/f$$



Tulloch & Smith 2006

finite depth SQG system

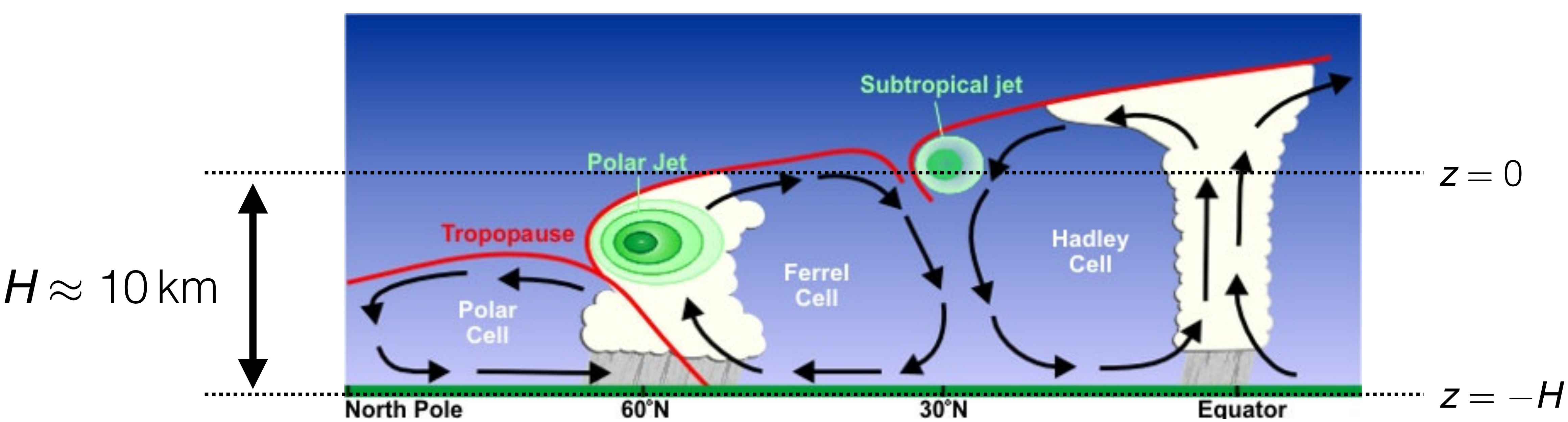
$$\partial_t \theta + \mathrm{J}(\psi, \theta) = 0\ , \quad z = 0$$

$$\theta = \partial_z \psi$$

$$q = (\partial_x^2 + \partial_y^2 + \sigma^{-2}\partial_z^2)\psi = 0\ , \quad z < 0$$

$$\partial_z \psi = 0\ , \quad z = -H$$

$$\sigma = N/f$$



Tulloch & Smith 2006

plus a 2/3 filter
for numerical stability

finite depth
SQG system

$$\partial_t \theta + \mathrm{J}(\psi, \theta) = \ F \quad z = 0$$

$$\theta = \partial_z \psi$$

$$q = (\partial_x^2 + \partial_y^2 + \sigma^{-2}\partial_z^2)\psi = 0\,, \quad z < 0$$

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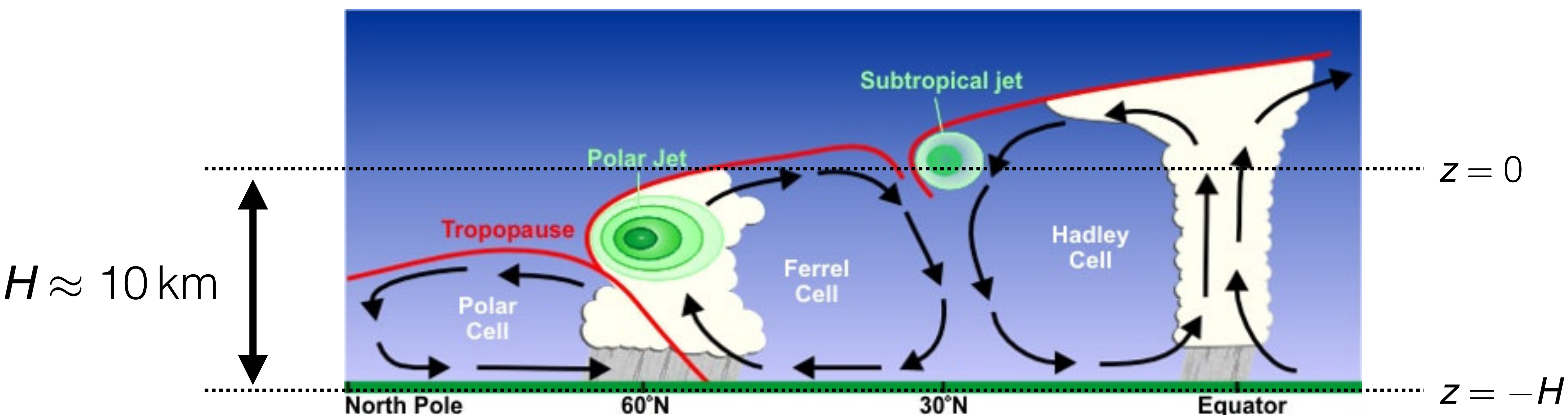


Tulloch & Smith 2006

relation between θ and ψ in fSQG

$$\theta = \partial_z \psi \Rightarrow \hat{\psi}(\mathbf{k}, z=0) = \frac{\hat{\theta}(\mathbf{k}, 0)}{\sigma k \tanh\left(\sigma H k\right)}$$

$$k \ll (\sigma H)^{-1} \Rightarrow \hat{\theta}(\mathbf{k}, 0) \approx \sigma^2 k^2 H \hat{\psi}(\mathbf{k}, 0) \quad \text{(2D-like)}$$

$$k \gg (\sigma H)^{-1} \Rightarrow \hat{\theta}(\mathbf{k}, 0) \approx \sigma k \hat{\psi}(\mathbf{k}, 0) \quad \text{(SQG-like)}$$

Tulloch & Smith 2006

what does that imply for energy spectra?

$$\frac{1}{2}\overline{\psi^2} \equiv \int \mathcal{P}(k)\,\mathrm{d}k$$

$$E = \int k^2\mathcal{P}(k)\,\mathrm{d}k \equiv \int \mathcal{E}(k)\,\mathrm{d}k$$

$$T = \frac{1}{2}\overline{\theta^2} \equiv \int \mathcal{T}(k)\,\mathrm{d}k = \int [\sigma k \tanh(k/k_t)]^2\mathcal{P}(k)\,\mathrm{d}k$$

$$\varepsilon \approx \frac{k\mathcal{T}(k)}{[k^5\mathcal{P}(k)]^{-1/2}} = \text{const.}$$

some
fiddling
around

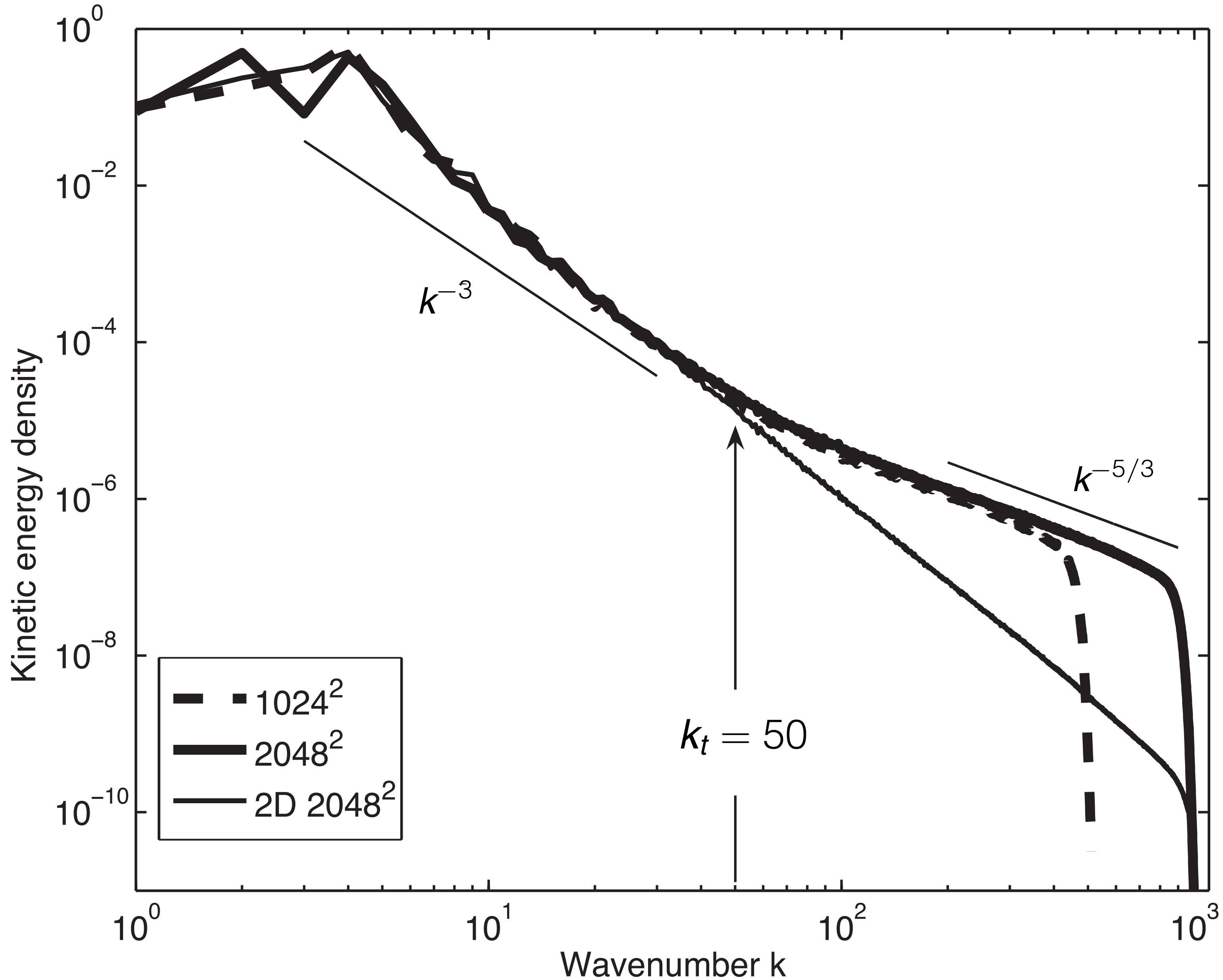
$$\mathcal{E}(k) \sim \varepsilon^{2/3}[\sigma \tanh(\sigma h k)]^{-4/3} k^{-5/3}$$

$$\sim k^{-3} \quad k \ll (\sigma H)^{-1}$$

$$\sim k^{-5/3} \quad k \gg (\sigma H)^{-1}$$

regime transition at scale $k_t \approx f/(NH)$

Tulloch & Smith 2006



and they checked their prediction of k_t by doing several simulations with varying σ

Discussion

- The Nastrom & Gage observed atmospheric spectrum is essentially captured by the very simple models presented
- TO03 predict that $k_t = \sqrt{\eta/\varepsilon}$ while TS06 that $k_t = f/(NH)$ (known params)
- Nastrom & Gage spectrum is a bit less steep than -3 at synoptic scales while TO03 and TS06 are more steep there
- There is also the work of Cho & Lindborg *J Geophys Res* 2001 that computed 3-correllators from observations (4/5-law)

$$\langle \delta u_L \delta u_L \delta u_L \rangle = -\frac{4}{5}\varepsilon(\xi - \xi')$$

the sign of this term determines whether there is forward or inverse energy cascade

- TS06 speculate that perhaps fully a continuously stratified QG model might resolve any inadequacies their simple model has — see paper by Deusebio & Lindborg *JFM* 2013

part of the whole story…

- **Tung & Orlando *JAS* 2003**
- Smith comment on TO03, *JAS* 2004
- Tung reply, *JAS* 2004
- Gkioulekas & Tung 2004 + later papers by Gkioulekas…
- **Tulloch & Smith *PNAS* 2006**
- Tulloch & Smith 2009 (baroclinic+SQG)
- Lindborg comment on TS06 & TS09, *JAS* 2009
- Smith & Tulloch reply, *JAS* 2009

thanks