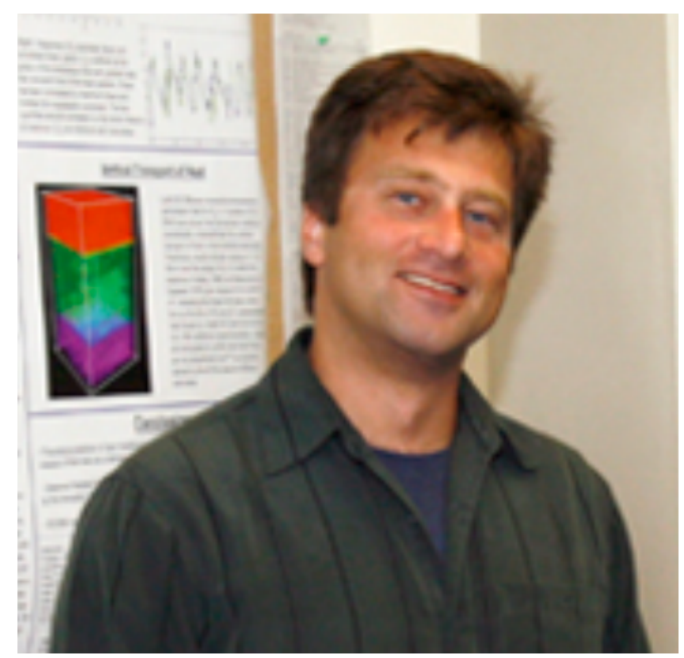
Residual-mean theory in 3D

Veronica Tamsitt
November 4th 2016

Radko and Marshall 2006

The Antarctic Circumpolar Current in Three Dimensions

John Marshall and Timour Radko



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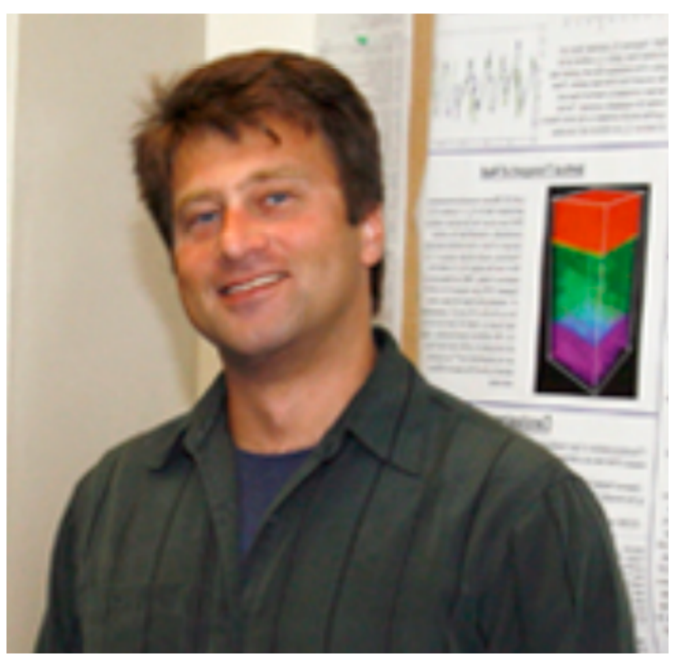
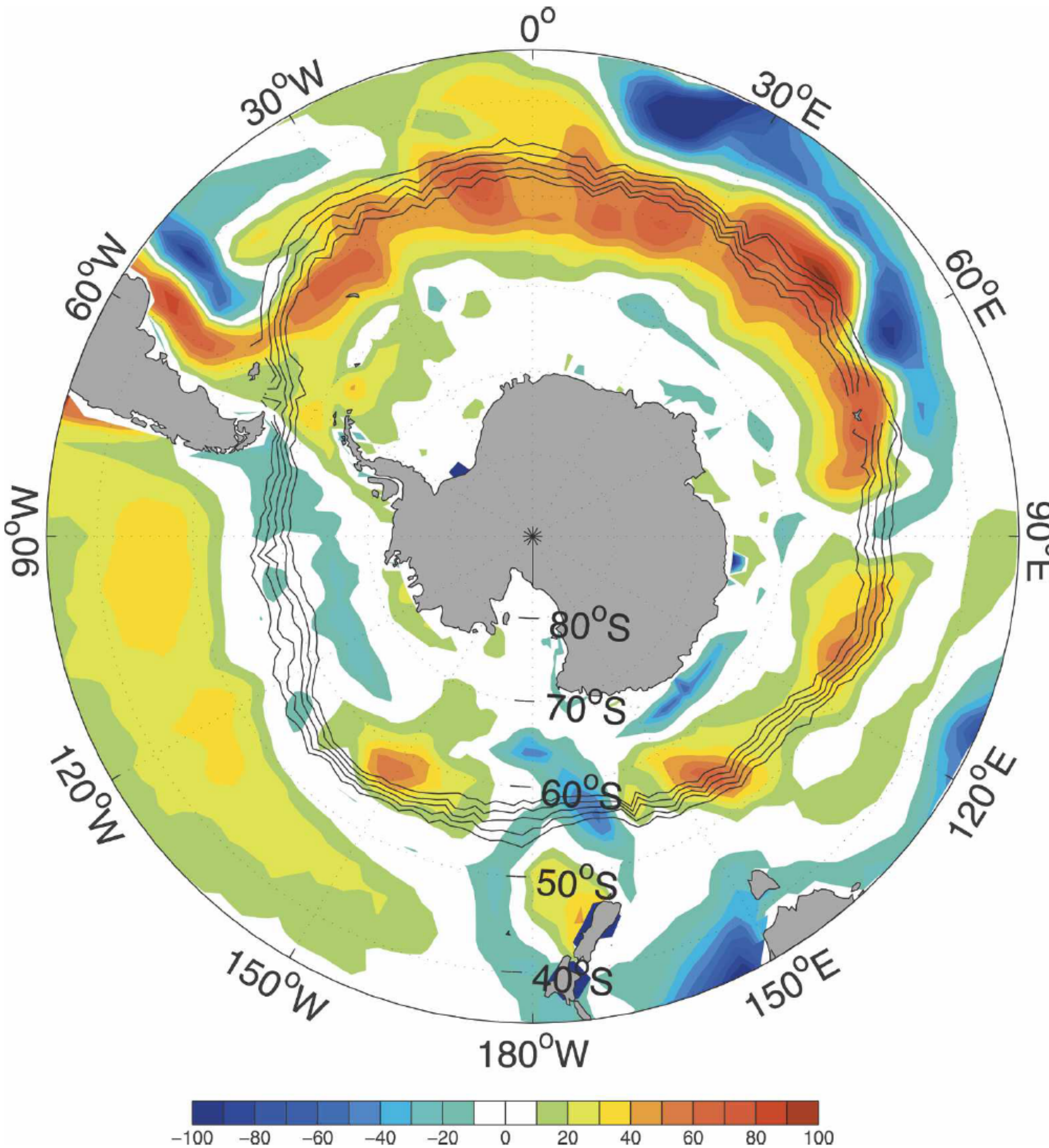


Motivation

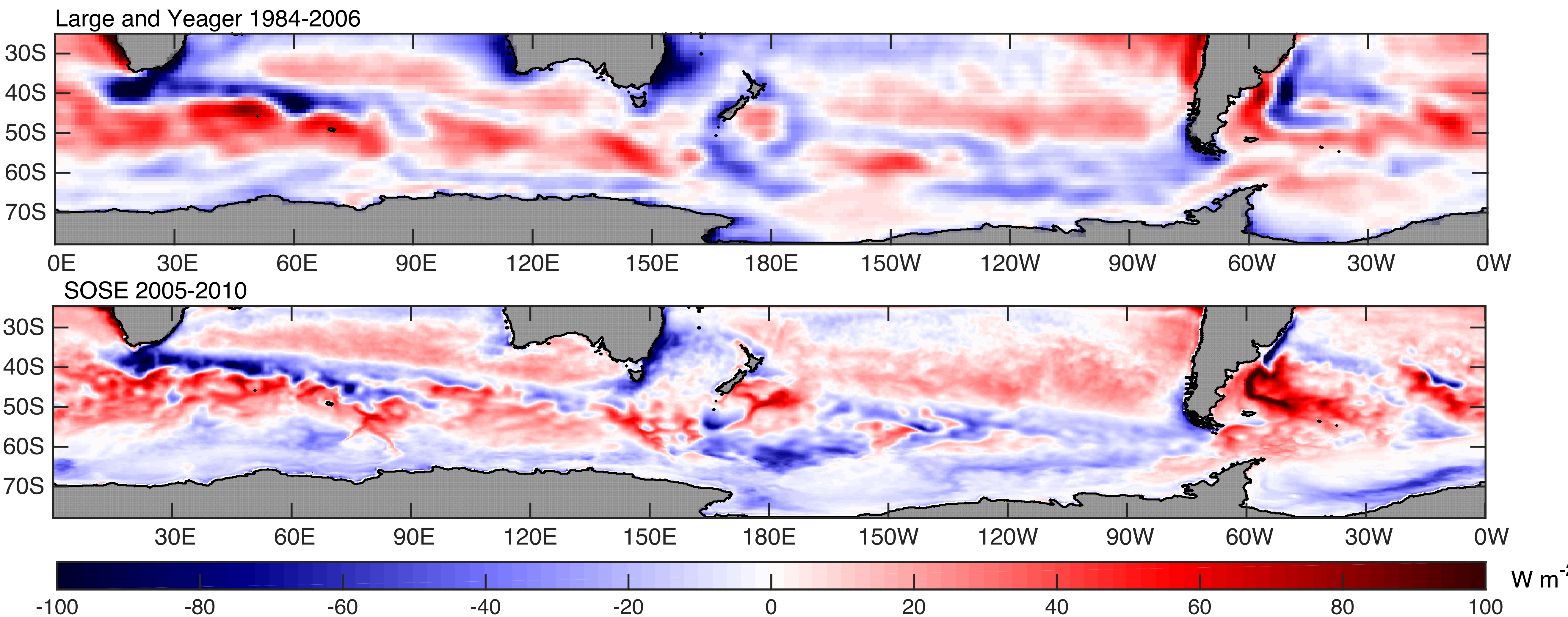
ACC buoyancy fluxes and wind stress are not zonally uniform, and have a large scale pattern

NCEP-NCAR reanalysis



RM2006 Fig. 1

Motivation



Observations and data assimilation show the ACC along-stream air-sea flux variations are large, and robust

Motivation

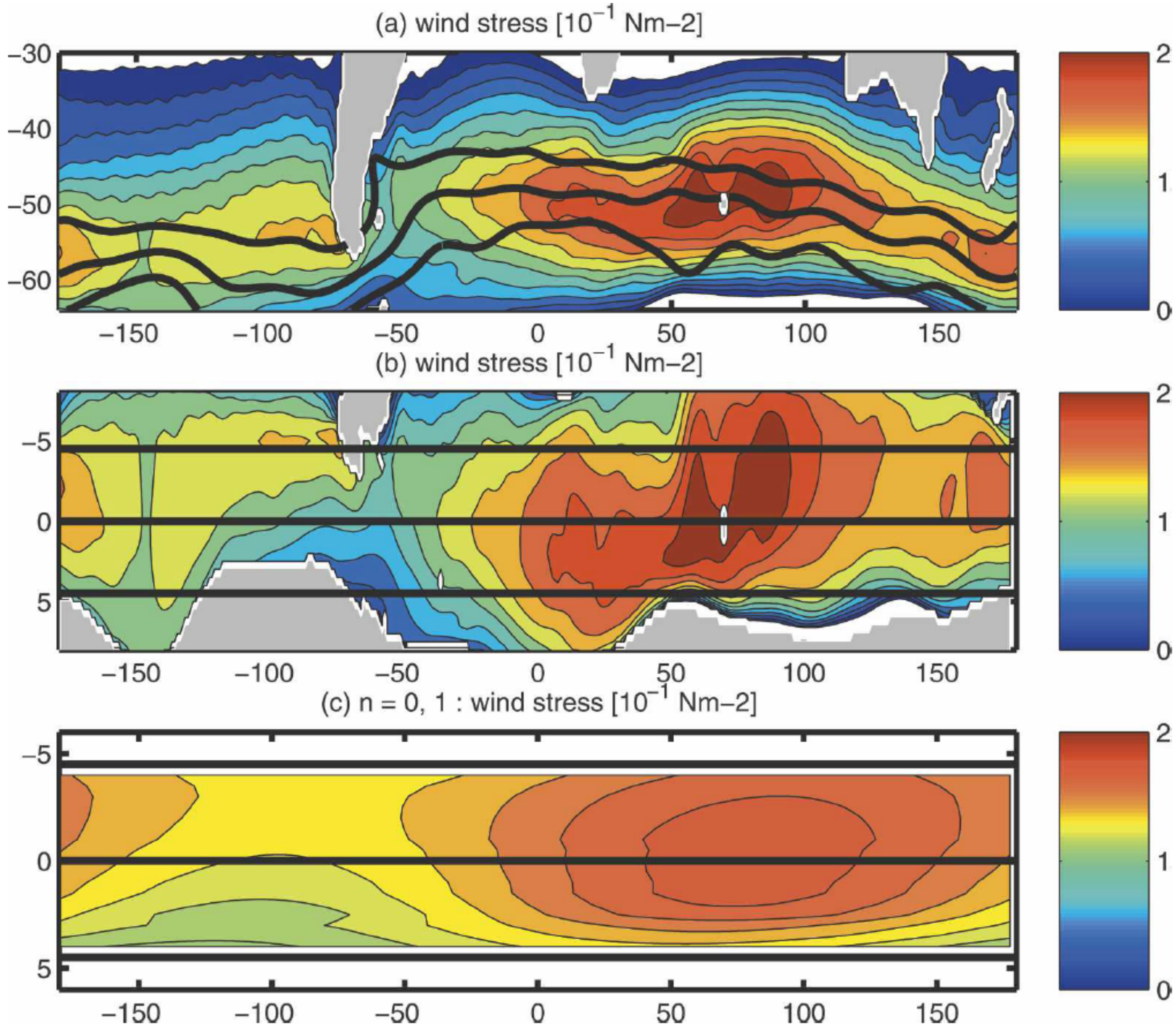
Q: What are the implications of large scale along-stream variations in buoyancy and wind forcing on the residual overturning of the ACC?

Goal

Take 2d residual overturning from MR2003

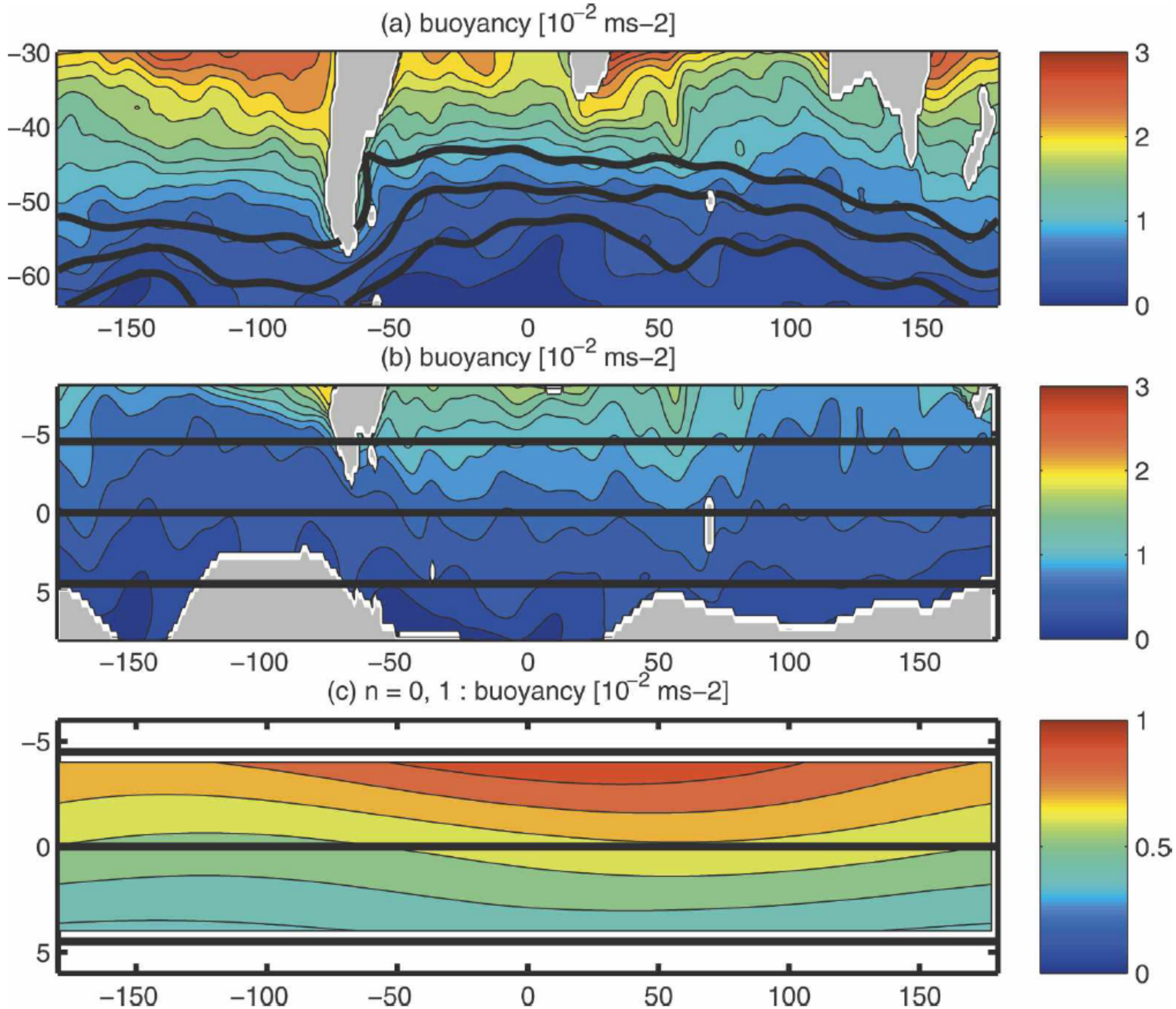
Perturbation expansion about zonal average to look at first order zonal variations in residual circulation

Observations of forcing



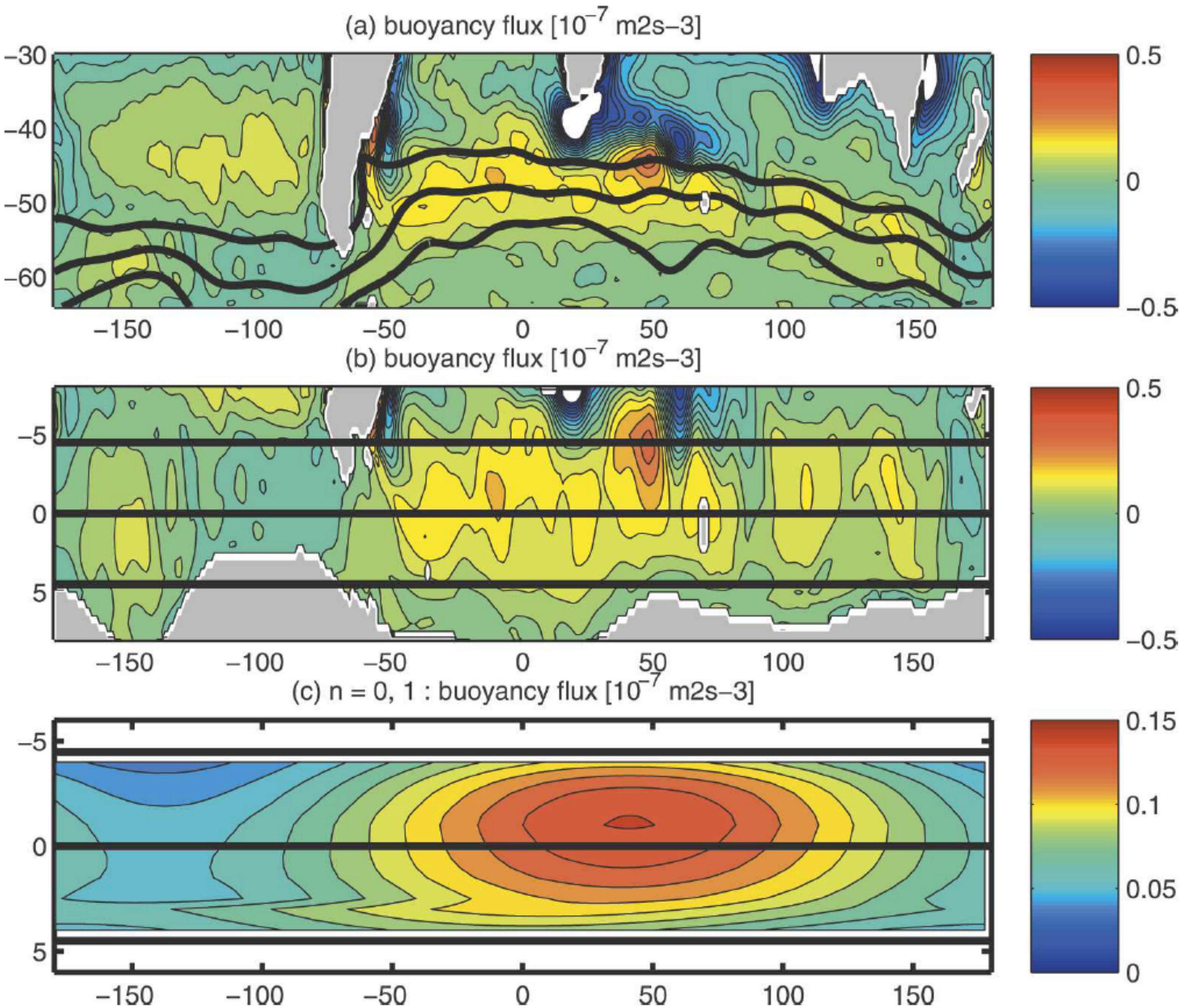
RM2006 Fig. 2

Observations of forcing



RM2006 Fig. 2

Observations of forcing



RM2006 Fig. 3

Formulation

Starts out similar to MR 2003:

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time-mean momentum

$$-fv = -\frac{\partial P}{\partial x} + \frac{\partial \tau_x}{\partial z} \quad \text{and}$$
$$fu = -\frac{\partial P}{\partial y} + \frac{\partial \tau_y}{\partial z},$$

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time-mean buoyancy

$$\mathbf{v} \cdot \boldsymbol{\nabla} b = -\boldsymbol{\nabla} \cdot (\overline{\mathbf{v}'b'}) + \frac{\partial B}{\partial z},$$

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residual velocity

$$\mathbf{v}_{\text{res}} = \mathbf{v} + \mathbf{v}^*,$$

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streamfunction

$$(u^*, v^*) = -\frac{\partial}{\partial z}\boldsymbol{\Psi}^* \quad \text{and}$$

$$w^* = \nabla_h \cdot \boldsymbol{\Psi}^*,$$

$$\boldsymbol{\Psi}^* = (\boldsymbol{\Psi}_u^*, \boldsymbol{\Psi}_v^*)$$

Formulation

Buoyancy

$$\mathbf{v}_{\text{res}} \cdot \nabla b = \frac{\partial \tilde{B}}{\partial z},$$

- Interior flow assumed fully adiabatic
- Assume diabatic eddy fluxes in the mixed layer are secondary to direct fluxes

Formulation

Assume that departure of the solution from its along-stream average is asymptotically small

$$\varepsilon = \frac{\langle b_1 \rangle}{\langle b_0 \rangle} \ll 1, \qquad \frac{\text{along stream}}{\text{cross stream}} \sim 0.1$$

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$$\frac{\text{along stream}}{\text{cross stream}} \sim 0.1$$

Asymptotic expansion, separate variables into 2d zonal average and weak fundamental harmonic

$$\mathbf{v} = \mathbf{v}_0(y, z) + \mathbf{v}_1(x, y, z) + \cdots, \quad \mathbf{v}_1 = \mathrm{Re}[\hat{\mathbf{v}}_1(y, z)\exp(ikx)],$$

$$\mathbf{v}^* = \mathbf{v}_0^*(y, z) + \mathbf{v}_1^*(x, y, z) + \cdots, \quad \mathbf{v}_1^* = \mathrm{Re}[\hat{\mathbf{v}}_1^*(y, z)\exp(ikx)], \quad \text{and}$$

$$b = b_0(y, z) + b_1(x, y, z) + \cdots, \quad b_1 = \mathrm{Re}[\hat{b}_1(y, z)\exp(ikx)], \tag{8}$$

Formulation

Same thing for forcing (assuming $\sim \varepsilon$ 1st harmonic??)

$$B = B_0(y, z) + B_1(x, y, z) + \cdot \cdot \cdot, \quad B_1 = \mathrm{Re}[\hat{B}_1(y, z) \exp(ikx)], \quad \text{and}$$

$$\tau_x = \tau_{0x}(y) + \tau_{1x}(x, y) + \cdot \cdot \cdot, \quad \tau_1 = \mathrm{Re}[\hat{\tau}_{1x}(y) \exp(ikx)],$$

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Now rewrite buoyancy equation with these definitions

$$\mathbf{v}_{\mathrm{res}} \cdot \boldsymbol{\nabla} b = \frac{\partial \tilde{B}}{\partial z},$$

$$\downarrow$$

$$(\mathbf{v}_0 + \mathbf{v}_1 + \cdot\ \cdot\ \cdot) \cdot \boldsymbol{\nabla}(b_0 + b_1 + \cdot\ \cdot\ \cdot) + (\mathbf{v}_0^* + \mathbf{v}_1^* + \cdot\ \cdot\ \cdot)$$

$$\cdot \boldsymbol{\nabla}(b_0 + b_1 + \cdot\ \cdot\ \cdot) = \frac{\partial B_0}{\partial z} + \frac{\partial B_1}{\partial z} + \cdot\ \cdot\ \cdot.$$

Formulation

Zero-order balance: $\mathbf{v}_{0\,\mathrm{res}} \cdot \nabla b_0 = \frac{\partial B_0}{\partial z}\,,$

Same as 2D MR 2003

Formulation

Zero-order balance: $\mathbf{v}_{0\,\mathrm{res}} \cdot \nabla b_0 = \dfrac{\partial B_0}{\partial z}\,,$ **Same as 2D MR 2003**

First-order balance: $\boldsymbol{\varepsilon}$ $\mathbf{v}_{1\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_0 + \mathbf{v}_{0\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_1 = \dfrac{\partial B_1}{\partial z}\,.$

Formulation

Zero-order balance: $$\mathbf{v}_{0\,\mathrm{res}} \cdot \nabla b_0 = \frac{\partial B_0}{\partial z},$$

Same as 2D MR 2003

First-order balance: $\boldsymbol{\varepsilon}$ $$\mathbf{v}_{1\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_0 + \mathbf{v}_{0\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_1 = \frac{\partial B_1}{\partial z}.$$

Do the same for momentum, continuity and eddy closure.

Formulation

First-order correction (3D)

$$\mathbf{v}_{1\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_0 + \mathbf{v}_{0\,\mathrm{res}} \cdot \boldsymbol{\nabla} b_1 = \frac{\partial B_1}{\partial z}\,.$$

$$\Downarrow$$

$$\hat{\boldsymbol{v}}_{1\,\mathrm{res}} \frac{\partial b_0}{\partial y} + \hat{w}_{1\,\mathrm{res}} \frac{\partial b_0}{\partial z} + u_0 ik\hat{b}_1 + \boldsymbol{v}_{0\,\mathrm{res}} \frac{\partial \hat{b}_1}{\partial y} + w_{0\,\mathrm{res}} \frac{\partial \hat{b}_1}{\partial z} = \frac{\partial \hat{B}_1}{\partial z}\,. \tag{17}$$

Formulation

Transform equations to variable l = distance along zero-order isopycnals

$$\frac{dy}{dl} = v_c = 1 \quad \text{and}$$

$$\frac{dz}{dl} = w_c = s_0 = -\frac{b_{0y}}{b_{0z}} = -\sqrt{-\frac{\tau_0}{fk_0} - \frac{\Psi_{0\,\mathrm{res}}}{k_0}}\,.$$

$$\frac{d\hat{b}_1}{dl} - \frac{(\psi_{1\,\mathrm{res}} + \hat{\tau}_{1x}/f)b_{0z}}{2k_0 s_0} = ik\frac{b_{0z}\int_0^z \hat{P}_1\,dz}{2k_0 s_0 f} \quad \text{and}$$

$$\frac{d\psi_{1\,\mathrm{res}}}{dl} - \left(\frac{d\Psi_{0\,\mathrm{res}}}{db_0}\right)\frac{d\hat{b}_1}{dl} + ik\frac{u_0\hat{b}_1}{b_{0z}} = ik\int_0^z \hat{u}_1\,dz,$$

$$\psi_{1\,\mathrm{res}}(-h_m) = \frac{\hat{B}_1 - \Psi_{0\,\mathrm{res}}(-h_m)\dfrac{\partial}{\partial y}\hat{b}_{1m} - iku_0\hat{b}_{1m}h_m}{\dfrac{\partial}{\partial y}b_{0m}}\,.$$

Base of ML

2 model solutions

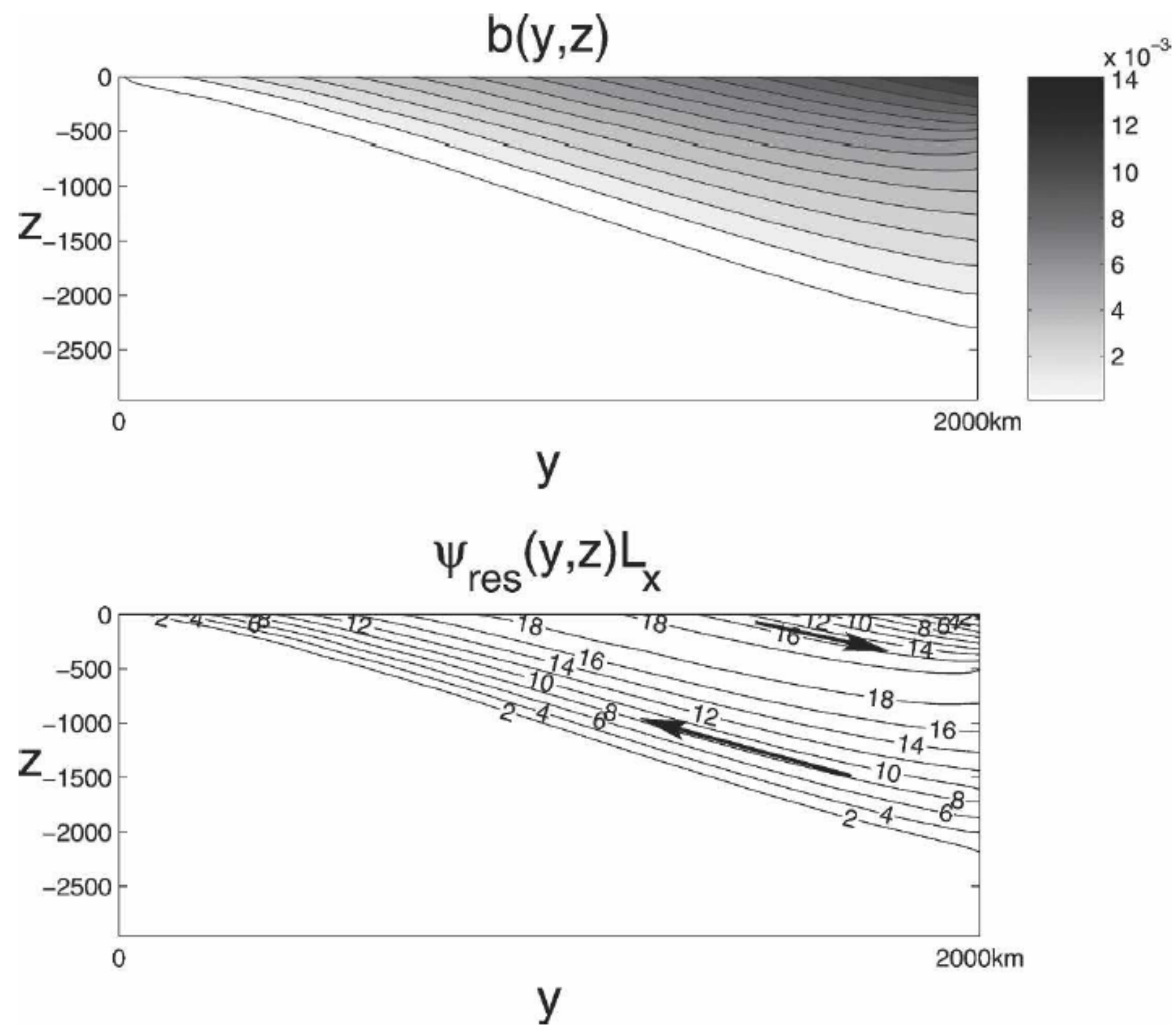
4. Diagnostic model (buoyancy and air-sea fluxes prescribed)

5. Prognostic model (solve for buoyancy and air-sea fluxes, with northern boundary condition and relaxation to buoyancy distribution)

Solution

Zero-order (2D)

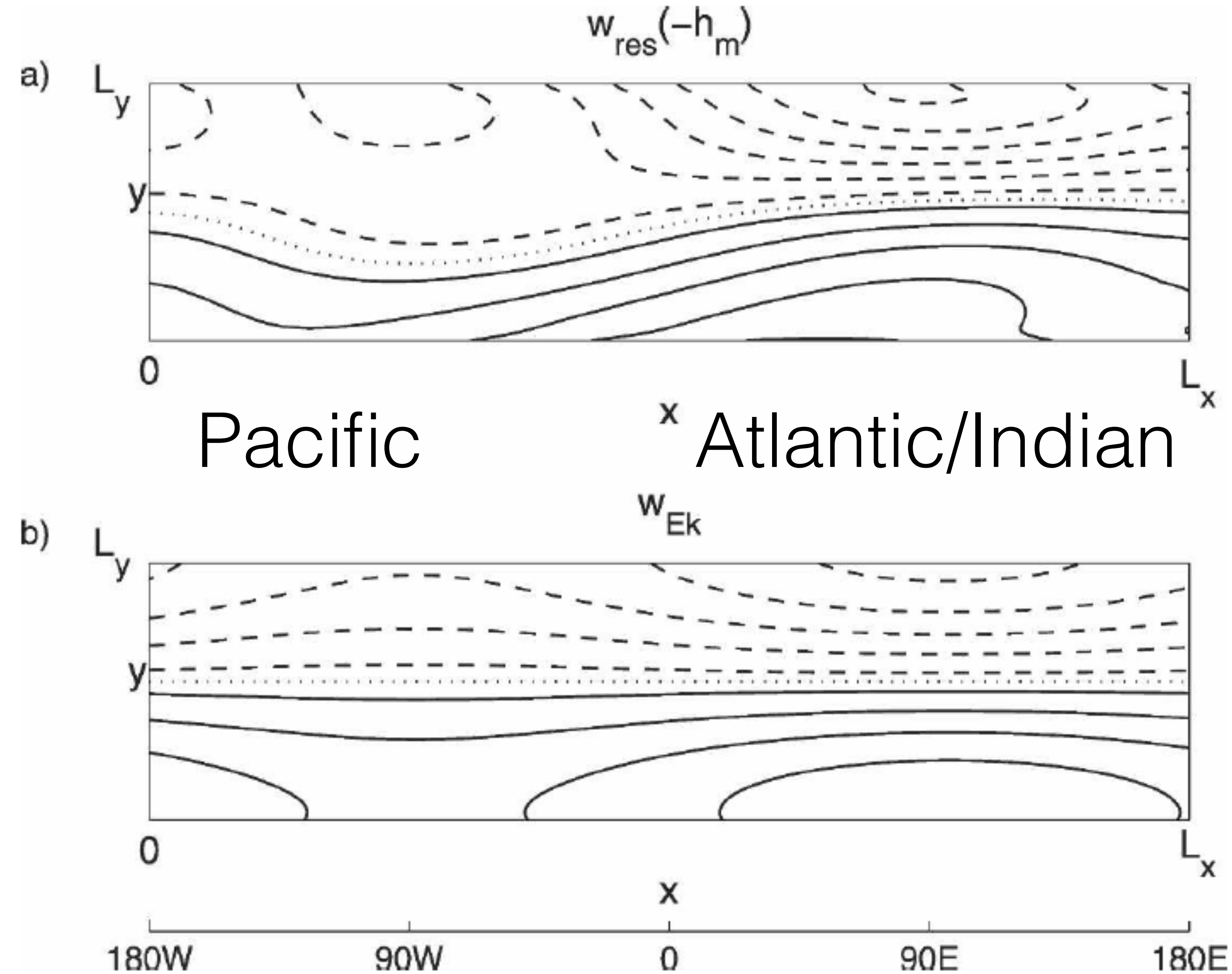
= same as MR2003



RM2006 Fig. 5

Diagnostic solution

First-order (3D)



Overturning is enhanced in the Atlantic-Indian sector and reduced in the Pacific sector

RM2006 Fig. 6

Prognostic solution

Relax mixed layer buoyancy to target distribution

Prescribe stratification at northern boundary (**No x variation!**)

Buoyancy flux parameterization: $B = -\lambda(b_m - b^*).$

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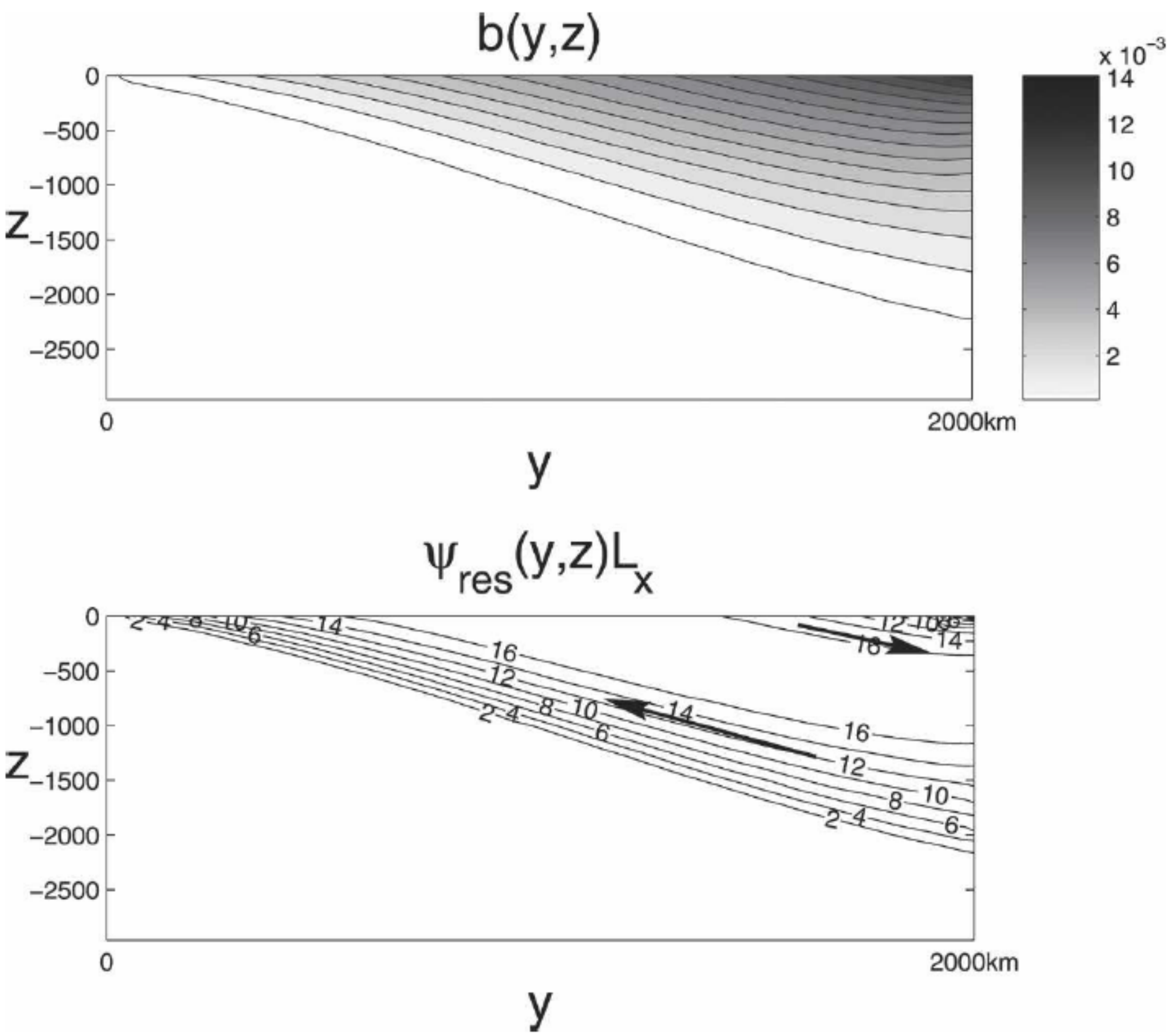
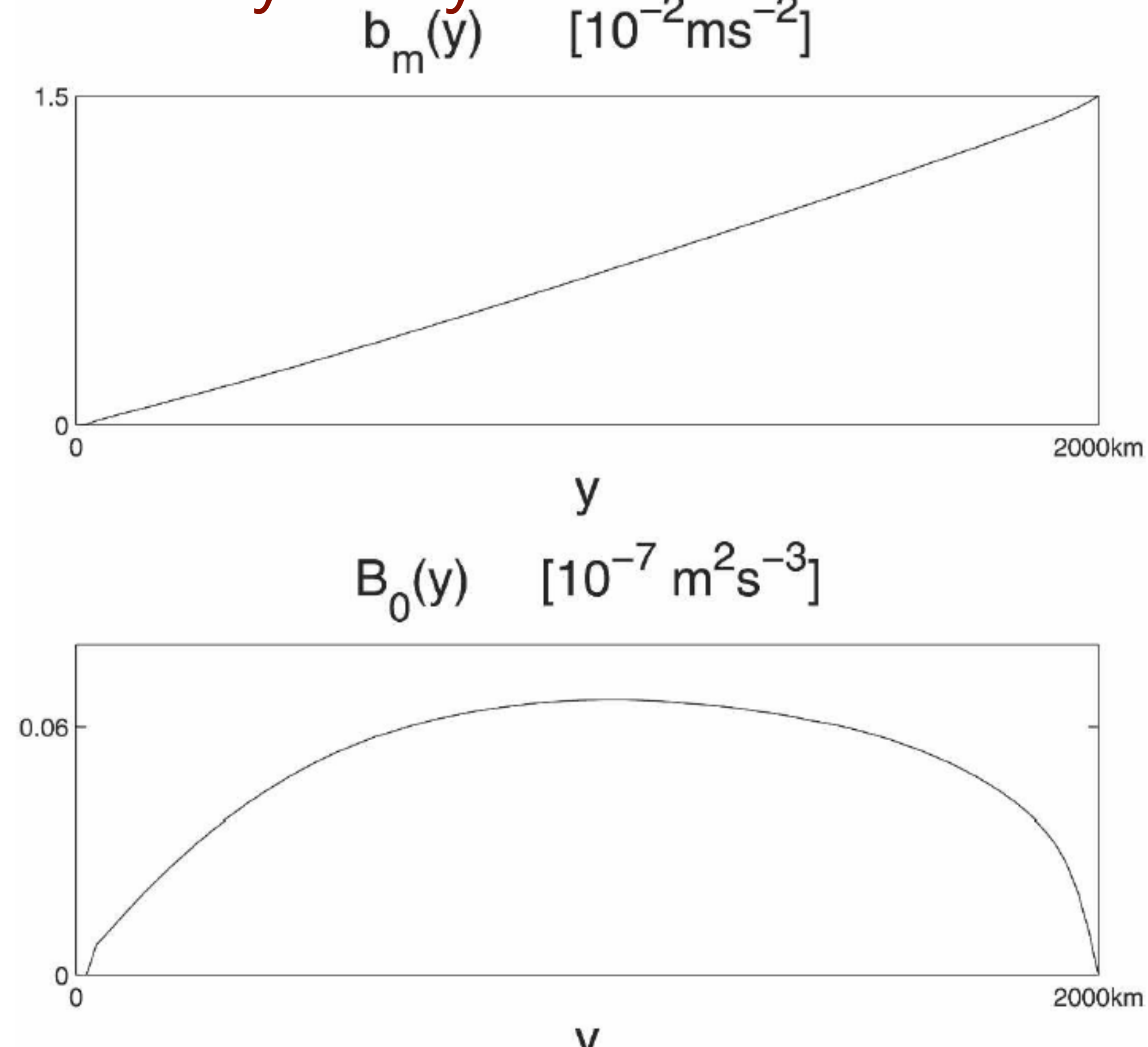


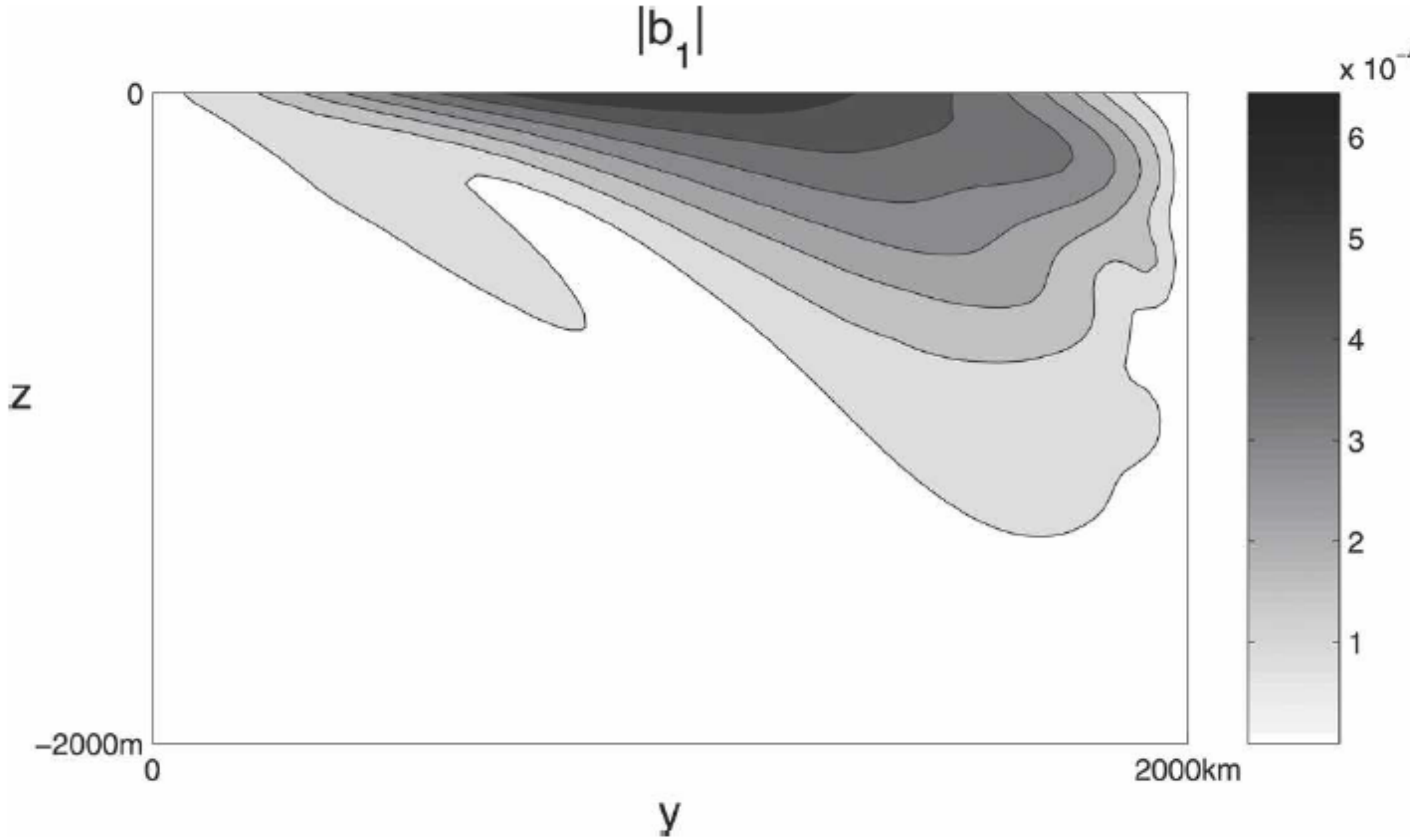
RM2006 Fig. 7

buoyancy



RM2006 Fig. 8

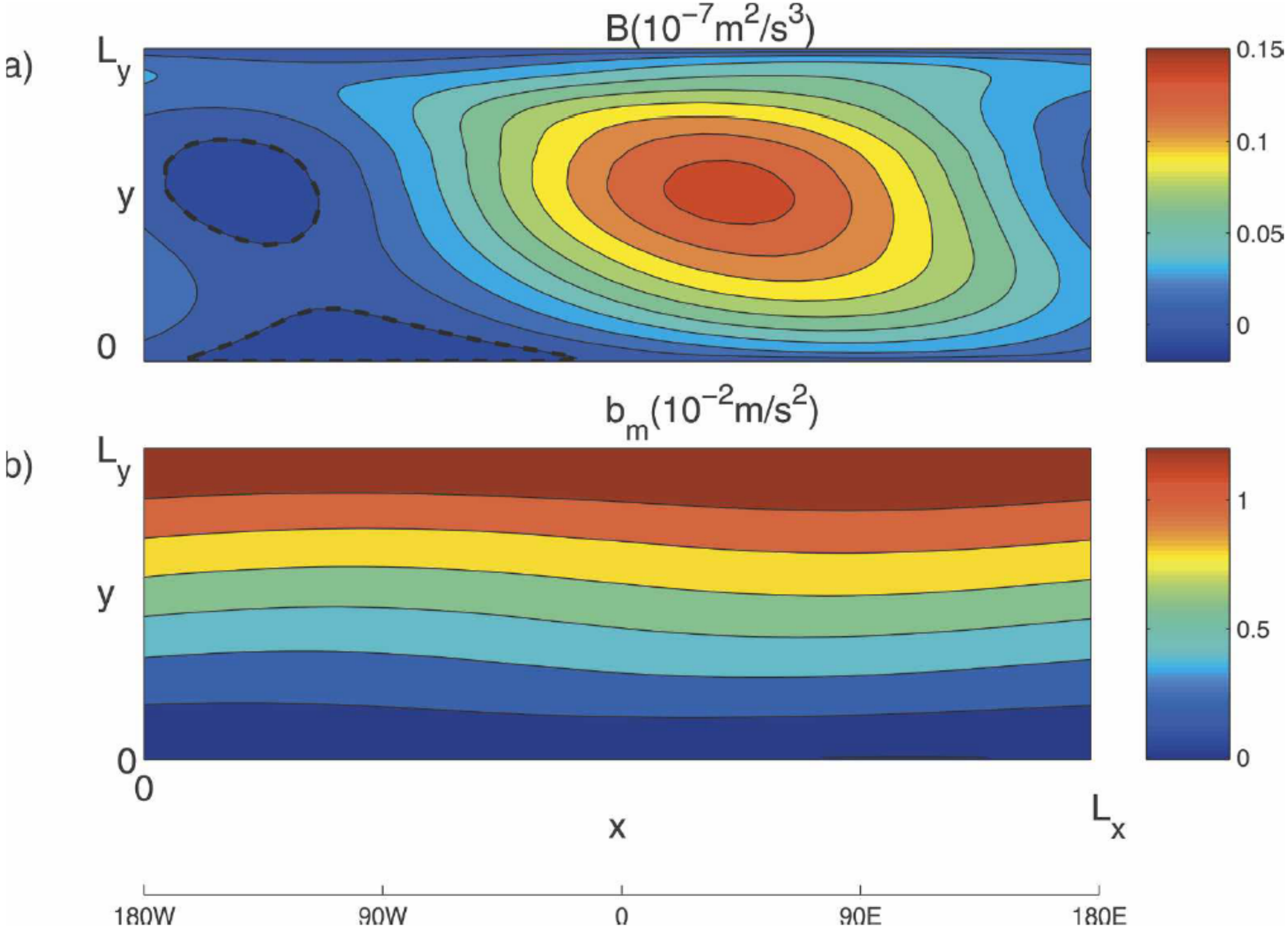
Prognostic solution



amplitude of fundamental harmonic of buoyancy, weak relative to mean

RM2006 Fig. 10

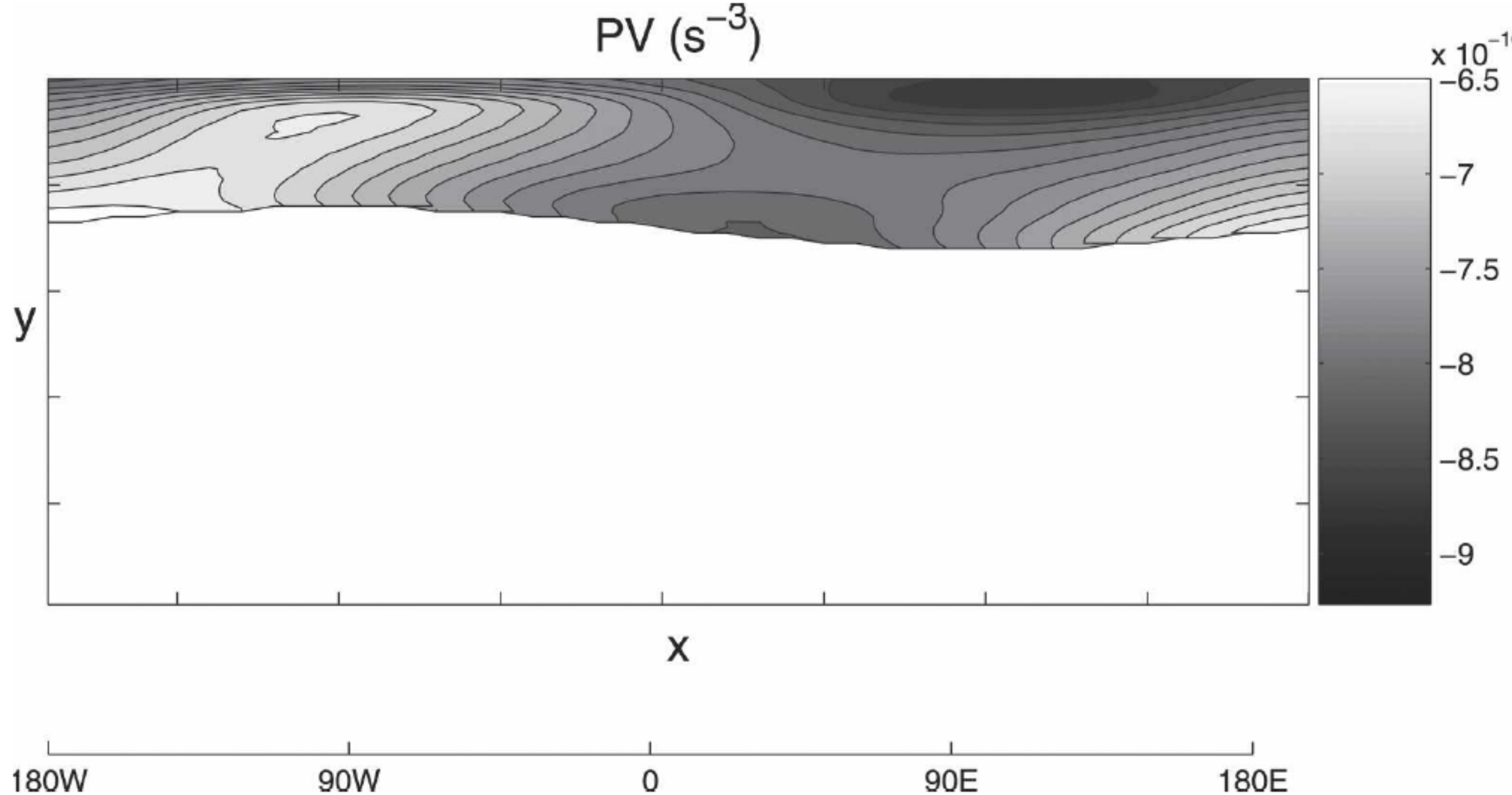
Prognostic solution



Implied buoyancy and buoyancy flux

RM2006 Fig. 11

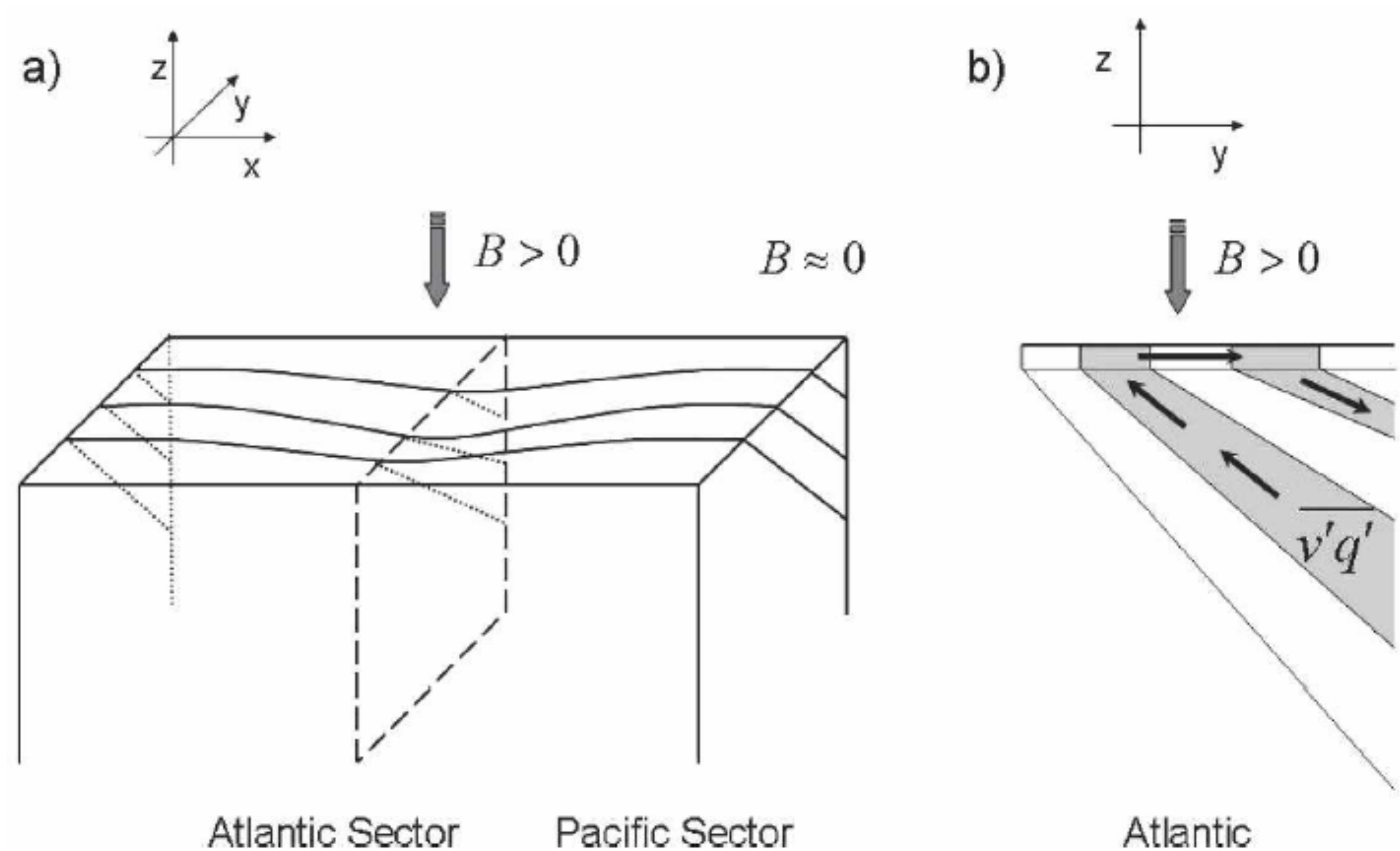
Prognostic solution



Decreased isopycnal PV gradient in Pacific reduces downgradient PV flux -> weaker residual circulation

RM2006 Fig. 12

Physical interpretation



Increase in buoyancy AND buoyancy gradient (on poleward side) in the Atlantic leads to strengthened isopycnal PV gradients

RM2006 Fig. 13

Current approaches

Focus on 3-dimensionality through looking at differences between Atlantic and Indo-Pacific basins

-Only sinking in one basin

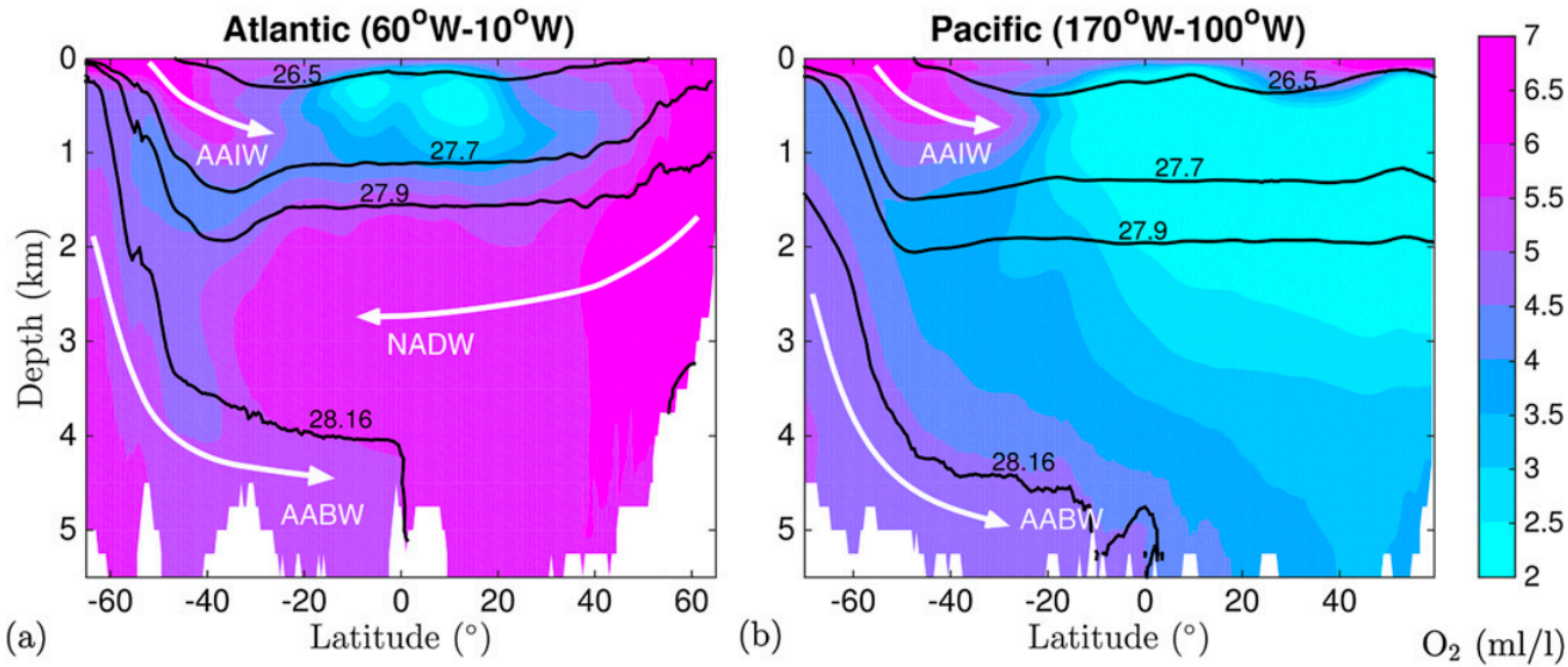
-width of basins differ

Thompson, Stewart and Bischoff

A Multibasin Residual-Mean Model for the Global Overturning Circulation

Motivation

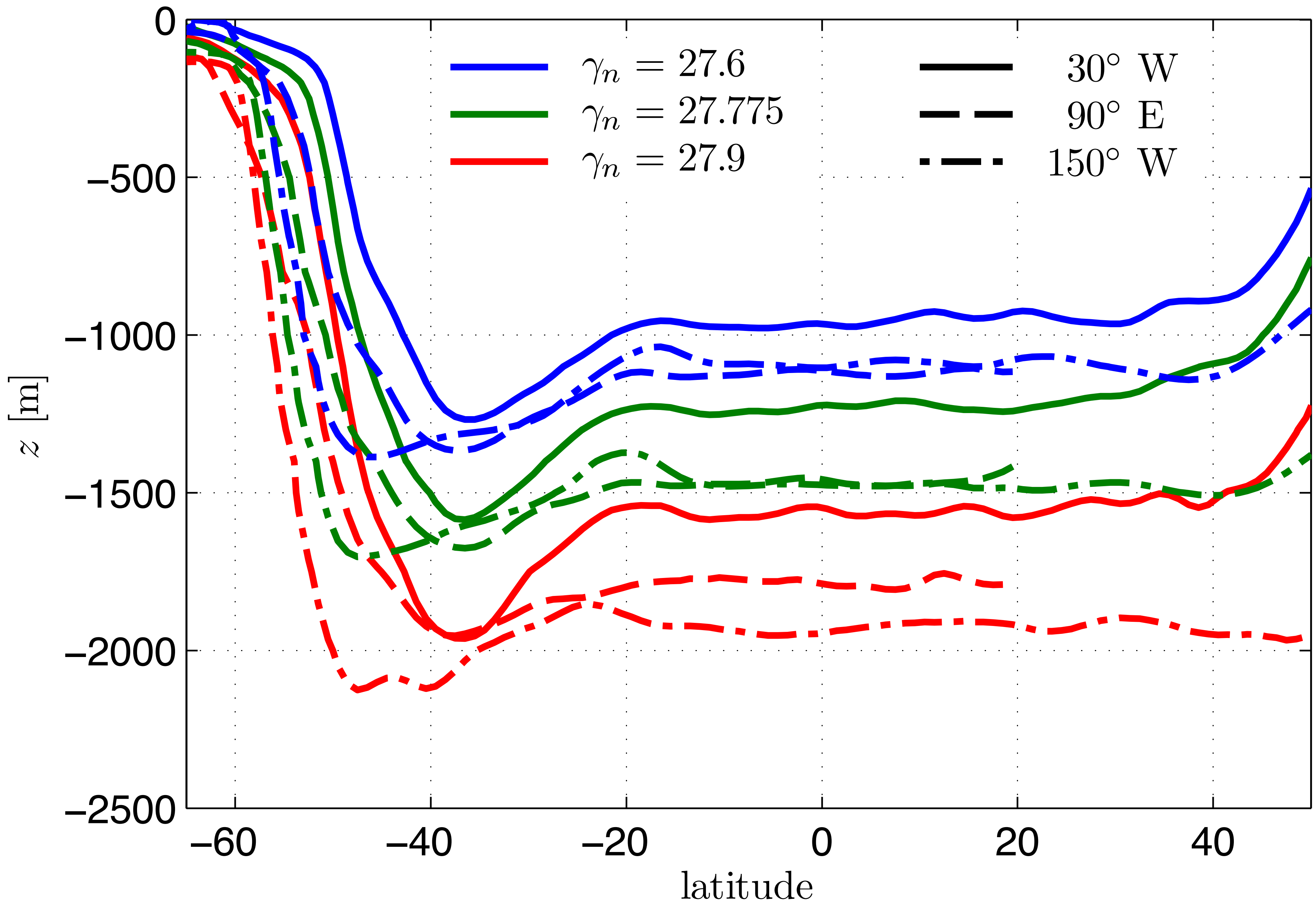
Different overturning circulation and stratification in Atlantic vs Pacific



Thompson et al. Fig. 1

Motivation

Observed isopycnal depth differences

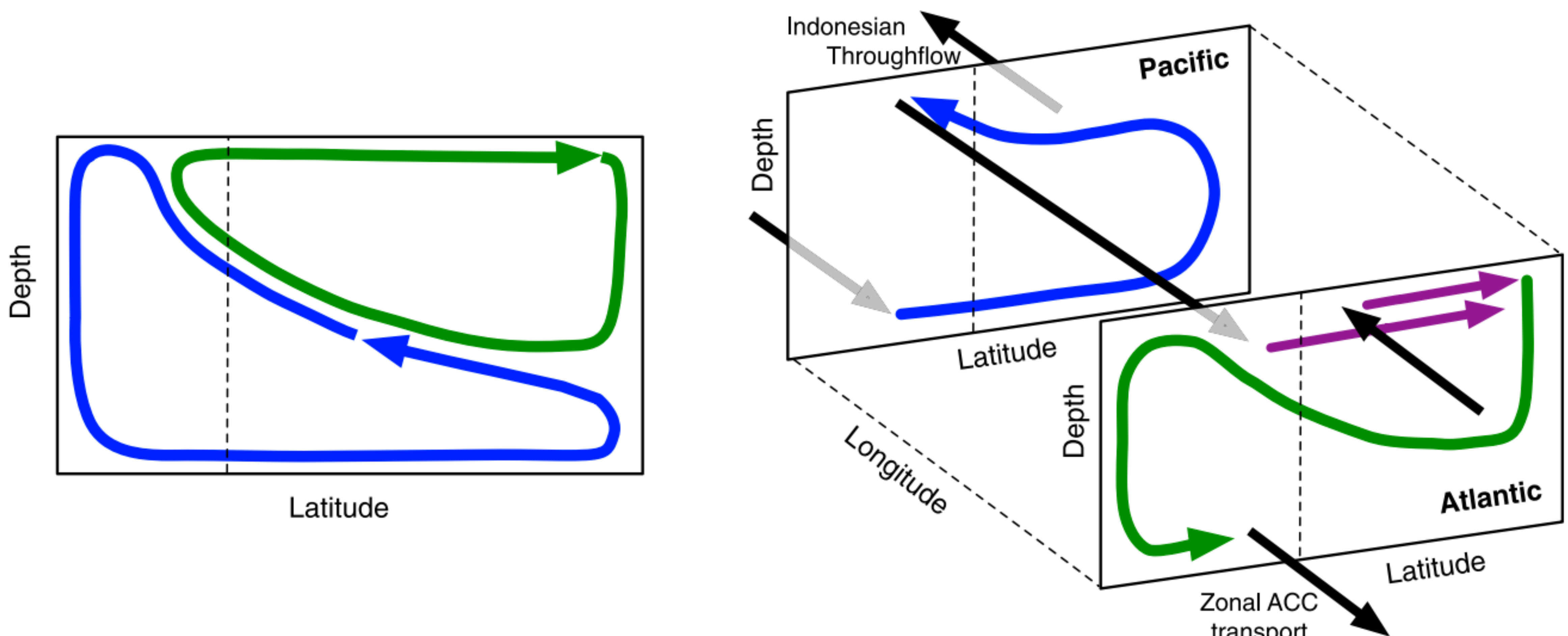


Jones and Cessi 2016 Fig. 1

Motivation

Goal: bridge gap between idealized two-dimensional, residual-mean overturning and complex, fully three-dimensional models

Approach: Two separate two-dimensional basins that can exchange properties through the AC or ITF.



Thompson et al. Fig. 2

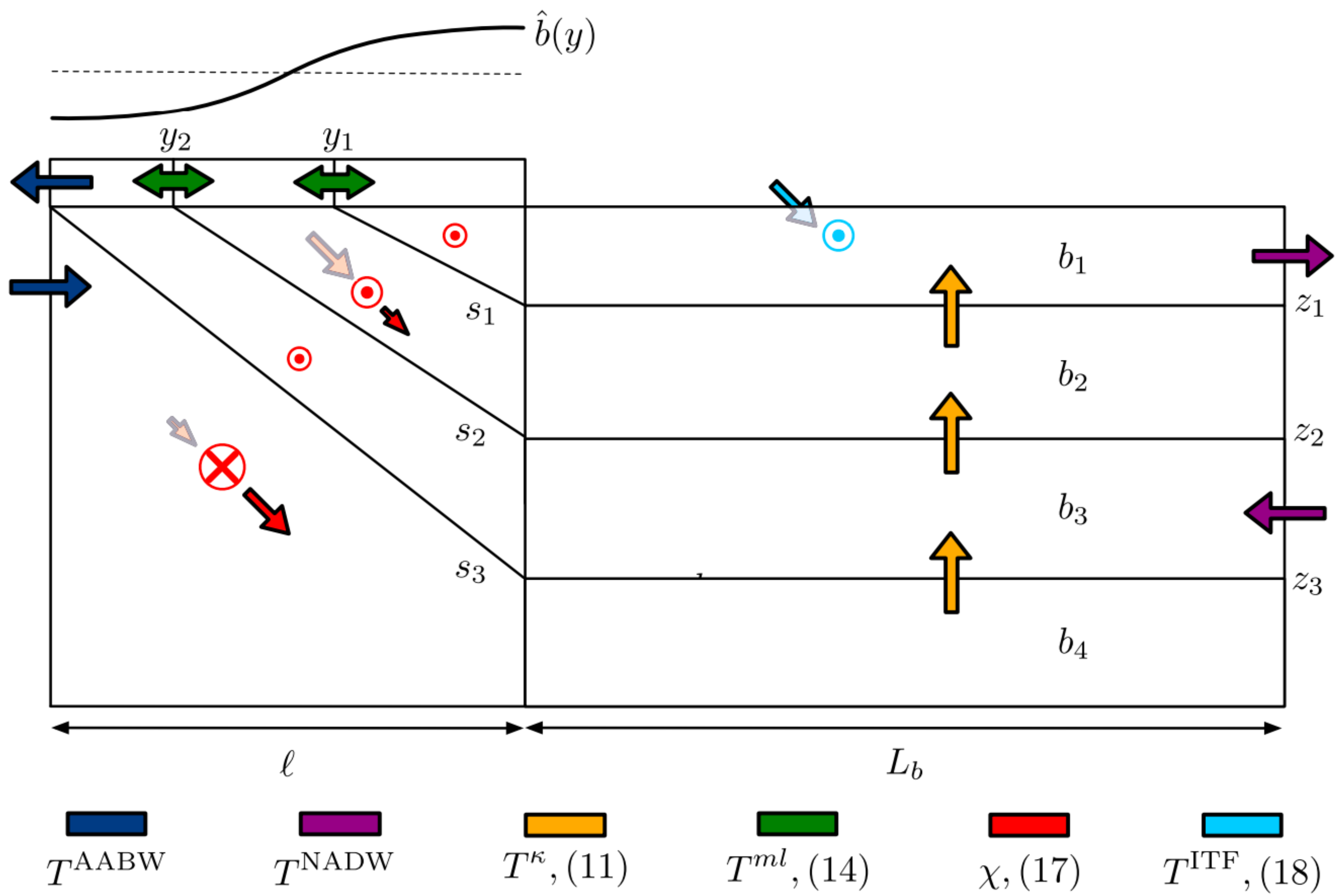
Model

1. Apply coarse discretization in the ACC, with diabatic processes (buoyancy forcing and diffusive upwelling in basins).

2. Prescribe high latitude dense water formation

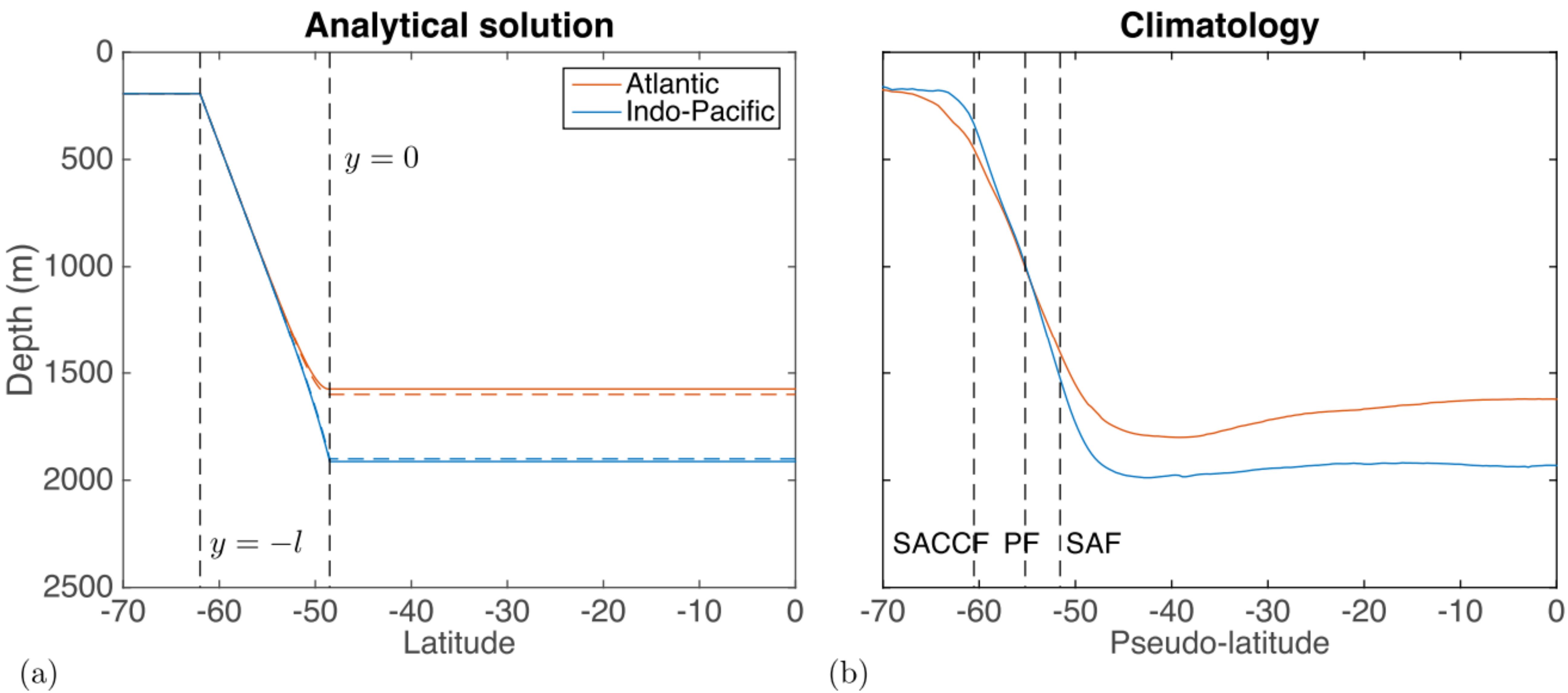
3. Consider case with 2 sectors only, and look at exchanges between Atlantic and “Pacific”

4-layer model



Thompson et al. Fig. 4

Model vs observations



Questions?