Residual-mean solutions for the Antarctic Circumpolar Current and its associated overturning circulation

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ACC and MOC
Buoyancy

Streamwise-averaged buoyancy equation:

\[
\frac{\bar{v}}{\bar{v}} \frac{\partial \bar{b}}{\partial y} + \bar{w} \frac{\partial \bar{b}}{\partial z} + \frac{\partial}{\partial y} (v'b') + \frac{\partial}{\partial z} (w'b') = \frac{\partial B}{\partial z}
\]

where,

\[
J_{y,z}(\Psi_{res}, \bar{b}) = \frac{\partial B}{\partial z} - \frac{\partial}{\partial y} [(1 - \mu) v'b']
\]

where,

\[\Psi_{res} = \bar{\Psi} + \Psi^*, \quad \Psi^* = -\frac{w'b'}{b_y}, \quad \text{and} \quad \mu = \left( \frac{w'b'}{v'b'} \right) \left( \frac{1}{s_\rho} \right) = \begin{cases} 1, & \text{adiabatic} \\ 0. & \end{cases}\]
Buoyancy

Below the mixed layer:

\[ J(\Psi_{\text{res}}, \bar{b}) = 0. \quad \Psi_{\text{res}} = \Pi(\bar{b}) \]

Two assumptions: buoyancy forcing vanishes; eddy flux along isopycnal surfaces.

Within the mixed layer:

\[ -\frac{\partial \Psi_{\text{res}}}{\partial z} \frac{\partial b_o}{\partial y} = \frac{\partial B}{\partial z} - \frac{\partial}{\partial y} [(1 - \mu)\nu'b'] \]

At the base of mixed layer:

\[ \Psi_{\text{res}|z=-h_m} \frac{\partial b_o}{\partial y} = \tilde{B} \quad \Psi_{\text{res}|z=-h_m} = \frac{\tilde{B}}{\partial b_o/\partial y} \]

\[ \tilde{B} = B_o - (1 - \mu) \int_{-h_m}^{0} \frac{\partial}{\partial y} \nu'b' \, dz \]
Closure for $\overline{\Psi}$ and $\Psi^*$

$$f \frac{\partial \overline{\Psi}}{\partial z} = \frac{\partial \tau}{\partial z} + \frac{\Delta P}{\rho L_x} - \frac{\partial w'v'}{\partial y} \quad \rightarrow \quad f \overline{\Psi} = -\tau_0$$

Interior region

$$\Psi_{\text{res}} = -\tau_0/f + \Psi^*$$

Eddies assumed to be adiabatic in the interior:

$$\Psi^* = -\frac{w'b'}{b_y} = \frac{v'b'}{b_z}$$

$$v'b' = -Kb_y \quad \rightarrow \quad \Psi^* = \frac{v'b'}{b_z} = -K \frac{b_y}{b_z} = Ks_\rho.$$
Interior:

\[ \Psi_{\text{res}} = \Pi(b) \]

\[ \Psi_{\text{res}}(b) = k |s_\rho| s_\rho - \tau_0 / f \Rightarrow s_\rho(b, y) = - \left[ -\frac{\tau_0(y)}{f k} - \frac{\Psi_{\text{res}}(b)}{k} \right]^{1/2} \]

Base of Mixed layer:

\[ \Psi_{\text{res}|z=-h_m}(b) = \frac{\tilde{B}}{\partial b_0 / \partial y}, \quad \tilde{B} = B_0 - (1 - \mu) \int_{-h_m}^{0} \frac{\partial}{\partial y} v' v' \, dz \]

\[ b(y, z = -h_m) = b_0(y) \]

Can be solved with \( b_o, \tilde{B}, \) and \( \tau_o \) known.
Examples

Buoyancy structure when:

\[ \tau_o(y) = \frac{\tau_s y}{L_y} \]
\[ \tilde{B} = 0 \Rightarrow \Psi_{res} = 0 \]
\[ b_o(y) = \Delta b_0 \frac{y}{L_y} \]

\[ s_p(b, y) = -\left[-\frac{\tau_0}{f k}\right]^{1/2} \]

Baroclinic transport \( \sim \frac{\tau_o L^2 \Delta b}{f^2 k} \)
Examples

\[ b(y) = b_0 \frac{y}{L_y} \]

\[ \tilde{B} = B_0 \sin\left(\frac{2\pi y}{L_y}\right) \]

\[ \tau_o(y) = \tau_s \left[ 0.3 + \sin\left(\frac{\pi y}{L_y}\right) \right] \]

\[ b_o(y) = \Delta b_0 \frac{y}{L_y} \]

\[ \Psi_{\text{res}} = \frac{\tilde{B}_r L_y L_z}{\Delta b_o} = 12 \text{ Sv} \]
Role of dyapycnal eddy buoyancy fluxes

\[ \tilde{B} = B_0 - (1 - \mu) \int_{-h_m}^{0} \frac{\partial}{\partial y} v' b' \, dz \]

**Fig. 11.** Total buoyancy flux \( \tilde{B} \), (solid line) computed for a given air-sea flux \( B_0 \) (dashed line). (a) The \( \tilde{B} \) when \( B_0 = 0 \): in this case diabatic eddy fluxes redistribute buoyancy within the mixed layer. (b) The \( \tilde{B} \) when \( B_0 = B_0 \sin[(2\pi y)/L] \), with \( B_0 = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-3} \).

**Fig. 12.** (a) The buoyancy field and (b) the residual circulation for the buoyancy forcing shown in Fig. 11a. The wind stress is given by Eq. (24). Contour intervals are \( \tilde{B} = 10^{-3} \text{ m s}^{-2} \) and \( \Psi_{\text{res}} = 2 \text{ Sv} \), respectively. Here the residual overturning streamfunction, Fig. 11b, is driven entirely by diabatic eddy fluxes in the mixed layer.
Another example

Constructing the residual circulation of the ACC from observations

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Base of Mixed layer:

\[
\Psi_{res|z=-h_m}(b) = \frac{\tilde{B}}{\partial b_o/\partial y}, \quad \tilde{B} = B_o - (1 - \mu) \int_{-h_m}^{0} \frac{\partial}{\partial y} v'b' \, dz
\]

\[
\Psi_{res} = \overline{\Psi} + \Psi^*,
\]

Here, a different approach:

Base of Mixed layer:

\[
\overline{\Psi} = -\frac{\overline{\tau}}{\rho_0 f}
\]

\[
\Psi^* = \frac{v'b'}{b_z} = -K \frac{\overline{b_y}}{\overline{b_z}}
\]

\[
K = \alpha \frac{g}{|f|} (\overline{h'/2})^{1/2}
\]

\(b\) estimated from Levitus and Boyer (1994), sea surface height from satellite.
Fig. 3. The Ekman transport, $\overline{\Psi}$, given by (2): dash–dot. The eddy induced transport, $\Psi^*$, given by (11): dashed. The residual transport, $\Psi_{res}$, given by (9): solid. The thin dash–dot and solid lines are based on the HR winds; the thick dash–dot and solid lines are based on SOC winds. The error bars on the eddy-induced transport and residual circulation are calculated from the errors in the eddy diffusivity.
Interior:

\[ J(\Psi_{\text{res}}, \bar{b}) = \kappa \frac{\partial^2 \bar{b}}{\partial z^2} \quad \Rightarrow \quad \frac{d\Psi_{\text{res}}}{ds} = \kappa \frac{\bar{b}_{zz}}{\sqrt{(\bar{b}_y)^2 + (\bar{b}_z)^2}} \]

\( \kappa = 1.5 \times 10^{-5} \text{ m}^2/\text{s} \)

\( \bar{b} \) known from observations; Residual-mean streamfunction at the base of the mixed layer also estimated from observations.
Residual-mean overturning circulation and salinity

Fig. 7. The thin lines are contours of mean salinity. The region of no shading marks fresh AASW and AAIW, salinity <34.4 psu; the darkest shading marks salty NADW, salinity >34.7 psu. The dark solid lines are contours of the residual circulation with the arrows showing the direction of flow.
Questions?