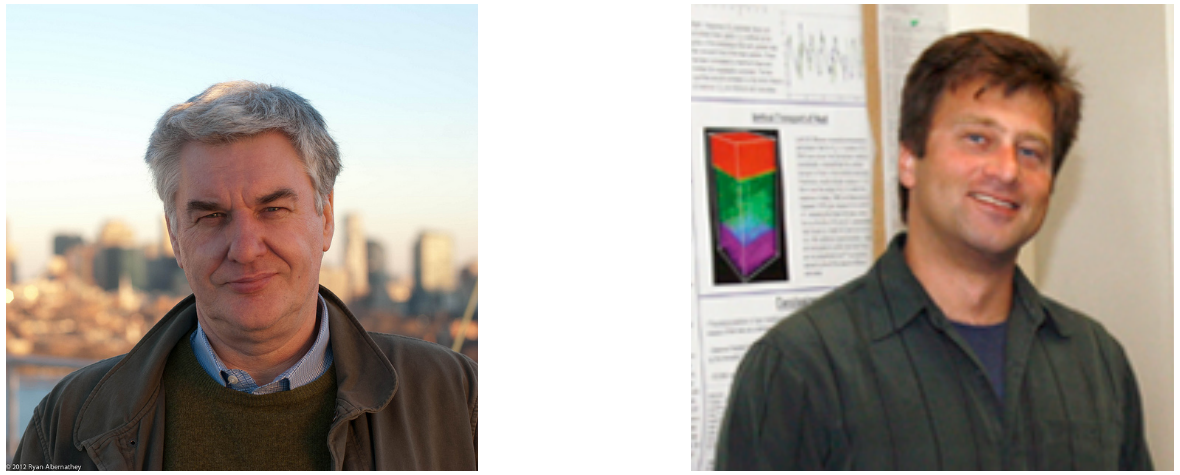
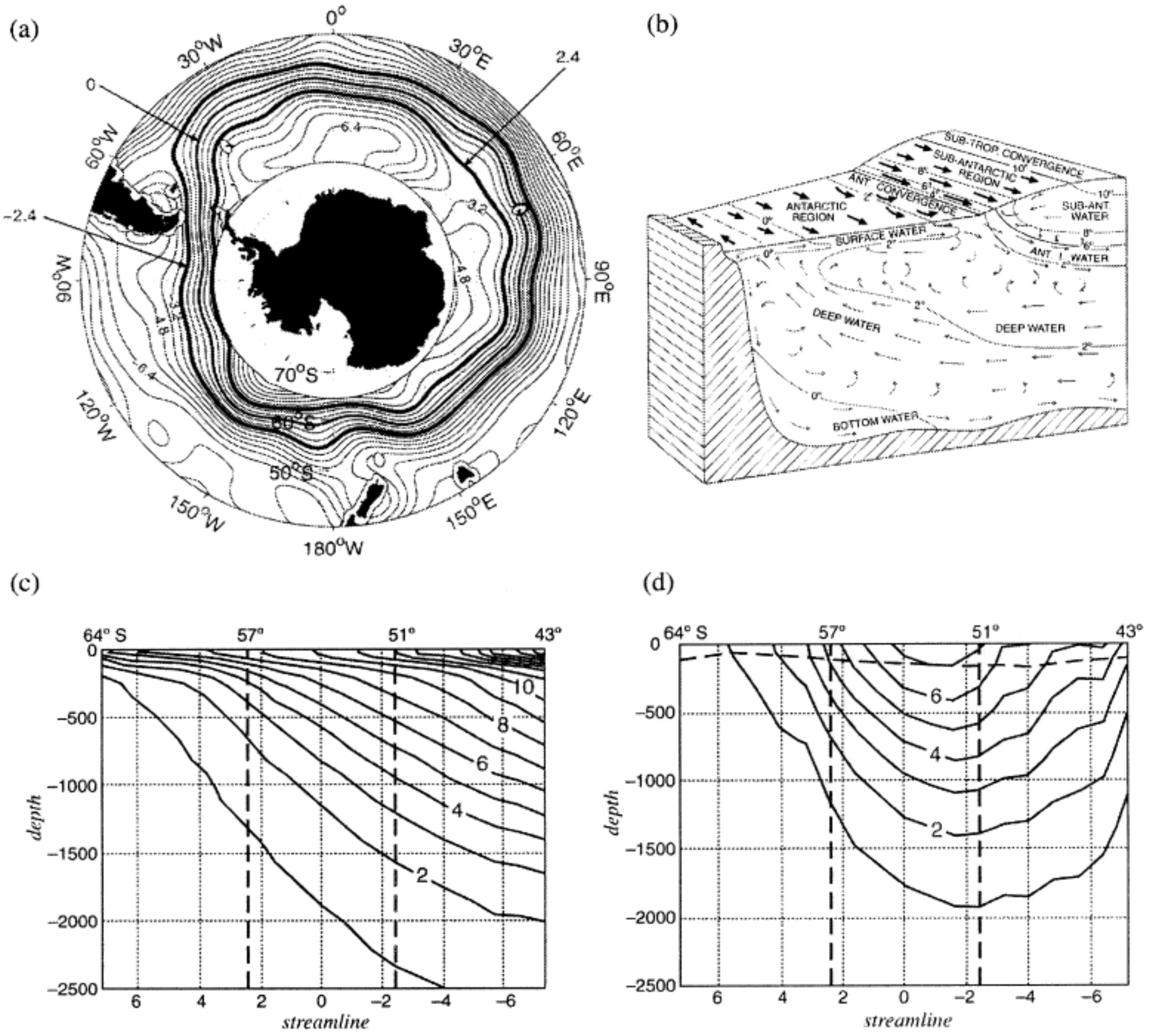
Residual-mean solutions for the Antarctic Circumpolar Current and its associated overturning circulation

John Marshall and Timour Radko

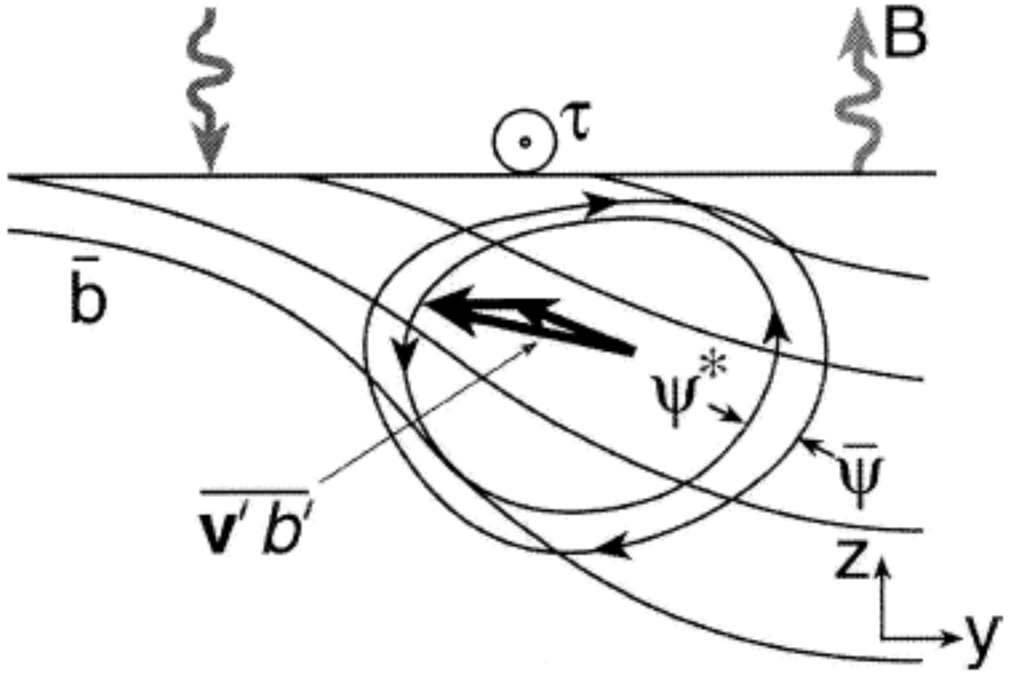


presented by Shantong Sun

ACC and MOC


(a)
0°
30°W
30°E
60°W
60°E
90°W
90°E
120°W
120°E
150°W
150°E
180°W
70°S
60°S
50°S
0
2.4
−2.4
6.4
3.2
4.8
(b)
SUB-TROP. CONVERGENCE
SUB-ANTARCTIC REGION
ANT. CONVERGENCE
ANTARCTIC REGION
SURFACE WATER
SUB-ANT WATER
ANT. WATER
DEEP WATER
DEEP WATER
BOTTOM WATER
10°
8°
6°
4°
2°
0°
(c)
64° S
57°
51°
43°
10
8
6
4
2
0
−500
−1000
−1500
−2000
−2500
depth
streamline
(d)
64° S
57°
51°
43°
6
4
2
0
−500
−1000
−1500
−2000
−2500
depth
streamline

Buoyancy



Streamwise-averaged buoyancy equation:

$$\overline{v}\frac{\partial \overline{b}}{\partial y} + \overline{w}\frac{\partial \overline{b}}{\partial z} + \frac{\partial}{\partial y}(\overline{v'b'}) + \frac{\partial}{\partial z}(\overline{w'b'}) = \frac{\partial B}{\partial z}$$

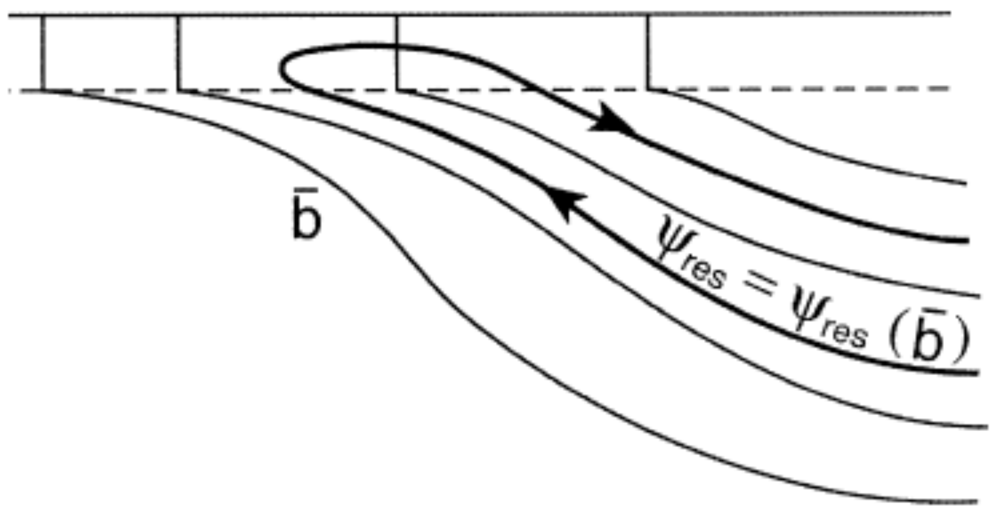
$$\Big\downarrow$$

$$J_{y,z}(\Psi_{\mathrm{res}}, \overline{b}) = \frac{\partial B}{\partial z} - \frac{\partial}{\partial y}[(1-\mu)\overline{v'b'}]$$

where,

$$\Psi_{\mathrm{res}} = \overline{\Psi} + \Psi^*, \quad \Psi^* = -\frac{\overline{w'b'}}{\overline{b}_y}, \quad \text{and} \quad \mu = \left(\frac{\overline{w'b'}}{\overline{v'b'}}\right)\left(\frac{1}{s_\rho}\right) = \begin{cases} 1, & \text{adiabatic} \\ 0. & \end{cases}$$

Buoyancy



Below the mixed layer:

$$J(\Psi_{\mathrm{res}}, \bar{b}) = 0. \longrightarrow \Psi_{\mathrm{res}} = \Pi(\bar{b})$$

Two assumptions: buoyancy forcing vanishes; eddy flux along isopycnal surfaces.

Within the mixed layer:

$$-\frac{\partial \Psi_{\mathrm{res}}}{\partial z}\frac{\partial b_o}{\partial y} = \frac{\partial B}{\partial z} - \frac{\partial}{\partial y}[(1-\mu)\overline{v'b'}]$$

$$\Big\downarrow$$

At the base of mixed layer:

$$\Psi_{\mathrm{res|z=-h_m}}\frac{\partial b_o}{\partial y} = \tilde{B} \longrightarrow \Psi_{\mathrm{res|z=-h_m}} = \frac{\tilde{B}}{\partial b_o/\partial y}$$

$$\tilde{B} = B_o - (1-\mu)\int_{-h_m}^{0}\frac{\partial}{\partial y}\overline{v'b'}\,dz$$

Closure for $\overline{\Psi}$ and Ψ^*

$$f\frac{\partial\overline{\Psi}}{\partial z} = \frac{\partial\overline{\tau}}{\partial z} + \cancel{\frac{\Delta P}{\rho L_x}} - \cancel{\frac{\partial\overline{u'v'}}{\partial y}} \longrightarrow f\overline{\Psi} = -\tau_0$$

$$\Big\downarrow$$

Interior region

$$\Psi_{\text{res}} = -\tau_o/f + \Psi^*$$

Eddies assumed to be adiabatic in the interior:

$$\Psi^* = -\frac{\overline{w'b'}}{\overline{b}_y} = \frac{\overline{v'b'}}{\overline{b}_z}$$

$$\overline{v'b'} = -K\overline{b}_y \longrightarrow \Psi^* = \frac{\overline{v'b'}}{\overline{b}_z} = -K\frac{\overline{b}_y}{\overline{b}_z} = Ks_\rho.$$

Visbeck et al. (1997): $K = k|s_\rho|$ $\Psi^* = Ks_\rho = k|s_\rho|s_\rho$

$$\boxed{\Psi_{\text{res}} = -\tau_o/f + k|s_\rho|s_\rho}$$

Interior:

$$\Psi_{\text{res}} = \Pi(\bar{b})$$

$$\Psi_{\text{res}}(b) = k|s_\rho|s_\rho - \tau_o/f \Rightarrow s_\rho(b,y) = -\left[-\frac{\tau_0(y)}{fk} - \frac{\Psi_{\text{res}}(b)}{k}\right]^{1/2}$$

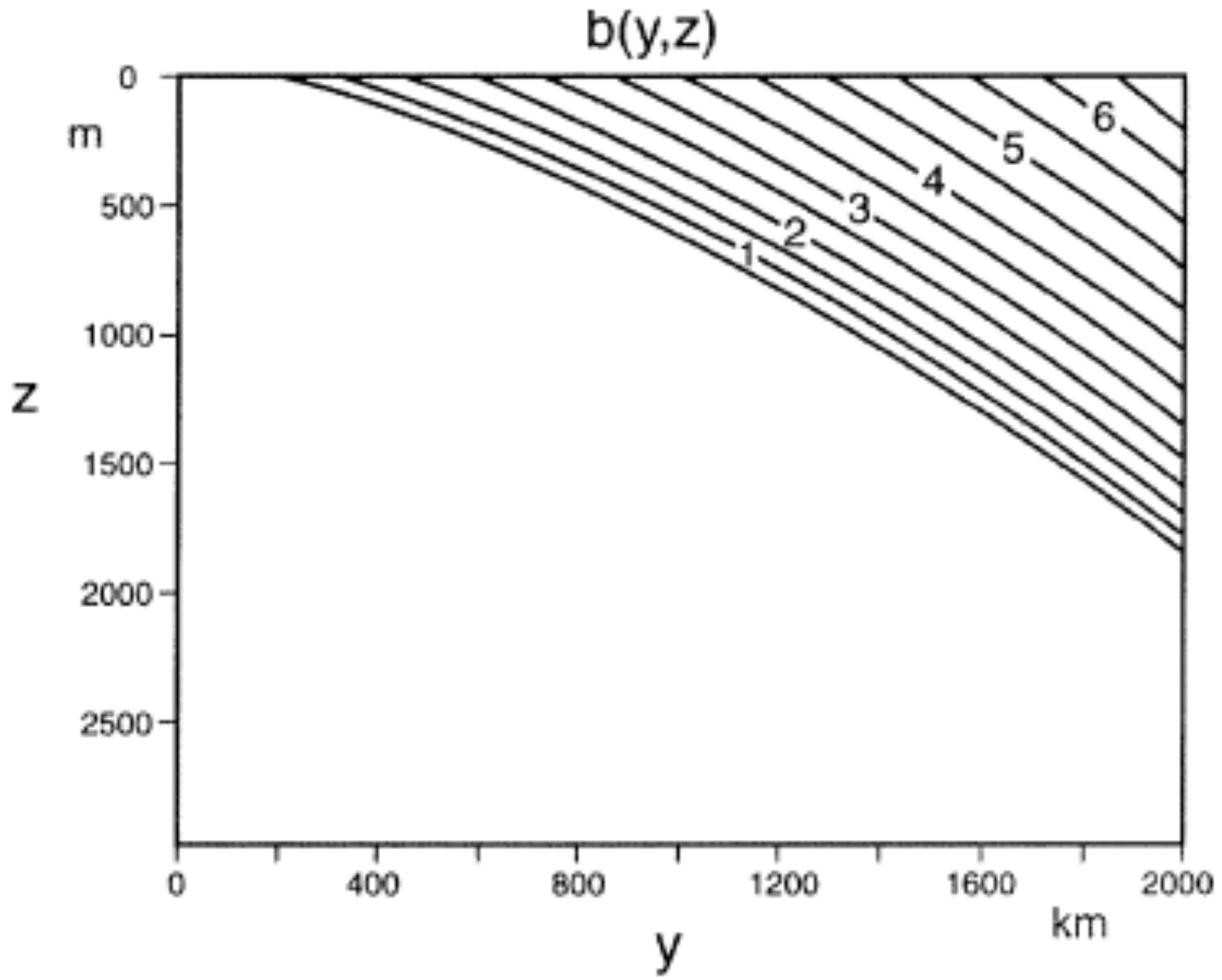
Base of Mixed layer:

$$\Psi_{\text{res}|\text{z}=-\text{h}_\text{m}}(b) = \frac{\tilde{B}}{\partial b_o/\partial y}, \qquad \tilde{B} = B_o - (1-\mu)\int_{-h_m}^{0} \frac{\partial}{\partial y}\overline{v'b'}\,dz$$

$$b(y, z=-h_m) = b_0(y)$$

Can be solved with b_o, $\tilde{B}$, and τ_o known.

Examples



Buoyancy structure when:

$$\tau_o(y) = \tau_s y/L_y$$

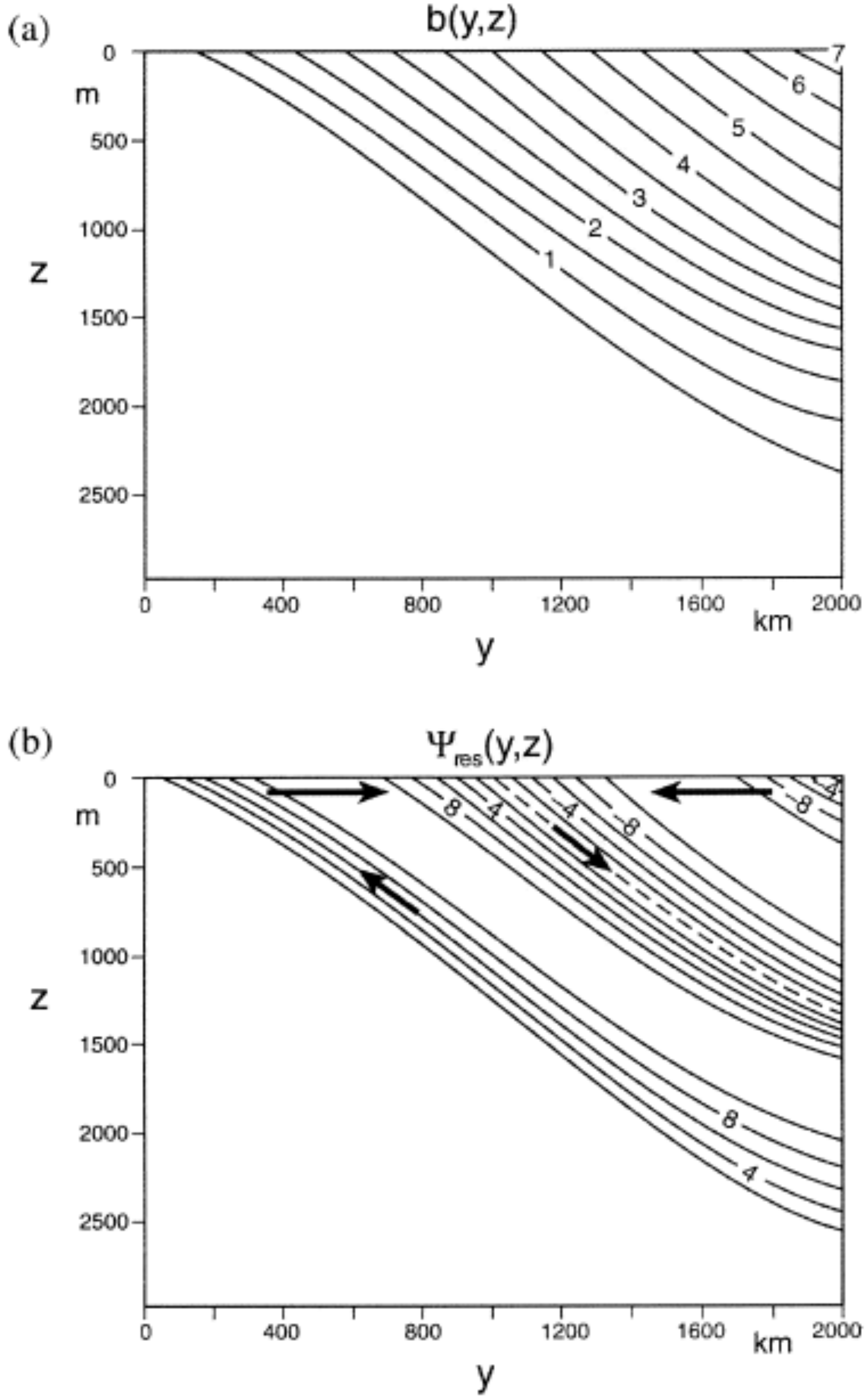
$$\tilde{B} = 0 \Rightarrow \Psi_{\mathrm{res}} = 0$$

$$b_o(y) = \Delta_{b_0}\frac{y}{L_y}$$

$$\longrightarrow \quad s_\rho(b,y) = -\left[-\frac{\tau_0}{fk}\right]^{1/2}$$

Baroclinic transport $\sim \dfrac{\tau_o L^2 \Delta b}{f^2 k}$

Examples



$$\tilde{B} = \tilde{B}_o \sin[(2\pi y)/L_y]$$
$$\tau_o(y) = \tau_s \left[0.3 + \sin\left(\frac{\pi y}{L_y}\right)\right]$$
$$b_o(y) = \Delta_{b_0} \frac{y}{L_y}$$

$$\Psi_{\text{res}} = \frac{\tilde{B}_o L_y L_x}{\Delta b_o} = 12 \text{ Sv}$$

Role of dyapycnal eddy buoyancy fluxes

$$\tilde{B} = B_o - (1-\mu)\int_{-h_m}^{0} \frac{\partial}{\partial y}\overline{v'b'}\,dz$$

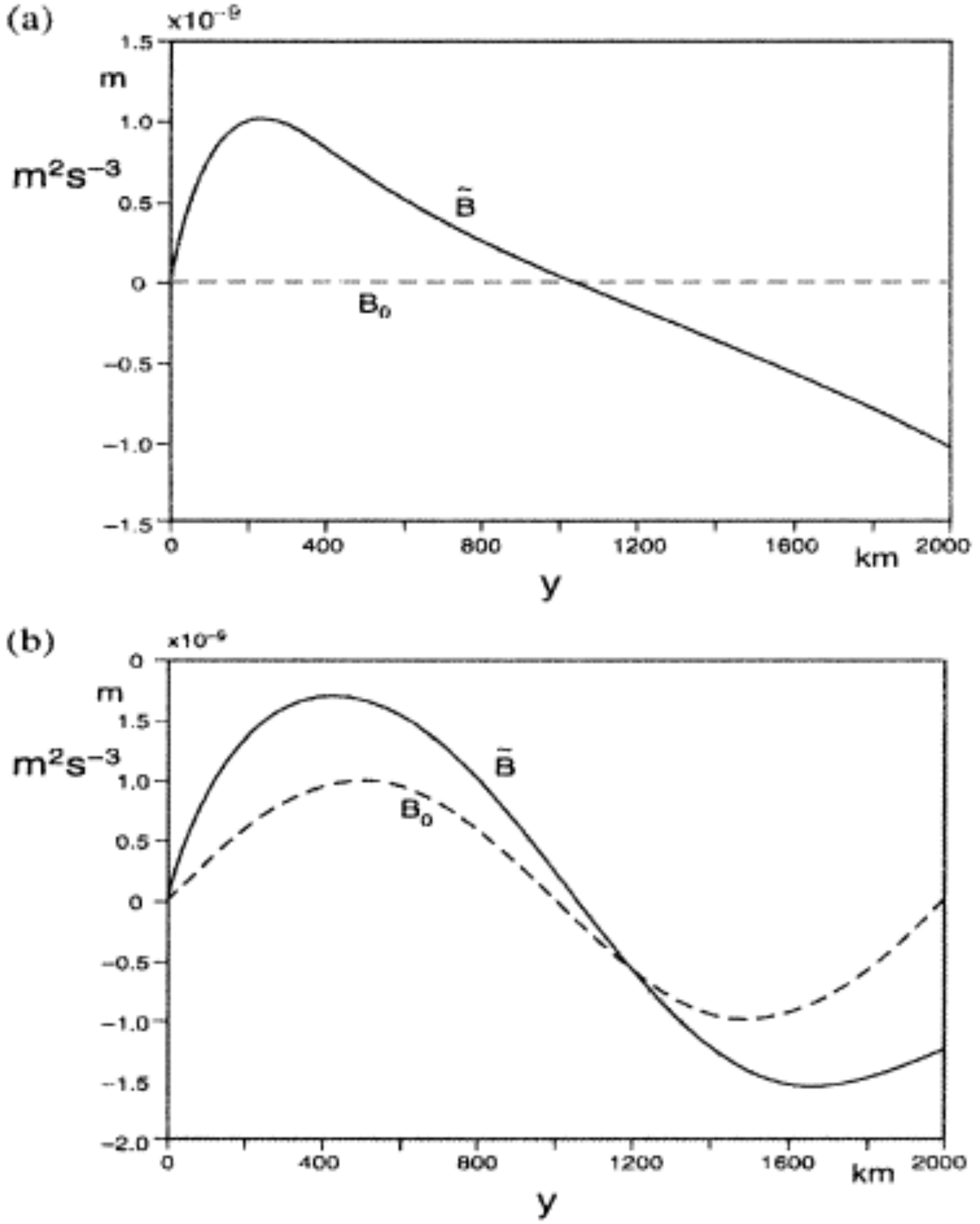


FIG. 11. Total buoyancy flux $\tilde{B}_o$ (solid line) computed for a given air–sea flux B_o (dashed line). (a) The $\tilde{B}_o$ when $B_o = 0$: in this case diabatic eddy fluxes redistribute buoyancy within the mixed layer. (b) The $\tilde{B}_o$ when $B_o = B_o \sin[(2\pi y)/L_y]$, with $B_o = 1 \times 10^{-9}$ m^2 s^{-3}.

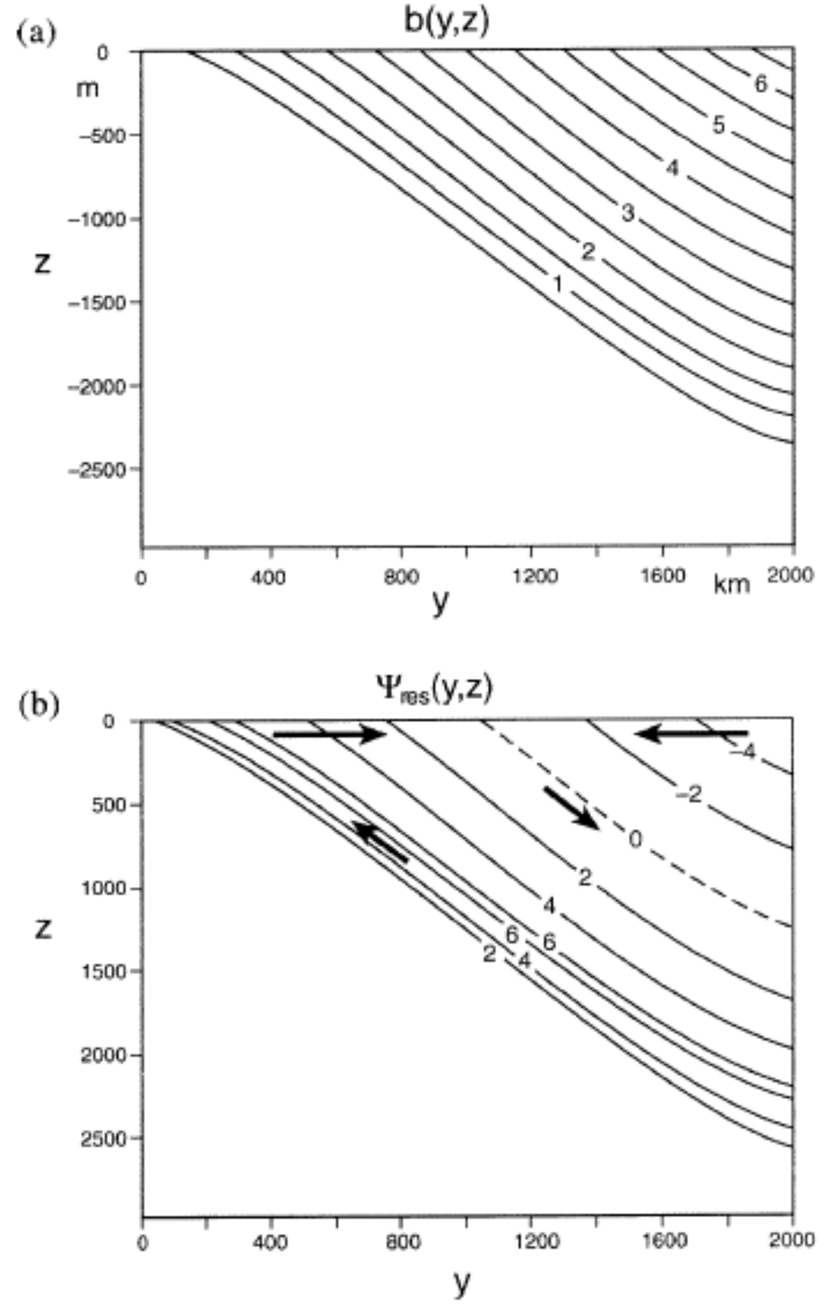


FIG. 12. (a) The buoyancy field and (b) the residual circulation for the buoyancy forcing shown in Fig. 11a. The wind stress is given by Eq. (24). Contour intervals are $\overline{b} = 10^{-3}$ m s^{-2} and $\Psi_{res} = 2$ Sv, respectively. Here the residual overturning streamfunction, Fig. 11b, is driven entirely by diabatic eddy fluxes in the mixed layer.

Another example

Constructing the residual circulation of the ACC from observations

Richard H. Karsten and John Marshall

From Marshall and Radko (2003)

Base of Mixed layer:

$$\Psi_{\mathrm{res}|\mathrm{z}=-\mathrm{h_m}}(b) = \frac{\tilde{B}}{\partial b_o/\partial y}, \qquad \tilde{B} = B_o - (1-\mu)\int_{-h_m}^{0} \frac{\partial}{\partial y}\overline{v'b'}\,dz$$

$$\Psi_{\mathrm{res}} = \overline{\Psi} + \Psi^*,$$

Here, a different approach:

Base of Mixed layer:
$$\overline{\Psi} = -\frac{\overline{\tau}}{\rho_0 f}$$

$$\Psi^* = \frac{\overline{v'b'}}{\overline{b}_z} = -K\frac{\overline{b}_y}{\overline{b}_z}$$

$$K = \alpha\frac{g}{|f|}(\overline{h'^2})^{1/2}$$

b estimated from Levitus and Boyer (1994), sea surface height from satellite.

Base of Mixed layer:

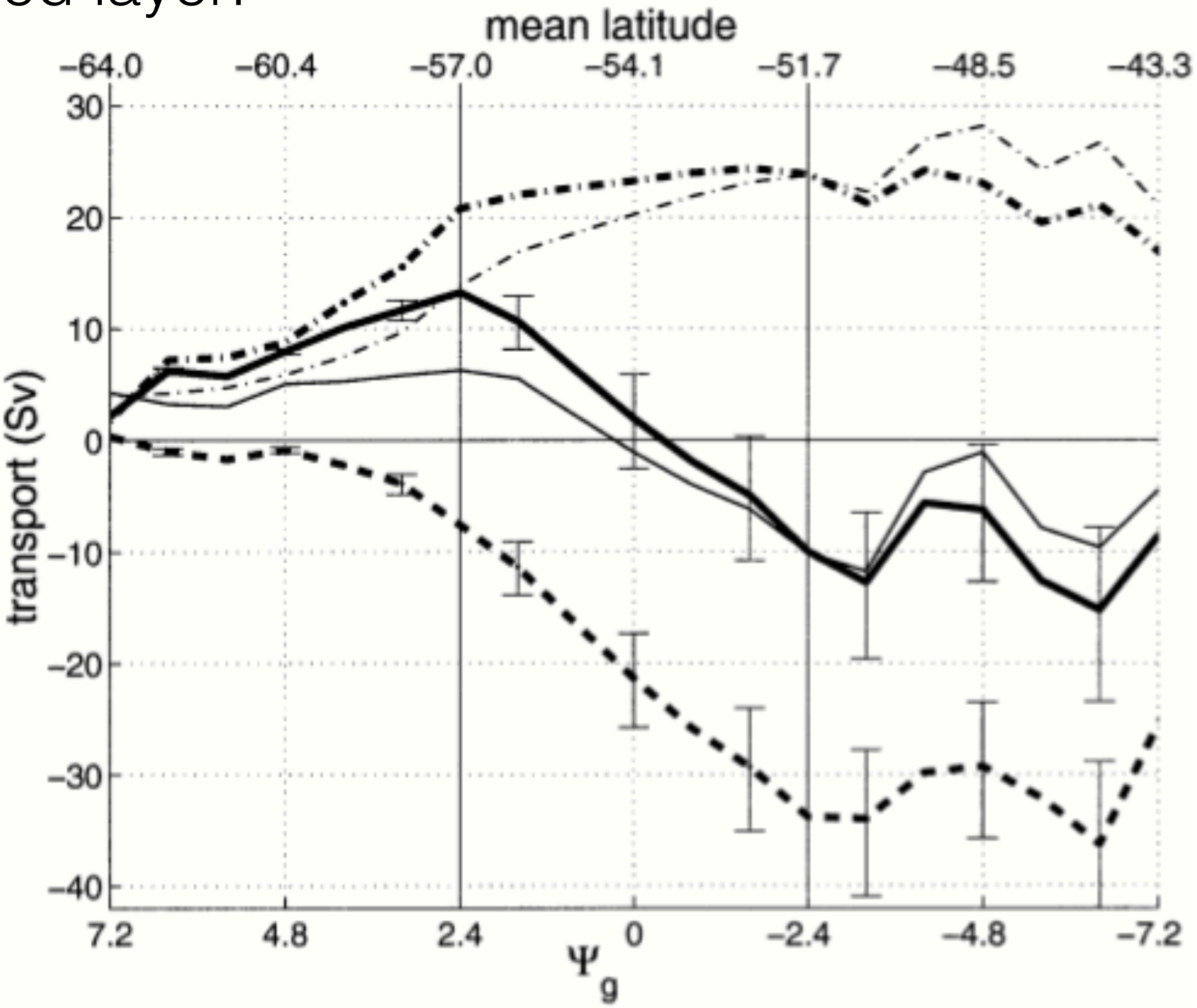
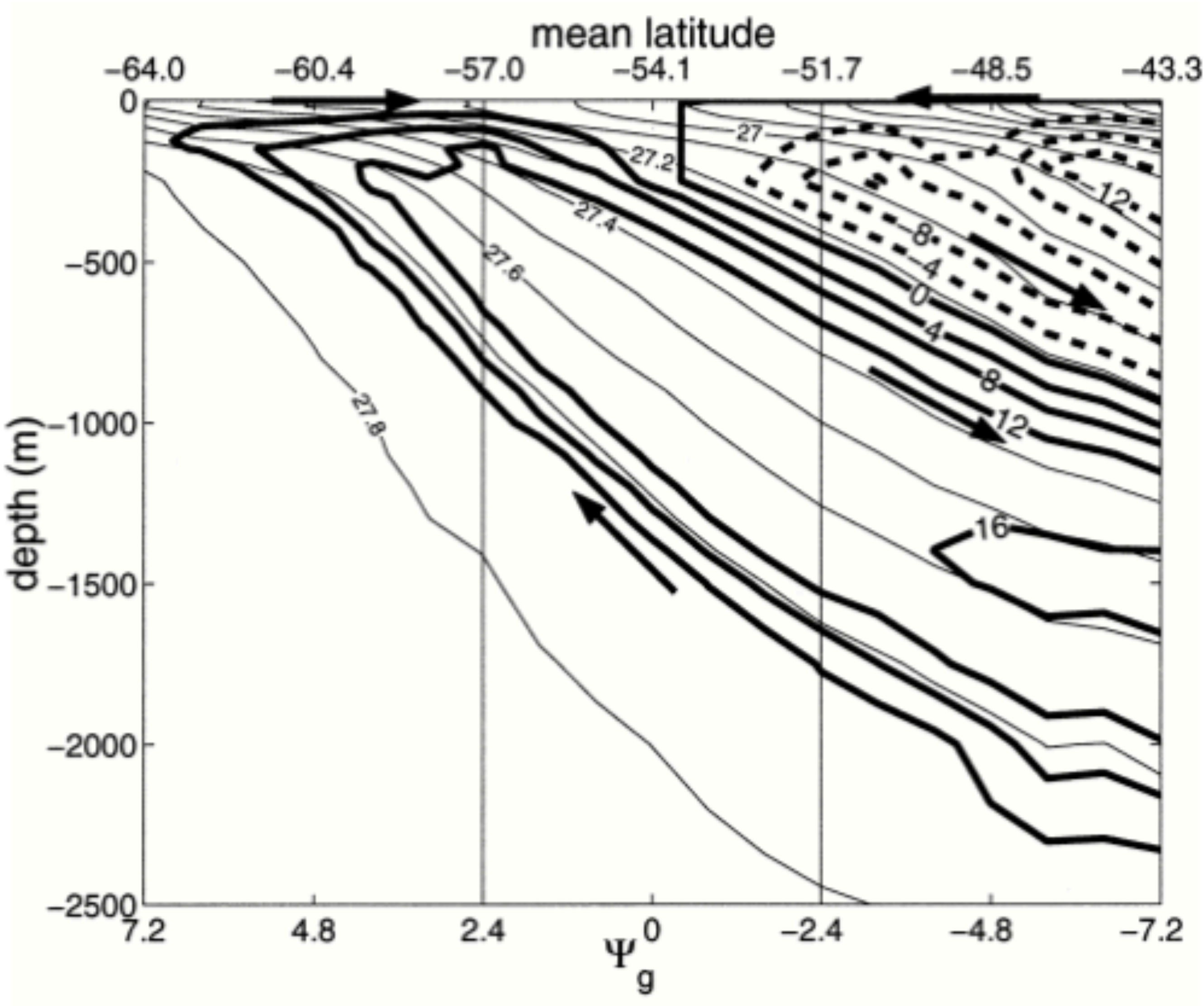


FIG. 3. The Ekman transport, $\overline{\Psi}$, given by (2): dash–dot. The eddy induced transport, Ψ^*, given by (11): dashed. The residual transport, Ψ_{res}, given by (9): solid. The thin dash–dot and solid lines are based on the HR winds; the thick dash–dot and solid lines are based on SOC winds. The error bars on the eddy-induced transport and residual circulation are calculated from the errors in the eddy diffusivity.

Interior:

$$J(\Psi_{\mathrm{res}},\bar{b})=\kappa\frac{\partial^2\bar{b}}{\partial z^2}\quad\longrightarrow\quad\frac{d\Psi_{\mathrm{res}}}{ds}=\kappa\frac{\bar{b}_{zz}}{\sqrt{(\bar{b}_y)^2+(\bar{b}_z)^2}}$$



$$\kappa=1.5\times10^{-5}\mathrm{m}^2/\mathrm{s}$$

b known from observations; Residual-mean streamfunction at the base of the mixed layer also estimated from observations.

Residual-mean overturning circulation and salinity

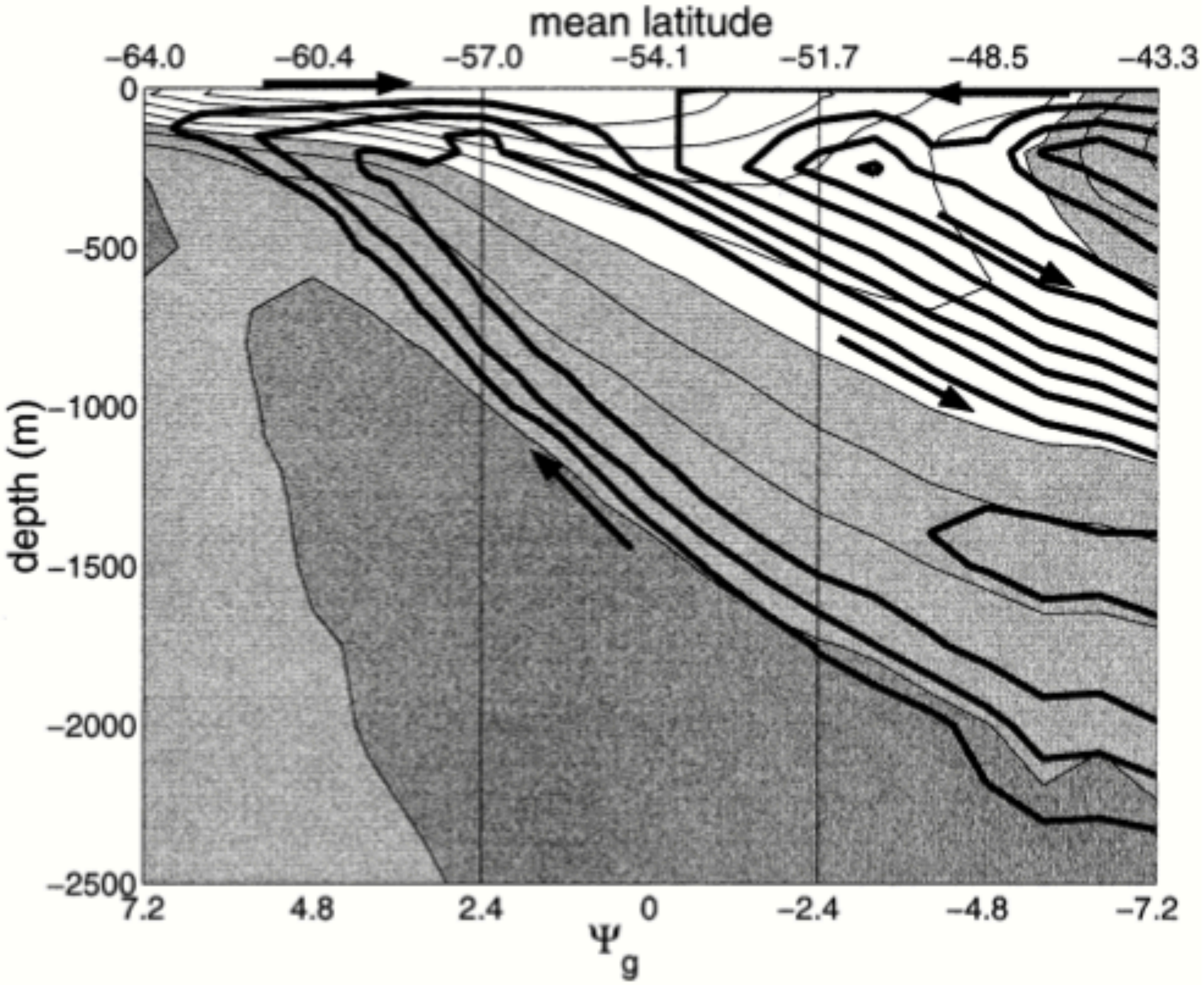


FIG. 7. The thin lines are contours of mean salinity. The region of no shading marks fresh AASW and AAIW, salinity <34.4 psu; the darkest shading marks salty NADW, salinity >34.7 psu. The dark solid lines are contours of the residual circulation with the arrows showing the direction of flow.

Questions?