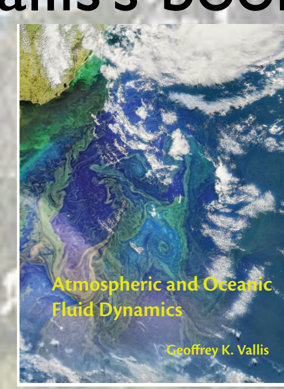
Parametrization of eddy fluxes in Eulerian formulation (Gent-McWilliams)

Paola Cessi
Scripps Institution of Oceanography
University of California, San Diego

With help from Cesar Rocha and Vallis's book (2nd ed.)



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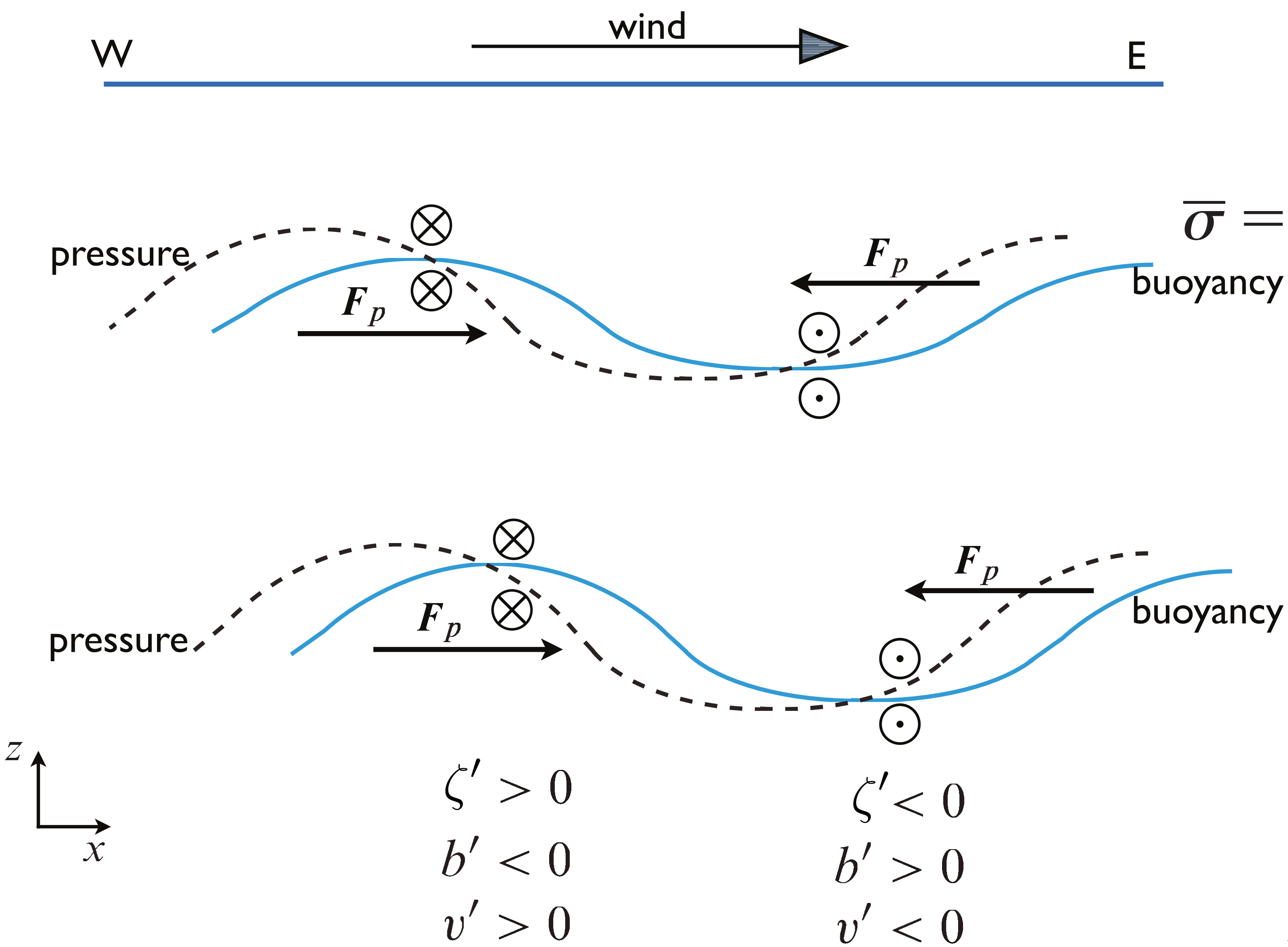
$\overline{\zeta' m'_x}$: form-stress



$$\overline{\sigma m_{\tilde{x}}} = \overline{\sigma}\,\overline{m}_{\tilde{x}} + (\overline{\zeta' m'_{\tilde{x}}})_{\tilde{b}} + \left(\frac{1}{2}\overline{\zeta'^2}\right)_{\tilde{x}}$$

$$\overline{\sigma} = \overline{\zeta}_{\tilde{b}} = -\overline{m}_{\tilde{b}\tilde{b}}.$$

In QG-TEM

$$f_0\, v' = p'_x$$

$$v'q' = -\frac{\partial}{\partial y}(u'v') + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}v'b'\right) + \frac{1}{2}\frac{\partial}{\partial x}\left((v'^2 - u'^2) - \frac{b'^2}{N^2}\right).$$

On a constant buoyancy surface $b = \bar{b}(z) + b'(x, y, z, t) \Longrightarrow \zeta' \approx -\frac{b'}{\bar{b}_z} = -\frac{b'}{N^2}$

Form-stress transfers momentum vertically (or across buoyancies)

Using residual velocities in a prognostic model: cartesian coordinates

$$\hat{u}_t + \hat{u}\hat{u}_x + \hat{v}\hat{u}_y + w^\sharp \hat{u}_z - f\hat{v} + p^\sharp_x + \nabla \cdot \mathbf{E}^u = 0,$$

$$\hat{v}_t + \hat{u}\hat{v}_x + \hat{v}\hat{v}_y + w^\sharp \hat{v}_z + f\hat{u} + p^\sharp_y + \nabla \cdot \mathbf{E}^v = 0,$$

$$p^\sharp_z = b^\sharp,$$

$$\hat{u}_x + \hat{v}_y + w^\sharp_z = 0,$$

$$b^\sharp_t + \mathbf{u}^\sharp \cdot \nabla b^\sharp = \hat{\varpi}.$$

diabatic effects

(1) Only residual velocity appears.

(2) All tracers are advected by the residual velocity.

(3) Eddy effects are confined to the momentum equations, and appear in EP vectors.

The residual velocity is: $\boldsymbol{u}^\sharp = \hat{u}\boldsymbol{i} + \hat{v}\boldsymbol{j} + \underbrace{(\bar{z}_t + \hat{u}\bar{z}_x + \hat{v}\bar{z}_y)}_{=w^\sharp}\boldsymbol{k}$

$$\hat{u} = \frac{\overline{\sigma u}}{\overline{\sigma}} \qquad \sigma = \frac{1}{b_z}$$

An eulerian observer at (x,y,z,t) is at the mean depth z of some buoyancy surface. This defines

$$b^\sharp(x, y, z, t)$$

Parametrization of EP fluxes

A model in terms of the TWA fields requires parametrizing the EP fluxes

$$\overline{\zeta' m'_x} = -\mu \bar{\sigma} \hat{u}_z$$
$$\overline{\zeta' m'_y} = -\mu \bar{\sigma} \hat{v}_z$$

Vertical viscosity of horizontal momentum (Rhines and Young, 1982)

This is equivalent to adding extra velocities to the Coriolis terms such that

$$f v^* \equiv \left(\frac{\overline{\zeta' m'_x}}{\bar{\sigma}} \right)_z \qquad f u^* \equiv - \left(\frac{\overline{\zeta' m'_y}}{\bar{\sigma}} \right)_z$$

If $\hat{u}$ $\hat{v}$ are in geostrophic balance, then $(u^*, v^*) = \left(\kappa_a \frac{\nabla \rho^\sharp}{\rho^\sharp_z} \right)_z$

With $\kappa_a = \mu f^{-2} \bar{\sigma}^{-1} = \mu f^{-2} \bar{N}^2$

The parametrization for the extra velocity is equivalent to Gent-McWilliams scheme with diffusivity κ_a

Prognostic Residual Mean Flow in an Ocean General Circulation Model and its Relation to Prognostic Eulerian Mean Flow
Saenz, J.A. et al. JPO 2017. https://doi.org/10.1175/JPO-D-15-0024.1

Comparison to a Eulerian model

A model in terms of the **Eulerian** fields requires parametrizing the eddy-fluxes: start with the buoyancy fluxes (no momentum fluxes)

$$\frac{D\overline{u}^z}{Dt} - f\overline{v}^z + \frac{\partial p}{\partial x} = \overline{R}_x,$$

$$\frac{D\overline{v}^z}{Dt} + f\overline{u}^z + \frac{\partial p}{\partial y} = \overline{R}_y,$$

$$\frac{\partial p}{\partial z} = b,$$

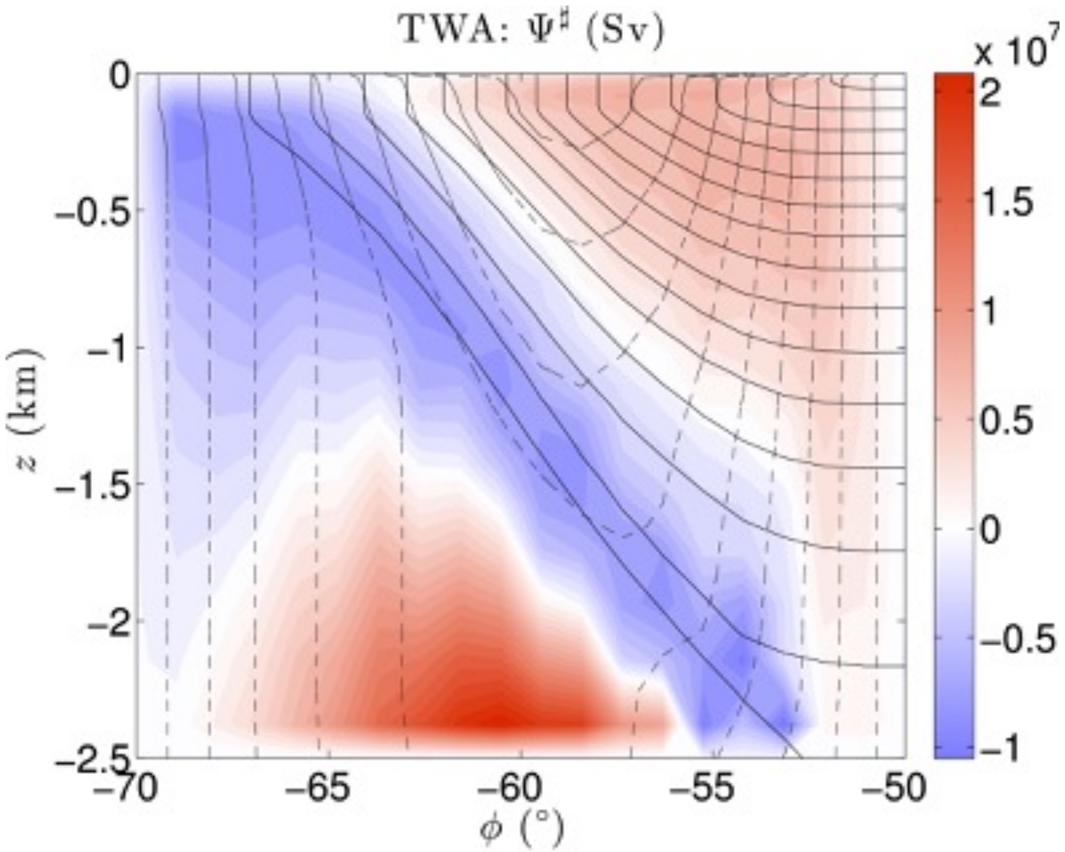
$$\frac{\partial(\overline{u}^z + u_*^z)}{\partial x} + \frac{\partial(\overline{v}^z + v_*^z)}{\partial y} + \frac{\partial(\overline{w}^z + w_*^z)}{\partial z} = 0,$$

$$\frac{Db}{Dt} + u_*^z\frac{\partial b}{\partial x} + v_*^z\frac{\partial b}{\partial y} + w_*^z\frac{\partial b}{\partial y} = 0,$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{u}^z\frac{\partial}{\partial x} + \overline{v}^z\frac{\partial}{\partial y} + \overline{w}^z\frac{\partial}{\partial z},$$

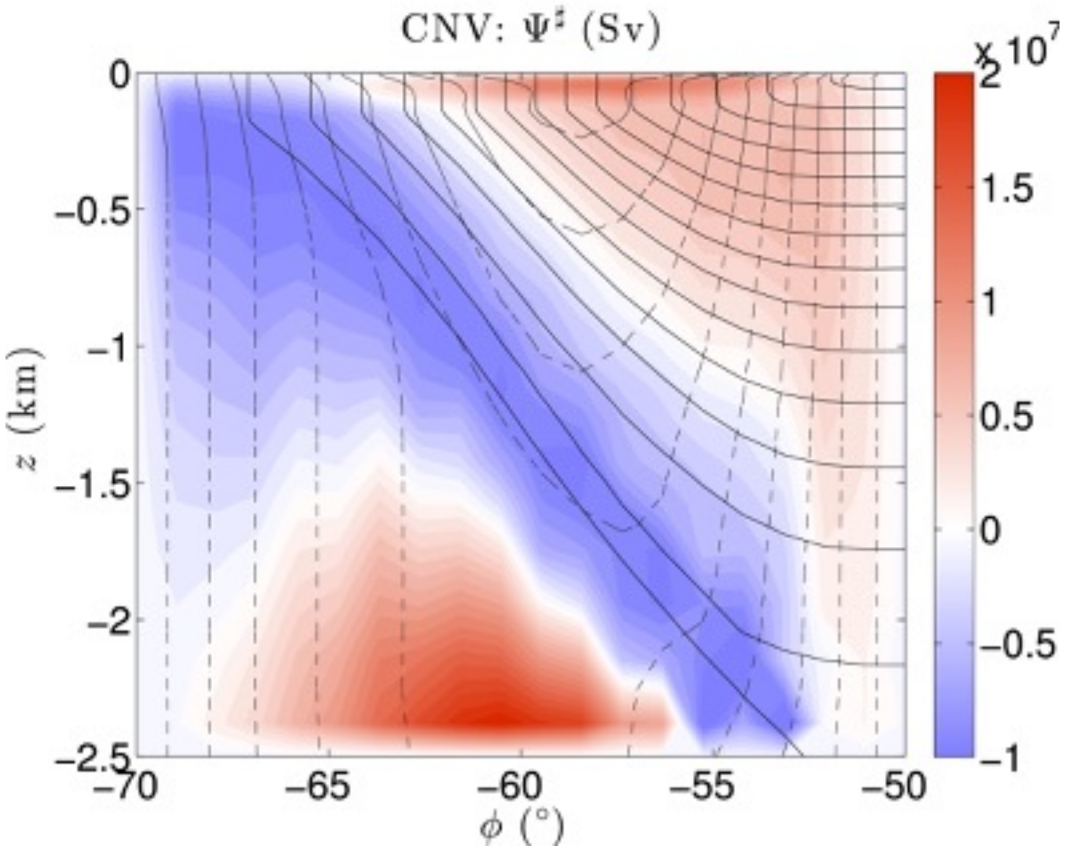
$$u_*^z = -\frac{\partial}{\partial z}\left(\kappa\frac{\overline{\rho}_x^z}{\overline{\rho}_z^z}\right), \quad \text{and} \quad v_*^z = -\frac{\partial}{\partial z}\left(\kappa\frac{\overline{\rho}_y^z}{\overline{\rho}_z^z}\right),$$

Implementation in a numerical model of the ACC



Residual overturning using TWA model EP fluxes parametrized as vertical viscosity

With $\kappa_a = \mu f^{-2} \bar{\sigma}^{-1} = \mu f^{-2} \bar{N}^2$



Residual overturning using conventional Eulerian mean, parametrized buoyancy fluxes, assuming $(\hat{u}, \hat{v}) = (\bar{u}, \bar{v}) + (u^*, v^*)$

Quantitative agreement, because eddy mom. flux is negligible.

Prognostic Residual Mean Flow in an Ocean General Circulation Model and its Relation to Prognostic Eulerian Mean Flow
Saenz, J.A. et al. JPO 2017. https://doi.org/10.1175/JPO-D-15-0024.1

Summary

Residual mean formalism is very useful to capture the effect of eddy-fluxes on buoyancy transport (and possibly other tracers).

TWA places eddy-effect in EP flux divergence in the momentum equation using a single velocity. Not clear how to get the Eulerian flow (is it needed?)

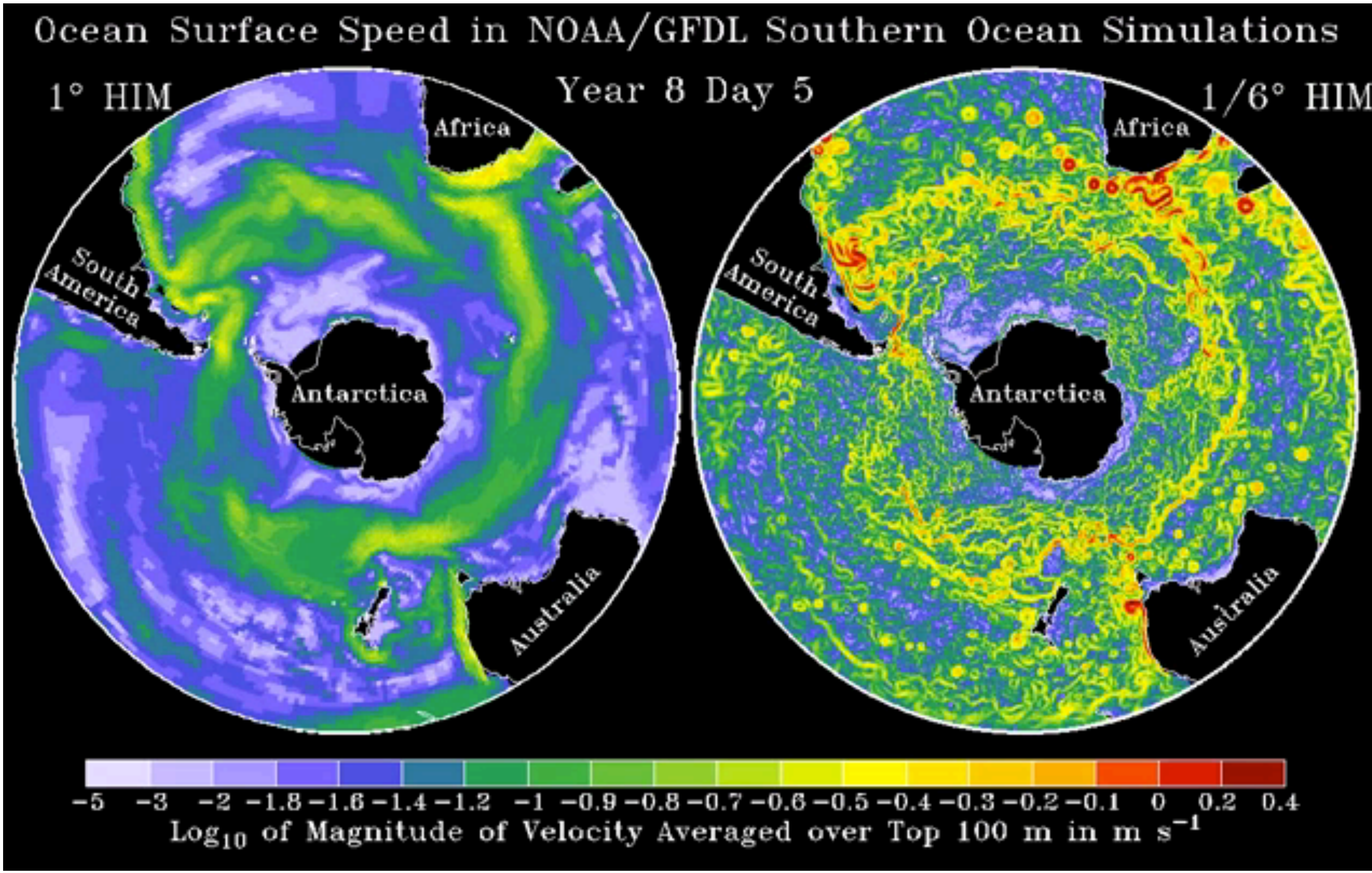
Not widely implemented yet, but it can and has been done in the ACC setting.

Agrees with parametrization of eddy-fluxes, if confined to buoyancy fluxes.

Not clear how to parametrize all of the EP fluxes (momentum), which are important for jets formation and maintenance.

Climate models do not resolve mesoscale eddies

Speeds are weaker and the eddy fluxes are missed: a serious problem in the ACC



Source: Hallberg @ GFDL

How can we parametrize eddy fluxes of tracers?

Beyond Fickian diffusion

A general approach to diffusive fluxes of a tracer φ :

$$\frac{D\varphi}{Dt} = D$$ The tracer is conserved, except for small diffusive processes

$$\frac{D\overline{\varphi}}{Dt} = -\nabla\cdot\overline{\boldsymbol{v}'\varphi'} + \bar{D}\bar{\varphi}$$ We are interested in large-scales, long times (the “mean”)

$$\frac{1}{2}\frac{\partial}{\partial t}\overline{\varphi'^2} + \overline{\boldsymbol{v}'\varphi'}\cdot\nabla\overline{\varphi} + \frac{1}{2}\overline{\boldsymbol{v}}\cdot\nabla\overline{\varphi'^2} + \frac{1}{2}\nabla\cdot\overline{\boldsymbol{v}'\varphi'^2} = \overline{D'\varphi'}$$ The variance equation

$$\overline{\boldsymbol{v}'\varphi'}\cdot\nabla\overline{\varphi} \approx \overline{D'\varphi'} < 0$$ for large diffusion eddy flux is downgradient $\overline{\boldsymbol{v}'\varphi'} = -\kappa\nabla\overline{\varphi}$

$$\overline{\boldsymbol{v}'\varphi'}\cdot\nabla\overline{\varphi} \approx 0$$ for weak diffusion eddy flux is **orthogonal** to the mean gradient

Generalize to $\overline{\boldsymbol{v}'\varphi'} = -\boldsymbol{K}\nabla\overline{\varphi}$ where $\boldsymbol{K} = \boldsymbol{S} + \boldsymbol{A}$ is a general tensor

$$\boldsymbol{S} = \begin{pmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{pmatrix}$$ diffusive $$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & -\kappa_a'^x \\ 0 & 0 & -\kappa_a'^y \\ \kappa_a'^x & \kappa_a'^y & 0 \end{pmatrix}$$ advective: $-(\boldsymbol{A}\nabla\bar{\varphi})\cdot\nabla\bar{\varphi} = 0$

Example for a passive tracer, c, in 2-D (I)

Steady state advection diffusion $J(\psi, c) = \kappa \nabla^2 c$ where $J(a,b) \equiv a_x b_y - a_y c_x$

Assume ψ is a cellular, eddy-like flow, and c has a large-scale gradient $\boldsymbol{G} \equiv (G_x, G_y)$

$$c(x,y,t) = \boldsymbol{G} \cdot \boldsymbol{x} + c'(x,y)$$

With diffusion and no advection, $c' = 0$

c' satisfies $\boldsymbol{u} \cdot \nabla c' - \kappa \nabla^2 c' = -u G_x - v G_y$

with solution $c' = -a(x,y) G_x - b(x,y) G_y$

satisfying $\boxed{(\boldsymbol{u} \cdot \nabla - \kappa \nabla^2)(a,b) = (u,v)}$

The total flux is $\boldsymbol{F} \equiv \langle \boldsymbol{u} c - \kappa \nabla c \rangle = -\kappa \boldsymbol{G} + \langle \boldsymbol{u} c \rangle$ Area average: $\langle \;\; \rangle$

Giving the flux-gradient relation:

$$\begin{pmatrix} F_x \\ F_y \end{pmatrix} = - \begin{bmatrix} \kappa + \langle ua \rangle & \langle ub \rangle \\ \langle va \rangle & \kappa + \langle vb \rangle \end{bmatrix} \begin{pmatrix} G_x \\ G_y \end{pmatrix}$$

or $\boldsymbol{F} = -\boldsymbol{K} \boldsymbol{G}$

$\boldsymbol{K}$ is the diffusion tensor

Example for a passive tracer, c, in 2-D (II)

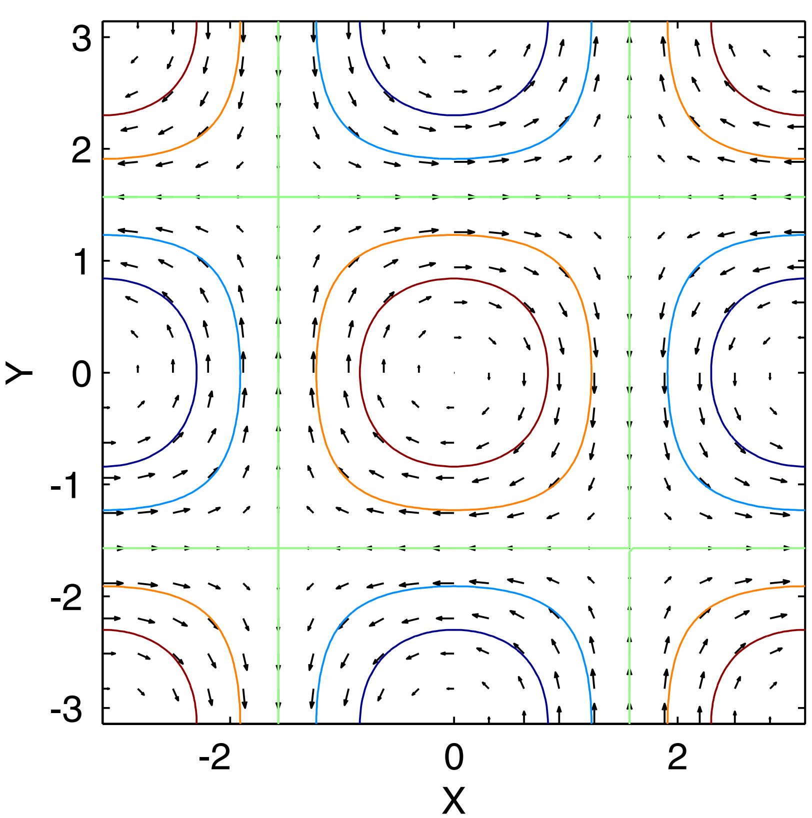
Multiply $\boxed{(\boldsymbol{u}\cdot\nabla - \kappa\nabla^2)(a,b) = (u,v)}$ by a or b and integrate over domain

to find $\kappa\langle\nabla a\cdot\nabla a\rangle = \langle ua\rangle,\qquad \kappa\langle\nabla b\cdot\nabla b\rangle = \langle vb\rangle$ and

$\langle\psi J(a,b)\rangle + \kappa\langle\nabla a\cdot\nabla b\rangle = \langle ub\rangle,\quad -\langle\psi J(a,b)\rangle + \kappa\langle\nabla a\cdot\nabla b\rangle = \langle va\rangle$ then

$\boldsymbol{K} = \boldsymbol{K}^{(s)} + \boldsymbol{K}^{(a)}$ with $\boldsymbol{K}^{(a)} = \begin{bmatrix} 0 & \langle\psi J(a,b)\rangle \\ -\langle\psi J(a,b)\rangle & 0 \end{bmatrix}$ and

$$\boldsymbol{K}^{(s)} = \begin{bmatrix} \kappa + \kappa\langle\nabla a\cdot\nabla a\rangle & \kappa\langle\nabla a\cdot\nabla b\rangle \\ \kappa\langle\nabla a\cdot\nabla b\rangle & \kappa + \kappa\langle\nabla b\cdot\nabla b\rangle \end{bmatrix}$$



Highly symmetric cellular flows like

$$\psi = \psi_{\max}\cos(kx)\cos(ky)$$

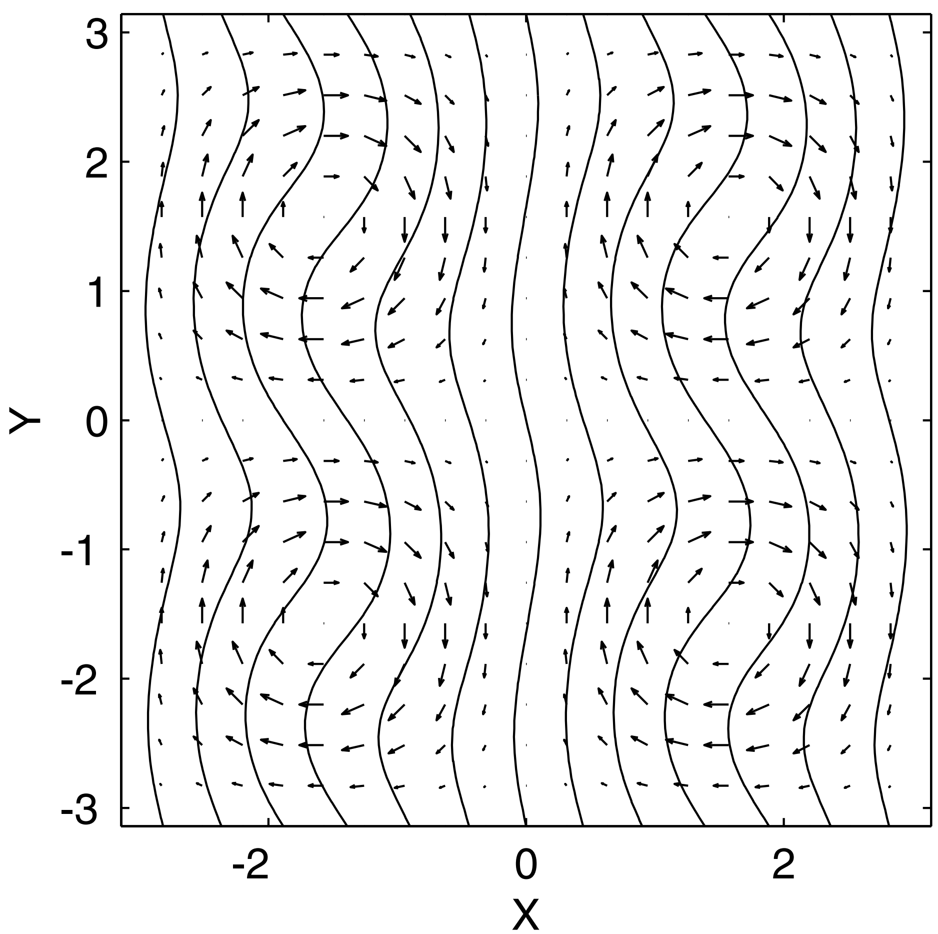
have $\boldsymbol{K}^{(a)} = \mathbf{0}$ regardless of the Peclet number

$$p \equiv \frac{\psi_{max}}{\kappa}$$

Example for a passive tracer, c, in 2-D (III)

A cellular flow with co-rotating eddies $\psi = \sin^2 x \sin^2 y$

has eddy-induced velocities and $\boldsymbol{K}^{(a)} \neq 0$



We can solve approximately, for small p

$$\nabla^2(a,b) = -(u,v) + p\,\boldsymbol{u}\cdot\nabla(a,b)$$

To obtain $\boldsymbol{K}$

This example shows that the eddy-diffusivity is a property of the flow, equal for all passive tracers

For small Peclet, the symmetric component dominates the diffusion tensor
It is not clear which dominates for large Peclet

Desirable properties of advective flux in oceanography

Mimic quasi-adiabatic baroclinic instability:

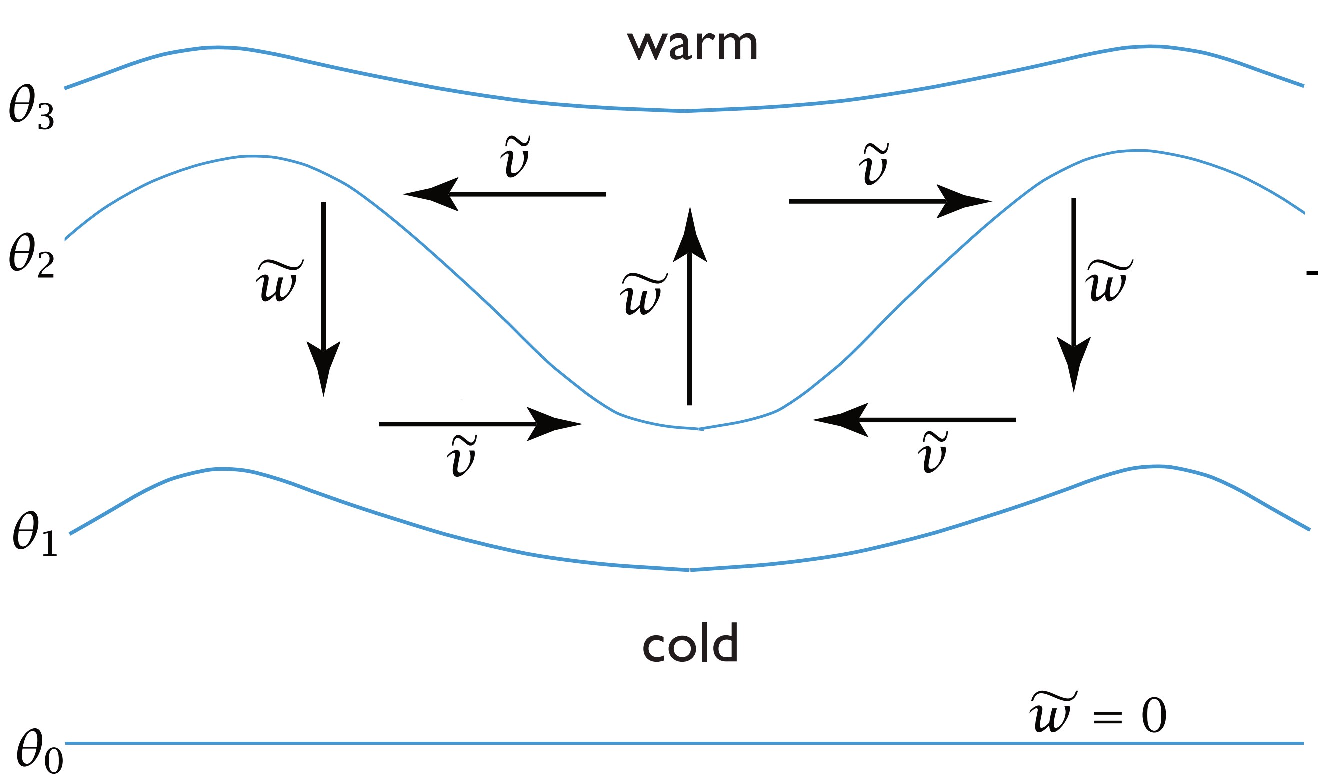
1) The tracers and its moments should be preserved: guaranteed with skew flux

2) The skew flux should decrease the mean available potential energy

$$(\kappa_a'^x, \kappa_a'^y) = \kappa_a (z_{\tilde{x}}, z_{\tilde{y}})|_\rho = -\kappa_a \frac{\nabla_h \bar{\rho}}{\bar{\rho}_z}$$

$$\overline{\mathbf{u}'\rho'} = -\kappa_a \nabla_h \bar{\rho}$$ Downgradient horizontal diffusion of density

$$\overline{w'\rho'} = \kappa_a \left(\frac{\nabla_h \bar{\rho}}{\bar{\rho}_z} \right)^2 \bar{\rho}_z$$ Upgradient vertical diffusion of density



The total flux is neither up- nor downgradient

The total flux tends to flatten isopycnals: mimicking baroclinic instability

Gent & McWilliams, JPO (the most cited JPO paper ⋆), 1990 Griffies, JPO, 1998

Equivalence to a large-scale flow

An antisymmetric tensor diffusivity is equivalent to an incompressible velocity

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & -\kappa_a'^x \\ 0 & 0 & -\kappa_a'^y \\ \kappa_a'^x & \kappa_a'^y & 0 \end{pmatrix}$$

$$\boxed{\tilde{v}_n = -\partial_m A_{mn},}$$

HMWK: prove this equivalence.

In the GM case we have

$$\tilde{\mathbf{u}} = \left(\kappa_a \frac{\nabla_h \rho}{\rho_z}\right)_z \qquad \tilde{w} = -\nabla_h \cdot \left(\kappa_a \frac{\nabla_h \rho}{\rho_z}\right)$$

The associated “eddy” velocity is proportional to the gradients of the isopycnal slope

Much work has gone into finding the dependence of κ_a on mean quantities

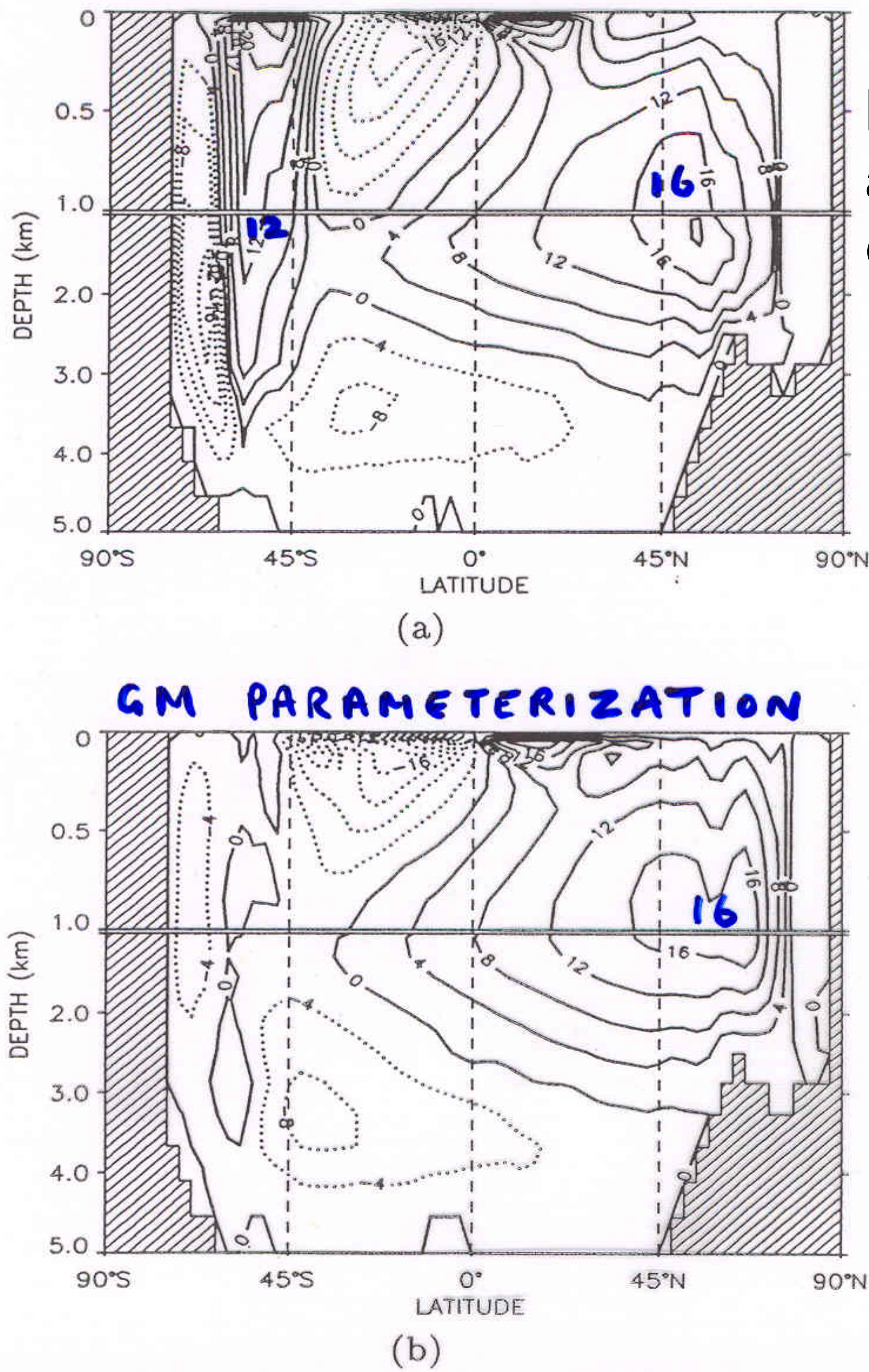
M. Visbeck et al., 1997: Specification of eddy transfer coefficients in coarse-resolution ocean circulation models. JPO

P. Cessi, 2008: An energy-constrained parametrization of eddy buoyancy flux. JPO

J. Mak et al., 2017: Emergent eddy saturation from an energy constrained eddy parameterisation. Ocean Modeling

NCOM - NCAR CSM OCEAN MODEL

HORIZONTAL TRACER MIXING



Pre-GM, buoyancy eddy-flux modeled as horizontal diffusion, with large diapycnal mixing for sloping isopycnal.

With GM Almost complete cancellation of overturning in ACC region, and less diapycnal mixing

There once was an ocean model called MOM,
That occasionally used to bomb,
But eddy advection, and much less convection,
Turned it into a stable NCOM.

(Limerick by Peter Gent)

Figure 4. Annual-mean zonally integrated meridional overturning streamfunction obtained with (a) Eulerian-mean velocity for H1 and (b) effective transport velocity for I1. Contour interval is $4 Sv$.

Summary so far

In geophysical flows the Peclet number is large and the expectation is that eddies transport tracers along, rather than across, isopycnals.

The breakthrough is to realize that this can be achieved by an antisymmetric diffusion tensor.

By making the component of the tensor proportional to the isopycnal slope, restratification of the fluid is achieved, mimicking the decrease in APE by baroclinic eddies.

Open questions remain on the coefficient in front of the slope, which is certainly not a constant.

Nothing about the eddy-momentum fluxes, yet.

The GM parametrization is useful for idealized theories of the deep and abyssal stratification and overturning circulation, and in coarse-resolution climate models.

Global stratification and overturning circulation

Paola Cessi
Scripps Institution of Oceanography
University of California, San Diego

With help from Spencer Jones, Christopher Wolfe and Vallis's book (2nd ed.)

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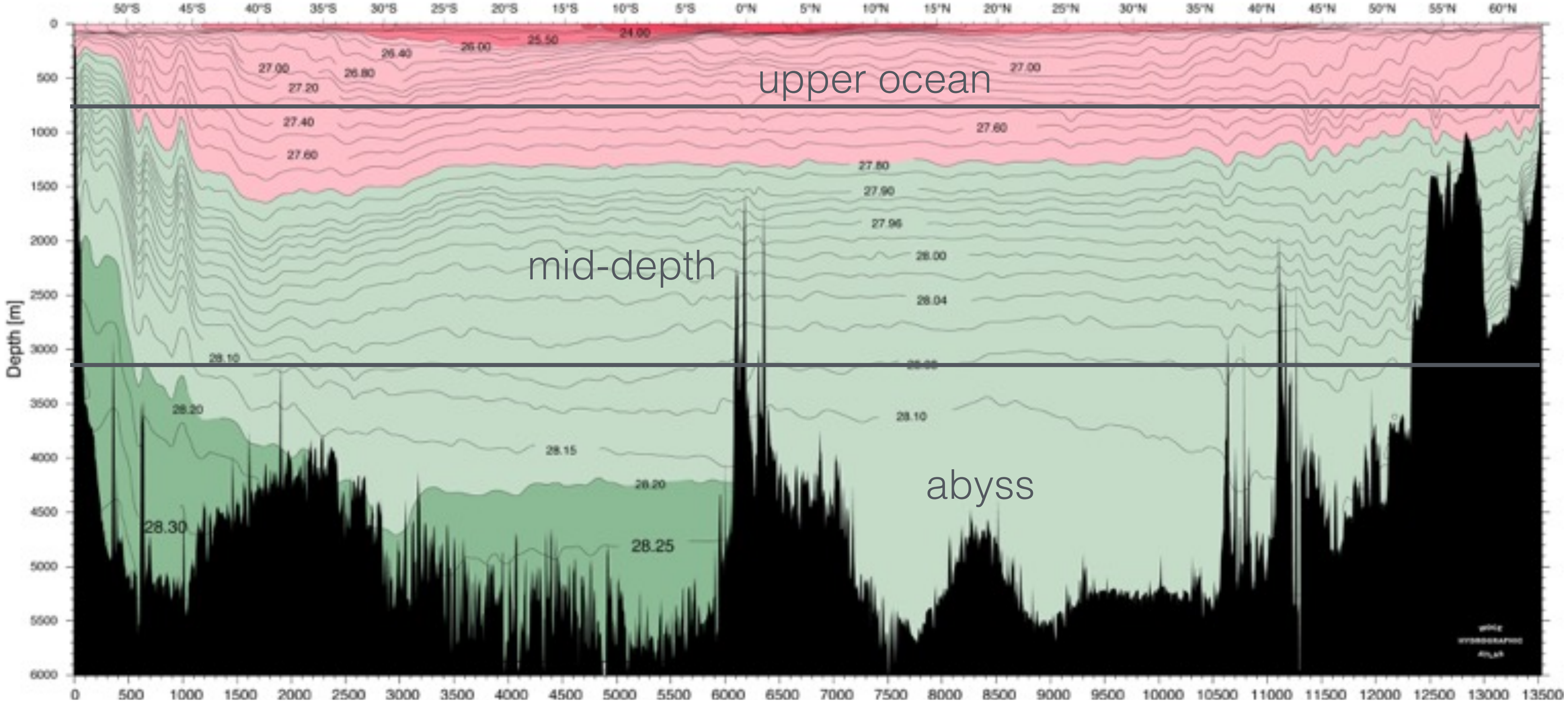
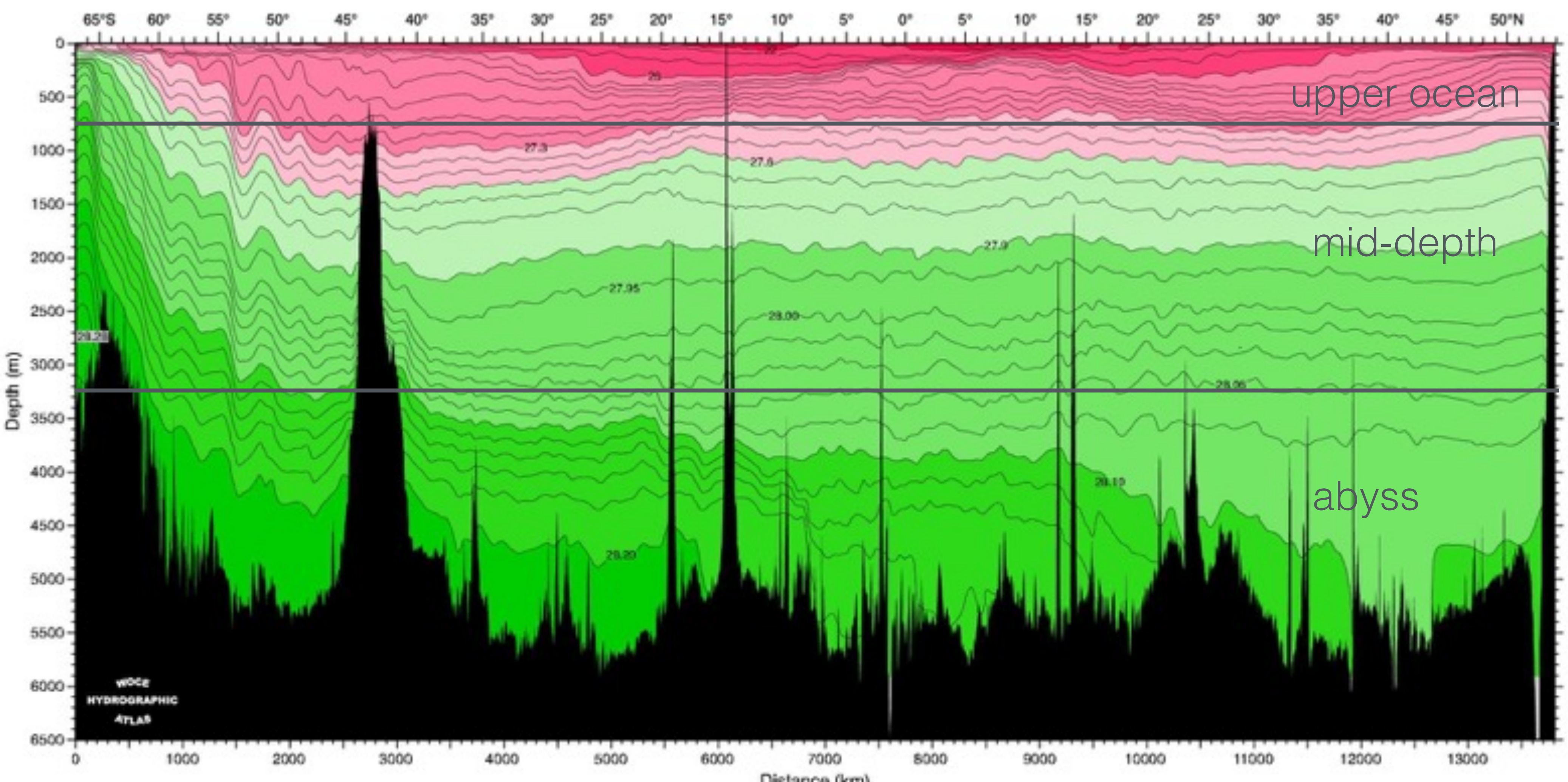
ACC is crucial for deep and abyssal global stratification



neutral density at 25°W (Atl.)



neutral density at 165°W (Pac.)

mid-depth and abyssal stratifications are set in the ACC

Stratification and MOC in the ACC: effect of wind

Buoyancy balance in the Antarctic Circumpolar Current

$$b_t + (ub)_x + (vb)_y + (wb)_z = (\kappa_v b_z)_z$$

Zonally and time-average

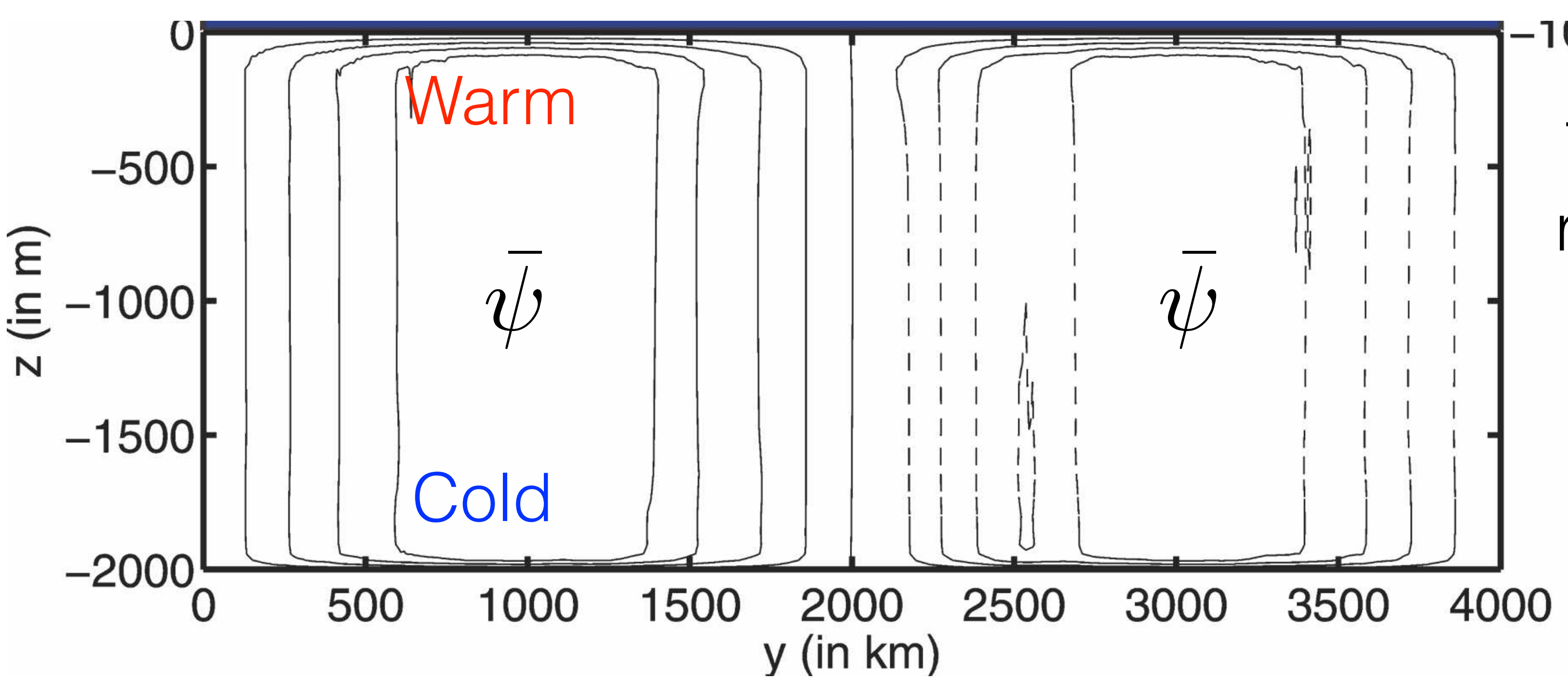
$$\bar{v}\bar{b}_y + \bar{w}\bar{b}_z + (\overline{v'b'})_y + (\overline{w'b'})_z = (\kappa_v \bar{b}_z)_z$$

In the top Ekman layer $\bar{V} = -\dfrac{\tau}{\rho f}$ In the bottom Ekman layer $\bar{V} = \dfrac{\tau}{\rho f}$

Below the Ekman layer and above the bottom Ekman layer

$$\bar{p}_x = 0 \Longrightarrow \quad \bar{v} = 0, \quad \bar{w} = -\left(\frac{\tau}{\rho f}\right)_y \quad \Longrightarrow \bar{\psi} = -\frac{\tau(y)}{\rho f}$$

$\tau > 0$ $\tau < 0$



The mean overturning circulation reaches the bottom

In a thermally stratified fluid it transports heat **equatorward** for westerly wind-stress $\tau > 0$

Stratification in the ACC: effect of eddies

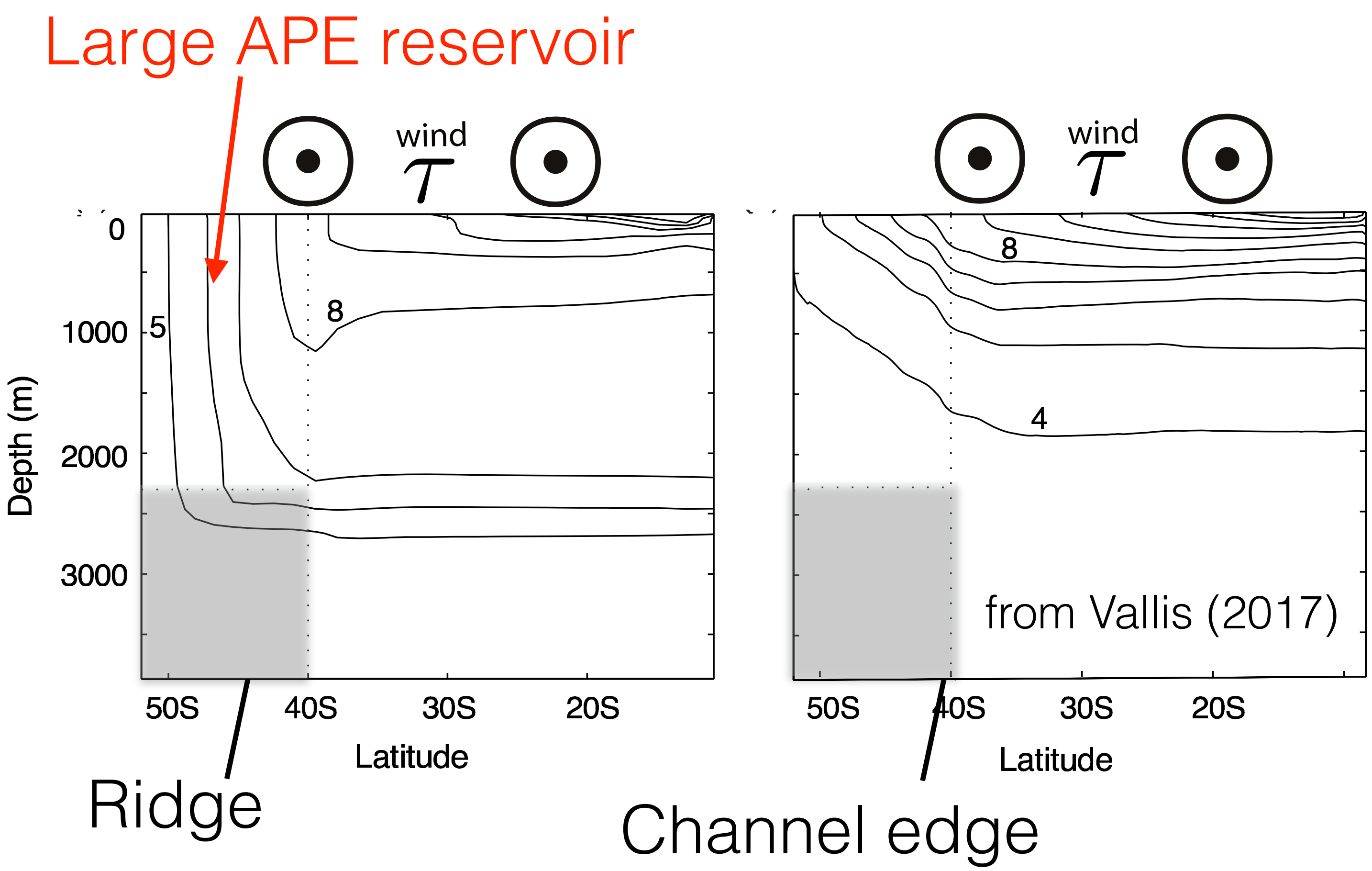
Buoyancy balance in the Antarctic Circumpolar Current

$$b_t + (ub)_x + (vb)_y + (wb)_z = (\kappa_v b_z)_z$$

Zonally and time-average

$$\bar{v}\bar{b}_y + \bar{w}\bar{b}_z + (\overline{v'b'})_y + (\overline{w'b'})_z = (\kappa_v \bar{b}_z)_z$$

For adiabatic eddies $-\dfrac{\overline{w'b'}}{\bar{b}_y} = \dfrac{\overline{v'b'}}{\bar{b}_z} \quad \Longrightarrow \quad \bar{\psi}^* = \dfrac{\overline{v'b'}}{\bar{b}_z}$



Advection by residual=eulerian + eddy

$$J_{y,z}(\bar{\psi} + \psi^*, \bar{b}) = (\kappa_v \bar{b}_z)_z$$

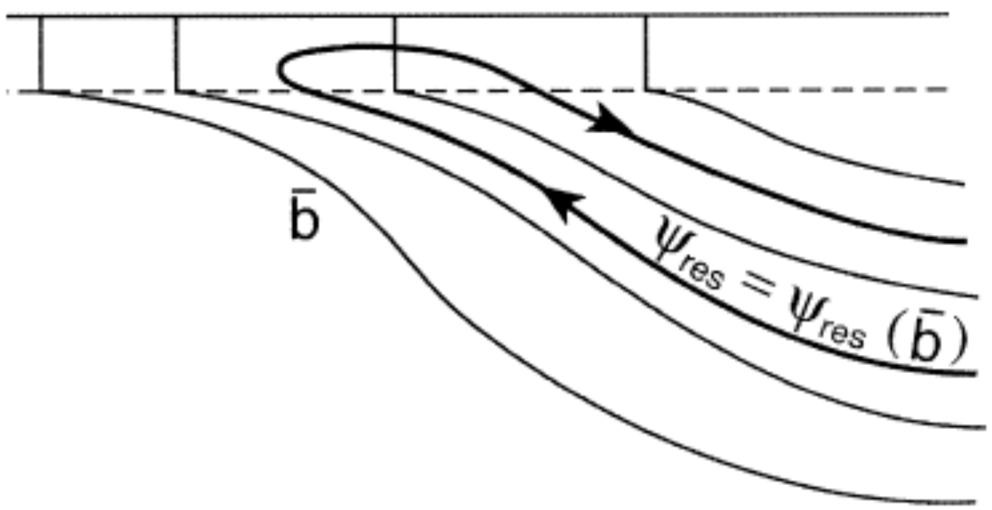
No eddies: large APE

With eddies: APE is converted

The adiabatic limit

Advection by eddy-fluxes **cancels** advection by mean flow

$J_{y,z}(\bar{\psi} + \psi^*, \bar{b}) = 0$ $\implies \bar{\psi} + \psi^* = \mathcal{F}(\bar{b})$



Residual flow is constant on mean buoyancy

Functional relation $\mathcal{F}$ is determined by diabatic processes and/or at open boundary

To make progress we need to specify $\mathcal{F}$ **and** relate ψ^* to $\bar{b}$

GM to the rescue! $\overline{v'b'} = -\kappa_a \bar{b}_y \implies \psi^* = -\kappa_a \frac{\bar{b}_y}{\bar{b}_z}$

The compensated solution

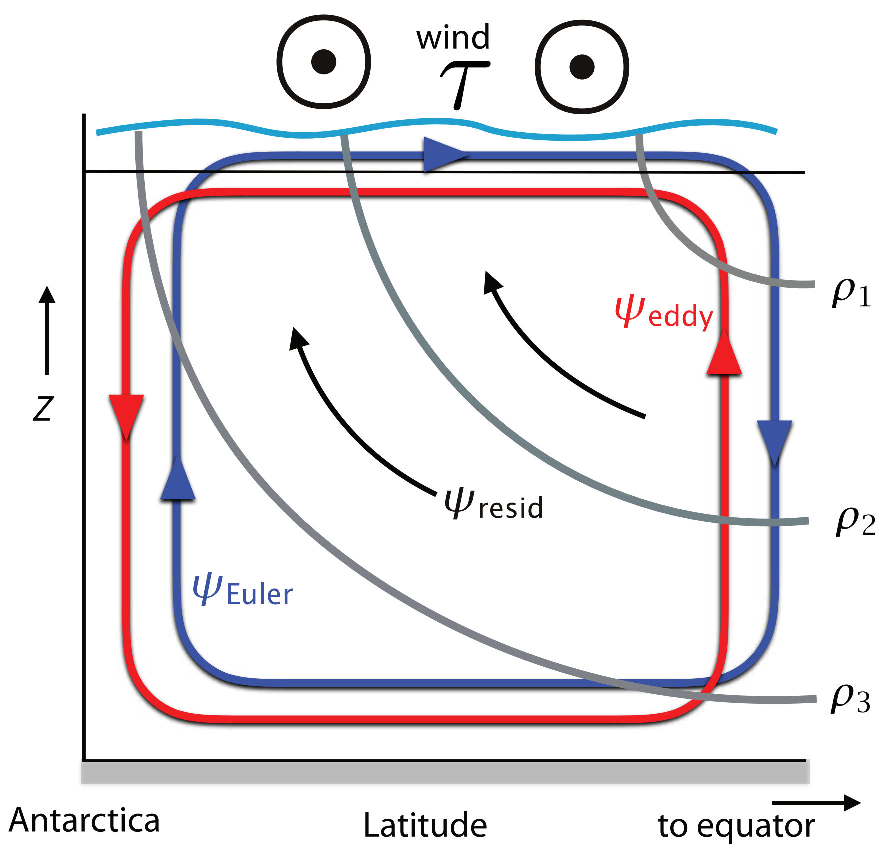
Zero residual flow $\bar{\psi} + \psi^* = 0$ Using Ekman and GM $\boxed{-\frac{\tau(y)}{\rho f} - \kappa_a \frac{\bar{b}_y}{\bar{b}_z} = 0}$

$\boxed{z_{\tilde{y}} = \frac{\tau(y)}{\kappa_a \rho f}}$ **Flatter for larger κ_a**

The slope of the isopycnals is independent of z

For constant κ_a linear characteristic equation for $\bar{b}$ with solution

$$\bar{b} = B\left(\kappa_a z - \int^y \frac{\tau(\hat{y})}{\rho f}\, d\hat{y}\right) \qquad @z = 0 \quad \bar{b}_s(y) = B\left(-\int^y \frac{\tau(\hat{y})}{\rho f}\, d\hat{y}\right)$$



from Vallis (2017)

The shapes of the wind and surface buoyancy give $\bar{b}$ everywhere

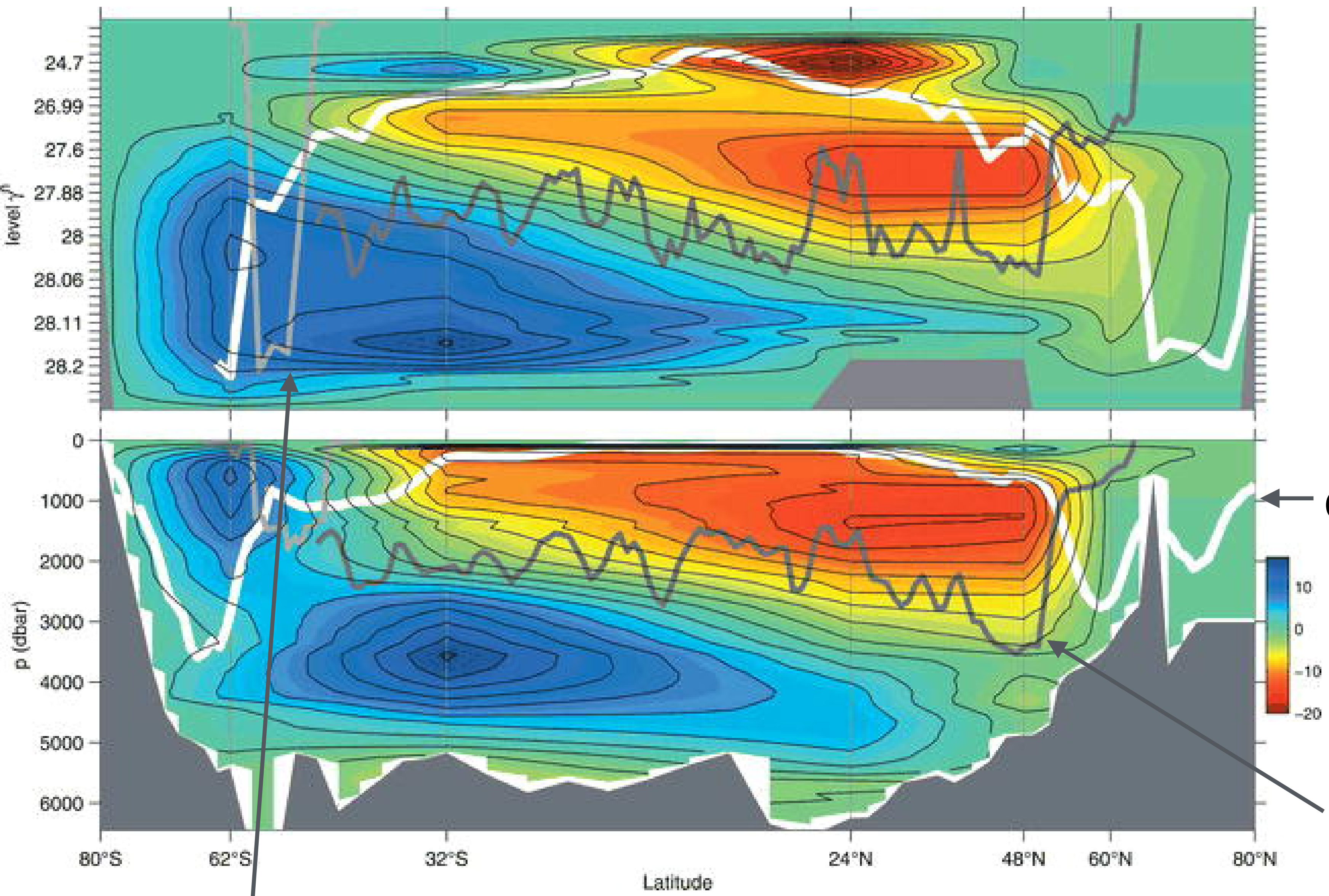
HMK: Work out an example given wind-stress and b_s

Marshall, J. and Radko, T., 2003. Residual-Mean Solutions for the Antarctic Circumpolar Current and Its Associated Overturning Circulation. J. Phys. Oceanogr.,

The ocean conveyor belt (NASA viz.)

The engine is in the Antarctic Circumpolar Current

ACC is crucial for meridional overturning circulation (MOC)



zonally integrated residual overturning (ROC)

depth of winter mixed layer

depth of ocean ridges

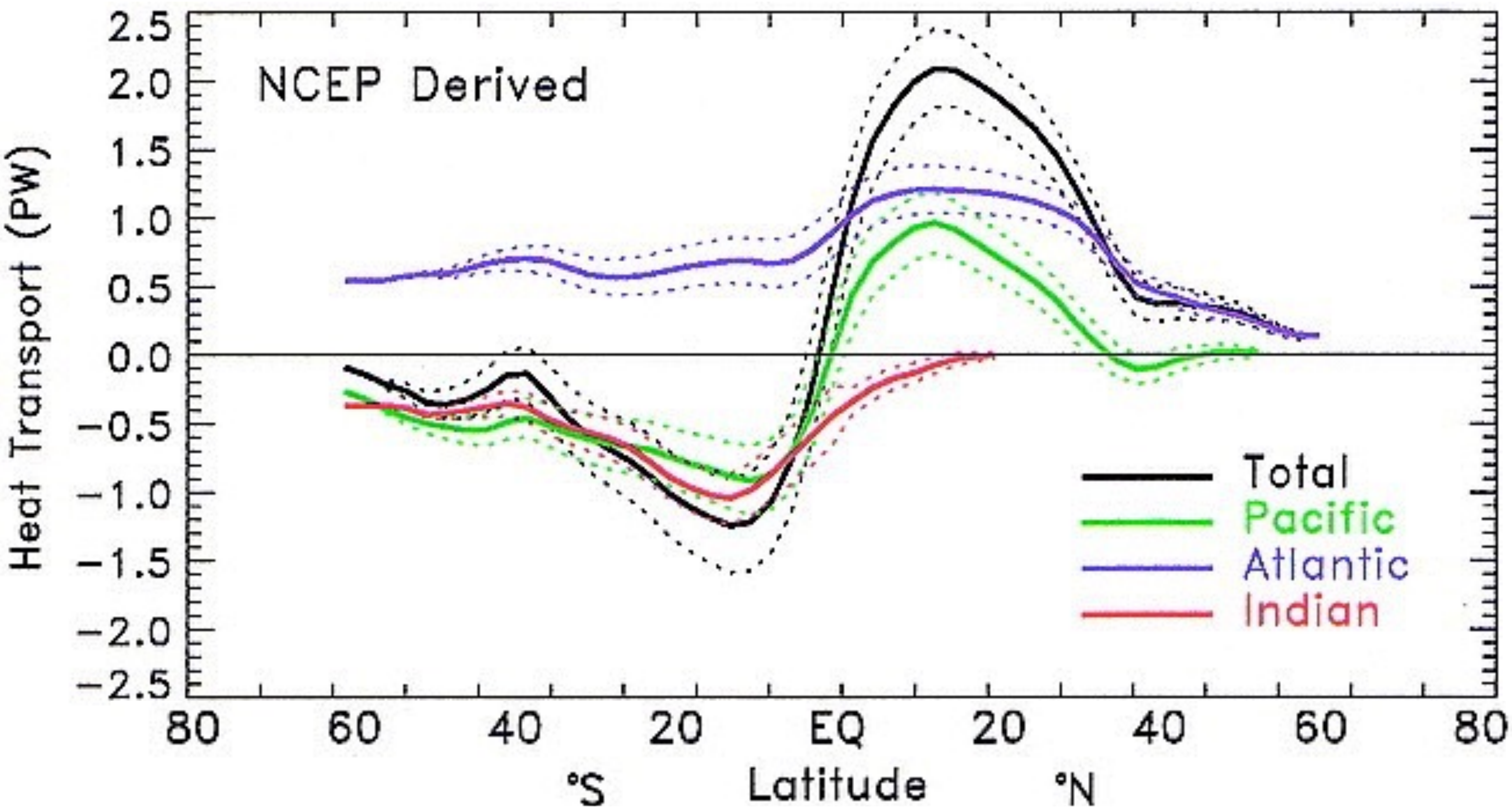
depth of circumpolar region

MOC and global intermediate/deep stratification problems are connected

MOC and global intermediate/deep stratification problems are forced in the ACC

Lumpkin, R. and Speer, K., 2007. Global ocean meridional overturning. J. Phys. Oceanogr., 37, 2550-2562.

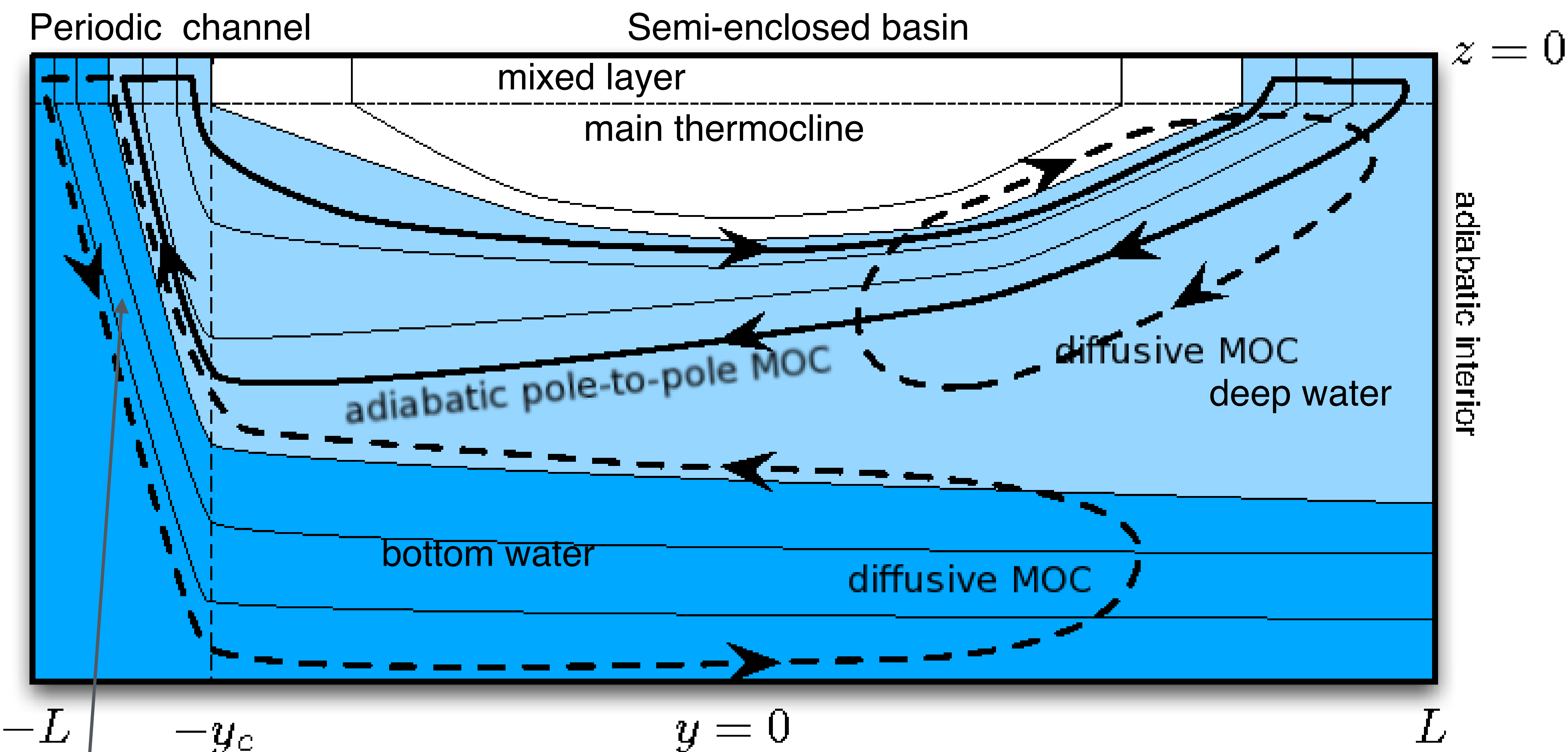
The ocean heat transport



Ocean heat transport is smaller in the Southern Hemisphere because the Atlantic transports heat northward everywhere, i.e. **equatorward** in the SH.

Thus the S. Atlantic is 1° colder than the N. Atlantic.

The quasi-adiabatic overturning in a single basin: Atlantic



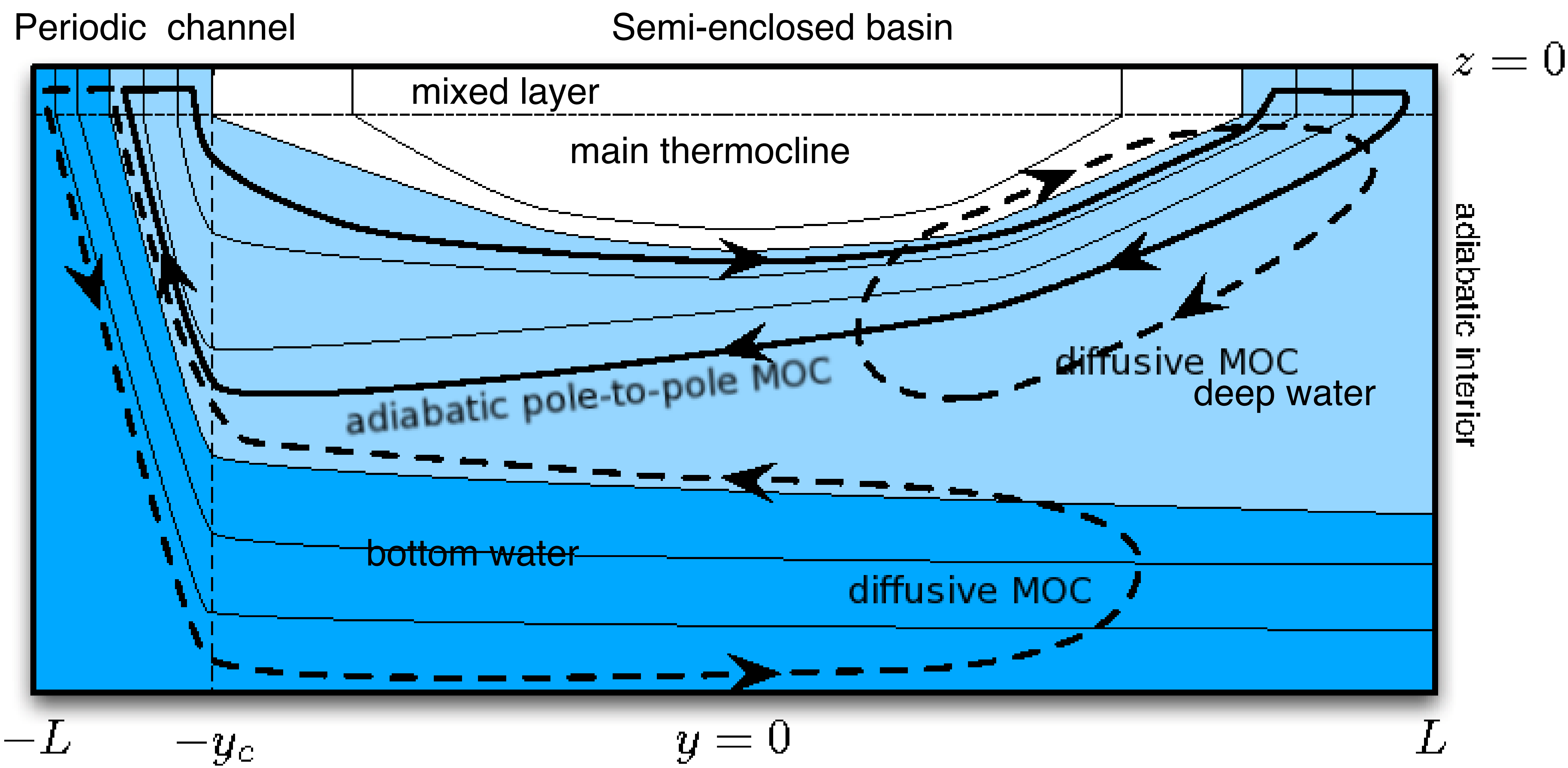
Competition between Ekman suction and eddy restratification determine stratification in ACC

Three types of isopycnals: outcrop in basin only (white); outcrop in ACC and basin (light blue); outcrop in ACC only (blue)

Three types of circulation: main thermocline (white); intermediate/deep OC (light blue); abyssal OC (blue)

Wolfe, C.L. and P. Cessi, JPO 2010

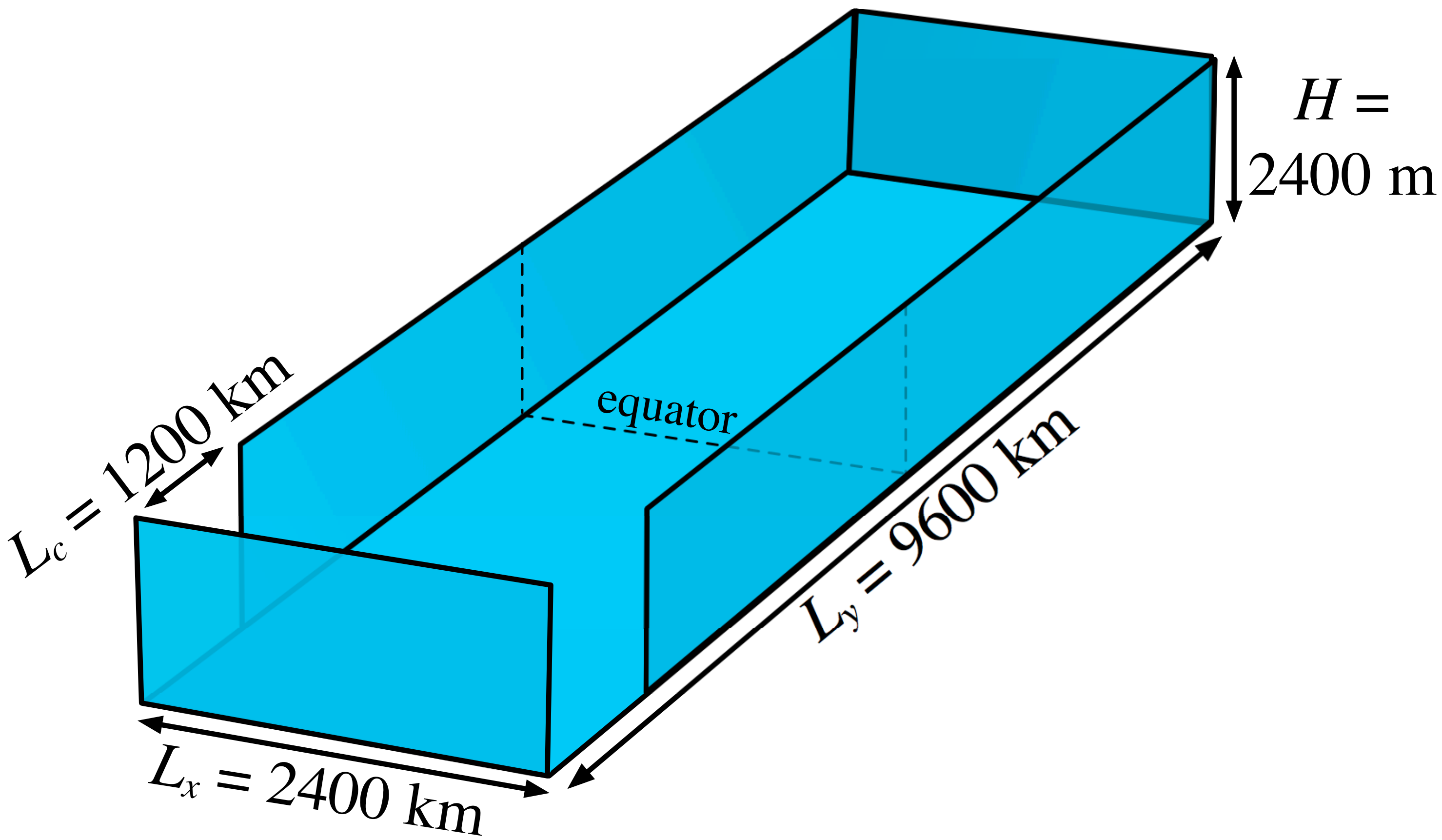
The quasi-adiabatic overturning in a single basin: Atlantic

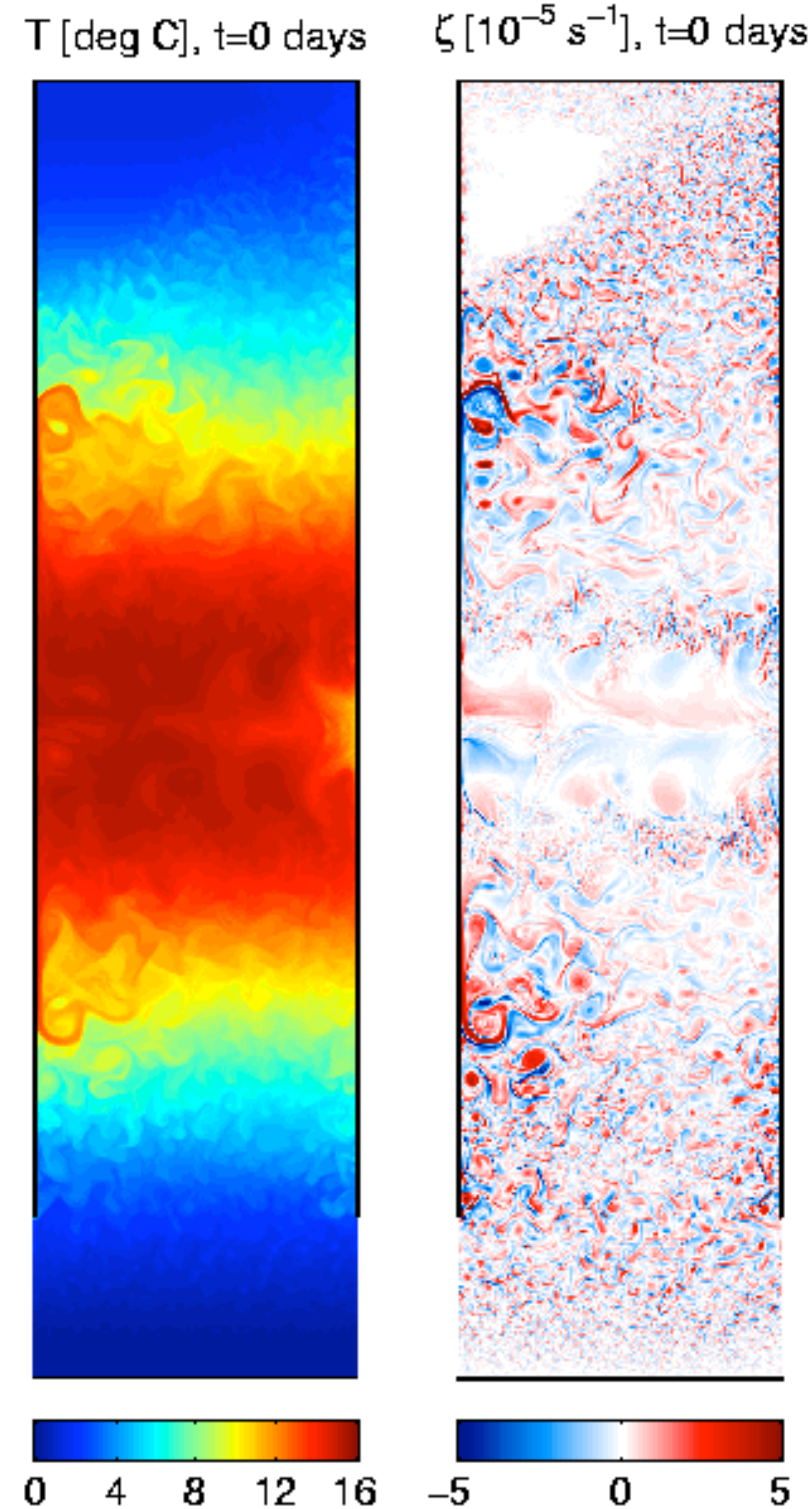


Necessary conditions for a quasi adiabatic pole-to-pole cell:
1) Isopycnal outcropping in channel and basin;
2) Diapycnal fluxes in the mixed layer

Ancillary features:
1) Weak diffusive cells in the abyss and in the NH
2) Abyssal and adiabatic cells share a boundary (Ferrari's lectures)

Testing the quasi-adiabatic MOC idea





Half-sized basin in a notched box

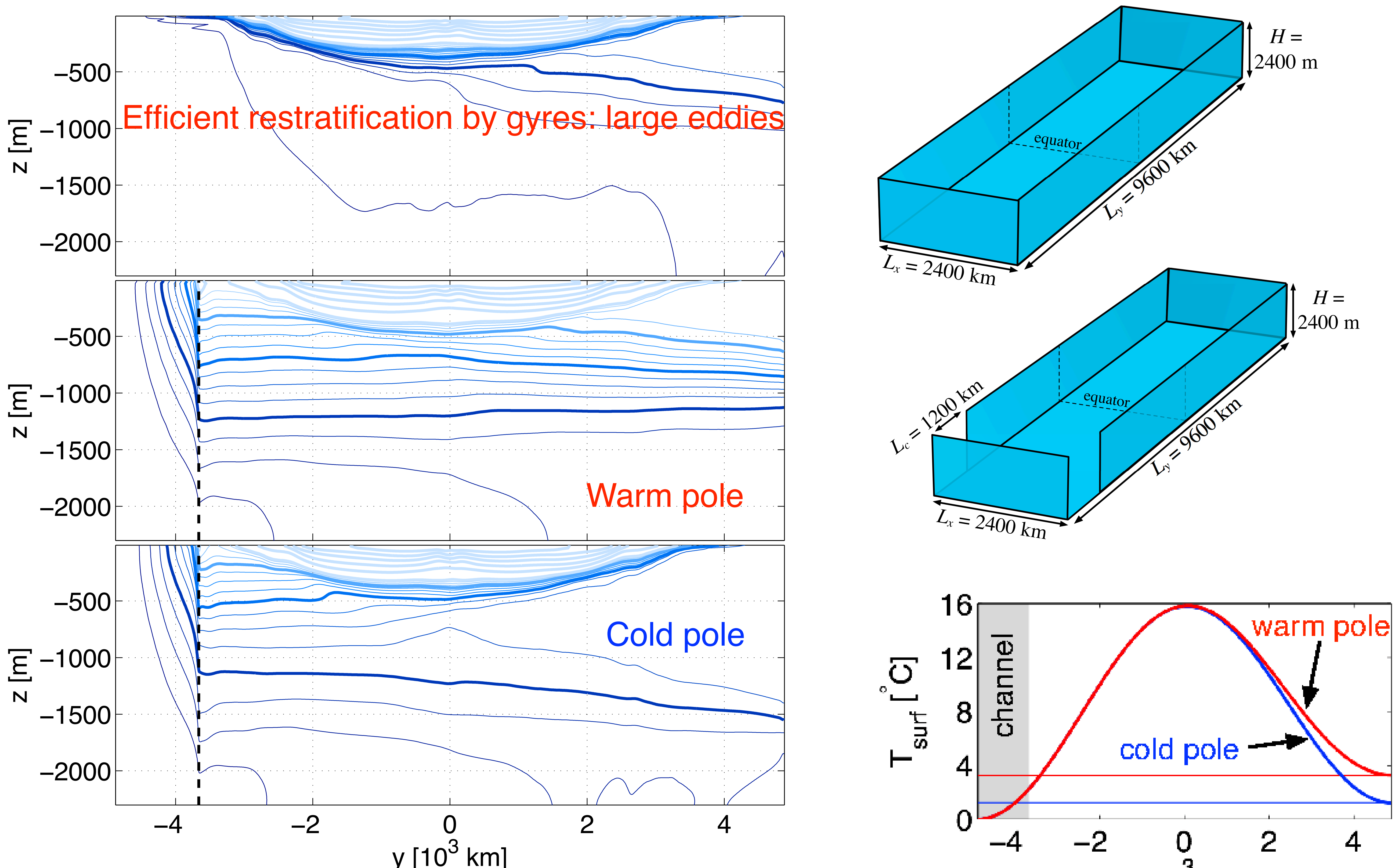
Hydrostatic MITGCM at 5.4km grid

No salt: $b \sim T$

Weak diapycnal diffusion $\kappa_v = 5 \times 10^{-5} m^2/s$

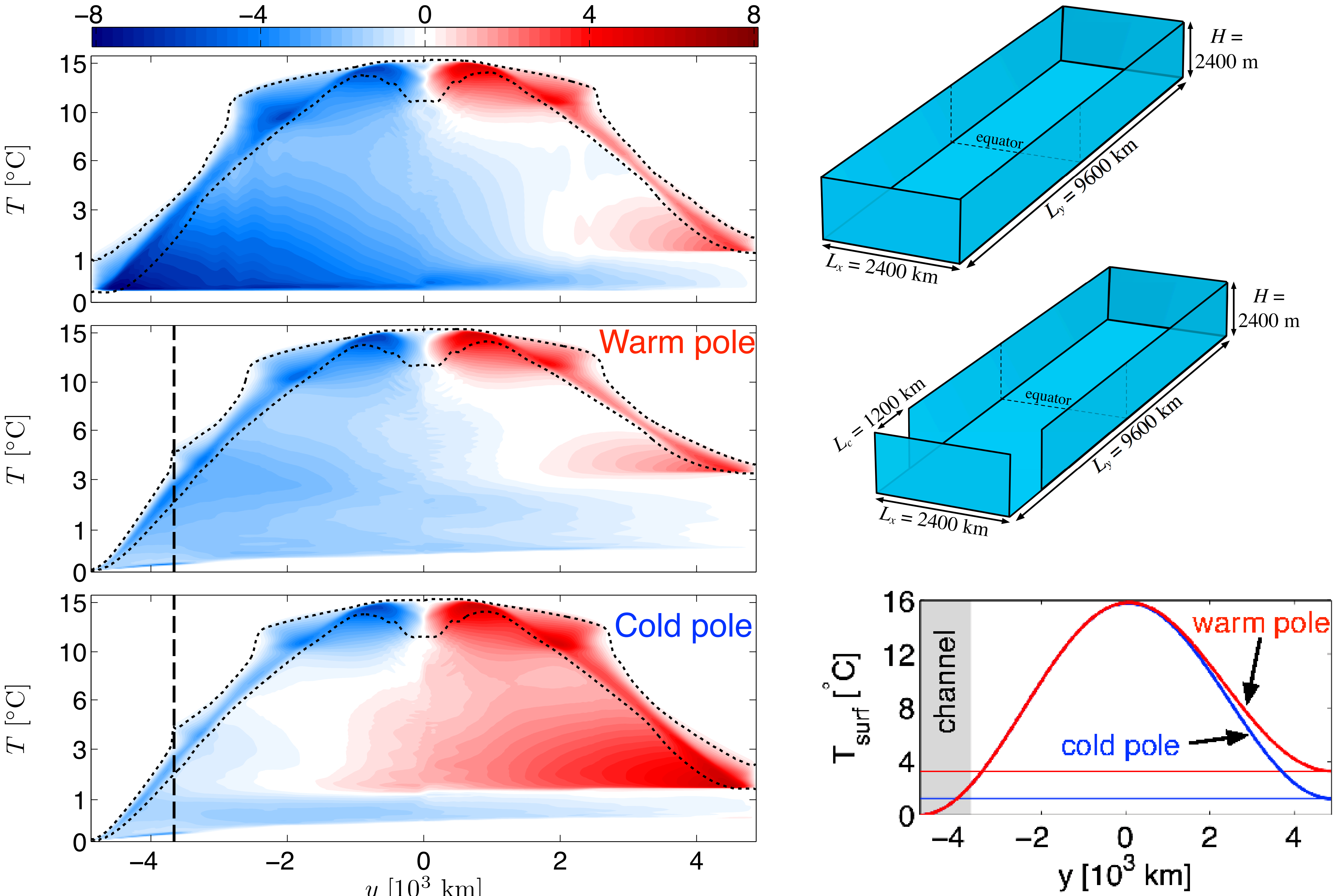
Forced by wind-stress and surface temperature

Stratification: effect of channel and buoyancy



No channel: no deep or abyssal stratification: gyres restratify very effectively.
No shared isopycnals: deep stratification, but no residual OC

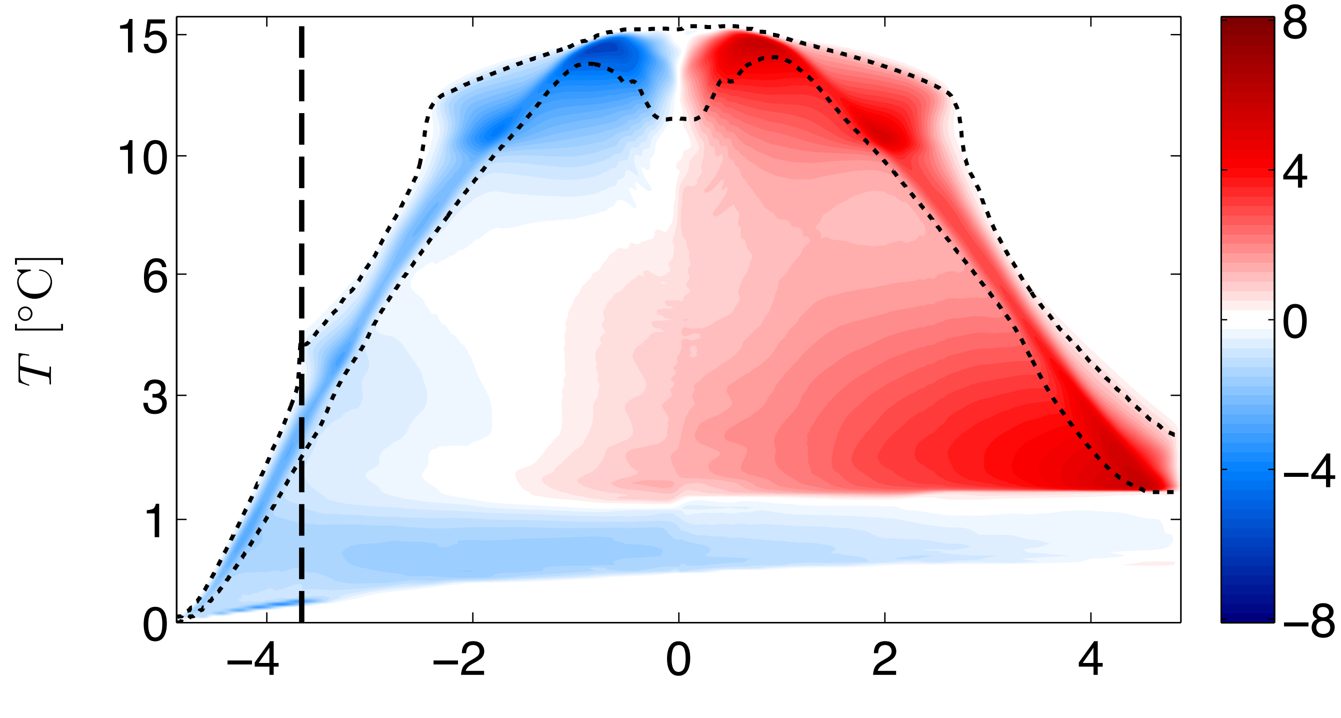
Residual OC: effect of channel and buoyancy

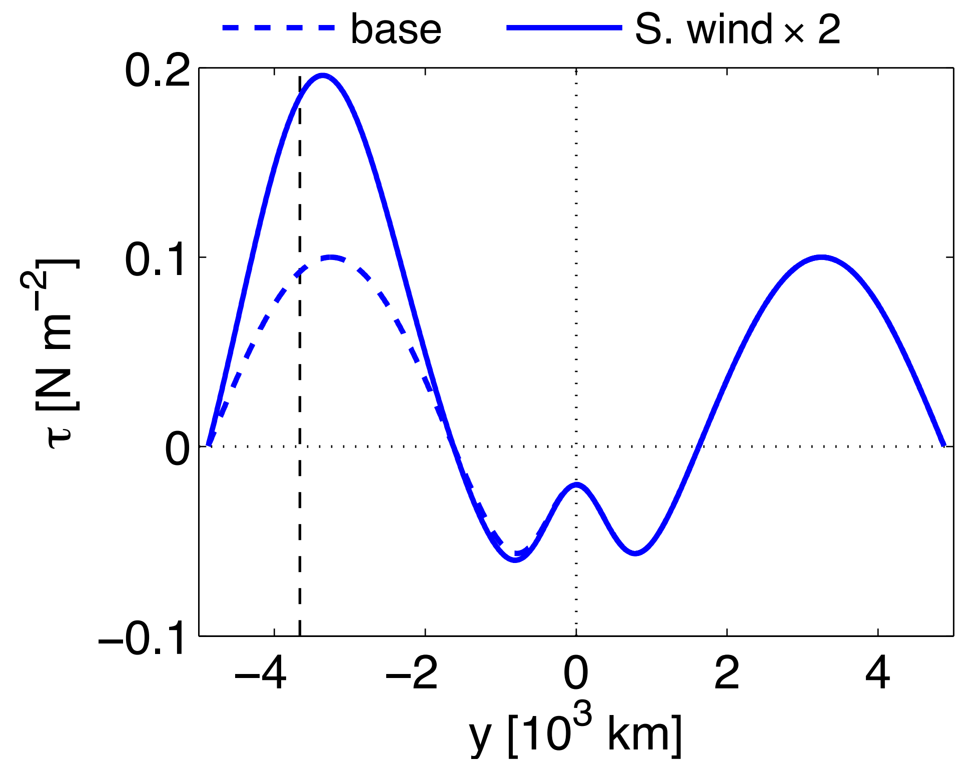


No channel: no deep overturning.

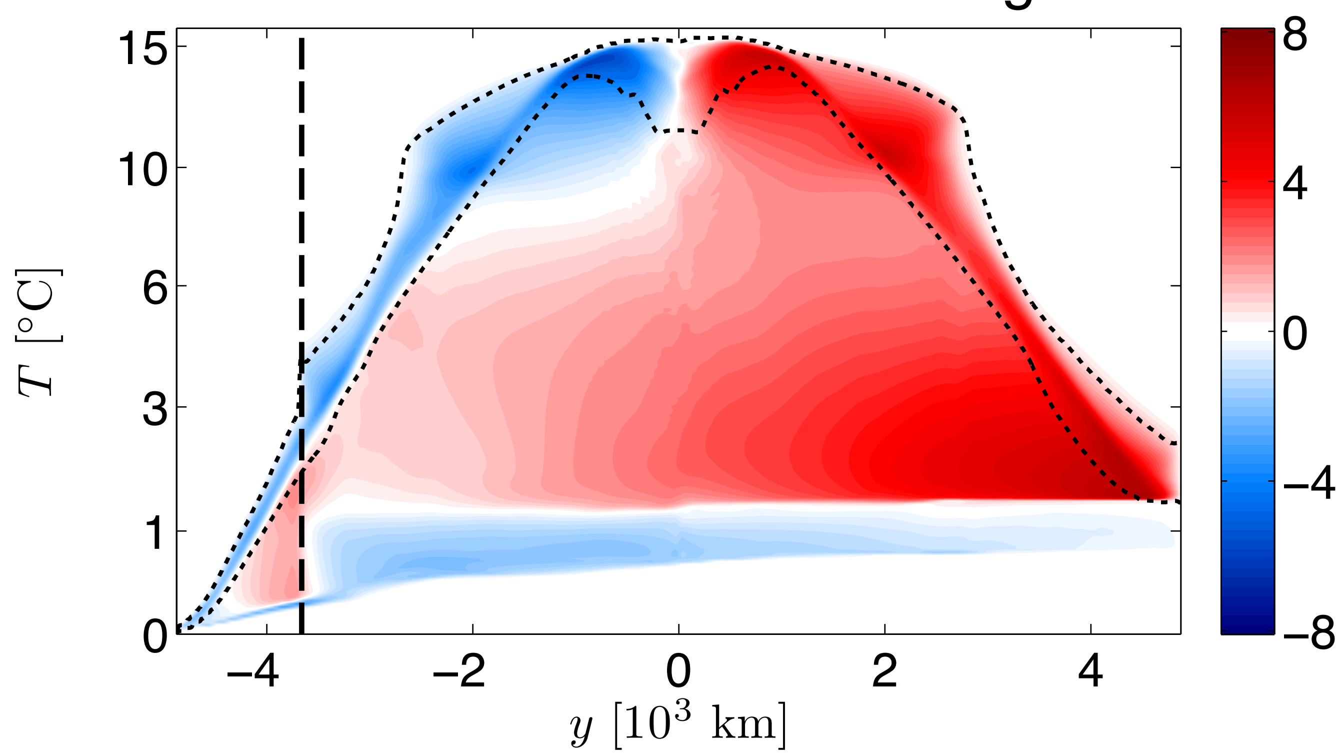
No shared isopycnals: no deep overturning

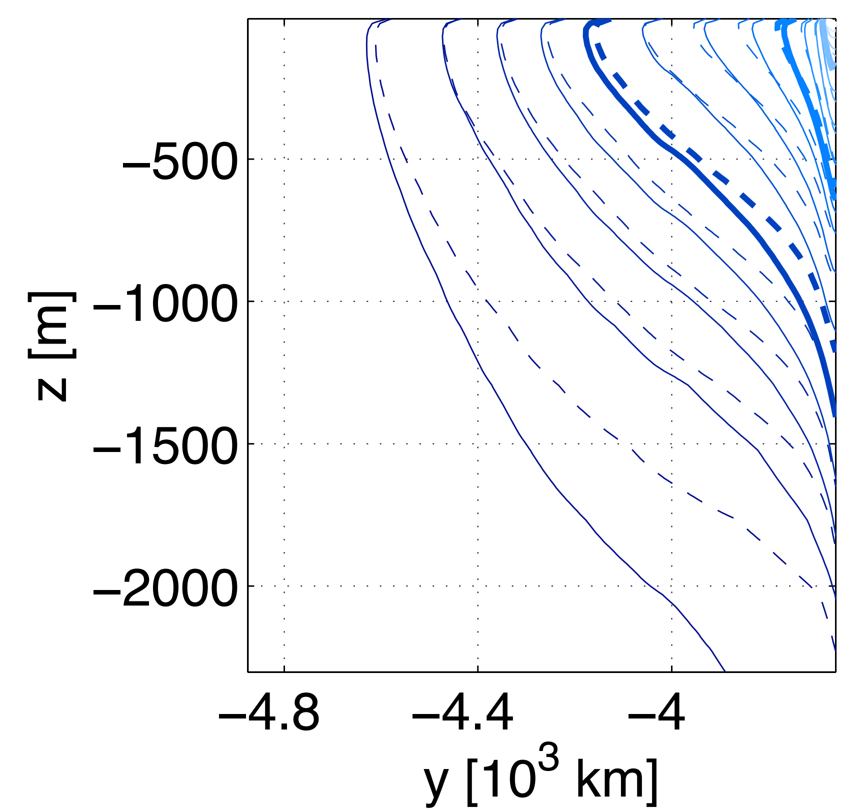
Residual OC: effect of winds in the ACC region





Double the wind in the ACC region





The overturning increases with increasing wind in the ACC
The slope increases only a little: partial eddy compensation

$$\frac{\bar{b}_y}{\bar{b}_z} \sim \frac{\tau}{\rho f}\frac{\bar{b}_y}{\overline{v'b'}}$$

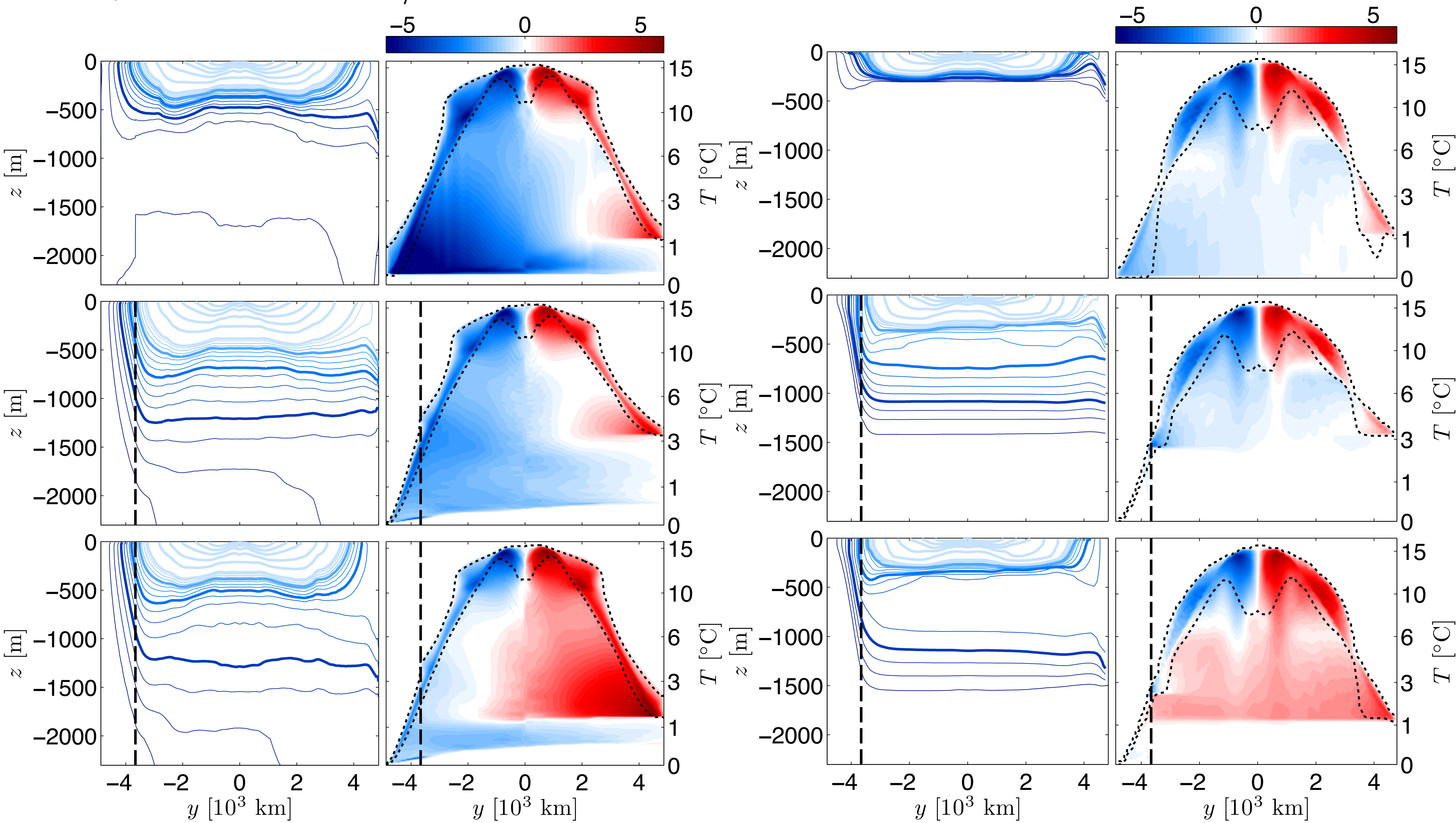
Comparison with low-res runs

5.4 km resolution - no mixed layer

$\kappa_v = 5 \times 10^{-5} m^2/s$

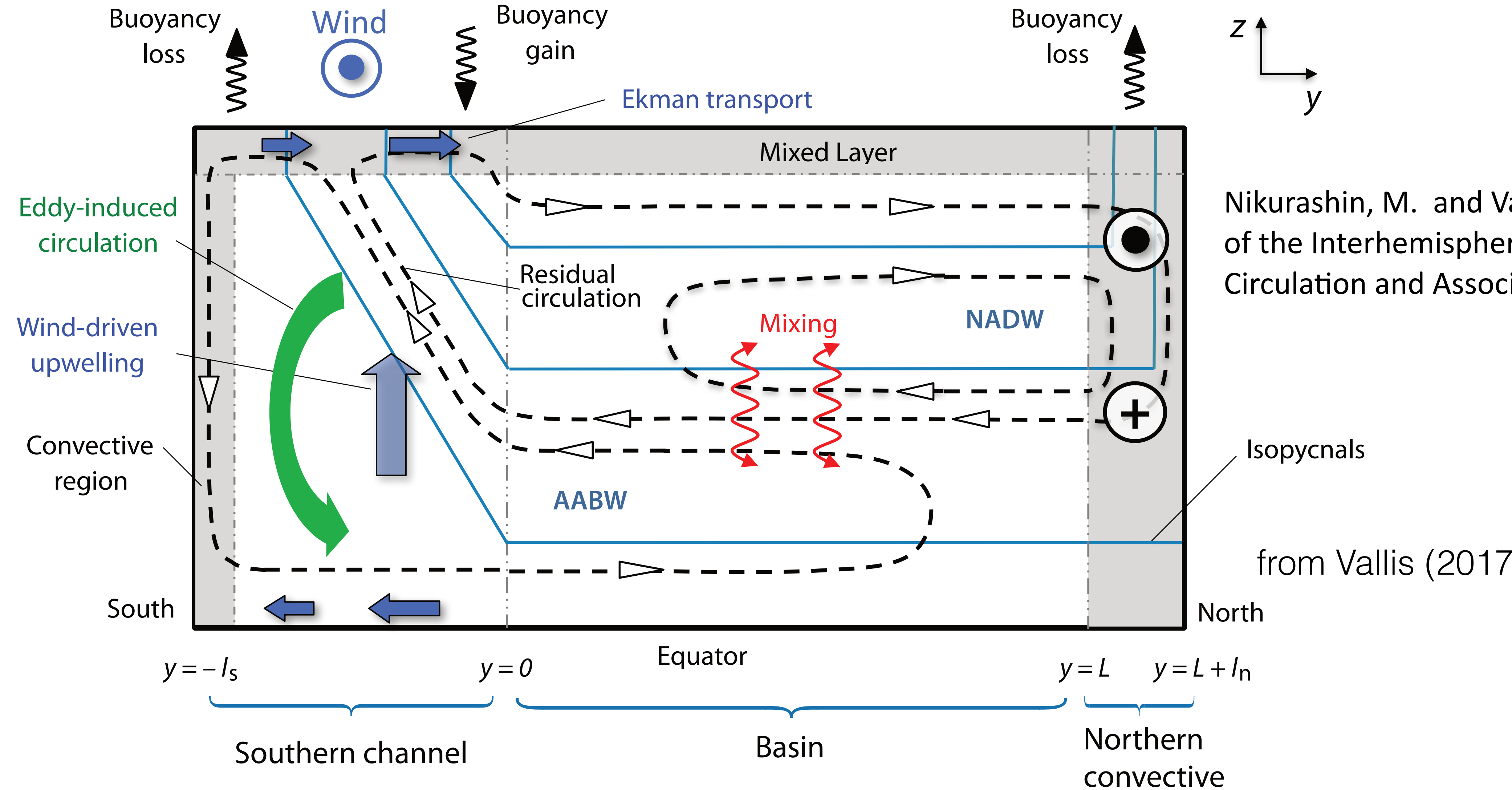
80 km resolution - 50 m mixed layer

$\kappa_v = 0,\ \kappa_a = 500\, m^2/s$



Qualitative, not quantitative agreement, even with optimal choice of constant eddy diffusivity

Connecting the channel region to the basin: theory



Nikurashin, M. and Vallis G.K. 2012 *JPO:* A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

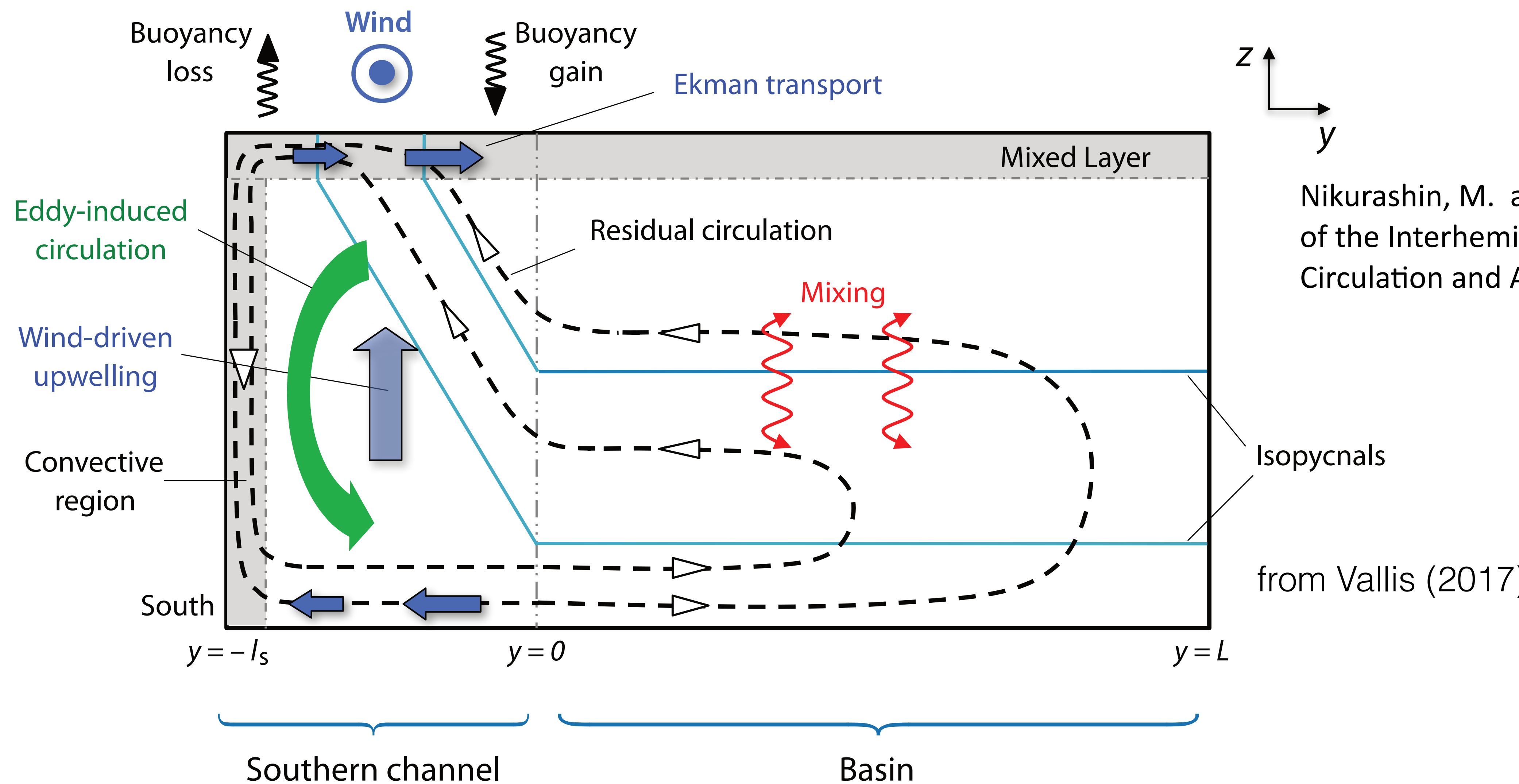
from Vallis (2017)

Dynamics in the Circumpolar channel $J_{y,z}(\bar{\psi} + \psi^*, \bar{b}) = (\kappa_v \bar{b}_z)_z \approx 0$

$\bar{\psi} = -\dfrac{\tau(y)}{\rho_o f(y)}$ $\psi^* = -\kappa_a \dfrac{b_y}{\bar{b}_z}$ B.C.: $\bar{b} = b_s(y)$ @$z = 0$ and patch $\bar{b}$ @ $y = 0$

Two B.C. in y: $\bar{\psi} + \psi^* = \mathcal{F}(\bar{b})$ Is part of the solution

A warm-up problem: the abyssal cell



Nikurashin, M. and Vallis G.K. 2012 *JPO:* A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

from Vallis (2017)

North of $y=0$ we have a diffusively-driven cell, where the isopycnals are almost flat

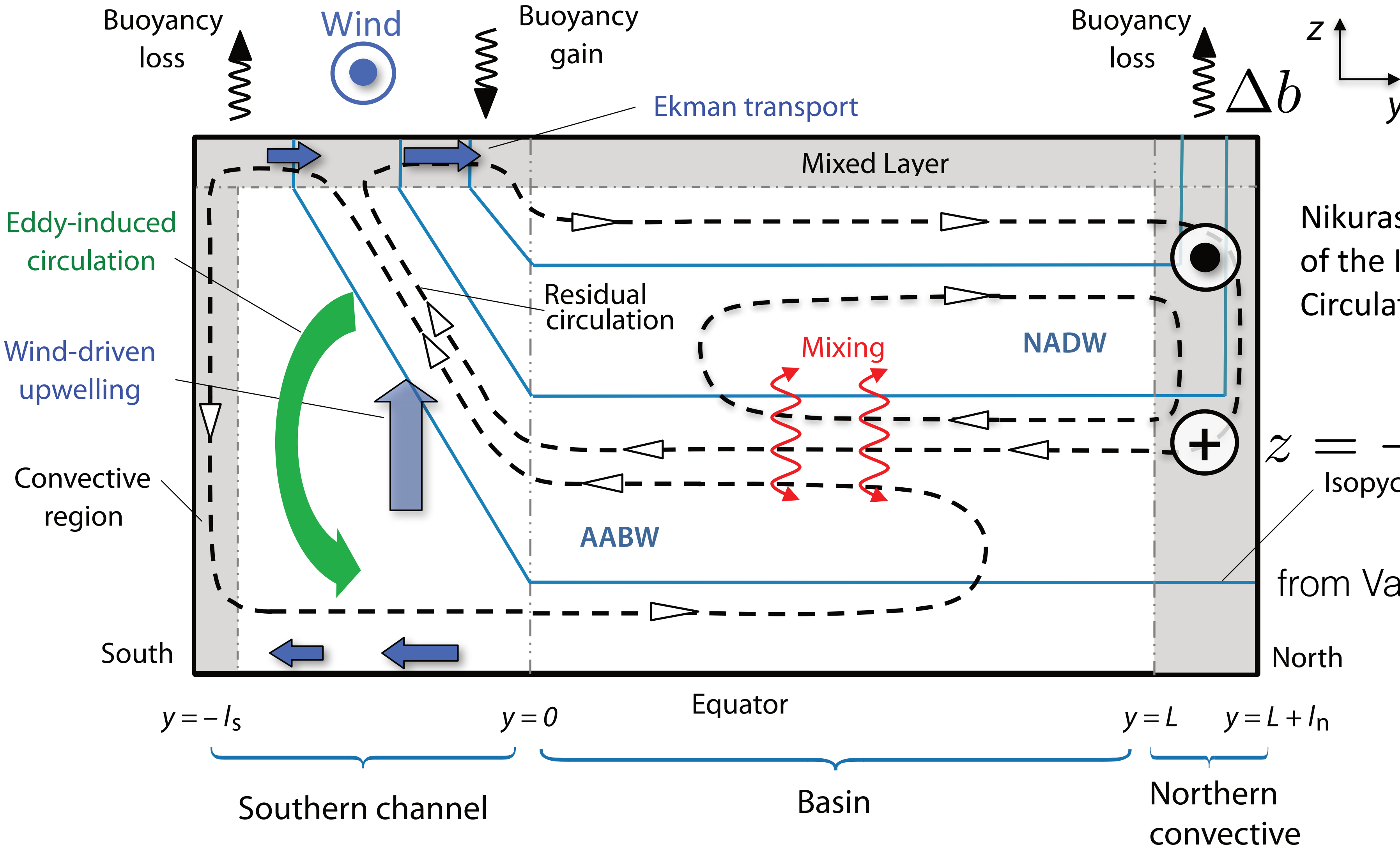
$J_{y,z}(\bar{\psi}+\psi^*,\bar{b}) = (\kappa_v \bar{b}_z)_z$ becomes $(\bar{\psi}+\psi^*)_y \bar{b}_z = (\kappa_v \bar{b}_z)_z$

Integrating in y from $y=0$ to $y=L$ we find $-(\bar{\psi}+\psi^*)|_{y=0} = L(\kappa_v \bar{b}_z)_z / \bar{b}_z$

Together with continuity of b, this gives the patching condition at $y=0$

HMK: Work out an example using epsilonics starting from the quasi-adiabatic limit

Dynamics of the upper cell in the basin



Nikurashin, M. and Vallis G.K. 2012 *JPO: A* Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

from Vallis (2017)

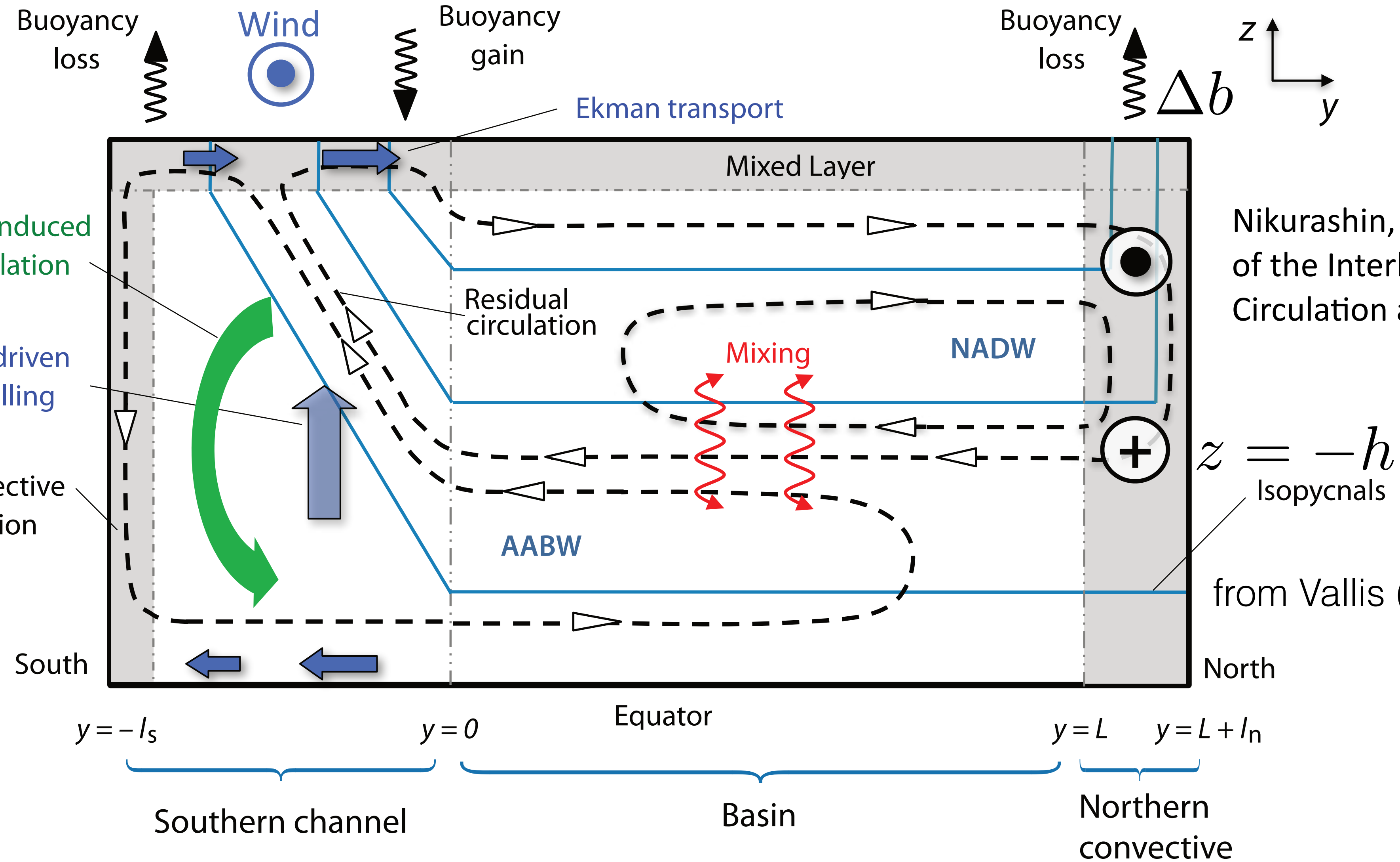
Dynamics in the basin **excluding** the convective region $(\bar{\psi} + \psi^*)_y \bar{b}_z = (\kappa_v \bar{b}_z)_z$

Integrating from $y{=}0$ to $y{=}L$ we find $(\bar{\psi} + \psi^*)|_{y=0} = (\bar{\psi} + \psi^*)|_{y=L} - L(\kappa_v \bar{b}_z)_z / \bar{b}_z$

In the convective region $\bar{b} = \bar{b}_y, \quad f u_z = -\bar{b}_y \implies u = -\frac{\bar{b}_y}{f}\left(z + \frac{h}{2}\right)$

u turns into a western boundary current with transport $\int_0^{L_x} \bar{\psi}_{zz}\, dx = -\frac{\Delta b}{f}$

Dynamics of the upper cell in the basin



Nikurashin, M. and Vallis G.K. 2012 *JPO:* A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

from Vallis (2017)

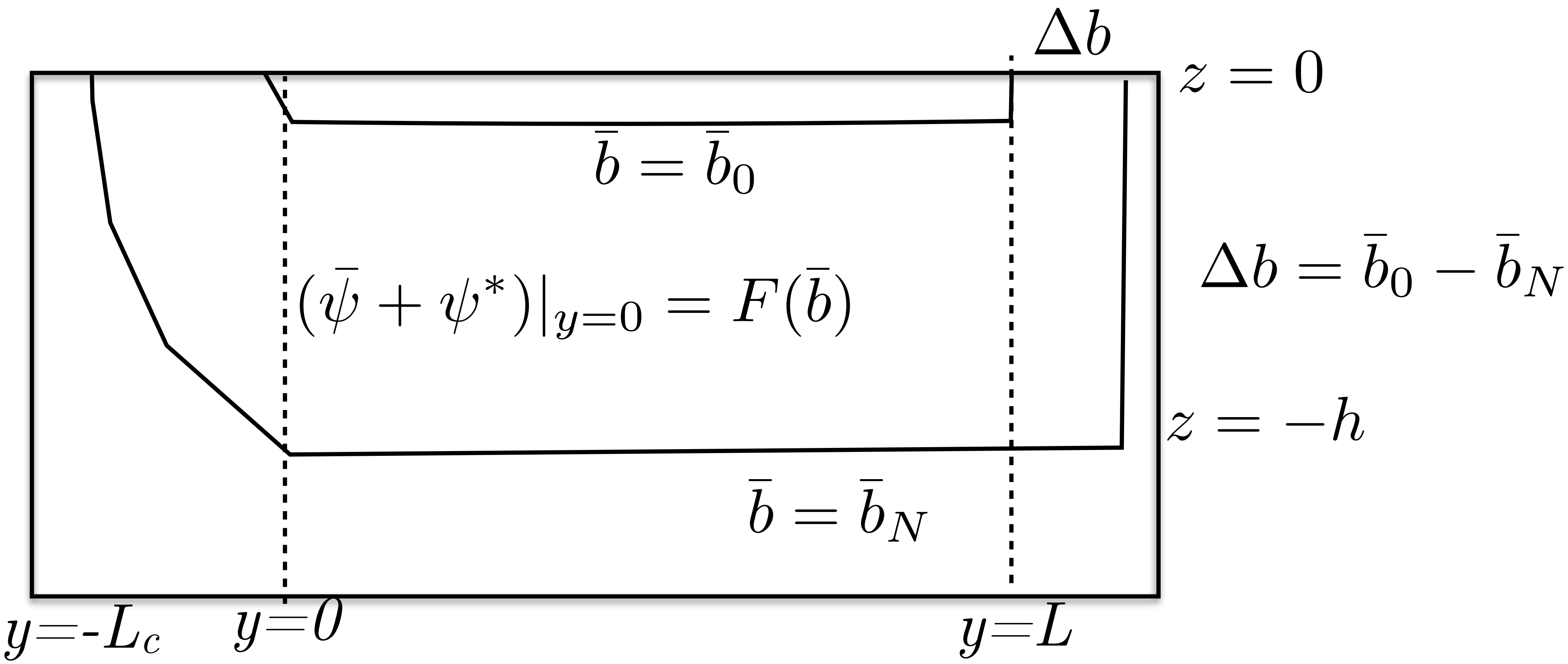
Finally $(\bar{\psi} + \psi^*)|_{y=0} = (\bar{\psi} + \psi^*)|_{y=L} - L(\kappa_v \bar{b}_z)_z / \bar{b}_z$

with $\bar{\psi} = -\dfrac{\Delta b}{2 f_N L_x} z(z+h)$ at $y{=}L$

Neglect ψ^* in the basin

Here h is unknown and it will be determined by patching with the channel solution at $y{=}0$. The residual circulation is given by the sinking at $y{=}L$, proportional to the parameters as given

Solution in the adiabatic limit



In the channel $J_{y,z}(\bar{\psi} + \psi^*, \bar{b}) = 0 \implies \bar{\psi} + \psi^* = \mathcal{F}(\bar{b})$

$$\underbrace{-\frac{\tau}{\rho f}}_{\bar{\psi}} \underbrace{-\kappa_a \frac{\bar{b}_y}{\bar{b}_z}}_{\psi^*} = F(\bar{b})$$

In buoyancy coordinates $-\frac{\tau}{\rho f} + \kappa_a z_y = F(\bar{b})$

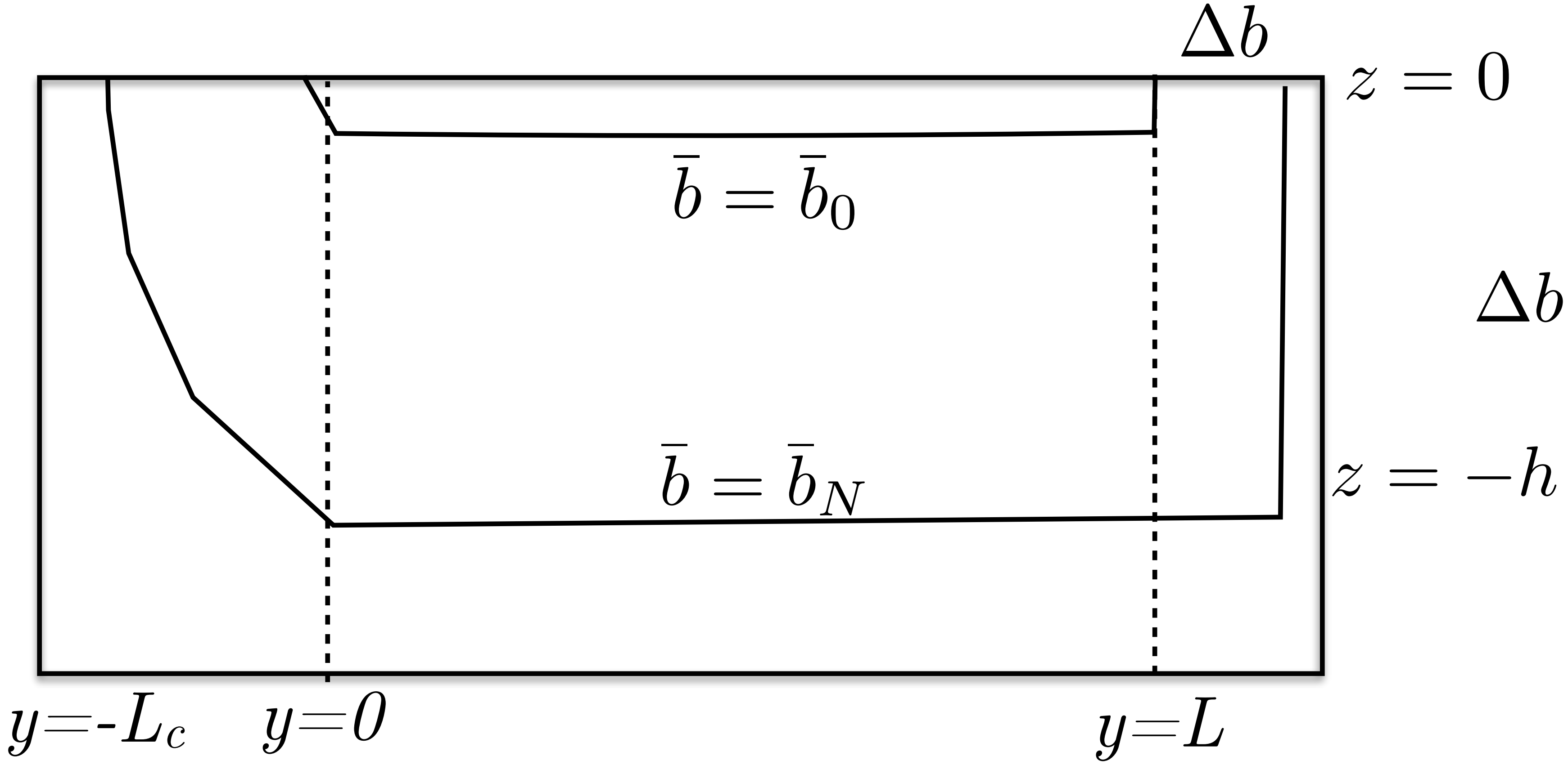
Shallower slopes than F=0

$$\kappa_a z(y, b) = \int_{y_s(\bar{b})}^{y} d\hat{y}\, \frac{\tau}{\rho f} + F(b)[y - y_s(\bar{b})]$$

Where $y_s(\bar{b}) = b_s^{-1}(y)$

is the inverse of the surface buoyancy

Solution in the adiabatic limit



$$\Delta b = \bar{b}_0 - \bar{b}_N$$

In the adiabatic limit $(\bar{\psi} + \psi^*)|_{y=0} = (\bar{\psi} + \psi^*)|_{y=L}$ Patch at y=0

$$F(b) = -\frac{\Delta b}{2 f_N L_x} z(z+h) \qquad \Longrightarrow \kappa_a z(y=0,\bar{b}) = \int_{y_s(\bar{b})}^{0} d\hat{y}\, \frac{\tau}{\rho f} - F(\bar{b}) y_s(\bar{b})$$

A quadratic equation for the stratification at y=0 $z(\bar{b})|_{y=0}$ determines $F(\bar{b})$

Where $-\kappa_a h = \int_{y_s(\bar{b}_N)}^{0} d\hat{y}\, \frac{\tau}{\rho f}$

The slope of the lowest shared buoyancy is the same as the zero residual overturning soln. Other isopycnals are shallower.

In the adiabatic limit

The maximum sinking is found at z=-$h/2$

$$F_{max} = \frac{\Delta b}{8 f_N L_x} h^2$$

Where $$-\kappa_a h = \int_{y_s(\bar{b}_N)}^{0} d\hat{y}\, \frac{\tau}{\rho f}$$

It depends on the winds over the ACC, the eddies in the ACC and the shared buoyancy range between the channel and the NH.

HMK: find the solution given wind-stress in the ACC and surface buoyancy.

Global diapycnal diffusion is neglected, but it adds to the total sinking: solve the 2-D advection diffusion numerically.

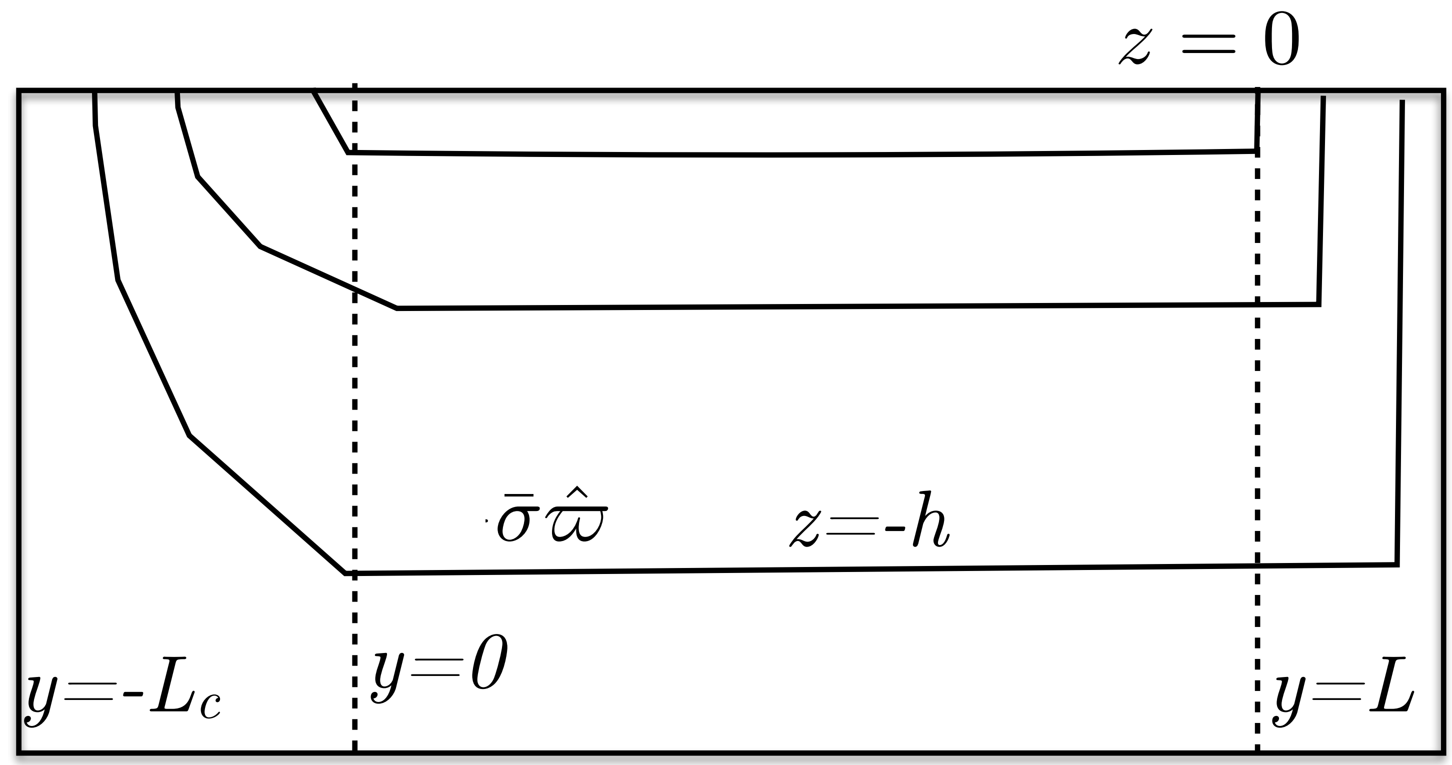
Research question: change fixed surface buoyancy to fixed surface flux, a much harder problem which requires consideration of diapycnal mixing.

Residual streamfunction budget including diapycnal mixing

Begin in buoyancy coordinates: $(\sigma u)_{\tilde{x}} + (\sigma v)_{\tilde{y}} + (\sigma \mathcal{D})_{\tilde{b}} = 0 \qquad \sigma = b_z^{-1}$

Zonally average at constant b:

$$\overline{(\sigma v)}_{\tilde{y}} + \overline{(\sigma \mathcal{D})}_{\tilde{b}} = 0$$



Define the residual streamfunction:

$$\psi^\dagger_{\tilde{b}} = -\overline{(\sigma v)} = -\bar{\sigma}\hat{v} \qquad \psi^\dagger_{\tilde{y}} = \overline{(\sigma \mathcal{D})} = \bar{\sigma}\hat{\varpi}$$

Diabatic term due to diapycnal mixing

Integrate from y=0 to y=L

$$\psi^\dagger|_{\tilde{y}=0}^{\tilde{y}=L} = \int_0^L \bar{\sigma}\hat{\varpi}\, d\tilde{y} \qquad \bar{\sigma}\hat{\varpi} = (\overline{\kappa_v \sigma^{-1}})_{\tilde{b}} \qquad \sigma = z_{\tilde{b}} \qquad -z = m_{\tilde{b}}$$

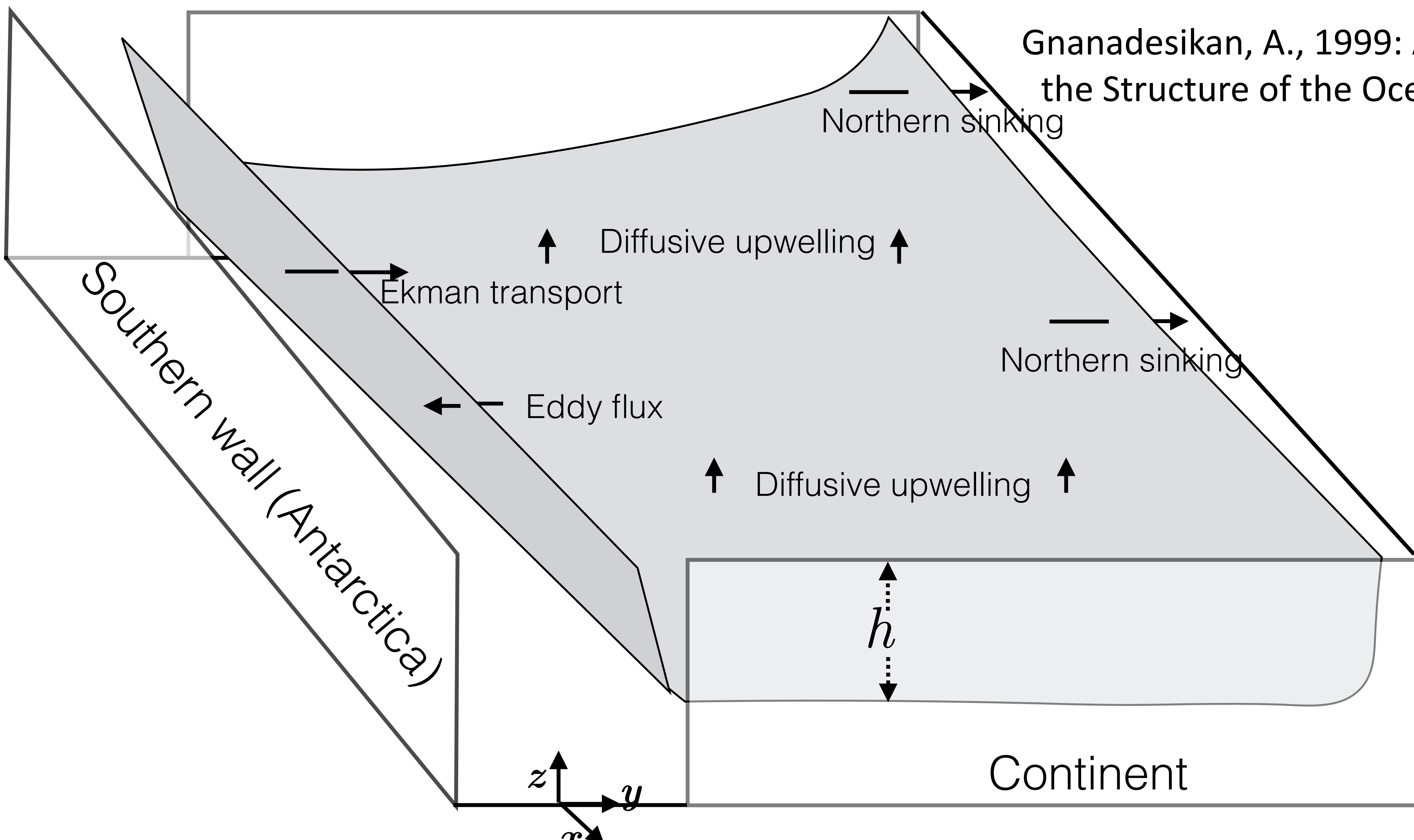
$$\psi^\dagger|_{\tilde{y}=0} = -\frac{\tau}{\rho f}|_{\tilde{y}=0} + \kappa_a z_y|_{\tilde{y}=0} \qquad \psi^\dagger|_{\tilde{y}=L} = \int_{b_N}^{b} d\tilde{b}\, \frac{\overline{m_{\tilde{b}\tilde{b}} m_{\tilde{x}}}}{f L_x}|_{\tilde{y}=L}$$

Geostrophic and hydrostatic balance

Scale analysis: *z~h*

Buoyancy budget on an isopycnal in the upper branch of the MOC

Gnanadesikan, A., 1999: A Simple Predictive Model for the Structure of the Oceanic Pycnocline Science,283



$$\underbrace{-\frac{\tau L_x}{\rho f_s}}_{\text{Ekman}} \underbrace{-\kappa_a L_x \frac{h}{L_c}}_{\text{Eddy flux}} \underbrace{+\kappa_v \frac{Area}{h}}_{\text{Diffusion}} = \underbrace{\epsilon \Delta b \frac{h^2}{2 f_n}}_{\text{Sinking}}$$

HMK: Show that adiabatic limit is the same as Nikurashin&Vallis eqn. for z if

$$\epsilon = 1/4$$

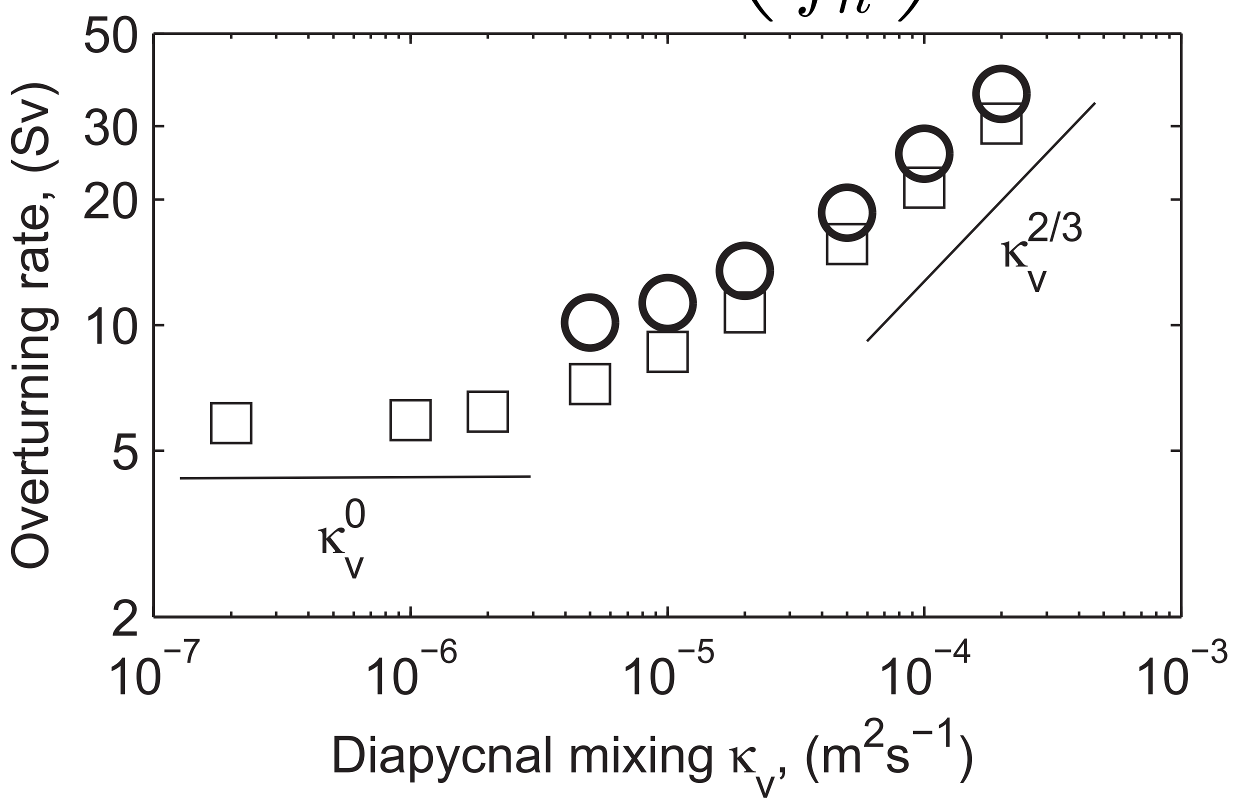
A cubic equation for the unknown h: h gets bigger adding diffusion

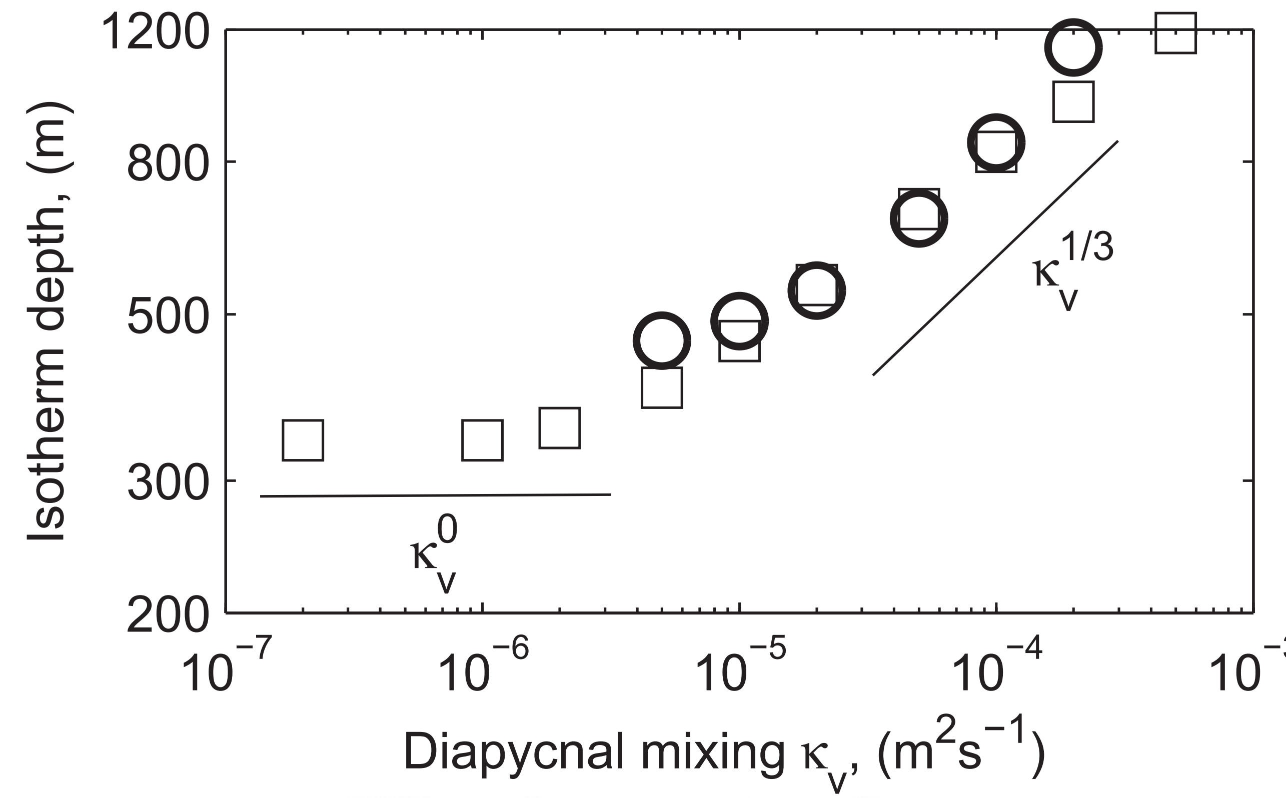
Solution in the diffusive limit

The dominant balance is

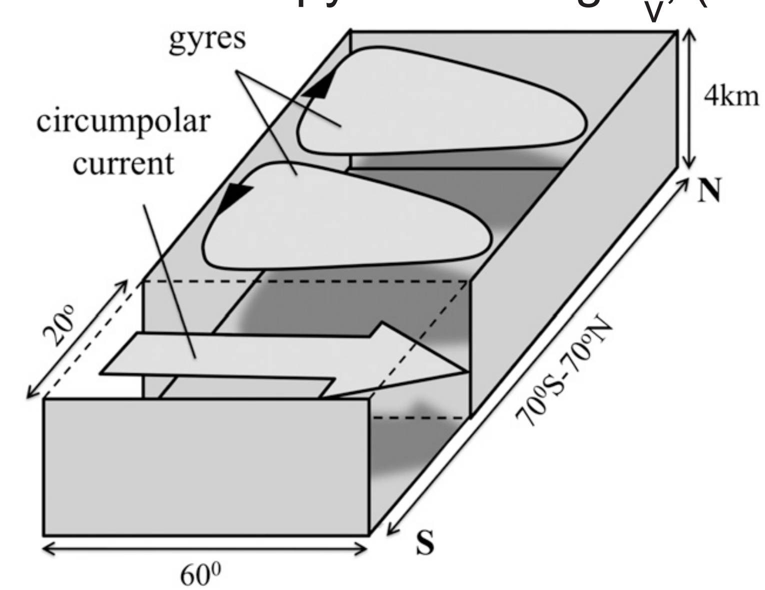
$$\underbrace{\kappa_v \frac{Area}{h}}_{\text{Diffusion}} \approx \underbrace{\epsilon \Delta b \frac{h^2}{2f_n}}_{\text{Sinking}}$$

With solution $MOC \sim \left(\frac{\Delta b}{f_n}\right)^{1/3} (\kappa_v Area)^{2/3}$ $\quad h \sim (f_n \kappa_v Area/\Delta b)^{1/3}$





Results from a 3-D numerical model with surface winds and buoyancy



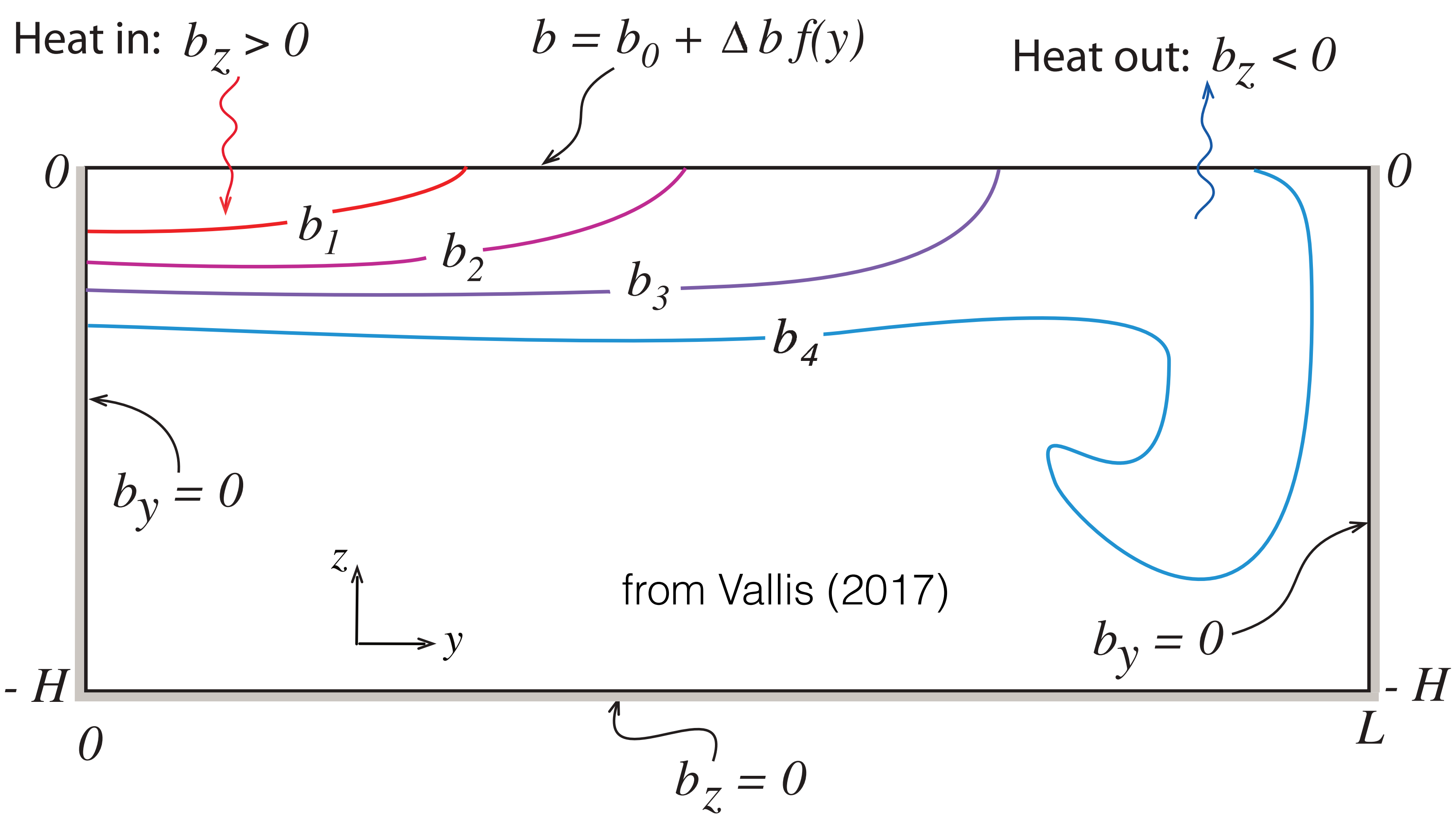
Nikurashin, M. and Vallis G.K. 2012 *JPO:* A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

The non-rotating case (horizontal convection)

Rotating diffusive scaling $h \sim (f_n \kappa_v Area/\Delta b)^{1/3}$ $\quad MOC \sim \left(\frac{\Delta b}{f_n}\right)^{1/3} (\kappa_v Area)^{2/3}$

Compare with non-rotating horizontal convection: viscosity instead of rotation

$$h \sim \left(\frac{\kappa_v \nu L_y^2}{\Delta b}\right)^{1/5} \qquad MOC \sim \left(\frac{\kappa_v^4 \Delta b L_y^3}{\nu}\right)^{1/5}$$



$$\frac{\partial \nabla^2 \psi}{\partial t} + J(\psi, \nabla^2 \psi) = \frac{\partial b}{\partial y} + \nu \nabla^4 \psi,$$

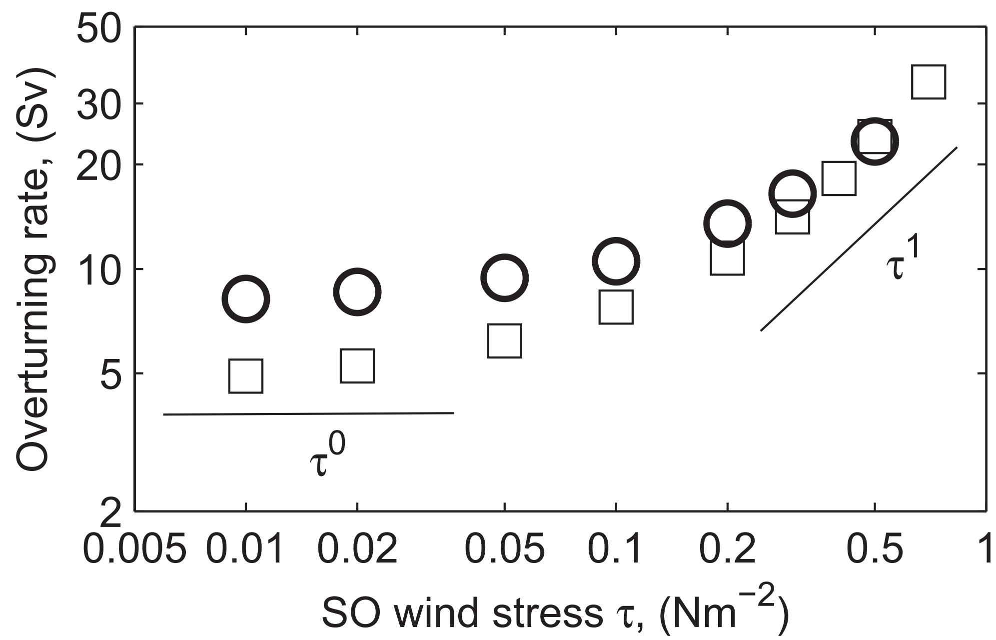
$$\frac{\partial b}{\partial t} + J(\psi, b) = \kappa \nabla^2 b,$$

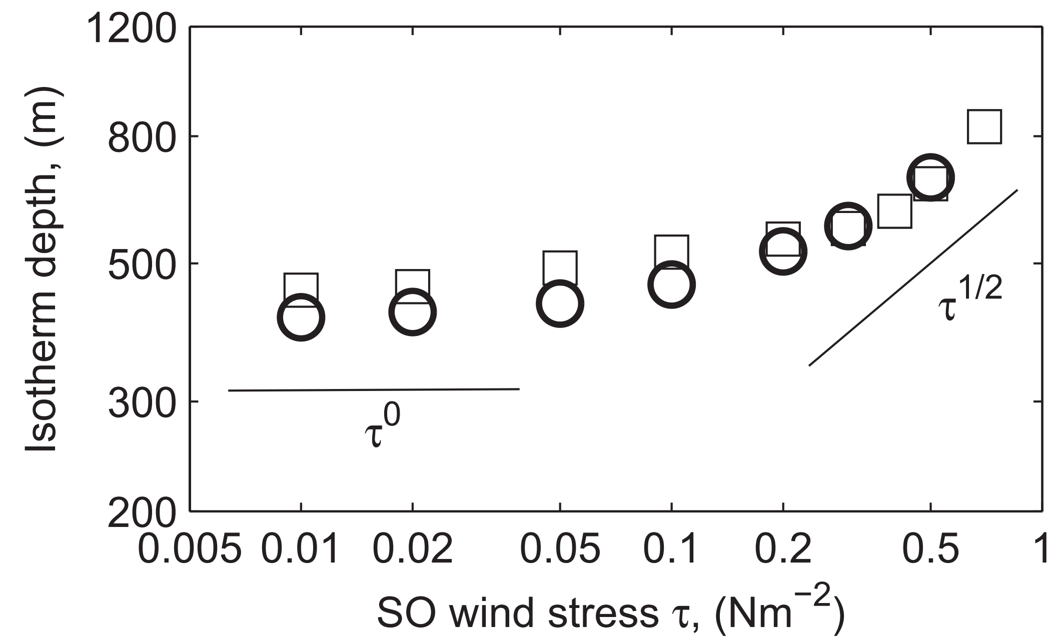
With no mechanical forcing the scale-height and transport —> 0 as diffusivity —> 0

Solution in the adiabatic limit with rotation and wind-stress

The dominant balance is $$\underbrace{-\frac{\tau_s L_x}{\rho f_s}}_{\text{Ekman}} \underbrace{-\kappa_a L_x \frac{h}{L_c}}_{\text{Eddy flux}} \approx \underbrace{\epsilon \Delta b \frac{h^2}{2 f_n}}_{\text{Sinking}}$$

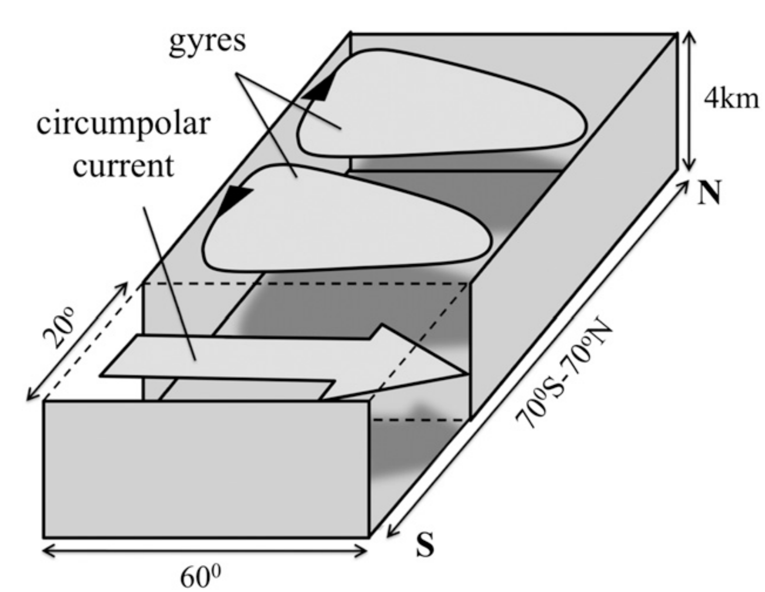
For weak eddies $$MOC \sim \frac{\tau_s L_x}{\Delta b \rho f_s} \qquad h \sim \left(\frac{f_n \tau_s L_x}{\Delta b \rho f_s} \right)^{1/2}$$





Results from a 3-D numerical model with surface winds and buoyancy

Nikurashin, M. and Vallis G.K. 2012 *JPO:* A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification



Summary so far

The deep and abyssal stratification of the world ocean is set in the ACC region.

The surface Ekman transport in the periodic channel is returned at the bottom: it overturns isopycnals until they are vertical, producing a large amount of APE.

Some APE is converted into baroclinic eddies, restratifying, but not as much as in basin.

In the weak-diffusion limit, residual overturning is along isopycnals shared with ACC.

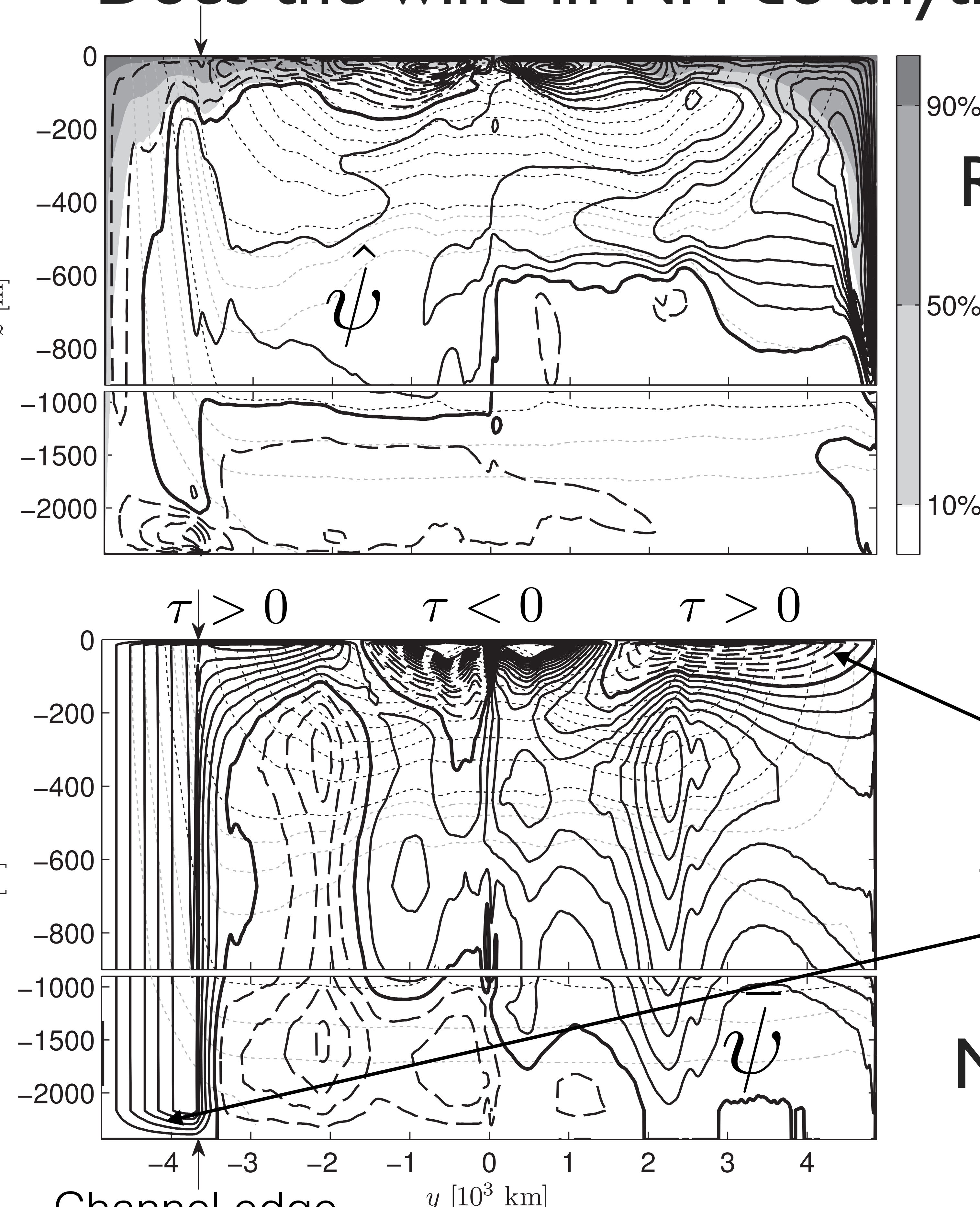
The strength of ROC is a competition between Ekman versus eddy transport in ACC.

Simple models with parametrized eddy fluxes of buoyancy are helpful.

Quantitive results depend on the details of parametrization.

The dominant driver of the MOC is the wind-stress in the ACC.

Does the wind in NH do anything to the MOC/ROC?



ROC

One pole-to-pole cell between 200 and 1000 m

The Ekman return flow is shallow in the basin region: gyres are very effective at restratifying

The Ekman return flow is at the bottom in the ACC region

MOC

Channel edge

Effect of wind-stress in the sinking region

Include wind-stress in vertically integrated momentum balance in the sinking region:

$$-f\,V = -\Delta b\,h\,h_x + \frac{\tau}{\rho}$$

Wind-stress forcing at the surface

The zonally integrated transport of northern sinking is

$$\psi^{\dagger}|_{\tilde{y}=L} = \Delta b\frac{h_e^2 - h_w^2}{2f}|_{\tilde{y}=L} - \frac{\tau L_x}{\rho f}|_{\tilde{y}=L}$$

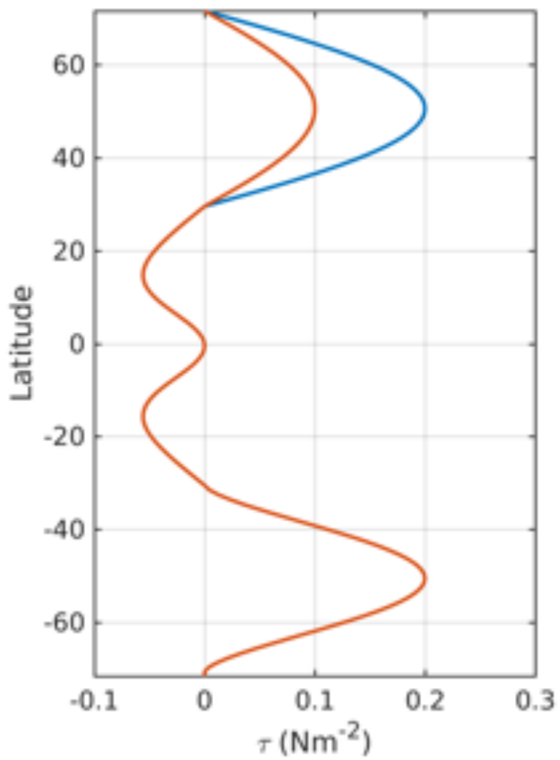
East-West pressure difference balances Ekman-transport return (in WBC) + MOC

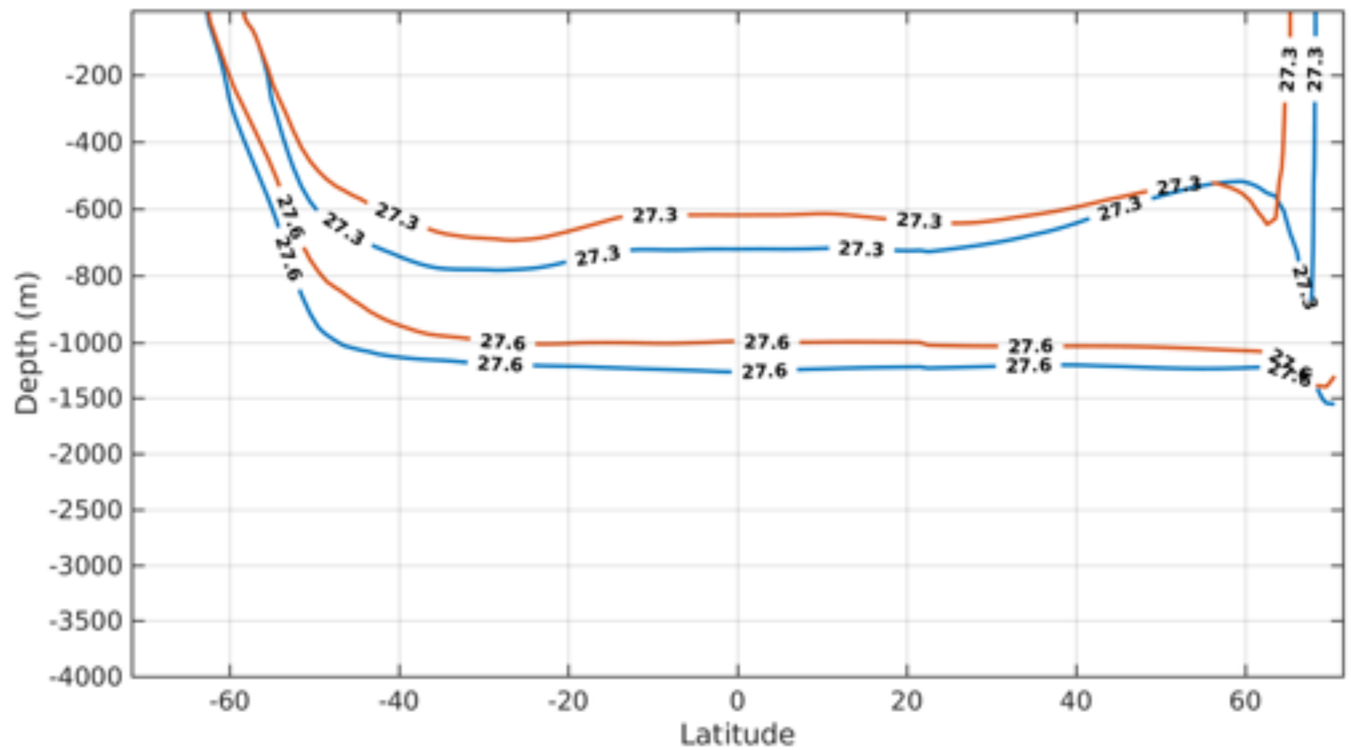
Modified buoyancy budget

$$\underbrace{-\frac{\tau_s L_x}{\rho f_s}}_{\text{Ekman}} \underbrace{-\kappa_a L_x \frac{h}{L_c}}_{\text{Eddy flux}} \underbrace{+\kappa_v \frac{Area}{h}}_{\text{Diffusion}} = \underbrace{\epsilon \Delta b \frac{h^2}{2f_n} - \frac{\tau_n L_x}{\rho f_n}}_{\text{Sinking}}$$

Additional term which increases h and reduces the overturning: wind-suction in sinking region suppresses downwelling.

Check these predictions with a GCM



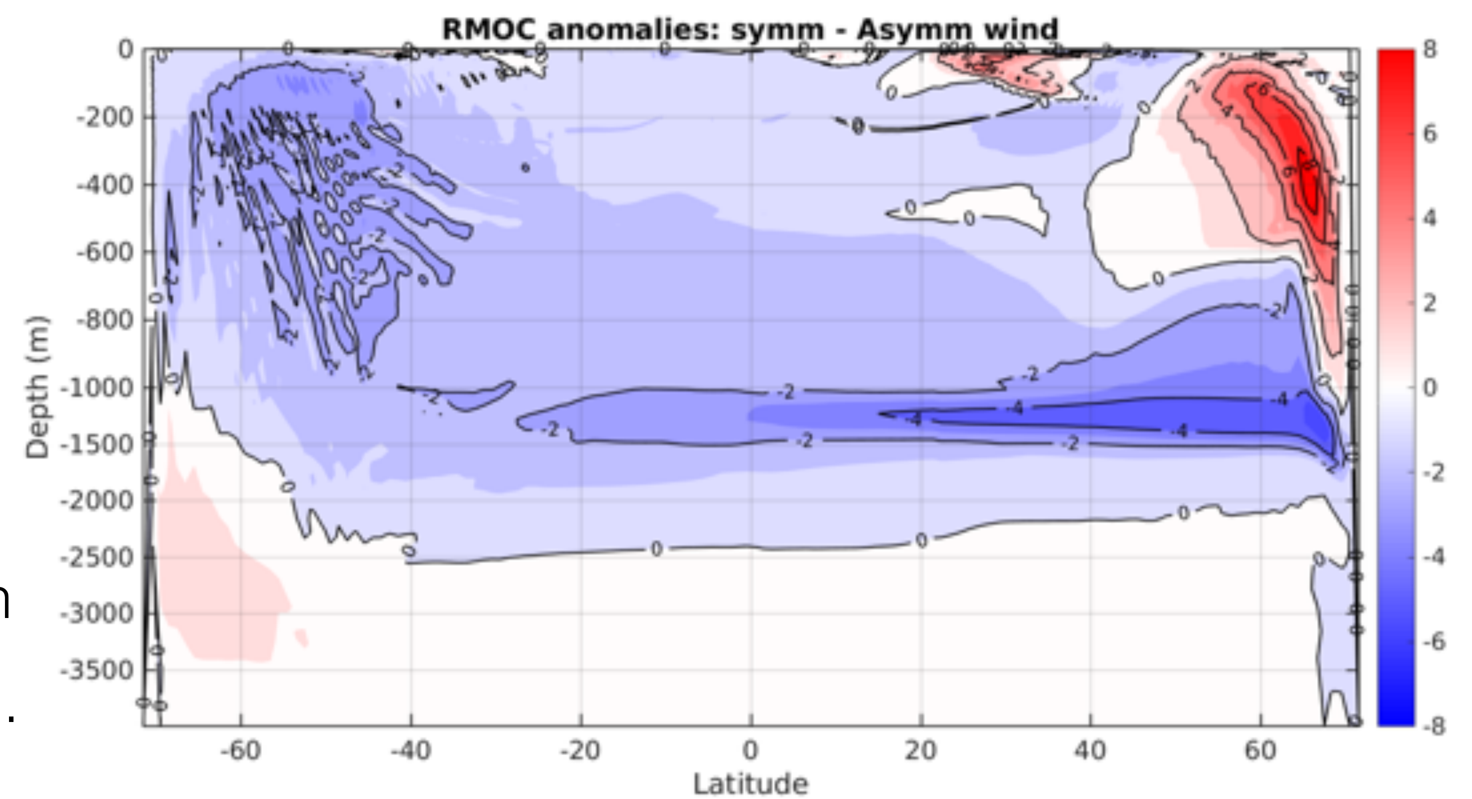


Depth of isopycnals increases and ROC decreases with increasing NH wind

Interhemispheric cell smaller with larger NH winds

Geometry: one basins + circumpolar region in SH:

- Continent is 1-grid space wide
- Zonally uniform surface forcing (wind, temperature, salt)
- Primitive equations on a sphere (MITgcm)
- Low-diffusivity regime
- 1 degree resolution with GM parametrization
- Domain is 60º wide, 140º long, 4000m deep.
- Submarine ridge south of continent's end

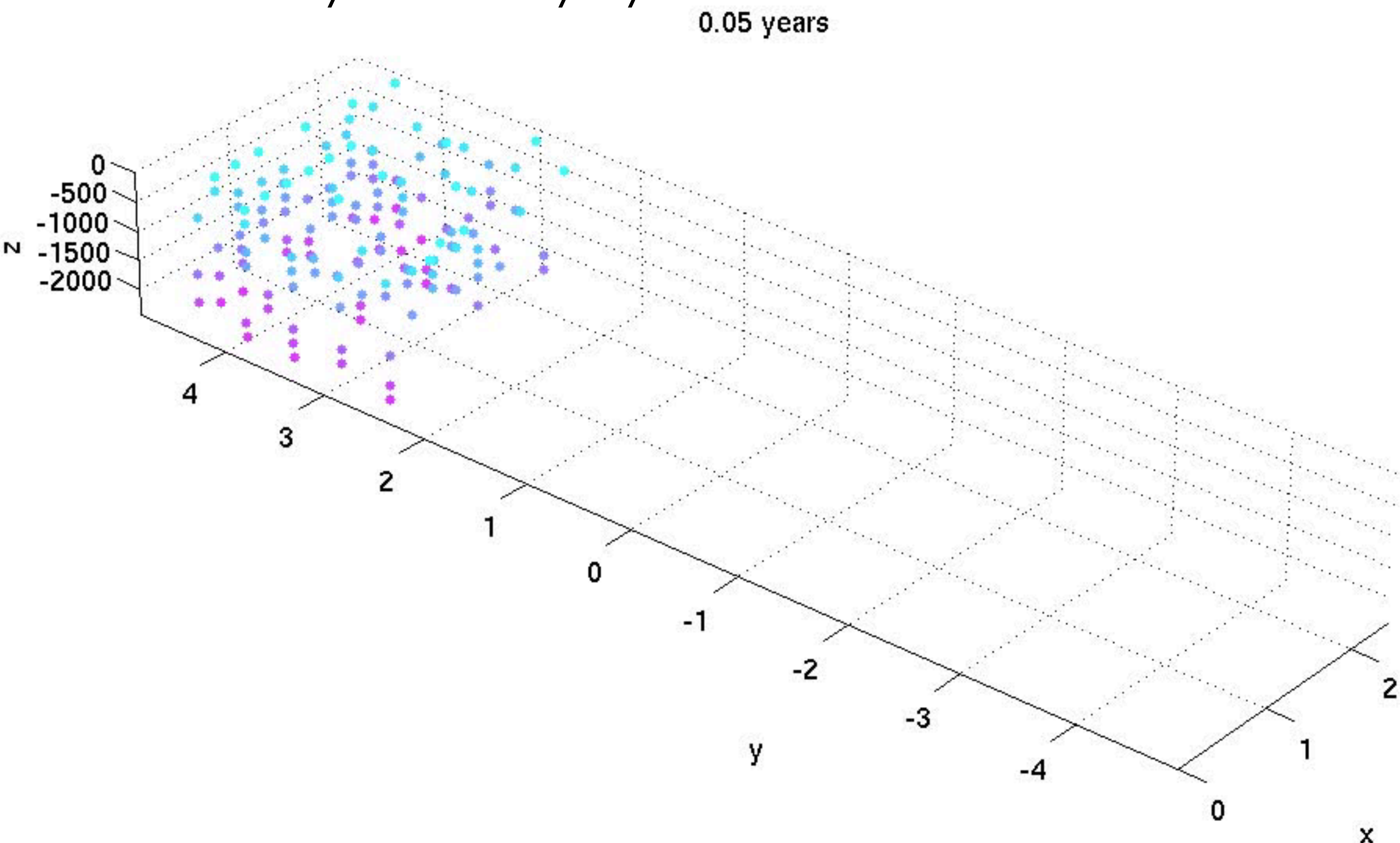


Preliminary results by Laura Cimoli, U Oxford

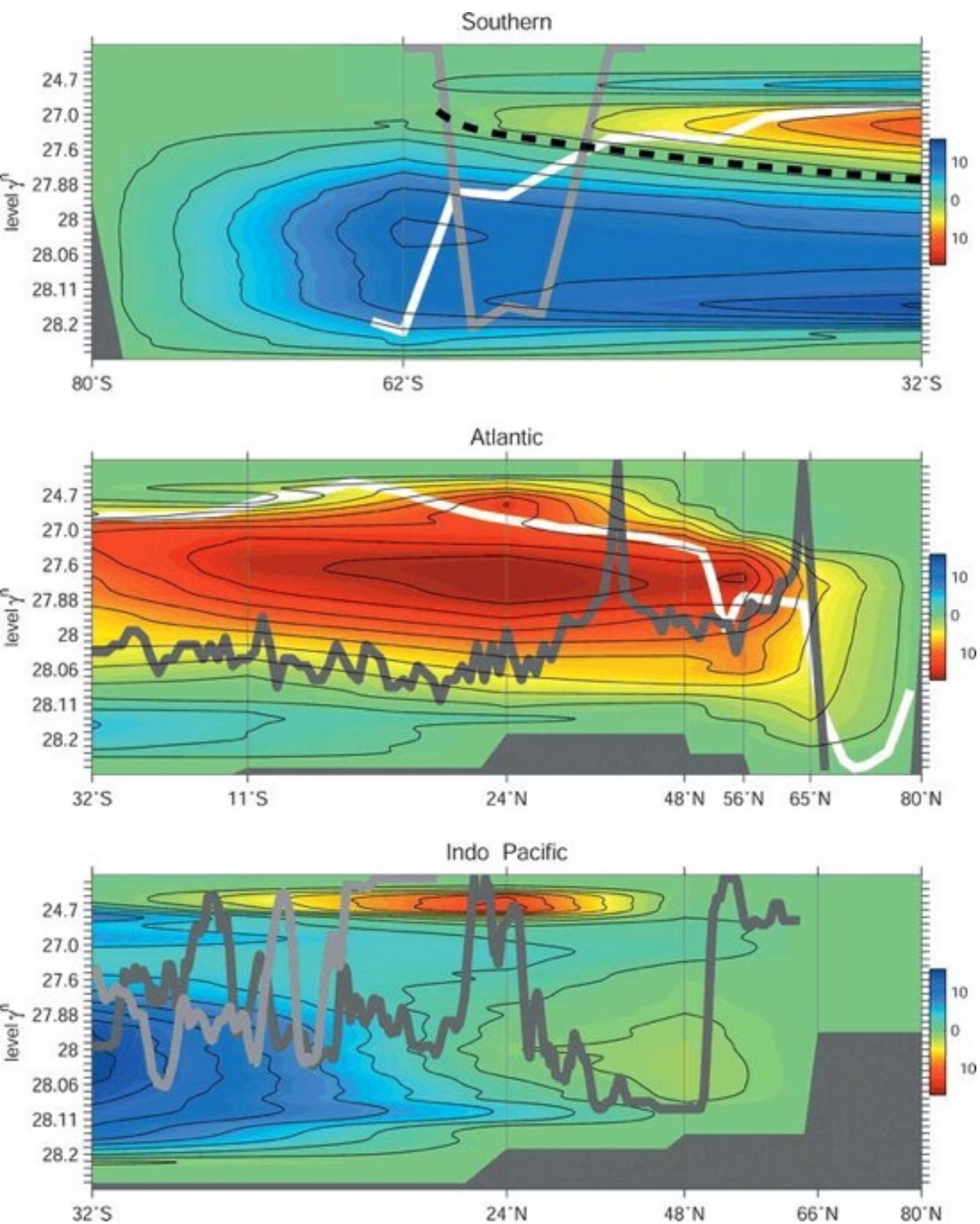
Summary so far

The deep and abyssal circulation and stratification in the world ocean is set in the ACC: competition between overturning by wind-stress and APE release by baroclinic eddies.

The MOC occurs primarily along isopycnals, with contributions from wind-stress at end-points, diapycnal diffusion and eddy-fluxes of buoyancy.



Global Meridional Overturning Circulation (3-D)



Zonally integrated in different sectors vertically cumulated in density

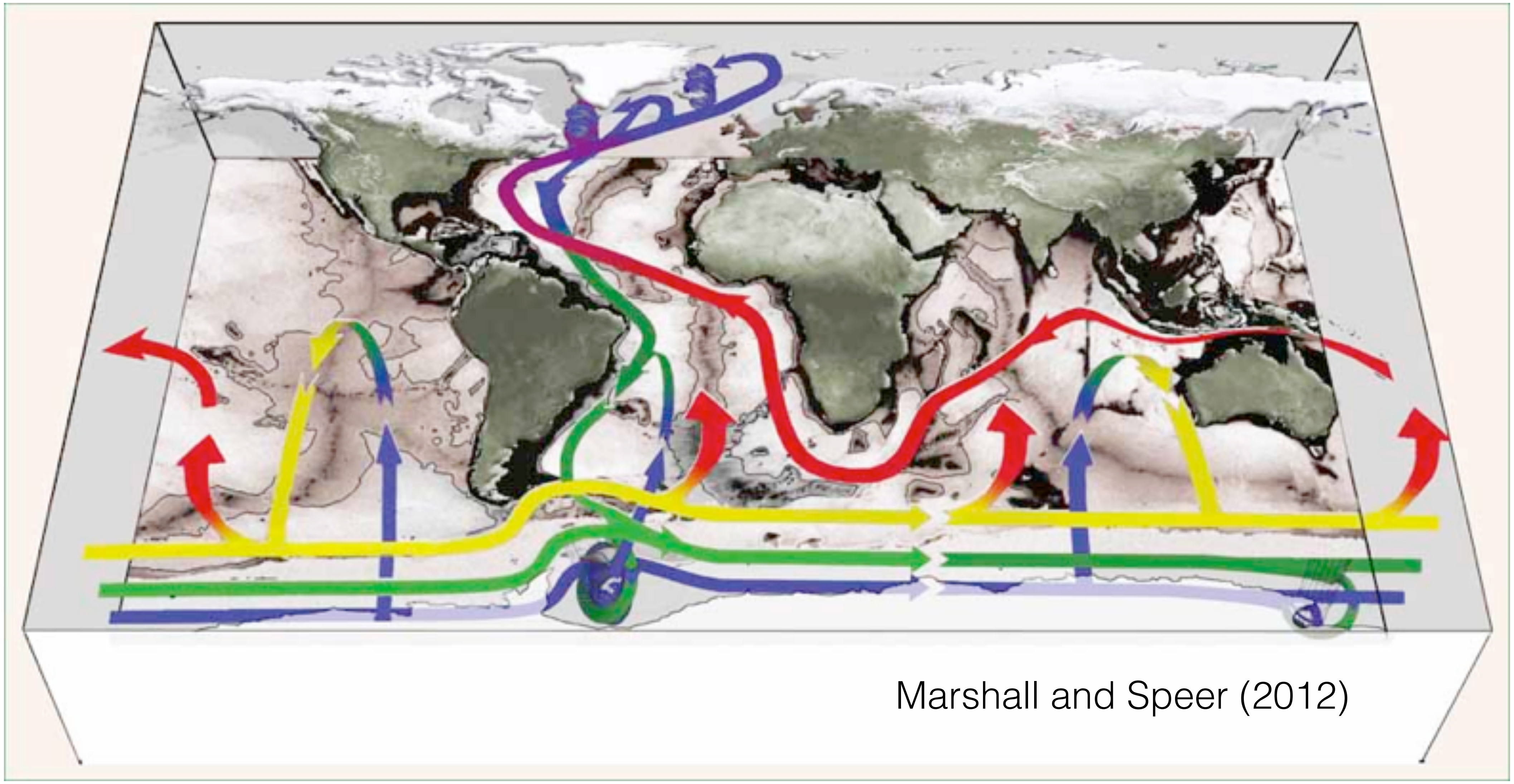
Southern Ocean sector: from 80S to 32S all longitudes. Dominated by abyssal thermally direct call, and a deep thermally indirect cell.

Deep cell in the Atlantic + Southern Ocean

Abyssal cell in Ind-Pacific + Southern Ocea

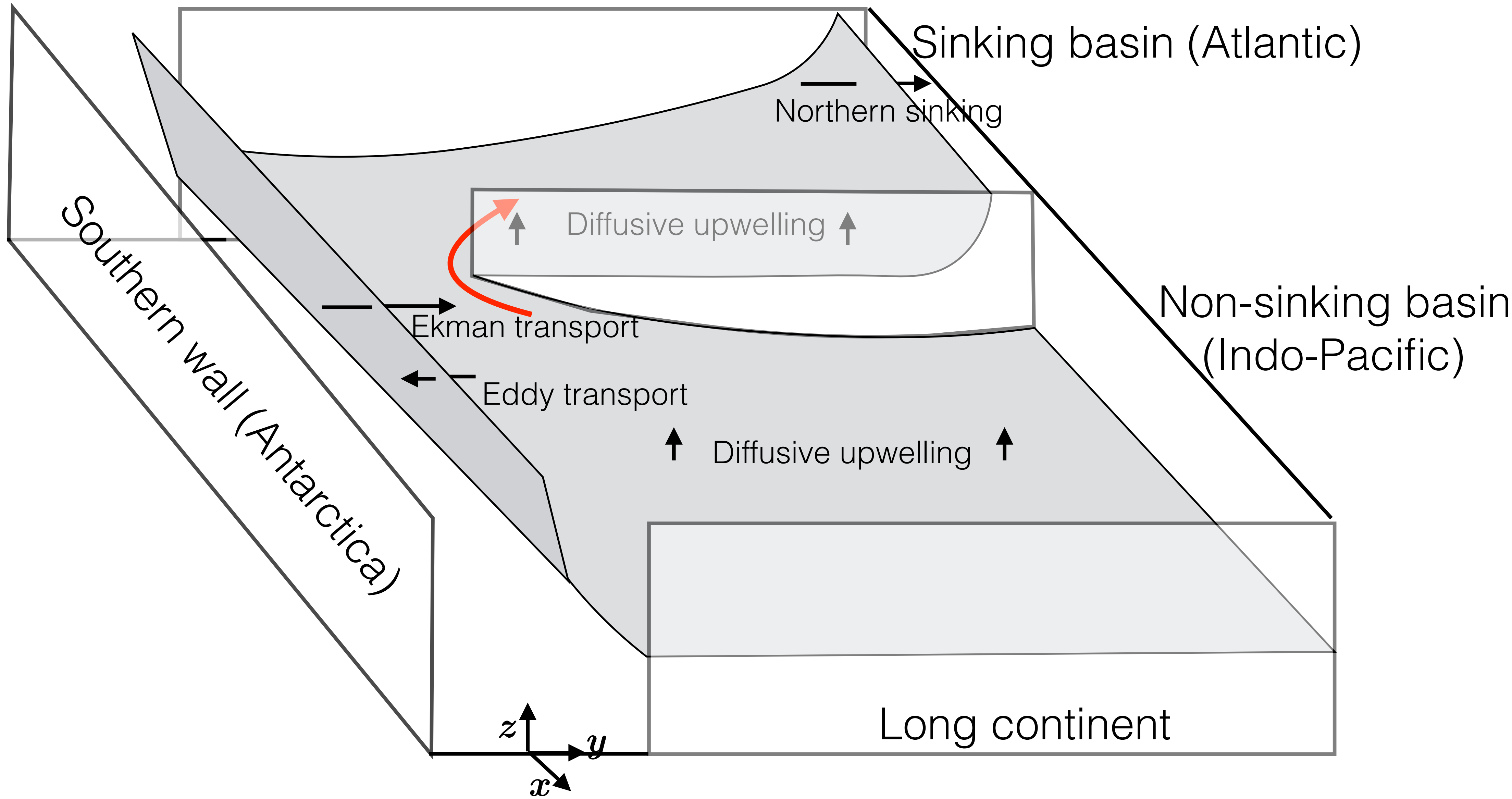
Lumpkin and Speer (2006)

A schematic of the **M**eridional **O**verturning **Circulation** in 3-D



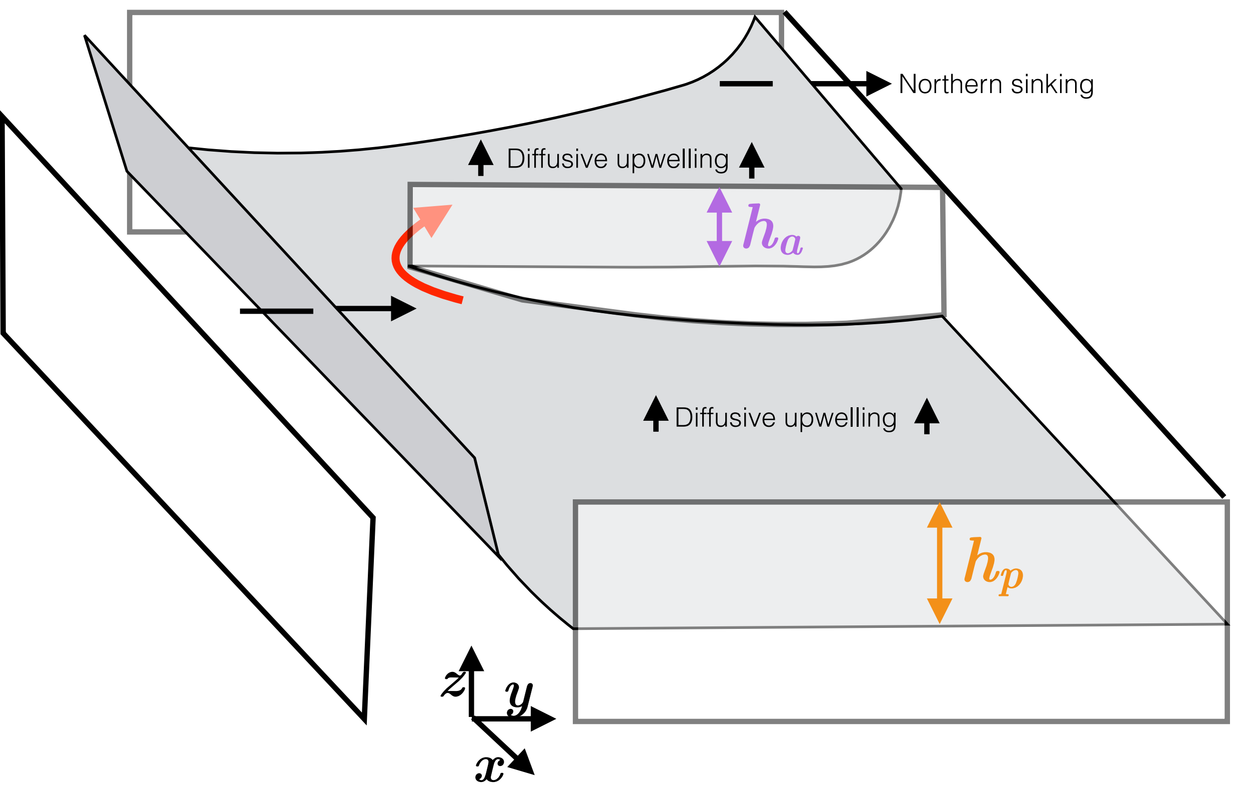
- Wind-driven upwelling in the Southern Ocean (SO) pumps waters up & north
- Baroclinic eddies transport buoyancy poleward in the SO
- Diffusive upwelling in all oceans pumps deep water up
- Deep water formation occurs in North Atlantic (saltier), but not North Pacific
- The sinking site is in the Atlantic
- **Water entering the Pacific returns to the Atlantic in the upper branch**

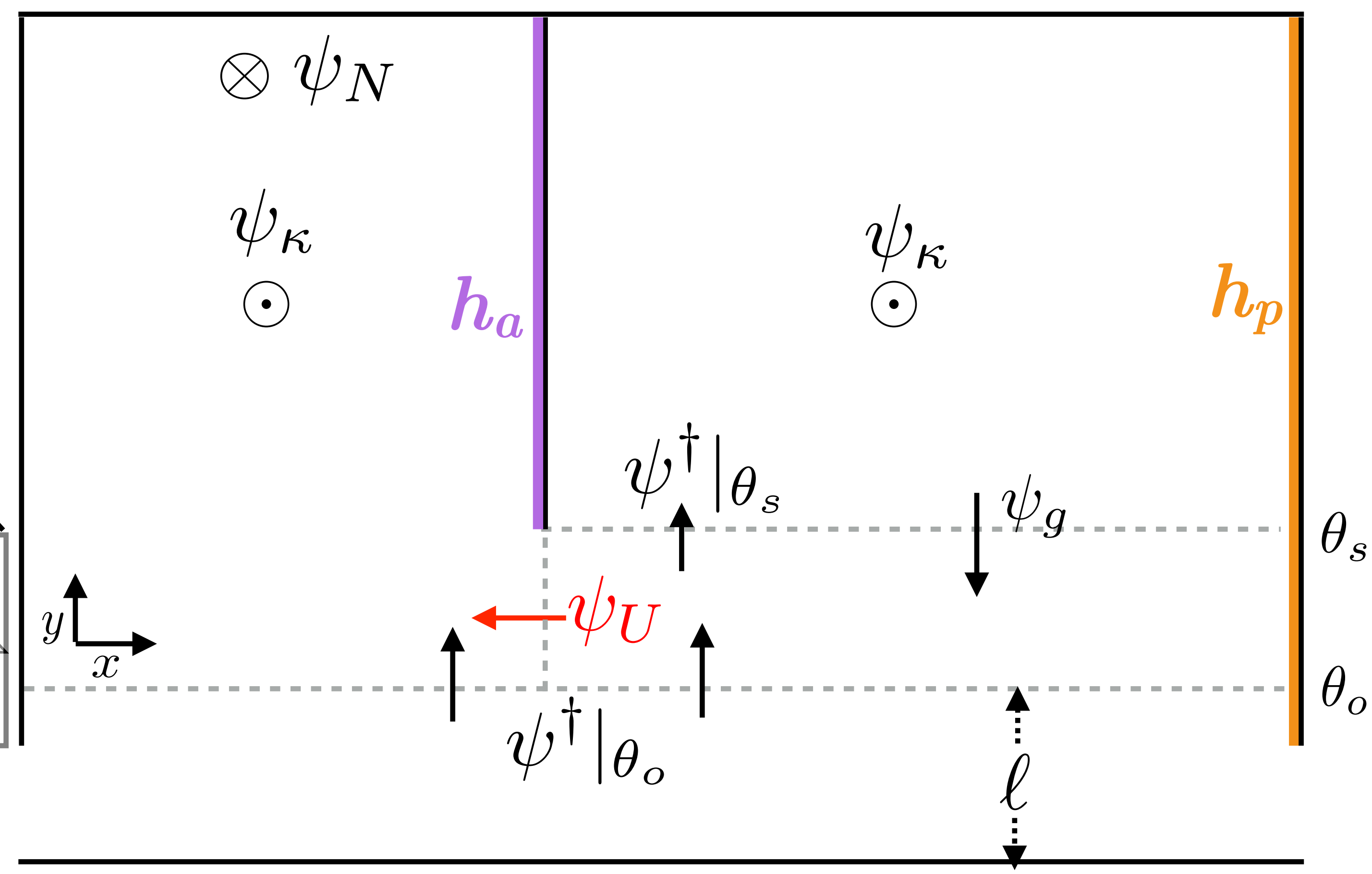
Add a non-sinking basin connected through a channel



No northern sinking in passive basin: Ekman transport from Southern Ocean + diffusive upwelling must transfer from non-sinking to sinking basin

A simple model of buoyancy transport budget





$\psi_g, \psi_N, \psi_\kappa$ can be related to h_a & h_p

$$\psi^\dagger|_{\theta_o} = -\frac{\tau}{\rho f}L_x - \kappa_a \frac{h_p}{\ell}L_x \quad \psi_N = \epsilon \Delta b \frac{h_a^2}{2f_n}$$

h_a Isopycnal depth on east of active basin

h_p Isopycnal depth on east of passive basin

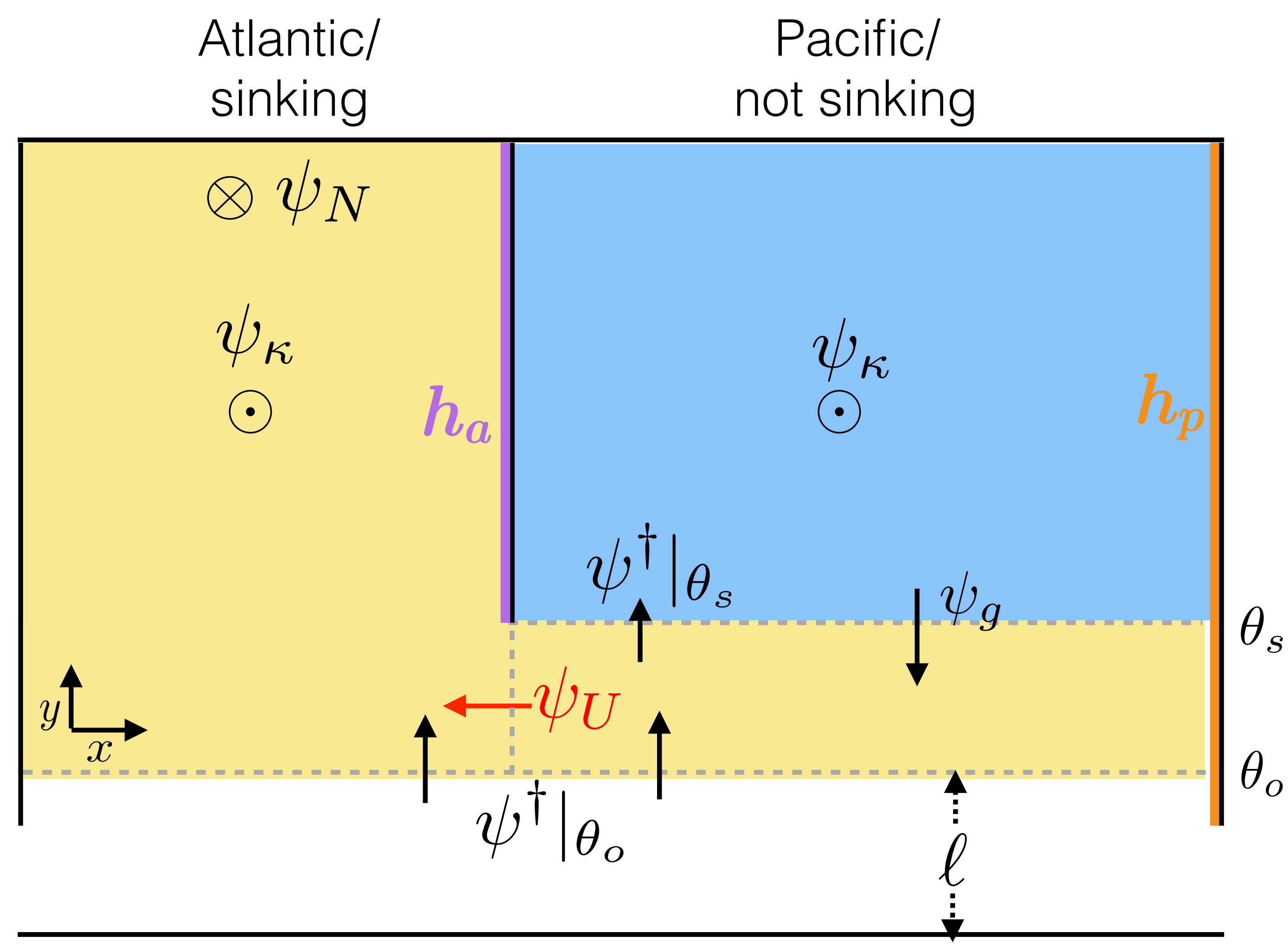
$$\psi_\kappa = \kappa_v \frac{Area}{h}$$

$$\boxed{\psi_g = \Delta b \frac{h_p^2 - h_a^2}{2f}}$$

New exchange term

Buoyancy budget for two regions: &

Budgets: 2 equations
in 2 unknowns, h_a & h_p



$\psi_g, \psi_N, \psi_\kappa$ can be related to h_a & h_p

$$\psi^\dagger|_{\theta_s} = -\frac{\tau}{\rho f} L_p - \kappa_a \frac{h_p}{\ell} L_p$$

$$\psi^\dagger|_{\theta_o} = -\frac{\tau}{\rho f} L_x - \kappa_a \frac{h_p}{\ell} L_x$$

$$\psi_N = \epsilon \Delta b \frac{h_a^2}{2 f_n}$$

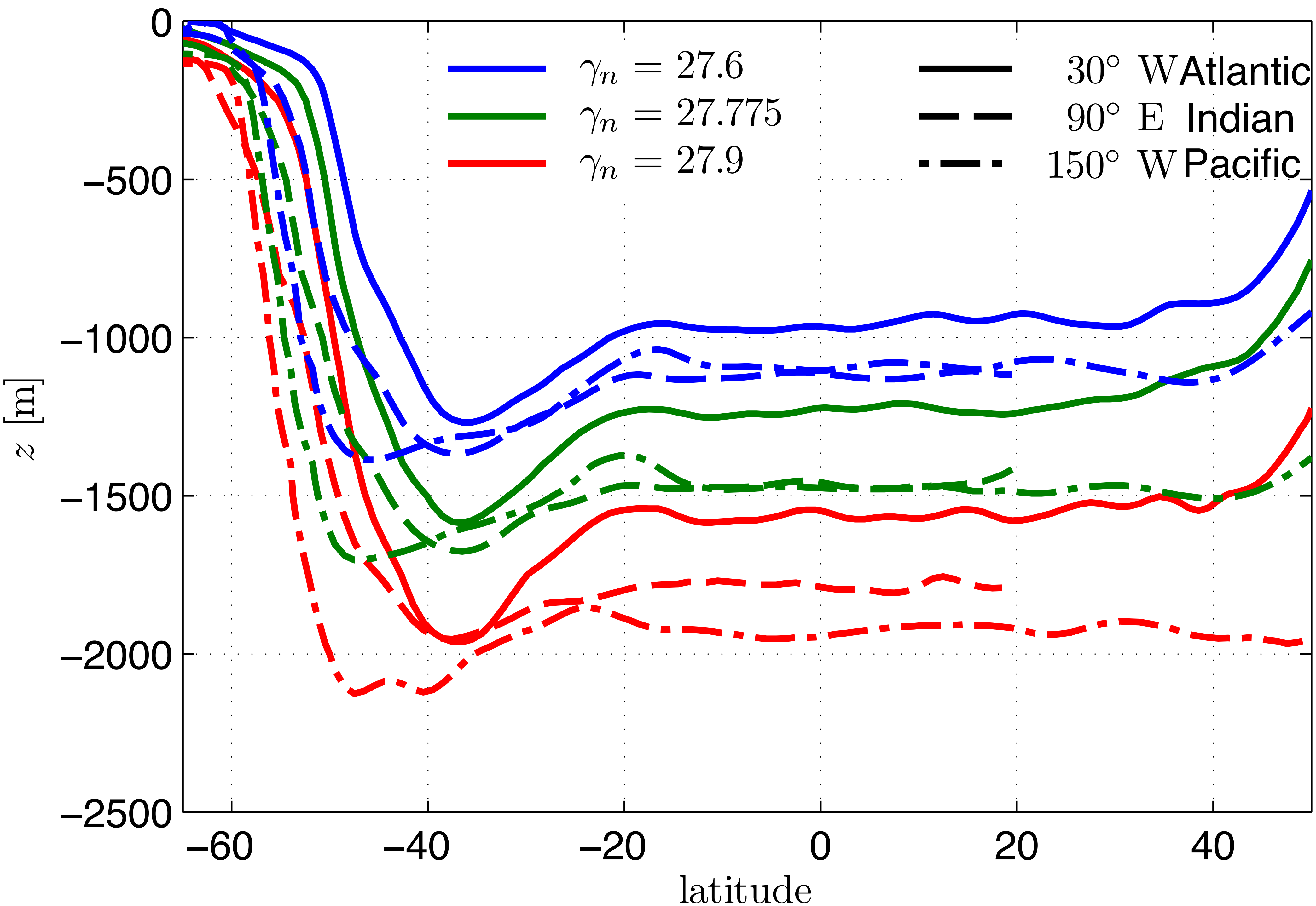
ψ_U calculated as a residual

$$\psi_\kappa = \kappa_v \frac{Area}{h}$$

$$\boxed{\psi_g = \Delta b \frac{h_p^2 - h_a^2}{2f}}$$

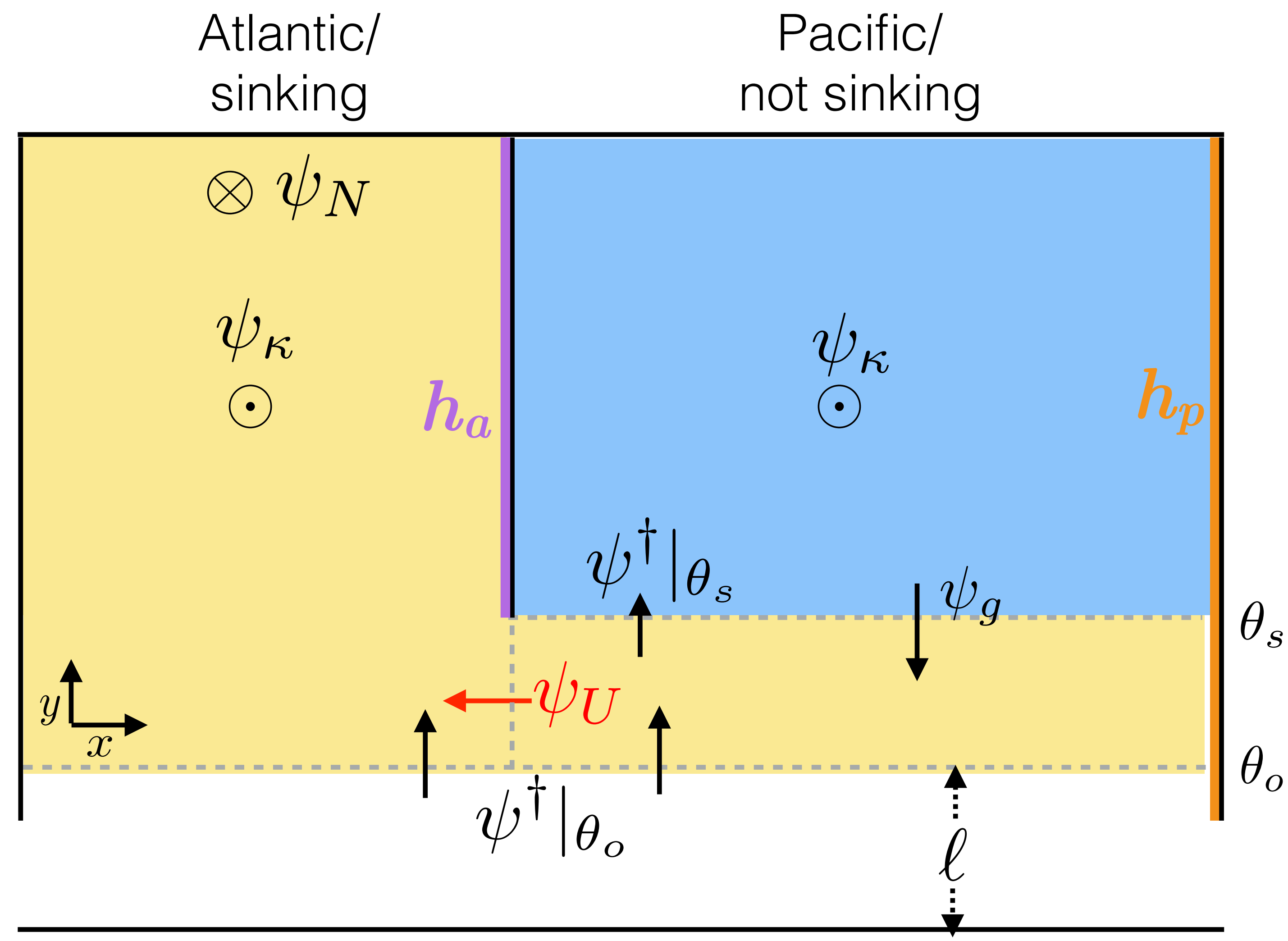
$$< 0 \implies h_p > h_a \quad (f < 0)$$

Isopycnals are deeper in non-sinking basin



Observed depths of isopycnals: shallower in Atlantic, especially at intermediate levels (upper branch of MOC) relative to Indo-Pacific

Zonal exchange from Indo-Pacific to Atlantic

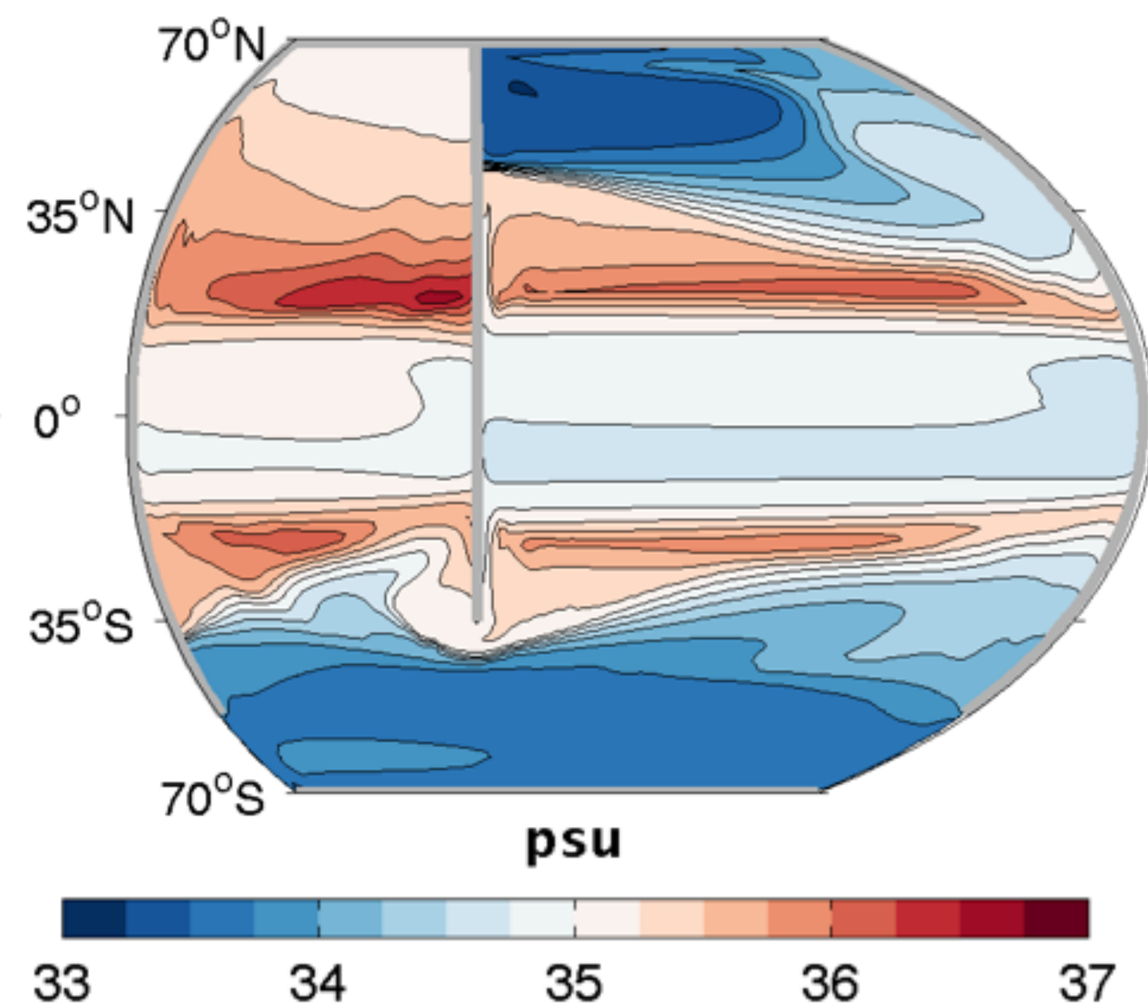


No net flow entering blue region

$$\psi_U = -\frac{\tau}{\rho f}|_{\theta_o} L_p + \kappa_a \frac{h_p}{\ell} L_p$$

The Ekman - eddy transport entering the passive sector must transfer to sinking sector

Check these predictions with a GCM



Geometry: two basins + circumpolar region in SH:

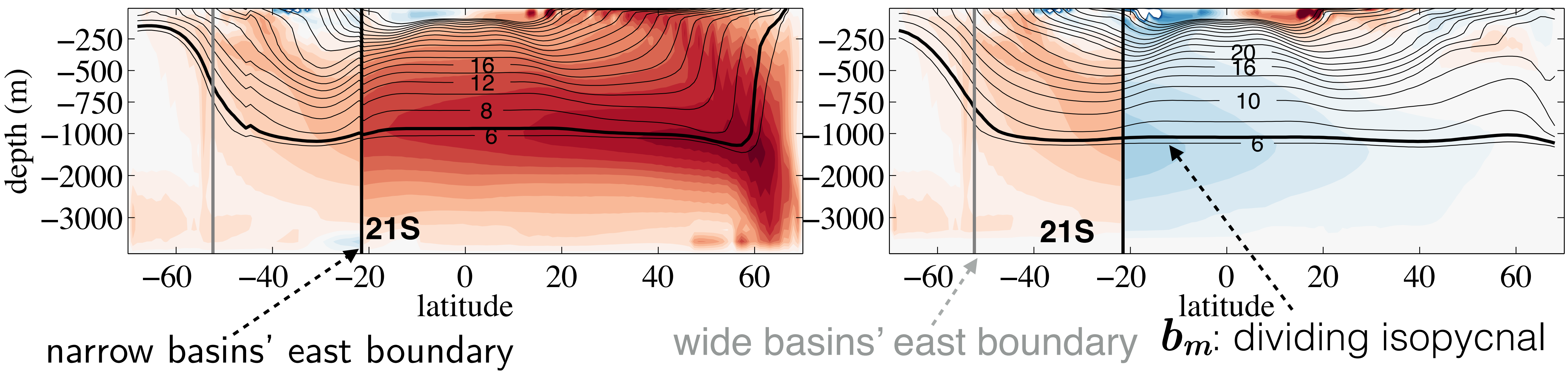
- Basins have same latitudinal extent
- Basins have different widths (narrow one is half as wide)
- Different continent lengths in the S.H.
- Continents are 1-grid space wide
- Zonally uniform surface forcing (wind, temperature, freshwater flux)
- Primitive equations on a sphere (MITgcm)
- Low-diffusivity regime
- 1 degree resolution with GM parametrization
- Domain is 210 degrees wide and periodic, 4000m deep.
- Submarine ridge south of long continent's end

Surface salinity is higher in sinking basin despite zonally uniform freshwater flux

The zonally integrated ROC

Atlantic/sinking basin

Pacific/non-sinking basin

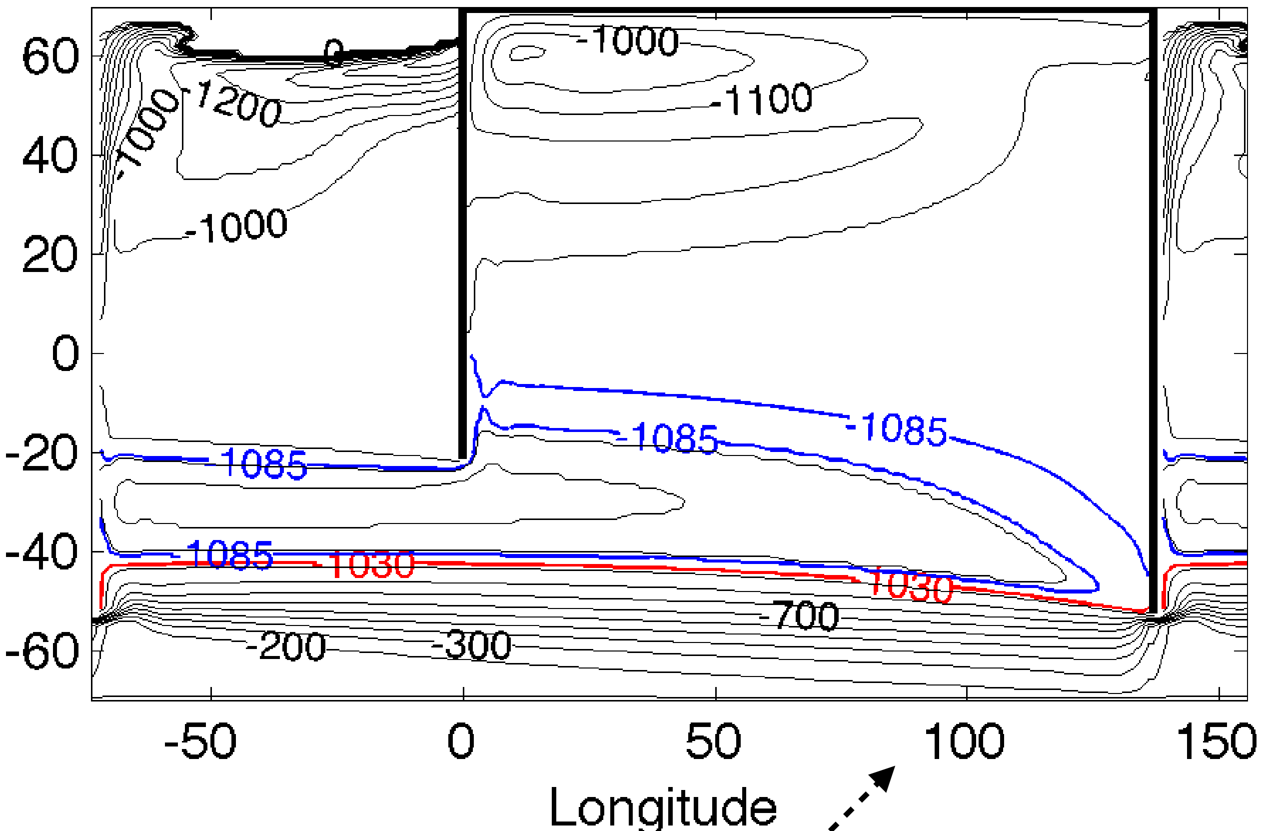


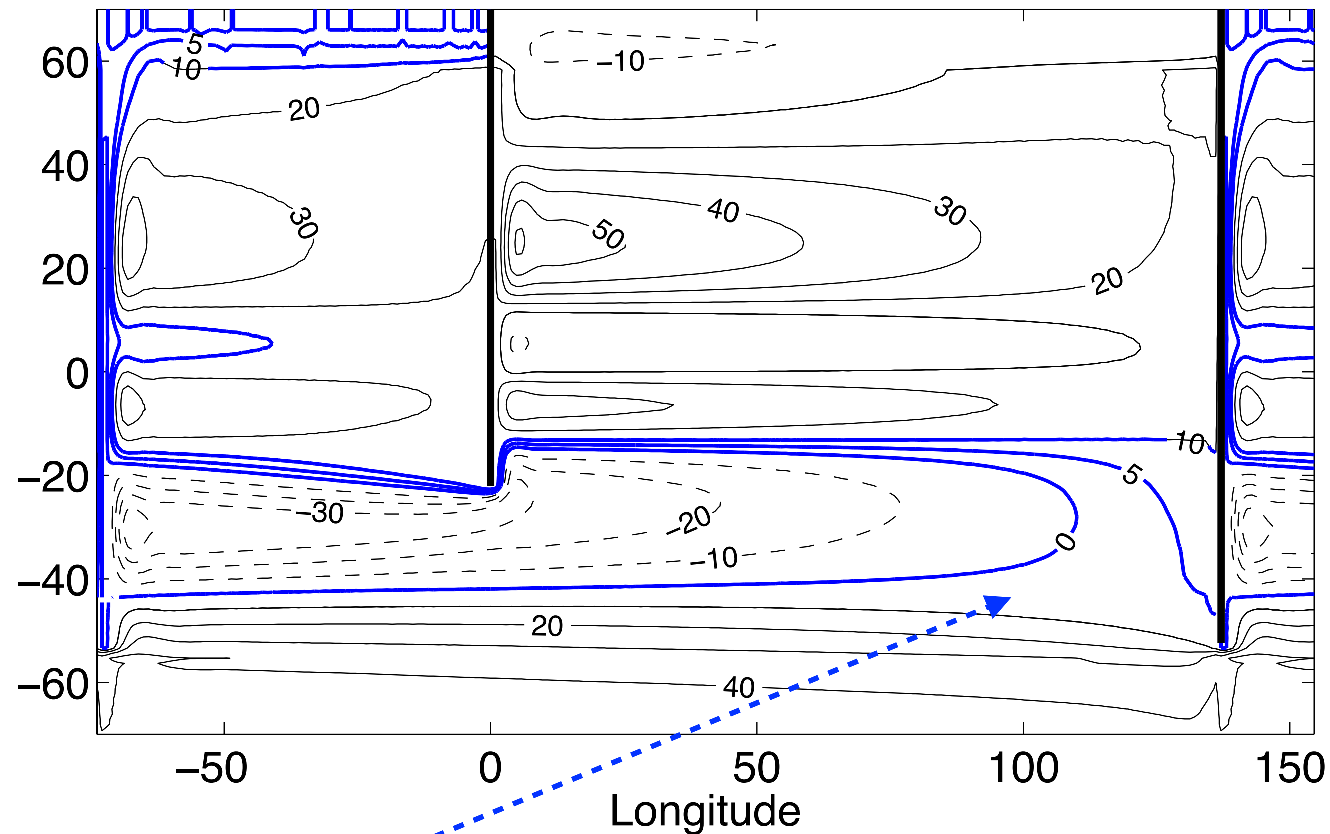
Color: transport in 10^6 m^3/s

Contours: buoyancy

- Sinking is in narrow basin, with short continent to the east
- Properties on upper branch of ROC, averaged above b_m

The upper branch of the MOC





Depth of target isopycnal dividing the upper and lower branches of the MOC: shallower in sinking basin

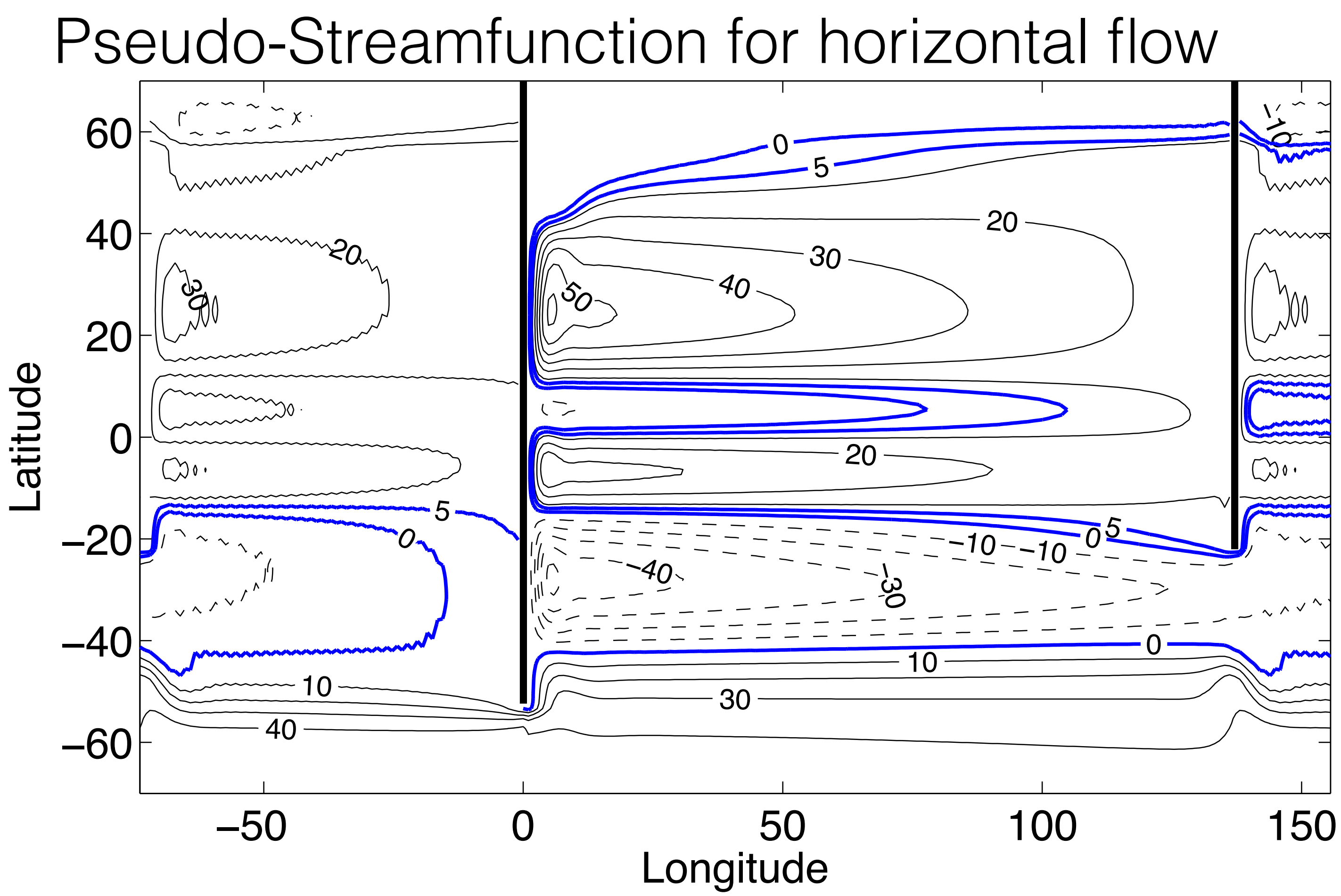
Residual zonal velocity integrated above target isopycnal

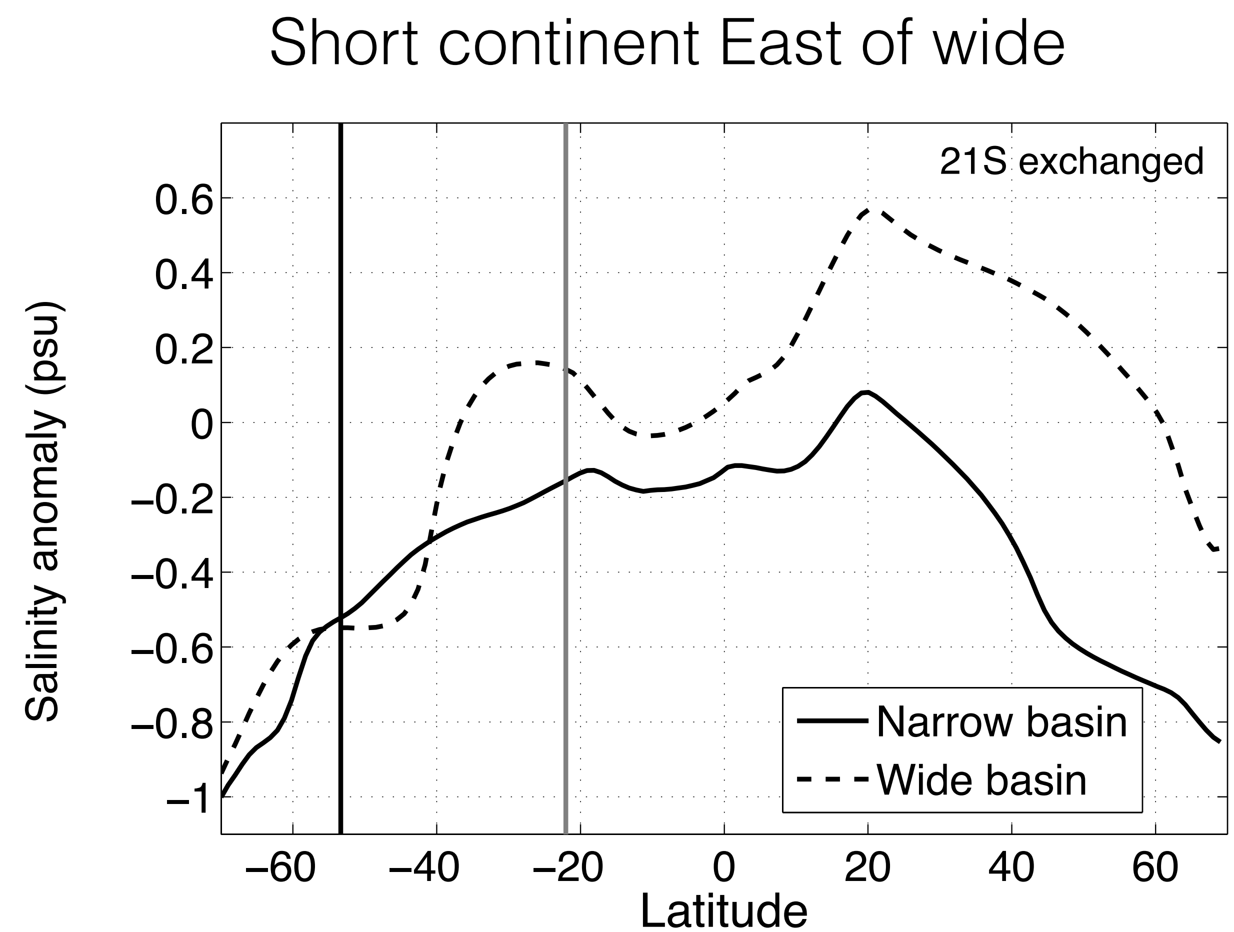
Cross-isopycnal velocity

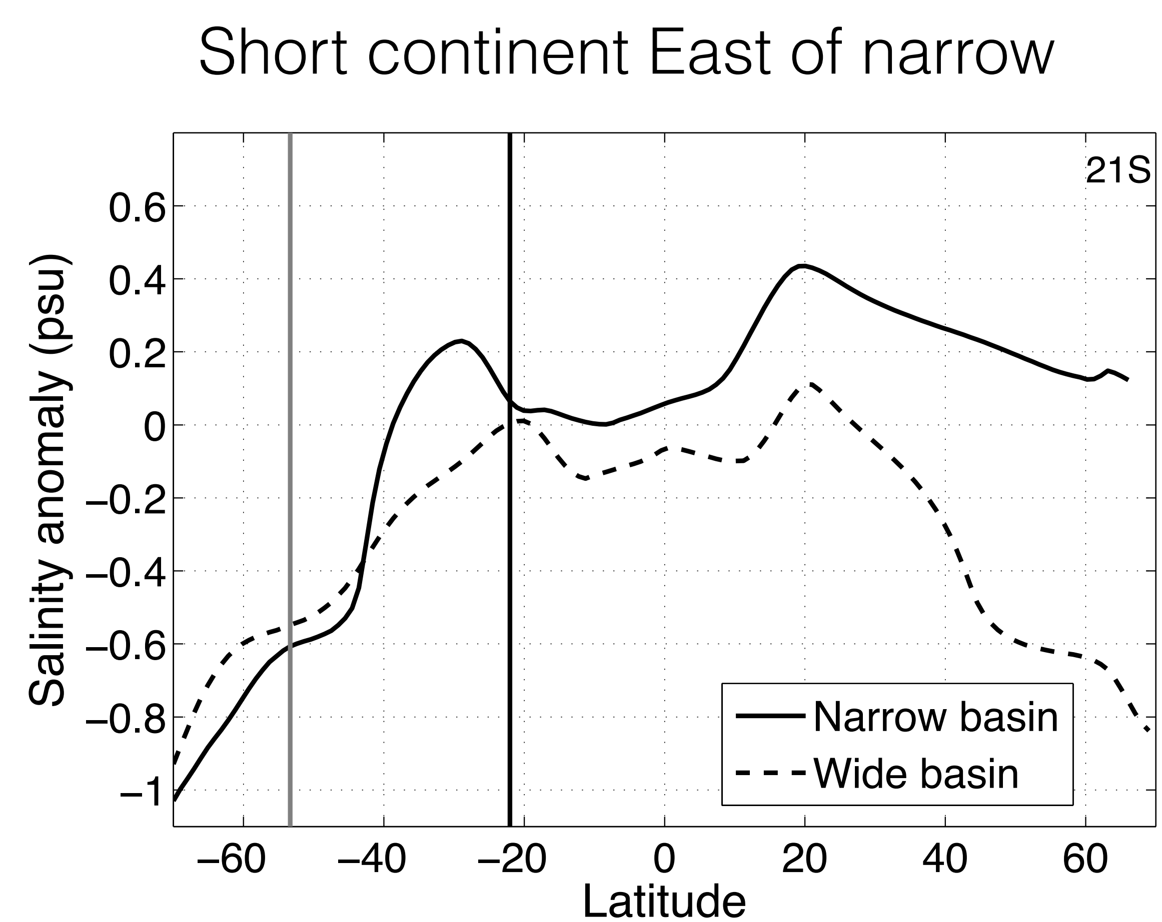
$$\phi = -\int_{-L}^{y} d\hat{y} \left[\hat{U}(x,\hat{y}) - \int_{0}^{x} d\hat{x} \varpi(\hat{x},\hat{y}) \right]$$

15 x 10^6 m^3/s sinking: 10 x 10^6 m^3/s are exchanged from non-sinking to sinking basin

Wide basin west of short continent: less salinity difference



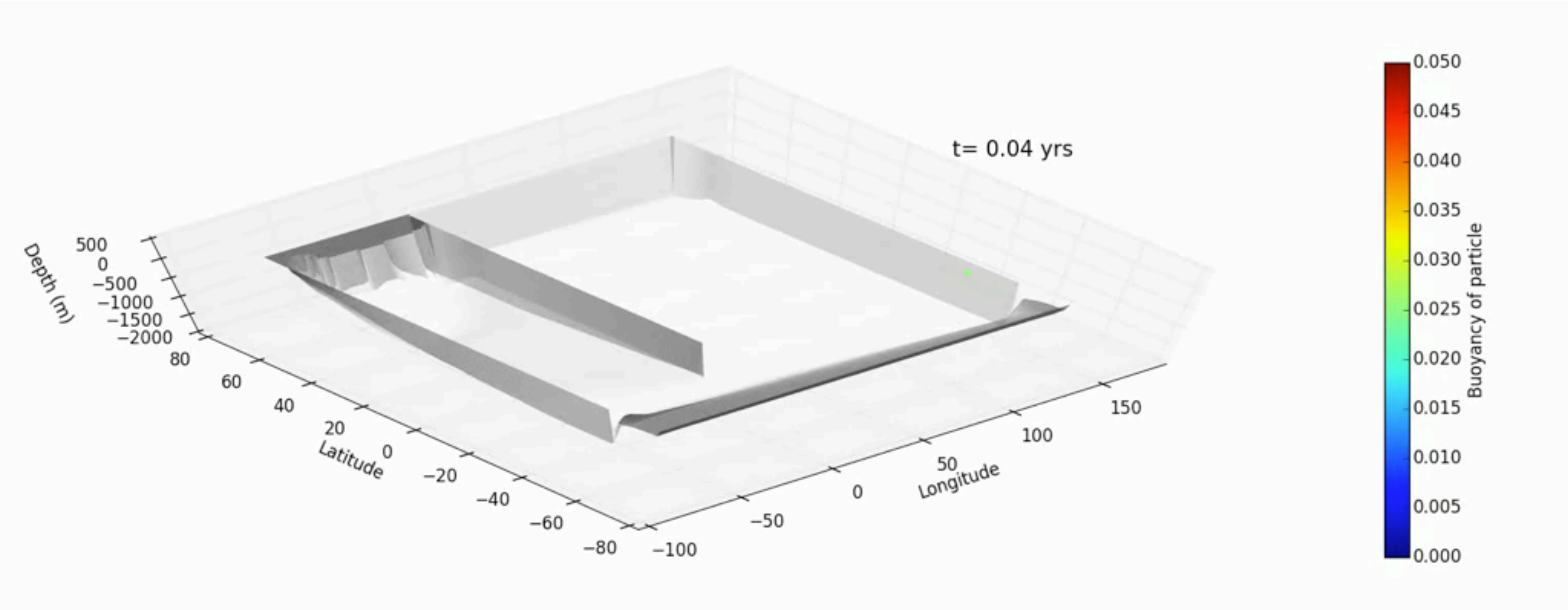




Interbasin transport is smaller for wide-basin sinking

Interbasin salinity difference is smaller for wide-basin sinking

A 50-years journey of a particle

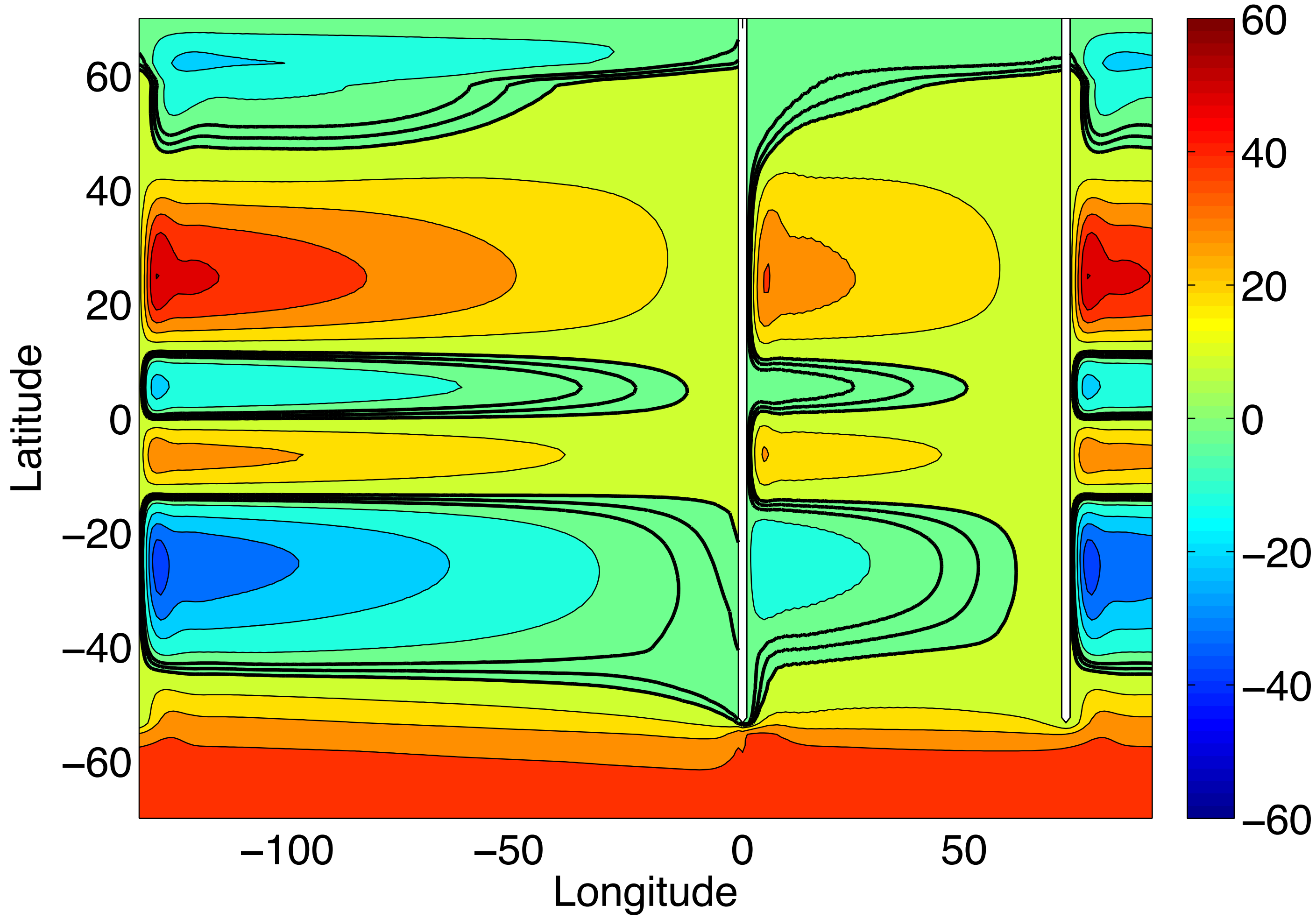


Particles stay mostly on isopycnals, except in tropics and sinking region

Horizontal structure of the flow above b^*

Visualize the 2-d flow integrating $\phi_y = -U + \int^x \varpi|_{-h}\, dx$

Narrow basin sinking



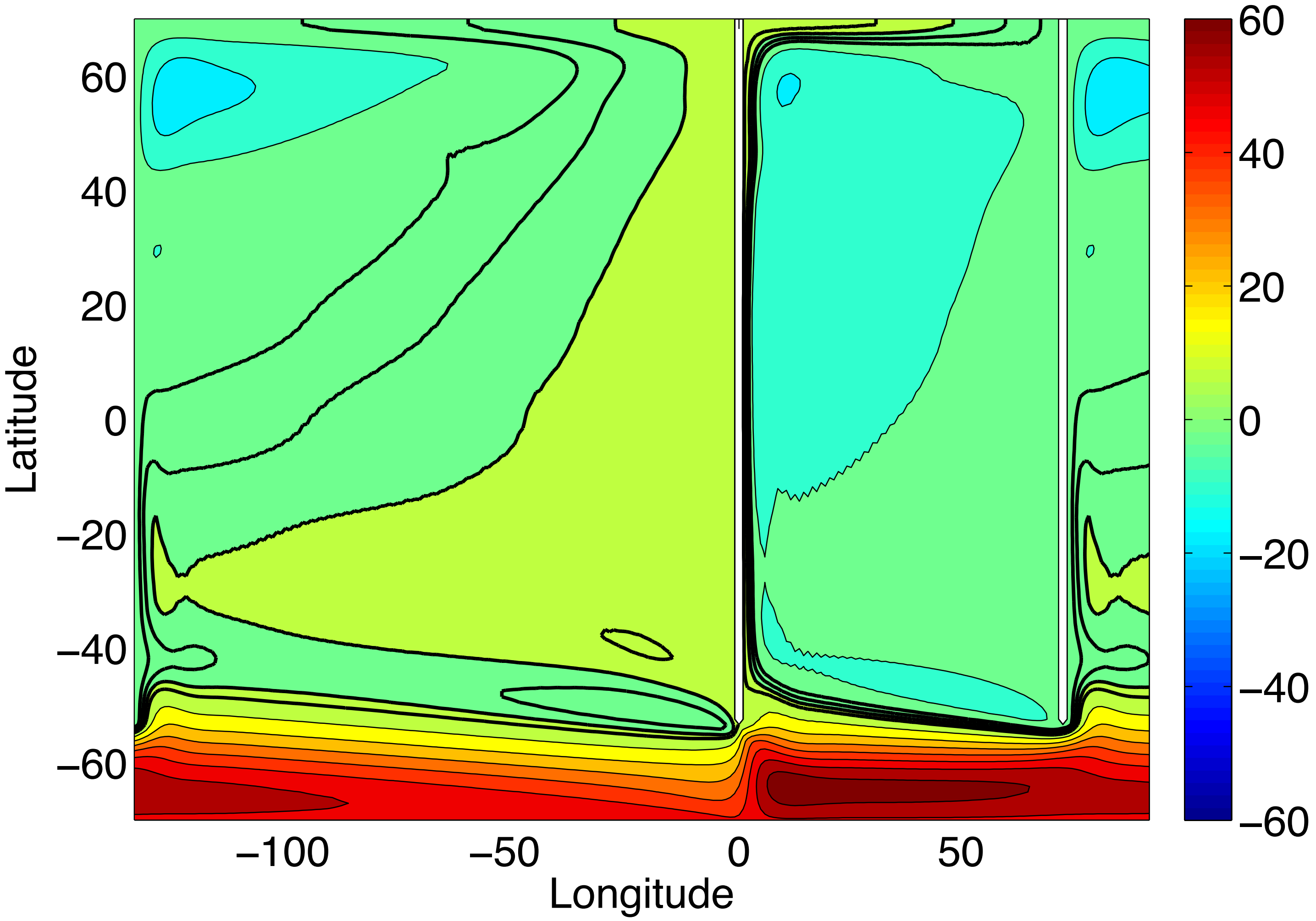
Thick contours: 2.5 Sv apart. Colors: 10 Sv apart

Exchange flow originates in SH of passive basin and enters active basin on western boundary

Horizontal structure of the flow below b^*

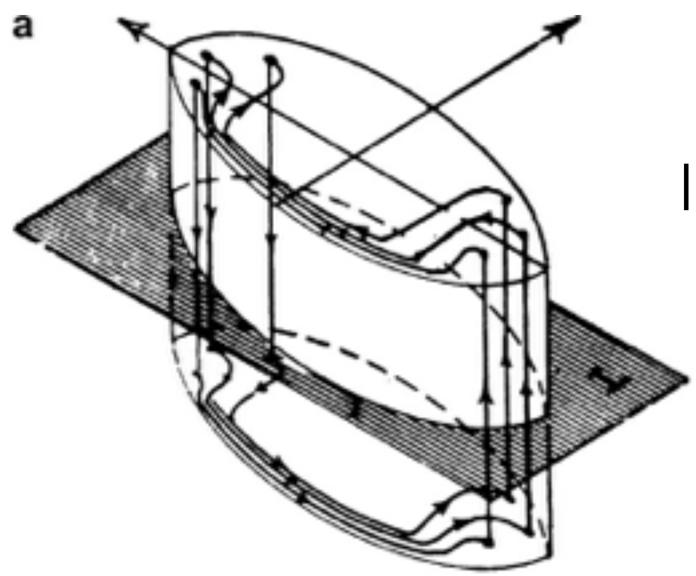
Visualize the 2-d flow integrating $-U$

Narrow basin sinking



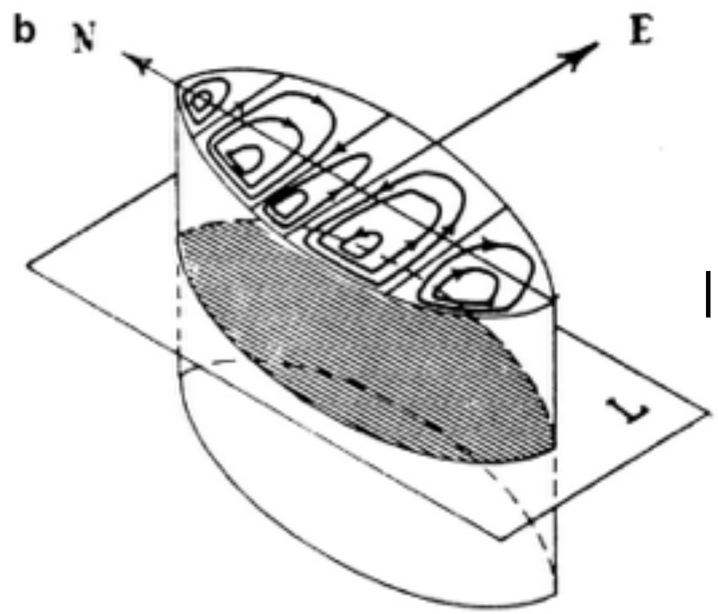
The lower branch of the ROC: deep western boundary current, Stommel-Arons gyre merging with wind-driven subpolar gyre in weakly stratified region.

Similar to Stommel's view (1957) of the OC



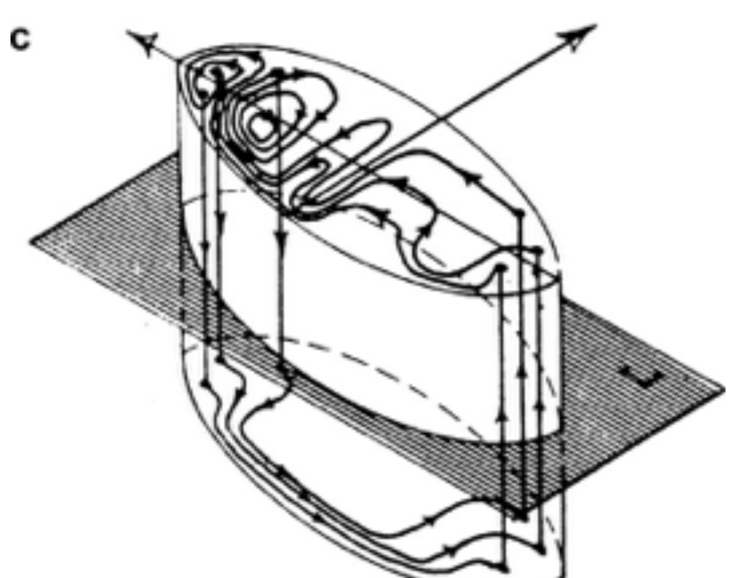
Inter-hemispheric circulation, confined to a boundary current

+



In the top branch we also have locally wind-driven gyres

=



Currents snaking around with open and closed streamlines