Three-Dimensional Propagation of Transient Quasi-Geostrophic Eddies and Its Relationship with the Eddy Forcing of the Time–Mean Flow

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(Manuscript received 5 February 1985, in final form 28 January 1986)

ABSTRACT

An approximate theory is developed of small-amplitude transient eddies on a slowly varying time-mean flow. Central to this theory is a flux M_T, which in most respects constitutes a generalization of the Eliassen–Palm flux to three dimensions; it is a conservable measure of the flux of eddy activity (for small amplitude transients) and is parallel to group velocity for an almost-plane wave train. The use of this flux as a diagnostic of transient eddy propagation is demonstrated by application of the theory to a ten-year climatology of the Northern Hemisphere winter circulation. Results show the anticipated concentration of eddy flux along the major storm tracks.

While, in a suitably transformed system, M_T may be regarded as a flux of upstream momentum, it is not a complete description of the eddy forcing of the mean flow; additional effects arise due to downstream transience (i.e., spatial inhomogeneity in the direction of the time-mean flow) of the eddy amplitudes.

The relation between M_T and the "E-vector" of Hoskins et al. is discussed.

1. Introduction

The development of the Eliassen–Palm (EP) flux concept (Eliassen and Palm, 1961; Andrews and McIntyre, 1976, 1978c) and its application as a diagnostic tool (e.g., Edmon et al., 1980) has greatly simplified the interpretation of some aspects of atmospheric circulation statistics and has thus led to rapid advances in the appreciation of the propagation of atmospheric eddies and their role in the general circulation. Briefly, the important properties of the EP flux **F** are

(i) For small amplitude waves on a zonal flow, **F** appears as the flux of wave activity in a conservation relation, relating $\nabla \cdot \mathbf{F}$ to wave transience and nonconservative effects.

(ii) In the WKB limit of almost-plane waves on a slowly varying mean flow, **F** is parallel to the group velocity.

(iii) for quasi-geostrophic flow, $\nabla \cdot \mathbf{F}$ is proportional to the northward eddy flux of quasi-geostrophic potential vorticity.

(iv) The quasi-geostrophic momentum and thermodynamic equations may be transformed in such a way that the only term describing eddy, mean flow interaction is an effective zonal force, proportional to $-\nabla \cdot \mathbf{F}$; thus **F** may be regarded as an effective flux of easterly momentum.

In addition to the practical benefits of **F** as a diagnostic tool, the conceptual framework embodied in (i)–(iv) has profound implications, linking as it does eddy transport properties with propagation characteristics. The general usefulness of this framework has thus far been limited, however, by the restriction of the theory to waves on zonal-mean flows. Many important questions relating to the role of eddies in the general circulation are more naturally addressed by partitioning the circulation into time-mean and transient eddy components. In fact, the EP formalism is intimately related to the concept of pseudomomentum conservation for eddies on a zonal-mean flow in generalized Lagrangian-mean (GLM) theory (Andrews and McIntyre, 1978a,b; McIntyre, 1980). For the case of transient eddies on a spatially nonuniform, time-mean flow, pseudomomentum conservation is replaced by pseudoenergy conservation in GLM theory and some attempts have been made to exploit this in an Eulerian framework. Thus, Andrews (1983) and Plumb (1985a) have derived conservation laws for small amplitude transients on a three-dimensional time-mean flow, while Dunkerton (1983) has discussed the mean pseudoenergy budget. However, as yet there is no complete theory of an Eulerian analogue of pseudoenergy.

In the absence of any such theory, diverse approaches have been followed to analyze the impact of transient eddies on the time-mean flow; a discussion of the advantages and shortcomings of these various techniques is given by Holopainen (1984). Interpretation of eddy momentum and heat fluxes is beset by the same problems that arise in the zonal-mean case (for a discussion

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of which, see McIntyre, 1980) although an interesting recent development has involved solution of the full time-mean system of equations to assess simultaneously the impact of both momentum and heat fluxes (Lau and Holopainen, 1984). A notable shortcoming of most such approaches, however, as compared with EP diagnostics, is the lack of any information on eddy propagation characteristics. Some information on the generation and dissipation of eddy activity is implicit in the potential vorticity flux, since the component of this flux down the mean potential vorticity gradient is related to nonconservative effects and to downstream (i.e., following the mean flow) eddy transience (Rhines and Holland, 1979). Further, Illari and Marshall (1983) have shown that, for an almost-conservative mean flow, the contribution to the flux associated with downstream eddy transience is nondivergent and they have thus defined a "residual" flux whose downgradient component is simply related to nonconservative sources and sinks of eddy enstrophy. However, these results, useful though they are in relating eddy transport processes to the properties of the eddies themselves (e.g., Illari and Marshall, 1983; Shutts, 1983), still fall short of a direct link with eddy propagation. While Young and Rhines (1980) have derived a conservation law for eddy enstrophy in a constant mean potential vorticity gradient, this latter restriction renders the approach unsuitable for diagnostic applications in the atmosphere.

Recently, Hoskins et al. (1983) have defined a "quasi-vector" E which under certain approximations may be regarded as an effective flux of easterly momentum. Under further approximations, they showed that it is also related, but not parallel, to the group velocity relative to the mean flow. However, unlike the EP flux, E does not appear to be associated with any conservation relation, and there is no obvious way of determining its relationship with the total group velocity (i.e., that relative to fixed coordinates). Further, it will be shown in section 3 that one of the assumptions which underlie the association with group velocity is rather poor for the synoptic-scale, baroclinic eddies.

In this study, the derivation of a conservation law for small-amplitude transient eddies is taken as a starting point. As already noted, one approach in this direction—the one which GLM theory suggests as the most appropriate—involves Eulerian analogues of pseudoenergy conservation. However, partly because of the difficulty of applying these results using conventional atmospheric circulation statistics and partly because, at present, there is no complete Eulerian theory of transient eddies and their interaction with the time-mean flow, this approach is not pursued here. Instead, an attempt is made to retain, as far as possible, the essentials of the EP formalism. While this approach loses its validity if the basic state is spatially nonuniform, it is shown here that an approximate conservation relation which is similar to the EP relation (and

which reduces to that relation in the appropriate limit) can be derived for sufficiently weak departures of the mean state from zonal uniformity. Thus, on the basis of an assumption that the time-mean flow is slowly varying in an appropriate sense, a flux M_T is defined in section 2 that is a conservable measure of the flux of eddy activity and that is parallel to the group velocity for an almost-plane wave. The applicability of the slowly varying assumption to real climatological states is addressed in section 3. Results for M_T in a ten-year climatology of the Northern Hemisphere are presented, showing the anticipated concentration of transient eddy fluxes within the oceanic storm tracks and, in fact, showing relatively little difference in propagation characteristics between the "band-pass" and "low-pass" transients. Some aspects of the results are, however, difficult to explain. Calculation of M_T places more severe demands on data quality than is the case with more conventional diagnostic procedures; the possible impact of data errors is discussed.

The thrust of this paper concerns eddy propagation characteristics. As noted earlier, however, the singular appeal of the EP flux in the zonally averaged problem derives from its dual interpretation as a flux of eddy activity and as a measure of eddy momentum transport. For the present problem, the partial relationship between M_T and transient eddy momentum transport is discussed briefly in section 4. It is shown that, for an almost conservative mean flow, $\nabla \cdot M_T$ is proportional to the downgradient component of Illari and Marshall's residual potential vorticity flux. This leads to the development of a transformed set of time-mean momentum and thermodynamic equations in which M_T appears as an effective flux of "pseudowestward" momentum. In these two respects, M_T represents a generalization of the EP flux; however in this three-dimensional time-mean problem additional transport terms (cross-gradient potential vorticity and cross-stream momentum fluxes) arise which are related not to eddy generation and dissipation as such but to spatial inhomogeneity (downstream transience) of the eddy amplitudes.

2. A conservation law for transient eddies on a slowly varying time-mean flow

For quasi-geostrophic flow whose streamfunction is $\psi = \Phi/f$ where Φ is geopotential and $f = 2\Omega \sin\phi$ the Coriolis parameter, the quasi-geostrophic potential vorticity[1] is

$$q = f + \frac{1}{a^2 \cos^2\phi} \frac{\partial^2\psi}{\partial\lambda^2}$$

$$+ \frac{1}{a^2 \cos\phi} \frac{\partial}{\partial\phi}\left(\cos\phi \frac{\partial\psi}{\partial\phi}\right) + \frac{f^2}{p}\frac{\partial}{\partial z}\left(\frac{p}{N^2}\frac{\partial\psi}{\partial z}\right) \quad (2.1)$$

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[1] Since quasi-geostrophic assumptions are invoked throughout this paper, q will be referred to simply as "potential vorticity."

where a is the earth's radius, λ and ϕ are longitude and latitude, p is pressure, $z = -H\ln p$, where H is a constant scale height and N is the buoyancy frequency. The potential vorticity satisfies

$$\frac{dq}{dt} = S \tag{2.2}$$

where d/dt is the derivative following the geostrophic flow and S represents nonconservative sources and sinks of potential vorticity.

Now, define by an overbar the time average over some interval $(0, \tau)$

$$\bar{a}(\lambda, \phi, z) = \frac{1}{\tau} \int_0^\tau a(\lambda, \phi, z, t). \tag{2.3}$$

It is assumed that $\partial\bar{a}/\partial t = 0$, i.e., that the time series is free of trends. We then denote by a prime the deviation from the average:

$$a'(\lambda, \phi, z, t) = a(\lambda, \phi, z, t) - \bar{a}(\lambda, \phi, z). \tag{2.4}$$

Note that $\overline{a'} = 0$. Suppose further that these deviations are small, so that $|a'| \sim O(\delta)$ in some appropriate dimensionless sense, where $\delta \ll 1$. Then, to $O(\delta)$, the linearized potential vorticity equation is, from (2.2),

$$\frac{Dq'}{Dt} + \mathbf{u}' \cdot \nabla_H \bar{q} = S' \tag{2.5}$$

where $\mathbf{u} = [-\partial\psi/(a\partial\phi), \partial\psi/(a\cos\phi\partial\lambda), 0]$ is the geostrophic velocity,

$$\nabla_H = \left(\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}, \frac{1}{a}\frac{\partial}{\partial\phi}, 0\right)$$

is the horizontal gradient operator and $D/Dt \equiv \partial/\partial t + \bar{\mathbf{u}} \cdot \nabla_H$ is the derivative following the time-mean geostrophic flow. Multiplying (2.5) by q' and taking the time average gives the eddy enstrophy equation

$$\frac{De}{Dt} + \overline{\mathbf{u}'q'} \cdot \nabla_H \bar{q} = \overline{S'q'} \tag{2.6}$$

(e.g., Rhines and Holland, 1979), where

$$e = \frac{1}{2}\overline{q'^2} \tag{2.7}$$

is the eddy potential enstrophy. Equation (2.6) is the time-mean equivalent of a similar expression for quasi-conserved quantities in the zonal-mean case (e.g., Plumb, 1979) and implies that the upgradient component of the eddy potential vorticity flux is related to nonconservative effects and to the time derivative *following the mean flow* of eddy enstrophy.

The eddy potential vorticity flux is a central component of the eddy enstrophy budget. In zonal flows, the northward component of this flux may be written as the divergence of another flux (Plumb, 1985b), which when zonally averaged, reduces to the EP flux (e.g., Edmon et al., 1980). In fact, this relationship may be generalized for zonally nonuniform flow to give

$$p\cos\phi\overline{u_i'q'} = \partial_j B_{ij} \tag{2.8}$$

where

$$\nabla. = (\partial_1, \partial_2, \partial_3) = \left(\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}, \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\cos\phi, \frac{\partial}{\partial z}\right)$$

is the divergence operator, and

$$\mathbf{B} = p\cos\phi \begin{bmatrix} -\dfrac{1}{a^2\cos\phi}\overline{\dfrac{\partial\psi'}{\partial\lambda}\dfrac{\partial\psi'}{\partial\phi}} & \epsilon - \dfrac{1}{a^2}\overline{\dfrac{\partial\psi'}{\partial\phi}}^2 & \dfrac{-f^2}{aN^2}\overline{\dfrac{\partial\psi'}{\partial\phi}\dfrac{\partial\psi'}{\partial z}} \\[12pt] \dfrac{1}{a^2\cos^2\phi}\overline{\left(\dfrac{\partial\psi'}{\partial\lambda}\right)^2} - \epsilon & \dfrac{1}{a^2\cos\phi}\overline{\dfrac{\partial\psi'}{\partial\phi}\dfrac{\partial\psi'}{\partial\lambda}} & \dfrac{f^2}{aN^2\cos\phi}\overline{\dfrac{\partial\psi'}{\partial\lambda}\dfrac{\partial\psi'}{\partial z}} \\[12pt] 0 & 0 & 0 \end{bmatrix} \tag{2.9}$$

where

$$\epsilon = \frac{1}{2}\left\{\frac{1}{a^2\cos^2\phi}\overline{\left(\frac{\partial\psi'}{\partial\lambda}\right)^2} + \frac{1}{a^2}\overline{\left(\frac{\partial\psi'}{\partial\phi}\right)^2} + \frac{f^2}{N^2}\overline{\left(\frac{\partial\psi'}{\partial z}\right)^2}\right\} \tag{2.10}$$

is the wave energy density. Equation (2.9) has been derived previously by Hoskins et al. (1983); it follows by substitution from (2.1) and some straightforward manipulation. In terms of geostrophic velocities and potential temperature θ, (2.9) and (2.10) may be written

$$\mathbf{B} = p\cos\phi \begin{bmatrix} \overline{u'v'} & \epsilon - \overline{u'^2} & \dfrac{f\overline{u'\theta'}}{d\bar{\theta}/dz} \\[12pt] \overline{v'^2} - \epsilon & -\overline{u'v'} & \dfrac{f\overline{v'\theta'}}{d\bar{\theta}/dz} \\[12pt] 0 & 0 & 0 \end{bmatrix} \tag{2.11}$$

$$\epsilon = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \frac{Rp^\kappa}{H}\frac{\overline{\theta'^2}}{d\bar{\theta}/dz}\right) \tag{2.12}$$

where R is the gas constant and $\kappa = R/c_p$.

Substituting from (2.8) into (2.6) the eddy enstrophy equation may then be written

$$p \cos\phi \frac{De}{Dt} + (\partial_j B_{ij})\nabla_{H,i}\bar{q} = p \cos\phi \overline{S'q'}. \quad (2.13)$$

For the zonal flow case $\bar{u} = (\bar{u}, 0, 0)$, $\bar{q} = \bar{q}(\phi, z)$, this immediately gives a conservation relation

$$\left(\frac{\partial}{\partial t} + \frac{\bar{u}}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\frac{pe\cos\phi}{\partial\bar{q}/(a\partial\phi)} + \partial_j B_{2j}$$

$$= p \cos\phi \frac{\overline{S'q'}}{\partial\bar{q}/(a\partial\phi)} \quad (2.14)$$

(Plumb, 1985b), which is a nonzonally averaged statement of the quasi-geostrophic EP relation. In fact, the relation (2.14) is related to the law of conservation of pseudomomentum in the generalized Lagrangian-mean (GLM) theory (Andrews and McIntyre, 1978a,b); this law is, in turn, a consequence of the translational invariance of the basic state. If the basic state is not uniform in any one direction, pseudomomentum is not conserved; this GLM statement is paralleled in the present formulation by the fact that (2.13) no longer reduces to conservation form.

However, an approximate conservation law can be derived from (2.13) on the assumption that the mean state is slowly varying in some appropriate sense. Here, (2.13) is first written

$$\frac{p\cos\phi}{|\nabla_H\bar{q}|}\frac{De}{Dt} + n_i\partial_j B_{ij} = p\cos\phi\frac{\overline{S'q'}}{|\nabla_H\bar{q}|} \quad (2.15)$$

where

$$\mathbf{n} = \nabla_H\bar{q}/|\nabla_H\bar{q}| \quad (2.16)$$

is the unit vector in the direction of the potential vorticity gradient. We now *assume* that the mean potential vorticity gradient is slowly varying, relative to the length scale of the transient eddy statistics. (This is a more severe assumption than the more usual one that the mean state is slowly varying on the length scale of the eddies themselves.) Specifically, we assume that we may write, in (2.15),

$$\frac{\cos\phi}{|\nabla_H\bar{q}|}\frac{De}{Dt} \approx \frac{D}{Dt}\left(\frac{e\cos\phi}{|\nabla_H\bar{q}|}\right) \quad (2.17)$$

$$n_i\partial_j B_{ij} \approx \partial_j(n_i B_{ij}). \quad (2.18)$$

Thus, in (2.17), it is assumed that the *magnitude* of the mean potential vorticity gradient is slowly varying in the direction of the mean flow as compared with the eddy enstrophy, while in (2.18), it is assumed that the *direction* of the mean potential vorticity gradient is slowly varying in all three directions as compared with B_{ij}. If the mean potential vorticity gradient is dominated by the planetary contribution, these assumptions have some basis. It is not, however, clear a priori to what extent they are justifiable for real climatological

flows; the errors involved for a Northern Hemisphere winter climatology will be discussed in section 3.

Under approximations (2.17) and (2.18), then, (2.15) takes the conservation form

$$\frac{DM}{Dt} + \nabla\cdot\mathbf{M}_R = S_M \quad (2.19)$$

where

$$M = pe\cos\phi/|\nabla_H\bar{q}|, \quad (2.20)$$

$$M_{R,i} = n_j B_{ji} \quad (2.21)$$

$$S_M = p\cos\phi\overline{S'q'}/|\nabla_H\bar{q}|. \quad (2.22)$$

As in the zonally averaged case (Andrews and McIntyre, 1976), note the need to divide e by the mean potential vorticity gradient in order to obtain a sufficiently general conservation law (cf. Young and Rhines, 1980). Equation (2.19) is, in fact, a straightforward extension to the transient eddy problem of the quasi-geostrophic EP relation for waves on a zonal flow (Eliassen and Palm, 1961; Edmon et al., 1980) as applied to the nonzonally averaged case by Plumb (1985b). This is most readily seen by rotating coordinates locally so that $\mathbf{n} = (0, 1, 0)$. Then, in this coordinate system $M = ape\cos\phi/(\partial\bar{q}/\partial\phi)$ and $M_{R,j} = B_{2j}$ whence, from (2.13),

$$\mathbf{M}_R = p\cos\phi\left(\overline{v'^2} - \epsilon, -\overline{u'v'}, \frac{f\overline{v'\theta'}}{d\bar{\theta}/dz}\right) \quad (2.23)$$

where the velocity components are evaluated in the rotated coordinates. Note that in the zonally averaged case the ϕ and z components of $\mathbf{M}_R$ reduce to the EP flux if $\partial\bar{q}/\partial\phi > 0$ but to the negative of the EP flux if $\partial\bar{q}/\partial\phi < 0$ [since (2.23) is then evaluated in coordinates rotated by 180° relative to fixed geographic coordinates].

A simpler expression for $\mathbf{M}_R$ in nonrotated coordinates may be obtained if the mean flow is almost conservative. Then the steady basic state satisfies $\bar{u}\cdot\nabla\bar{q} \approx 0$, and therefore $\mathbf{n} = \gamma(-\bar{v}, \bar{u})/|\bar{u}|$ where $\gamma = 1$ if the mean flow is "pseudoeastward" in the terminology of Andrews (1984) and $\gamma = -1$ if the flow is "pseudowestward." Therefore, from (2.21) and (2.11),

$$\mathbf{M}_R = \frac{p\cos\phi}{|\bar{u}|}\begin{bmatrix} \bar{u}(\overline{v'^2} - \epsilon) - \bar{v}\overline{u'v'} \\ \bar{v}(\overline{u'^2} - \epsilon) - \bar{u}\overline{u'v'} \\ \frac{f}{d\bar{\theta}/dz}(\bar{u}\overline{v'\theta'} - \bar{v}\overline{u'\theta'}) \end{bmatrix}. \quad (2.24)$$

Comparing (2.24) with Eq. (2.12) of Plumb (1985a), $\mathbf{M}_R$ is related to the conventional energy flux $\tilde{\mathbf{E}} = p\overline{\mathbf{u}'_a\phi'}$ (where $\mathbf{u}_a$ is the ageostrophic velocity) and the flux $\mathbf{E}_R$ defined by Plumb (1985a) as conservable flux of transient eddy activity (analogous to the GLM pseudoenergy flux) by the expression

$$\mathbf{E}_R = \tilde{\mathbf{E}} - \gamma|\bar{u}|\mathbf{M}_R; \quad (2.25)$$

in the limit of a zonally uniform mean flow, (2.25) is equivalent to the expression (4.14b) of Plumb (1983), defining eddy energy flux in a transformed Eulerian-mean formalism.

Perhaps more usefully, (2.19) may be rewritten

$$\frac{\partial M}{\partial t} + \nabla \cdot \mathbf{M}_T = S_M \qquad (2.26)$$

where

$$\mathbf{M}_T = \mathbf{M}_R + \bar{\mathbf{u}}M \qquad (2.27)$$

is the total (radiative plus advective) wave activity flux. Then it is $\mathbf{M}_T$, and not $\mathbf{M}_R$, whose divergence is related to local wave transience (which is zero for stationary eddy statistics) and to local nonconservative sources or sinks of wave activity. In the almost-plane wave limit,

$$\mathbf{M}_T = \mathbf{c}_g M. \qquad (2.28)$$

This was derived for waves on a zonal flow by Plumb (1985b) and is implicit in the WKB results of Hoskins et al. (1983); therefore its derivation is not repeated here. Since M is positive definite, $\mathbf{M}_T$ is everywhere parallel to the group velocity in this limit.

Despite the overall similarity there are a number of important differences between the radiative flux $\mathbf{M}_R$ in (2.23) and the "E-vector" derived by Hoskins et al. Hoskins et al. noted that, under a slowly varying assumption similar to that made here in (2.17) and (2.18), and for barotropic flow in which the mean vorticity contours make small angles with the zonal direction, the angle between the group velocity vector (relative to the mean flow), $\mathbf{c}_g - \bar{\mathbf{u}}$, and the x-axis is one-half of the angle between the E-vector and the x-axis. This is consistent with the present results since, from (2.27) and (2.28), $\mathbf{M}_R$ is parallel to $\mathbf{c}_g - \bar{\mathbf{u}}$. If the $\bar{q}$ contours make small angles with the zonal direction, (2.23) may be evaluated as it stands without rotation of coordinates. Then the ϕ- and z-components of $\mathbf{M}_R$ reduce to those of the E-vector, while, for barotropic eddies, the zonal component is $\frac{1}{2}(v'^2 - u'^2)$ which is one-half of the zonal component of the E-vector. In the baroclinic case where the eddy potential energy is not negligible (as will become apparent later), there is no clear relationship between the zonal components of $\mathbf{M}_R$ and the E-vector. Probably more importantly, the latter does not incorporate the advective contribution to $\mathbf{M}_T$. As we have seen, it is $\mathbf{M}_T$, and not $\mathbf{M}_R$, that is parallel to $\mathbf{c}_g$, and it is the divergence of $\mathbf{M}_T$ and not of $\mathbf{M}_R$ that is directly, through (2.26), related to the generation or dissipation of wave activity. Therefore, $\mathbf{M}_T$ should permit an appreciation of the geographical location of regions of eddy generation and dissipation and of the propagation between these regions. By contrast, the useful information contained in $\mathbf{M}_R$ is limited in these respects.

3. Eddy propagation in the winter Northern Hemisphere as determined from circulation statistics

The diagnostic quantities derived in section 2 have been evaluated for a ten-year climatology of Northern Hemisphere (north of 20°N) winters (in fact, the 120-day period beginning 15 November) based on twice-daily NMC analyses for the years 1965/6–1968/9 and 1970/1–1975/6. Details of this dataset are presented in Lau et al. (1981). The climatology comprises the time-mean fields of various quantities for the period and transient eddy statistics; the latter are subdivided into "band-pass" (BP; period $\sim$ 2.5–6 days) and "low pass" (LP; period > 10 days but with the seasonal cycle removed) filtered contributions (see Lau et al., 1981); the transients are defined as deviations from the winter mean from each year, and therefore exclude interannual variations. Some characteristics of the transient eddies in this dataset have been described by Blackmon et al. (1977), Lau (1978, 1978a,b), Lau and Wallace (1979) and Holopainen et al. (1982), among others.

The mean fields of geopotential height and potential vorticity are illustrated in Fig. 1. (Similar figures have been presented in the literature cited above; they are included here for completeness and convenience.) Of particular interest, given the assumptions of section 2, is the form of $\bar{q}$; the contours of Fig. 1d are predominantly zonal and variations are of planetary scale. This gives some a priori justification for the assumption of slowly varying mean flow; quantitative assessments of the validity of the approximations invoked in (2.17) and (2.18) will be given below.

The radiative flux $\mathbf{M}_R$ was evaluated from (2.21) and (2.11) for the BP and LP eddies separately. Before discussing these results, we consider their validity by assessing the errors involved in (2.18) [the definition of $\mathbf{M}_R$ does not invoke assumption (2.17)]. Note from (2.8) and (2.21) that (2.18) may be written, after division by $p \cos\phi$,

$$\mathbf{n} \cdot \overline{u'q'} \approx \nabla \cdot \mathbf{M}_R / p \cos\phi. \qquad (2.18')$$

The two sides of this equation are compared for the BP and LP eddies in Fig. 2. The comparison is made at 400 mb (which, as we shall see, is the level of maximum flux); at other levels the comparison is similar to that at this level. As can be seen from Fig. 2, the approximation (2.18) preserves the structure of the $\overline{u'q'} \cdot \mathbf{n}$ field quite well for both BP and LP eddies; the quantitative errors involved in (2.18') are typically 10 percent, though with maximum errors of about 20 percent for the BP eddies in the maxima at the upstream ends of the Pacific and Atlantic storm tracks.

In order to project a meaningful impression of the relative magnitudes of the components of a vector in plots such as those to be presented here it is important to scale the vector correctly, in such a way as to preserve the sign of its divergence (cf. Edmon et al., 1980). In
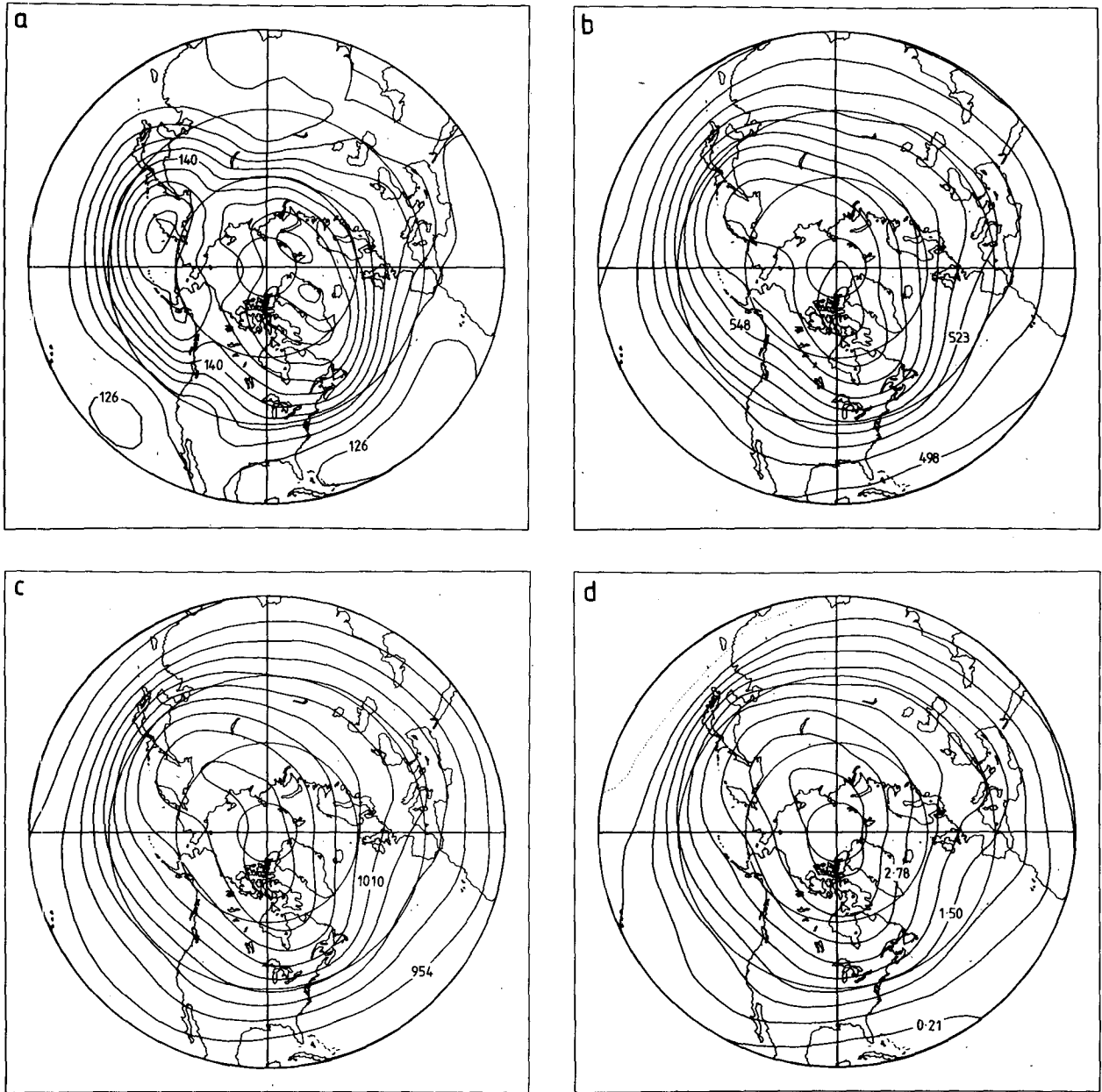
FIG. 1. Time-averaged structure of the Northern Hemisphere winter flow. Mean geopotential height (dam) at (a) 850 mb,
(b) 500 mb and (c) 250 mb; (d) Mean quasi-geostrophic potential vorticity (10^{-4} s^{-1}) at 250 mb.

the longitude–height plane, the longitudinal and vertical components of a three-dimensional vector (A_λ, A_ϕ, A_z) may be projected onto coordinates (ξ, ζ) $= (\sigma_x a\lambda, \sigma_z z)$ where σ_x and σ_z are constants. Then the two-dimensional divergence

$$\Delta = \frac{1}{a\cos\phi}\frac{\partial A_\lambda}{\partial\lambda} + \frac{\partial A_z}{\partial z}$$

may be written

$$\Delta = \frac{\sigma_x}{a\cos\phi}\left(\frac{\partial A_\lambda}{\partial\xi} + \frac{\sigma_z}{\sigma_x}\cos\phi\frac{\partial A_z}{\partial\zeta}\right).$$

If the range of λ and z represented on the plots is λ_1 to λ_2 and z_1 to z_2, respectively, then $\sigma_x = (\xi_2 - \xi_1)/a(\lambda_2 - \lambda_1)$ and $\sigma_z = (\zeta_2 - \zeta_1)/(z_2 - z_1)$. Therefore the correct vector to plot in (ξ, ζ) space is $(\tilde{A}_\xi, \tilde{A}_\zeta)$ where

$$\begin{pmatrix} \tilde{A}_\xi \\ \tilde{A}_\zeta \end{pmatrix} = \begin{pmatrix} A_\lambda \\ a\cos\phi\,\dfrac{(\lambda_2 - \lambda_1)(\zeta_2 - \zeta_1)}{(\xi_2 - \xi_1)(z_2 - z_1)}\cdot A_z \end{pmatrix} \quad (3.1)$$

since then

$$\Delta = \frac{\sigma_x}{a\cos\phi}\left(\frac{\partial\tilde{A}_\xi}{\partial\xi} + \frac{\partial\tilde{A}_\zeta}{\partial\zeta}\right)$$
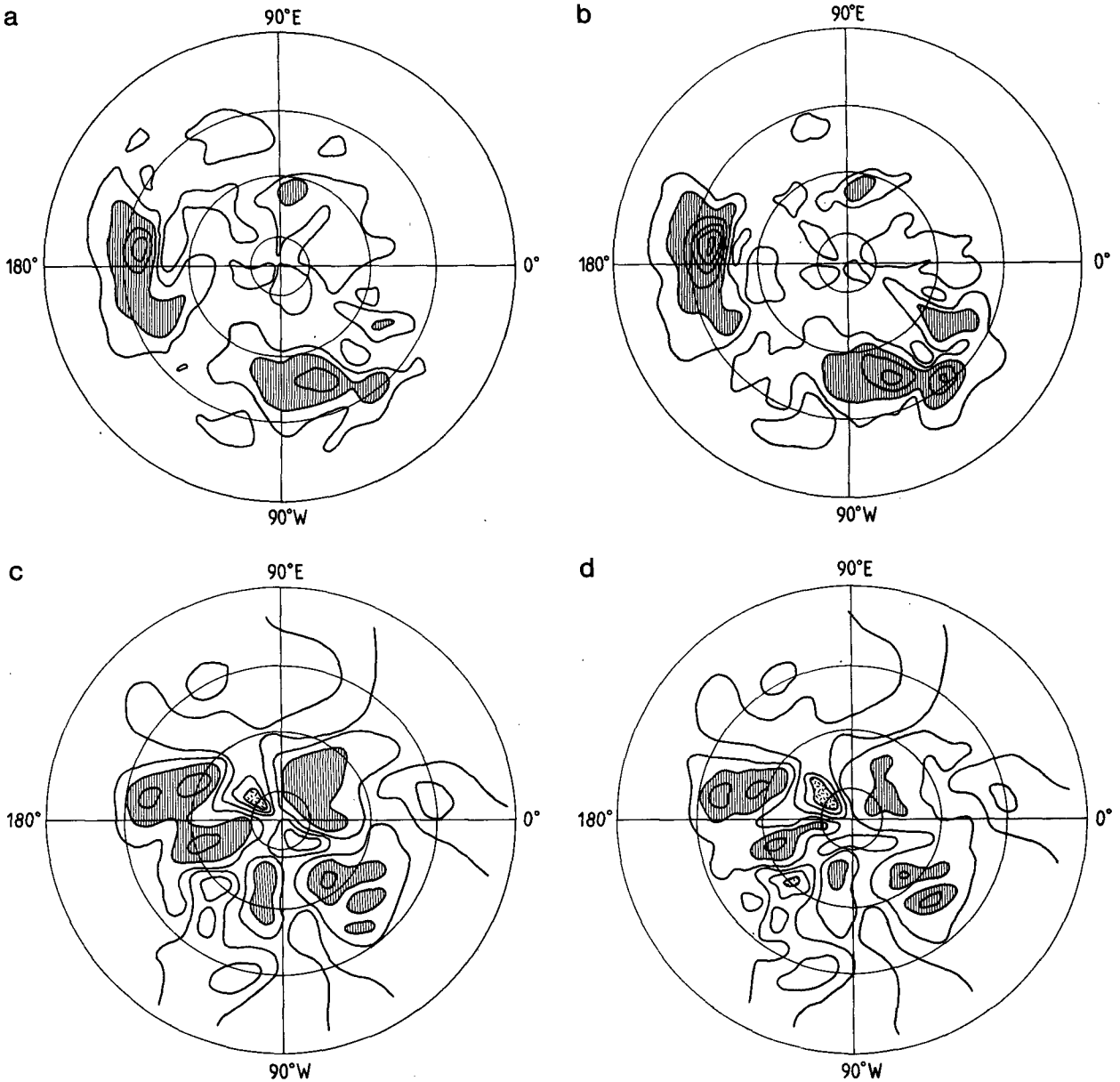
FIG. 2. Comparison of the downgradient potential vorticity flux at 400 mb as determined exactly [from the lhs of (2.18′)] and approximately [from the rhs of (2.18′)] involving slowly-varying assumptions. (a) $\overline{u'q'} \cdot \mathbf{n}$, BP eddies; (b) $\nabla \cdot \mathbf{M}_R/p \cos\phi$, BP eddies; (c) $\overline{u'q'} \cdot \mathbf{n}$, LP eddies; (d) $\nabla \cdot \mathbf{M}_R/p \cos\phi$, LP eddies. Polar stereographic projection; the outer circle is at 20°N. (a) and (b): Contour interval 2.10^{-5} m s^{-2}, values greater than 4.10^{-5} m s^{-2} shaded; (c) and (d): contour interval 5.10^{-5} m s^{-2}; values greater than 1.10^{-4} m s^{-2} shaded and less than -1.10^{-4} m s^{-2} stippled.

is proportional to the divergence of the plotted vector. (**A** may, of course, be rescaled by any scalar constant.)

In a meridional cross section where (A_ϕ, A_z) is to be plotted in coordinates $(\eta, \zeta) = (\sigma_y a\phi, \sigma_z z)$, the corresponding mapping is (Edmon et al., 1980)

$$\begin{pmatrix} \tilde{A}_\eta \\ \tilde{A}_\zeta \end{pmatrix} = \begin{pmatrix} \cos\phi A_\phi \\ a\cos\phi \dfrac{(\phi_2 - \phi_1)(\zeta_2 - \zeta_1)}{(z_2 - z_1)(\eta_2 - \eta_1)} A_z \end{pmatrix}. \quad (3.2)$$

Finally, for a horizontal cross section, (A_λ, A_ϕ) may be plotted onto polar stereographic coordinates (r, α) as $(\tilde{A}_r, \tilde{A}_\alpha)$ where (Plumb, 1985b)

$$\begin{pmatrix} \tilde{A}_r \\ \tilde{A}_\alpha \end{pmatrix} = \begin{pmatrix} -(1 + \sin\phi)A_\phi \\ (1 + \sin\phi)A_\lambda \end{pmatrix}. \quad (3.3)$$

Horizontal and longitude–height projections of $\mathbf{M}_R$, mapped according to (3.3) and (3.1), are shown in Fig. 3 for the BP eddies and in Fig. 4 for the LP eddies.
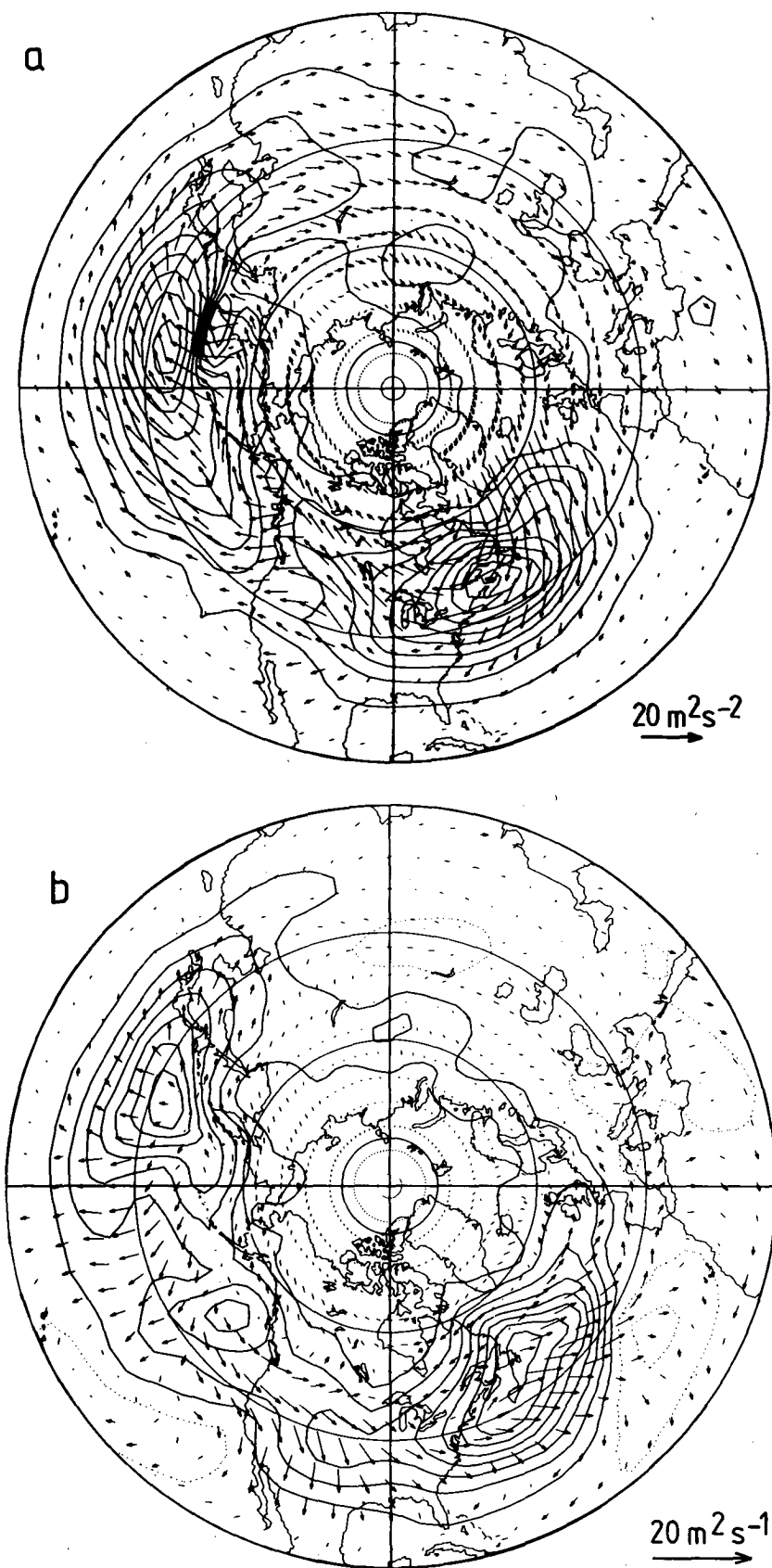
FIG. 3. Radiative eddy activity flux $\mathbf{M}_R$ for the BP eddies. Horizontal projections at (a) 500 and (b) 250 mb. Arrows: horizontal component. Note that the horizontal fluxes are mapped onto the polar stereographic projection according to Eq. (3.3). Scale on lower right is correct for $\mathbf{M}_R$
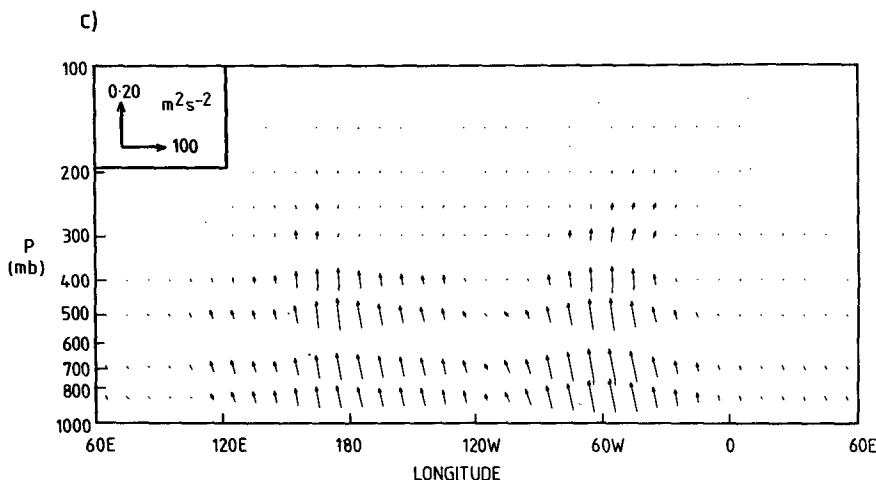
c)

FIG. 3. (Continued)

The BP eddies exhibit the expected strong vertical flux in the regions of maximum eddy heat flux at the upstream end of the major North Atlantic and North Pacific storm tracks. The horizontal component of $\mathbf{M}_R$ in the midtroposphere (Fig. 3a) is directed almost exactly upwind, while it is predominantly eastward and equatorward in the upper troposphere (Fig. 3b). However, these horizontal components are put into perspective by Fig. 3c, from which it can be appreciated that $\mathbf{M}_R$ is dominated by the vertical component of propagation. As will be seen below, the implied convergence of $\mathbf{M}_R$ in the middle and upper troposphere at the upstream end of the storm tracks is, in the context of the conservation law (2.19), largely balanced by the downstream advection of eddy activity.

The LP eddy flux is also mostly westward at 500 mb (Fig. 4a) but westward and equatorward in the upper troposphere (Fig. 4b). Even for these eddies, however, the flux is dominated by its vertical, baroclinic, component (Fig. 4c) although the zonal flux is relatively larger than for the BP eddies.

It is instructive to compare $\mathbf{M}_R$ as shown in Figs. 3 and 4 with the E-vector of Hoskins et al. (1983; their Fig. 7). They used a different dataset (ECMWF daily analyses of the 1979/80 Northern winter) and different time filters (high- and low-pass filters, both with a cutoff at about 10 days). The low-pass filter is therefore much the same in both studies; while the high-pass filter of Hoskins et al. has a broader window than the BP filter of the dataset used here, both should be dominated by the synoptic eddies and the characteristics should be broadly similar. Overall, the distinction noted by Hoskins et al. between the high- and low-pass eddies—that

the former exhibit eastward E-vectors at 250 mb while those of the latter are westward—is much less apparent in $\mathbf{M}_R$. While this distinction does exist, though less clearly, at 250 mb (where the fluxes are weak) it disappears altogether at 500 mb where, as noted above, $\mathbf{M}_R$ is directed upstream for both BP and LP eddies (the same is true of the vertical integral of $\mathbf{M}_R$). The main reason for this difference between $\mathbf{M}_R$ and the E-vector is the baroclinic term in $\mathbf{M}_R$ [the potential energy contribution to ϵ in (2.11)]. The other differences between the definitions of the two vectors—a factor of 2 in the longitudinal component and the fact that in (2.21) $\mathbf{M}_R$ is measured relative to the $\bar{q}$ contours rather than the zonal direction—are qualitatively less important.

In order to extend the analysis to evaluate the net flux $\mathbf{M}_T$ from (2.27) and (2.20), the eddy potential enstrophy must first be obtained. In fact, statistics of $\overline{q'^2}$ are not available in the dataset used; however, the quantity $\overline{P'^2}$, where

$$P = -(f + \zeta)\frac{\partial\theta}{\partial p} \tag{3.4}$$

is a "pseudo-potential vorticity," is included in the climatology. From this, it is possible to derive $\overline{q'^2}$; in the Appendix it is shown that, under quasi-geostrophic assumptions,

$$\overline{q'^2} = \frac{\overline{P'^2}}{(\partial\bar{\theta}/\partial p)^2} - \frac{2f}{p^\kappa(\partial^2\bar{\theta}/\partial p^2)}\left\{\frac{\partial}{\partial x}(\overline{v'T'}) - \frac{\partial}{\partial y}(\overline{u'T'})\right.$$
$$\left. - \frac{fH}{R}\left[\frac{\partial}{\partial z}(\overline{u'^2} + \overline{v'^2}) + Rp^{\kappa-1}\frac{\partial}{\partial z}\left(\frac{\overline{\theta'^2}}{\partial\bar{\theta}/\partial p}\right)\right]\right\}. \tag{3.5}$$

at 45°N. Contours: vertical components. Contours are plotted at values $(n + \frac{1}{2})\Delta$ where the contour interval is (a) $\Delta = 0.0139$ m^2 s^{-2}, (b) $\Delta = 0.0357$ m^2 s^{-2}. Contours are solid for positive values, dotted for negative values. (c) Longitude–height projection at 45°N, mapped according to (3.1). Scale at upper left.

a

$80 \text{ m}^2\text{s}^{-2}$
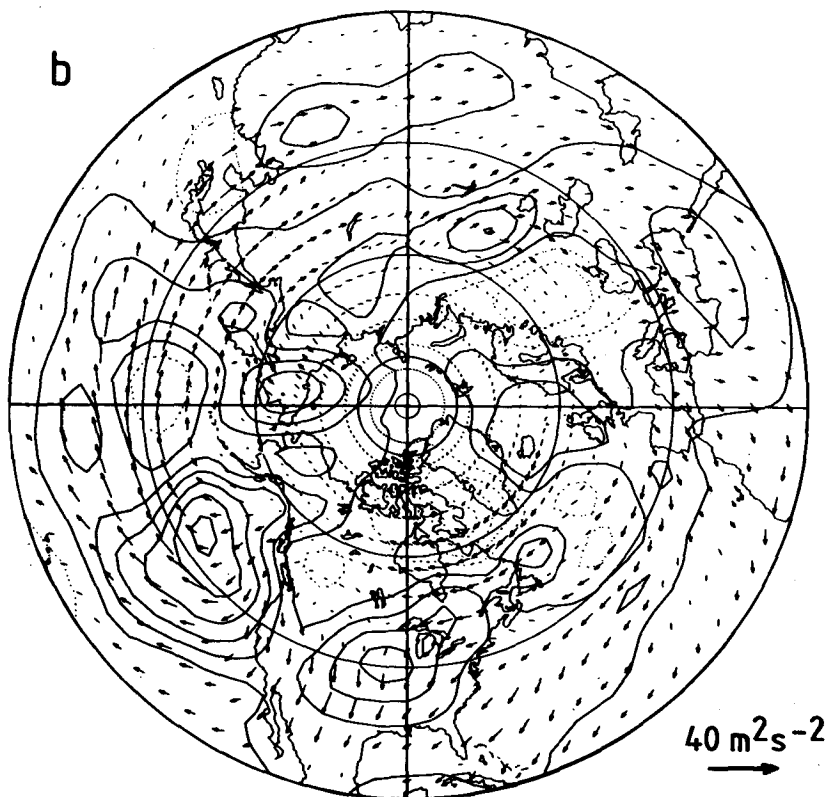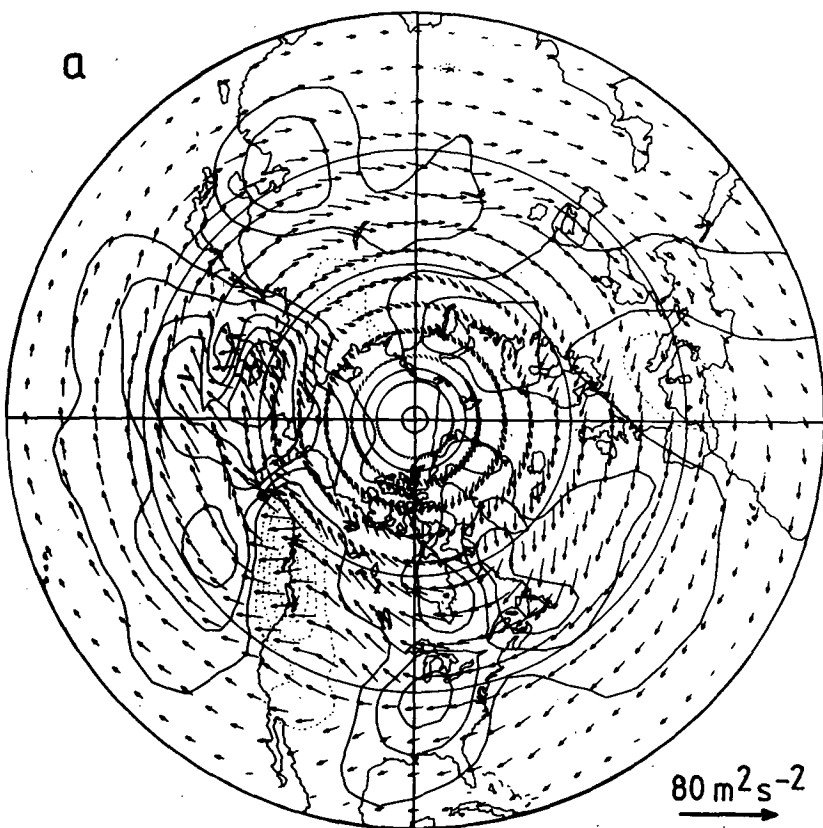
b

$40 \text{ m}^2\text{s}^{-2}$

FIG. 4. As in Fig. 2 but for the LP eddies. Contour interval
(a) $\Delta = 0.0268 \text{ m}^2 \text{ s}^{-2}$, (b) $\Delta = 0.00673 \text{ m}^2 \text{ s}^{-2}$.
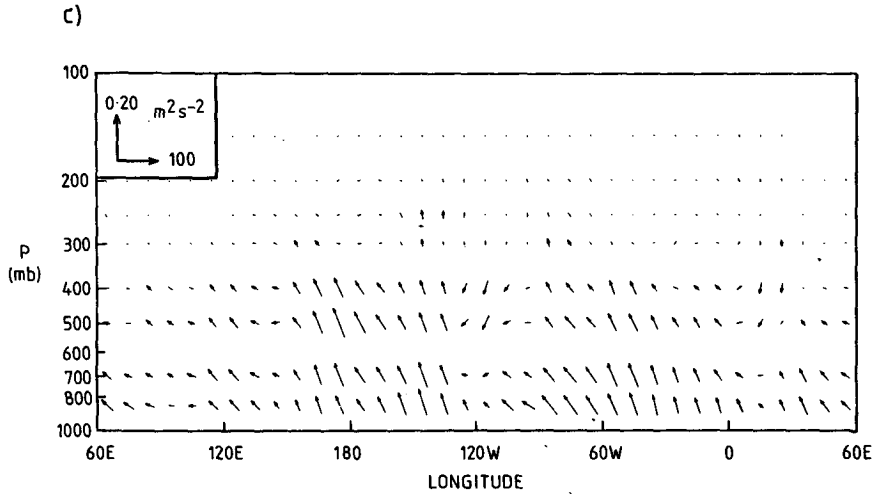
c)

FIG. 4. (*Continued*)

Thus $\overline{q'^2}$ could be evaluated from the available statistics (in fact, the first term proved to be dominant). Results of this procedure for the BP eddies are shown in Fig. 5. As might be expected the eddy enstrophy maximizes in the storm tracks, although it is rather less localized than most other quadratic eddy statistics. It should be borne in mind, of course, that the calculation of $\overline{q'^2}$ is prone to error, since spatial differentiation of the observed velocity field (which will amplify small-scale noise) is involved, followed by squaring and time-averaging (the last processes may have a smoothing effect and therefore the non-noisy appearance of Fig. 5 may be deceptive).

Using the statistics for $\overline{q'^2}$, the total flux $\mathbf{M}_T$ was evaluated from (2.27) and (2.20); again, before pre-

senting these fluxes, we consider the validity of the approximations involved. Equation (2.19) was replaced by (2.26) using (2.17) and (2.18), which together imply

$$p \cos\phi\left(\frac{1}{|\nabla_H \bar{q}|}\,\bar{\mathbf{u}}\cdot\nabla e + \mathbf{n}\cdot\overline{\mathbf{u}'q'}\right) \approx \nabla\cdot\mathbf{M}_T. \quad (3.6)$$

The expressions on the two sides of (3.6) are shown for the BP and the LP eddies in Fig. 6. As compared with the comparison of Fig. 2, while the gross patterns are reproduced reasonably well by the approximation (3.6), some more serious errors are apparent. The most obvious of these is a tendency of the region of positive values at about 135°E to extend too far east in Figs. 6b and d; this is occurring in the region of strong mean
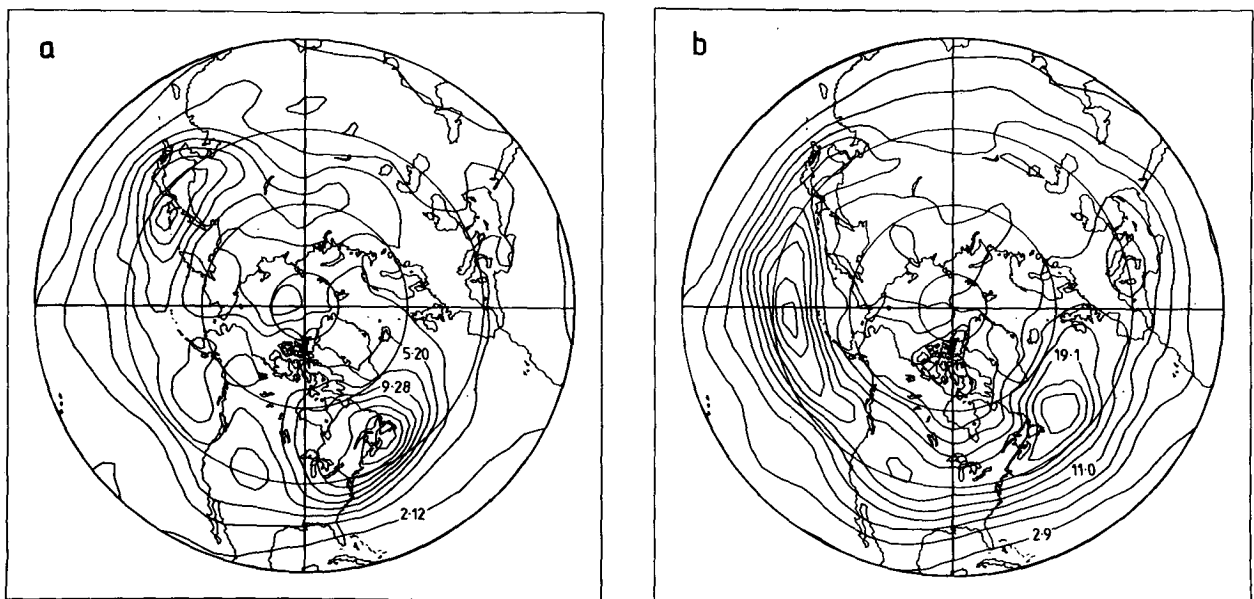

FIG. 5. Eddy potential enstrophy $\overline{q'^2}$ (10^{-10} s^{-2}) for the BP eddies as determined from (3.2). (a) 500 mb, (b) 250 mb.
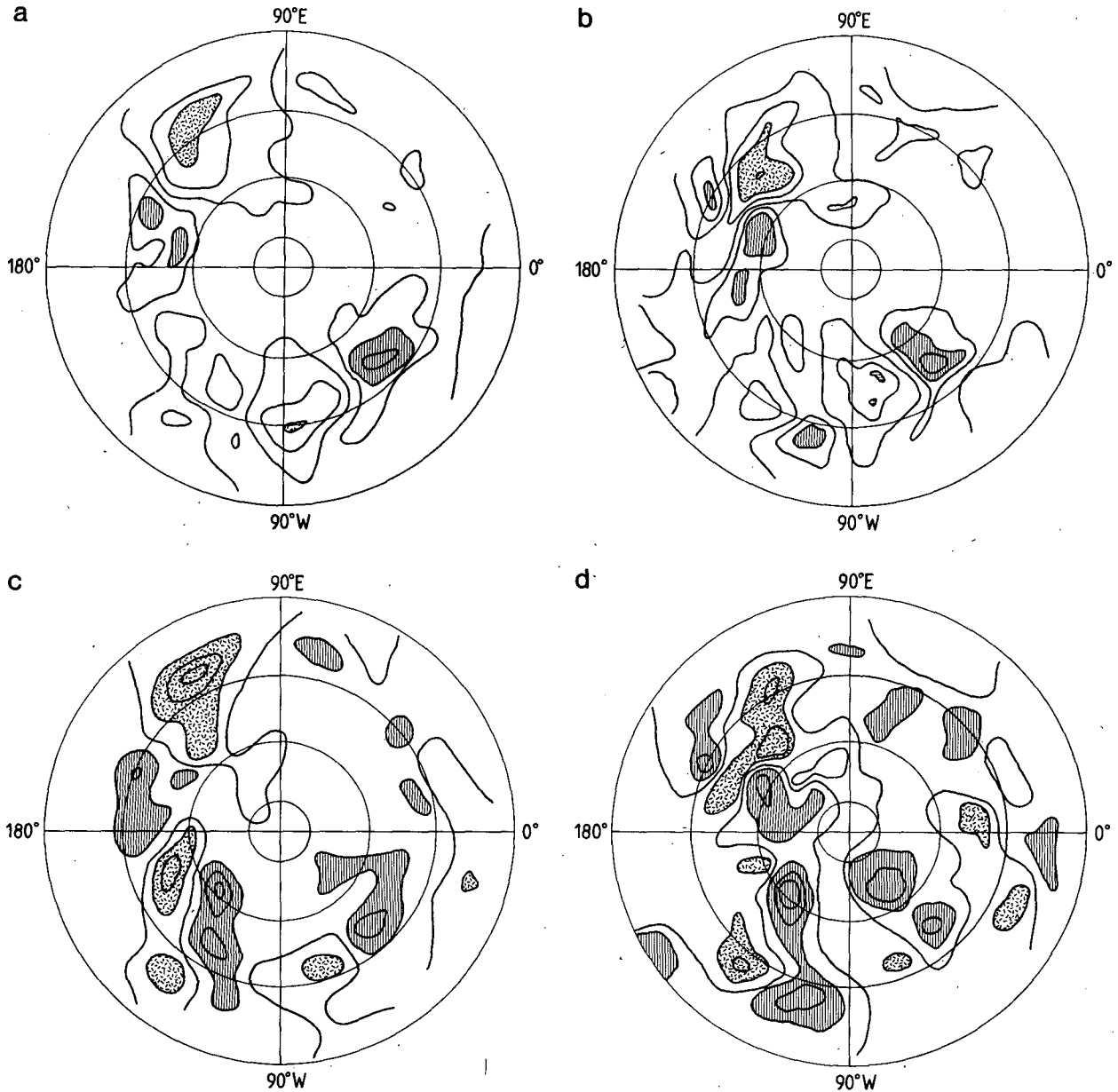
FIG. 6. Assessment of the validity of the slowly varying approximation (3.6). The lhs of (3.6) is shown in (a) for the BP eddies and (c) for the LP eddies, and the rhs in (b) for the BP eddies and (d) for the LP eddies. (a) and (b): contour interval 2.10^{-5} m s^{-2}, values greater than 4.10^{-5} m s^{-2} shaded and less -4.10^{-5} m s^{-2} stippled; (c) and (d): Contour interval 5.10^{-5} m s^{-1}, values greater than 1.10^{-4} m s^{-2} shaded and less than -1.10^{-4} stippled.

flow curvature (cf. Fig. 1) where $|\nabla_H \bar{q}|$ changes fairly rapidly downstream. The approximation is reasonably good for the BP eddies elsewhere, but is poorer for the LP eddies, especially in the East Atlantic/European region. On the whole, approximation (2.17) thus appears to be weaker for the present climatology than (2.18), particularly for the LP eddies.

Results for $\mathbf{M}_T$ are shown in Figs. 7 (BP eddies) and 8 (LP eddies). For the BP eddies, the total flux is predominantly eastward and upward at all levels (cf. $\mathbf{M}_R$ of Fig. 3). In fact, the horizontal component of $\mathbf{M}_T$ for

the BP eddies is completely dominated by the advective term $\bar{u}M$ in (2.27). This seems to be in accord with the fact that for Rossby-like eddies of synoptic scale, the group velocity relative to the local mean flow is much less than the zonal wind speed.[2] Because of the dom-

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[2] For Rossby waves on a zonal flow the zonal group velocity relative to the flow is $c_g - \bar{u} = \beta(k^2 - l^2)/(k^2 + l^2)$ where (k, l) is the horizontal wavenumber. Therefore $(c_g - \bar{u})/\bar{u} = K_s^2(k^2 - l^2)/(k^2 + l^2)^2$ where $K_s = (\beta/\bar{u})^{1/2}$ is the stationary wavenumber. For $k^2 + l^2 \gg K_s^2$, $|(c_g - \bar{u})/\bar{u}| \ll 1$.

inance of the advective term, the details of $\mathbf{M}_T$ depend to a large extent on $\overline{q'^2}$ and any errors in this quantity will therefore be reflected in $\mathbf{M}_T$. This may be the explanation for the appearance of the longitude–height cross section shown in Fig. 7c where, in addition to the surface sources associated with baroclinic instability at the storm track entrances, there is an apparent source in the upper troposphere upstream of the Pacific storm track. Since such a source seems unlikely, one is led to suspect the eddy enstrophy statistics, at least in this region. Note that the apparent source cannot be merely a consequence of the slowly varying approximation, since the region of positive $\nabla \cdot \mathbf{M}_T$ in this region (Fig. 6b) is equally apparent as an effective source of eddy enstrophy (Fig. 6a) as expressed by the lhs of (3.6), which invokes no such approximation. The implication of Fig. 7c that the downstream end of the Pacific storm track extends to merge with the Atlantic storm track may also be suspect.

The same caveat must apply to the LP eddy fluxes (Fig. 8). The vector $\mathbf{M}_T$ is predominantly upward and eastward for these eddies also, but again there is an apparent source of eddy activity over the Asian continent (Fig. 8c). In fact, the patterns for the BP and LP eddies are broadly similar except that the latter appear a little less baroclinic (i.e., $\mathbf{M}_T$ is a little more horizontal) and the vertical flux has a downward component over the west coast of North America.

The zonal means of $\mathbf{M}_R$ and $\mathbf{M}_T$ for the BP eddies are shown in Fig. 9. If we assume that the $\bar{q}$ contours are aligned zonally [and for the BP eddies the terms thus neglected in (2.21) are fairly small] then the meridional components of $\langle \mathbf{M}_R \rangle$ (where angle brackets denotes a zonal average) reduce to the EP flux. Thus Fig. 9a shows $\langle \mathbf{M}_R \rangle$ to be of the familiar form with upward and equatorward propagation (cf. Edmon et al., 1980). By contrast, $\langle \mathbf{M}_T \rangle$ shows a more symmetrical spreading of eddy propagation with height, with equatorward propagation south of about 45°N and poleward propagation at higher latitudes (Fig. 9b). The difference between $\langle \mathbf{M}_R \rangle$ and $\langle \mathbf{M}_T \rangle$ is, of course, that the latter incorporates advection of eddy activity by the mean flow, which is mostly poleward in the storm tracks. While, once again, interpretation of $\langle \mathbf{M}_T \rangle$ is subject to the quality of the eddy enstrophy statistics and to the slowly varying approximation, this comparison serves to emphasize that the characteristic upward/equatorward EP flux signature tells us that the eddies propagate equatorward *relative to the mean flow* and that the sense of propagation relative to geographical coordinates may differ from this picture.

4. Interaction of transients with the time-mean flow

As noted in the Introduction, the unique appeal of the zonally averaged EP flux derives from its significance both as a flux of eddy activity and as a measure

of the interaction of eddies with the zonal-mean flow. The latter relationship is expressed by

$$\nabla \cdot \mathbf{F} = p \cos\phi \langle v^* q^* \rangle \qquad (4.1)$$

—where ( )* means deviation from the zonal average $\langle ( ) \rangle$—relating the EP flux divergence to the northward potential vorticity flux and thus to the influence of the eddy motions on the zonal-mean potential vorticity budget. Further, one may derive a transformed Eulerian-mean set of equations describing the mean state in which (for quasi-geostrophic flow) the influence of eddy transport is described by an effective body force per unit mass of $-\langle v^* q^* \rangle$ acting on the mean zonal flow and a boundary term reflecting the influence of low-level heat fluxes in this formulation (Andrews and McIntyre, 1976; Edmon et al., 1980).

The main thrust of this paper is directed at the interpretation of $\mathbf{M}_R$ and $\mathbf{M}_T$ as fluxes of eddy activity. However, in appropriate formulations, both of these fluxes parallel the EP flux as a measure of interaction with the time-mean state, albeit in a less complete and slightly less obvious way. From (2.21) and (2.8), under the slowly varying assumption (2.18),

$$\nabla \cdot \mathbf{M}_R = p \cos\phi \overline{u'q'} \cdot \mathbf{n}. \qquad (4.2)$$

For a zonal flow with $\partial \bar{q}/\partial \phi > 0$, $\mathbf{n} = (0, 1, 0)$ and the parallel with (4.1) is obvious. According to (4.2), the BP eddy potential vorticity fluxes should, on the basis of their $\mathbf{M}_R$ fluxes shown in Fig. 3, exhibit strong downgradient components where $\mathbf{M}_R$ is strongly convergent in the middle troposphere at the upstream end of the storm tracks, with relatively weak downgradient components elsewhere. This is indeed apparent in the structure of the BP eddy fluxes shown in Fig. 10 (cf. the $\bar{q}$ structure of Fig. 1d). This component of $\overline{u'q'}$ is thus implicit in $\mathbf{M}_R$ (and hence in the eddy propagation characteristics). However, $\mathbf{M}_R$ has no obvious relationship with the components of $\overline{u'q'}$ in the direction normal to $\mathbf{n}$. Since there is no reason to neglect the contribution of this term to the mean potential vorticity budget (and inspection of Fig. 10 suggests that it is significant in practice) it appears that there is insufficient information implicit in $\mathbf{M}_R$ to specify completely the eddy potential vorticity transport.

Not surprisingly, this lack of completeness extends to the effects of the eddies on the mean momentum and heat budgets. Following Holopainen et al. (1982) and Hoskins (1983) these may be expressed, in Cartesian coordinates on an f-plane, as

$$\frac{D\bar{u}}{Dt} + f\mathbf{k} \times \tilde{\mathbf{u}}_a = \frac{1}{p} \mathbf{G} + \bar{\mathbf{X}}$$

$$\frac{D\bar{\theta}}{Dt} + \tilde{w}_a \frac{d\bar{\theta}}{dz} = \tilde{Q} \qquad (4.3)$$

where $\bar{\mathbf{X}}$ and $\bar{Q}$ are mean nonconservative terms (proportional to frictional forces and diabatic heating, re-
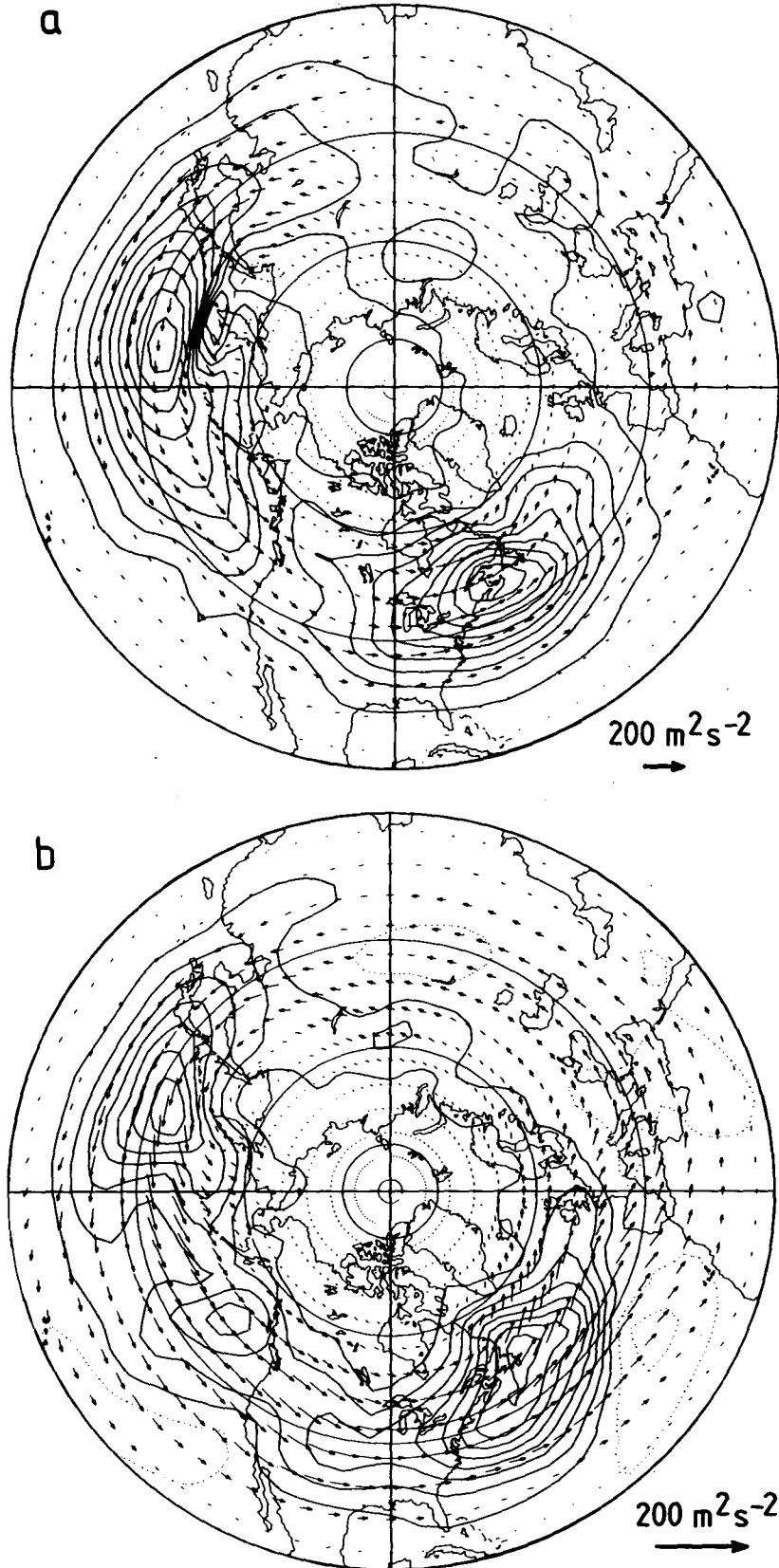
a

200 m²s⁻²
→

b

200 m²s⁻²
→

FIG. 7. Total eddy activity flux M_T for the BP eddies. Otherwise as Fig. 2,
with contour intervals: (a) $\Delta = 0.0139$ m² s⁻², (b) $\Delta = 0.00357$ m² s⁻².
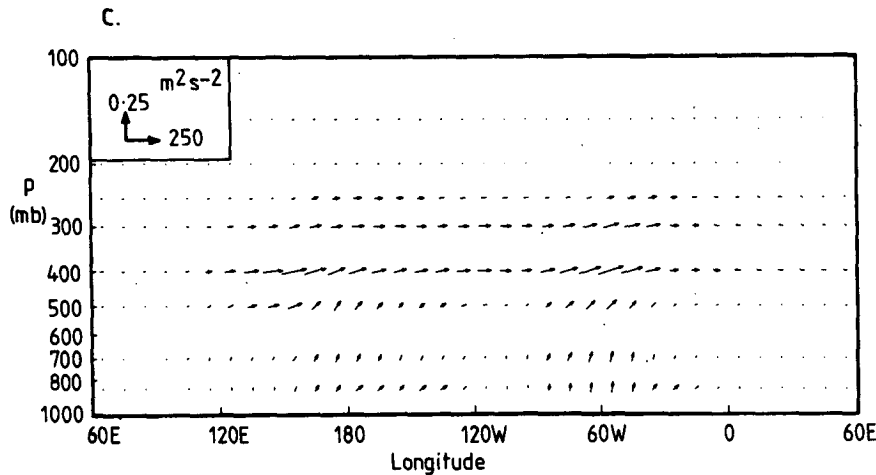
C.

FIG. 7. (Continued)

spectively), $\tilde{u}_a$ is a residual ageostrophic velocity defined by

$$\tilde{u}_a = \bar{u}_a + \frac{1}{p} \nabla \times p\mathbf{R} \qquad (4.4)$$

where

$$\mathbf{R} = \begin{pmatrix} -\overline{v'\theta'}/\bar{\theta}_z \\ \overline{u'\theta'}/\bar{\theta}_z \\ \frac{1}{f}(\overline{u'^2} + \overline{v'^2} - \epsilon) \end{pmatrix} \qquad (4.5)$$

and the effective eddy-induced force on the mean flow—the only explicit eddy term in (4.3)—is

$$\mathbf{G} = -p\mathbf{k} \times \overline{u'q'}. \qquad (4.6)$$

Defining a unit vector

$$\mathbf{s} = \mathbf{k} \times \mathbf{n} \qquad (4.7)$$

normal to the potential vorticity gradient, then

$$\overline{u'q'} = (\overline{u'q'} \cdot \mathbf{n})\mathbf{n} + (\overline{u'q'} \cdot \mathbf{s})\mathbf{s},$$

whence

$$\mathbf{G} = -p(\overline{u'q'} \cdot \mathbf{n})\mathbf{s} + p(\overline{u'q'} \cdot \mathbf{s})\mathbf{n}. \qquad (4.8)$$

Thus, only the s-component of $\mathbf{G}$ is determined by the downgradient flux [and thus, via (4.2), $\nabla \cdot \mathbf{M}_R$]. Note that for a conservative basic state in which eddy transports are weak $D\bar{q}/Dt = \bar{u} \cdot \nabla \bar{q} = 0$, so that the mean streamlines coincide with $\bar{q}$ contours. (This assumption has been discussed by Illari and Marshall, 1983.) In this case, as shown in Fig. 11, s points "pseudowestward," in the sense used by Andrews (1984), i.e., directly upstream in the usual case of pseudoeastward mean flow. Since (setting $\cos\phi = 1$ as we are working in Cartesian coordinates) $p\overline{u'q'} \cdot \mathbf{n} = \nabla \cdot \mathbf{M}_R$ and if we define a flux $N_{R,i} = -ps_j B_{ji}$ (such that $p\overline{u'q'} \cdot \mathbf{s} = -\nabla \cdot \mathbf{N}_R$, under the slowly varying assumption) then, from (2.8) (again setting $\cos\phi = 1$), (4.8) gives

$$\mathbf{G} = -\partial_i(sM_{R,i} + nN_{R,i}) \qquad (4.9)$$

within the slowly varying approximation. Thus $\mathbf{M}_R$ may be interpreted as the effective eddy flux of pseudowestward (s) momentum; a complete description of the eddy transport, however, requires knowledge of the flux $\mathbf{N}_R$ of cross-stream (n) momentum.

The presence of the extra effective cross-stream force, which is apparently unrelated to eddy propagation characteristics, weakens the link between $\mathbf{M}_R$ and the eddy–mean flow interaction. In terms of a parallel between this time-mean formulation and the zonal-mean EP formulation, it should be noted that a similar term (representing an effective northward force on the mean state) also appears in the latter; for the quasi-geostrophic problem, however, it is of little importance for the overall interaction, generating only an ageostrophic correction to the mean thermal wind balance (cf. Andrews and McIntyre, 1978a, §5.2).

Thus far, the properties of eddy transport have been discussed solely in terms of the radiative eddy activity flux $\mathbf{M}_R$, rather than the total flux $\mathbf{M}_T$. However, under the assumption of a conservative basic state the same interpretation as placed above on $\mathbf{M}_R$ can be put on $\mathbf{M}_T$ in an analogous system of equations. This is a consequence of the fact, demonstrated by Marshall and Shutts (1981) and Illari and Marshall (1983), that the contribution to $\overline{u'q'}$ associated with the advection of eddy enstrophy is, if $\bar{u} \cdot \nabla \bar{q} = 0$, purely rotational.

To be specific, consider

$$\Lambda = \frac{\nabla_H \bar{q} \cdot \nabla_H \bar{\psi}}{|\nabla_H \bar{q}|^2}. \qquad (4.10)$$

Then for a steady, conservative mean flow for which $\bar{u} \cdot \nabla \bar{q} = 0$, $\bar{\psi} = \bar{\psi}(\bar{q}, z)$ and hence

$$\Lambda = \frac{\partial \bar{\psi}}{\partial \bar{q}} = \Lambda(\bar{q}, z). \qquad (4.11)$$

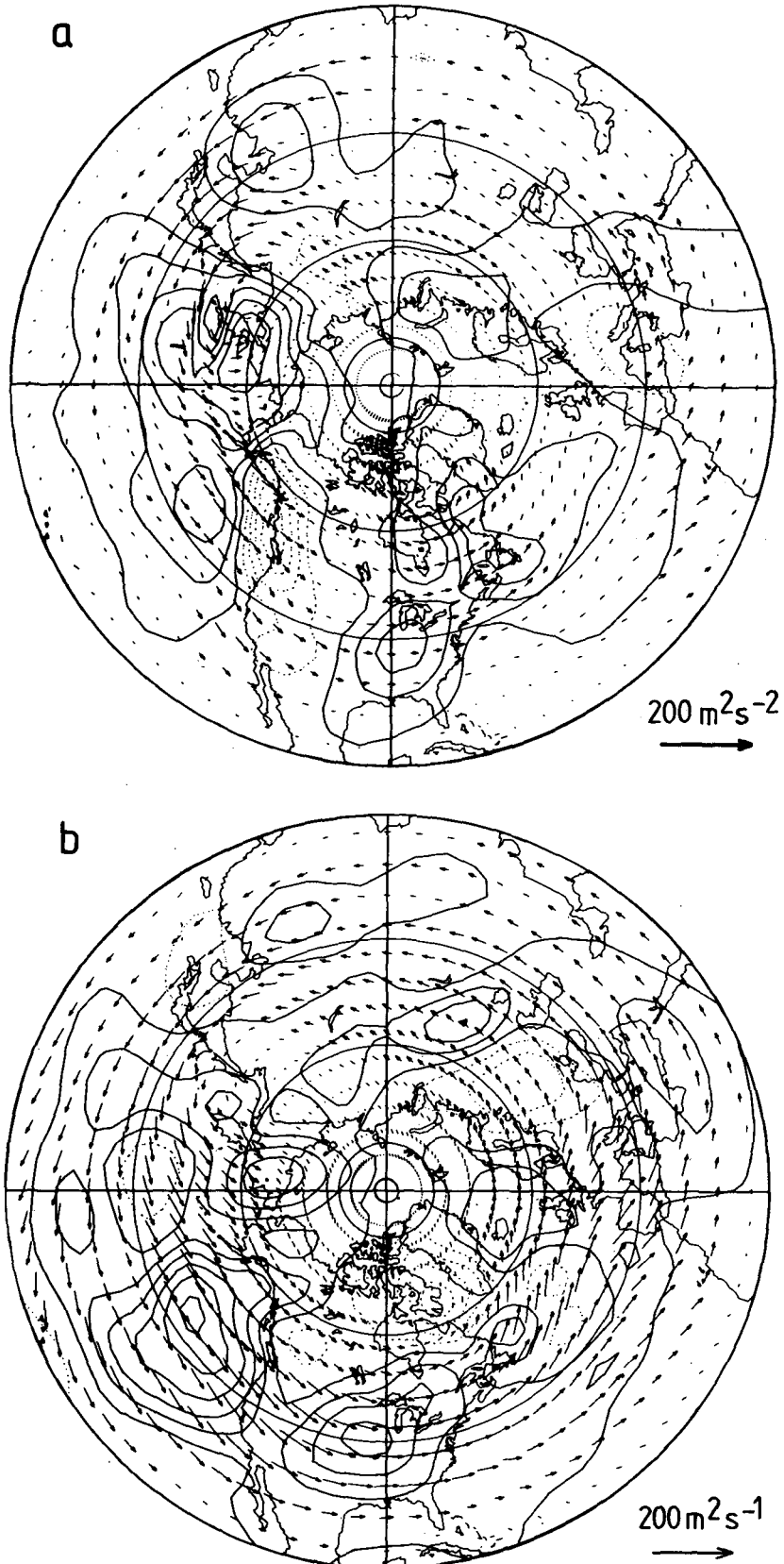Hence the enstrophy advection term in (2.6) may be written

a

$200\,m^2s^{-2}$

b

$200\,m^2s^{-1}$

FIG. 8. Total eddy activity flux M_T for the LP eddies. Otherwise as Fig. 2, with
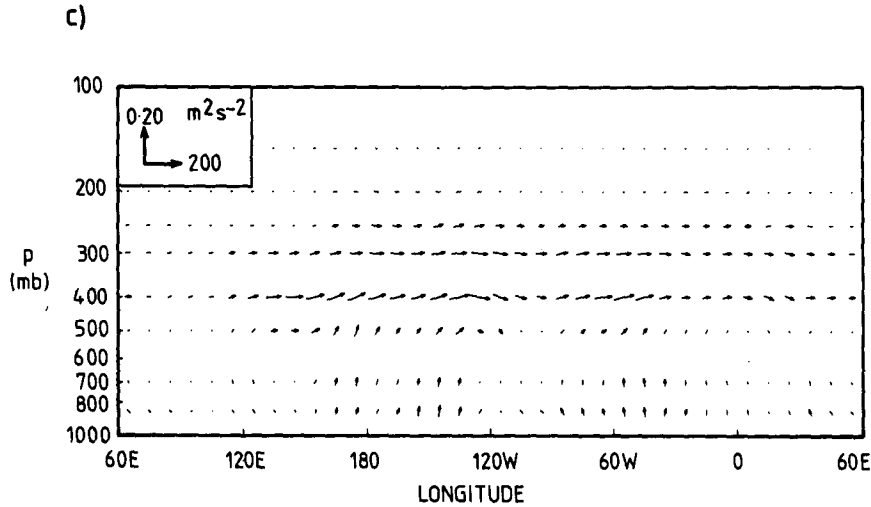contour intervals: (a) $\Delta = 0.0268$ m^2 s^{-2}, (b) $\Delta = 0.00673$ m^2 s^{-2}.

c)

FIG. 8. (Continued)

$$\frac{De}{Dt} = \bar{\mathbf{u}} \cdot \nabla e = \Lambda^{-1}\bar{\mathbf{u}} \cdot \nabla(\Lambda e),$$

since $\bar{\mathbf{u}} \cdot \nabla\Lambda = (\partial\Lambda/\partial\bar{q})\bar{\mathbf{u}} \cdot \nabla\bar{q} = 0$. Therefore,

$$\frac{De}{Dt} = \Lambda^{-1}(\mathbf{k} \times \nabla\bar{\psi}) \cdot \nabla(\Lambda e)$$

$$= -\Lambda^{-1}\nabla\bar{\psi} \cdot [\mathbf{k} \times \nabla(\Lambda e)]$$

$$= -\nabla\bar{q} \cdot (\overline{\mathbf{u}'q'})_R, \tag{4.12}$$

since $\nabla\bar{\psi} = \Lambda\nabla\bar{q}$, where

$$(\overline{\mathbf{u}'q'})_R = \mathbf{k} \times \nabla(\Lambda e) \tag{4.13}$$

is the rotational potential vorticity flux defined by Illari and Marshall (1983). From (2.27) and (2.20), invoking assumption (2.17),

$$\nabla \cdot \mathbf{M}_T = \nabla \cdot \mathbf{M}_R + \frac{p}{|\nabla_H\bar{q}|}\frac{De}{Dt} . \tag{4.14}$$

Therefore, from (4.2) and (4.12),

$$\nabla \cdot \mathbf{M}_T = p(\overline{\mathbf{u}'q'})_* \cdot \mathbf{n} \tag{4.15}$$

where

$$(\overline{\mathbf{u}'q'})_* = \overline{\mathbf{u}'q'} - (\overline{\mathbf{u}'q'})_R. \tag{4.16}$$

Shutts (1983) refers to $(\overline{\mathbf{u}'q'})_*$ as the "residual" potential vorticity flux.

Since, from (4.13), $\nabla \cdot \overline{(\mathbf{u}'q')_R} = 0$ the mean potential vorticity equation

$$\frac{D\bar{q}}{Dt} = -\nabla \cdot \overline{\mathbf{u}'q'} + \bar{S} \tag{4.17}$$

may be written

$$\frac{D\bar{q}}{Dt} = -\nabla \cdot (\overline{\mathbf{u}'q'})_* + \bar{S}, \tag{4.18}$$

so that statements made earlier regarding the relation-

ship of the downgradient component of the potential vorticity flux with $\mathbf{M}_R$ apply equally well to $\mathbf{M}_T$, given (4.15) and (4.18). [Of course, the s-component of the flux differs between the two formulations (4.17) and (4.18).] Moreover, $\mathbf{M}_T$ can also replace $\mathbf{M}_R$ in the mean momentum and thermodynamic budgets. First, we write

$$\mathbf{G}_* = -p\mathbf{k} \times (\overline{\mathbf{u}'q'})_* = \mathbf{G} - \nabla_H(p\Lambda e) \tag{4.19}$$

from (4.13) and (4.16). Then, since $\mathbf{G}_* - \mathbf{G}$ is the gradient of a scalar, it may be absorbed into (4.3) by a redefinition of the residual ageostrophic velocity to give

$$\frac{D\bar{\mathbf{u}}}{Dt} + f\mathbf{k} \times \tilde{\mathbf{u}}_{a*} = \frac{1}{p}\mathbf{G}_* + \bar{\mathbf{X}},$$

$$\frac{D\bar{\theta}}{Dt} + \tilde{w}_{a*}\frac{d\bar{\theta}}{dz} = \bar{Q}, \tag{4.20}$$

where

$$\tilde{\mathbf{u}}_{a*} = \tilde{\mathbf{u}}_a - \mathbf{k} \times \nabla(\Lambda e) \tag{4.21}$$

(note that $\tilde{w}_{a*} = \tilde{w}_a$). Similarly, we may also replace (4.9) by

$$\mathbf{G}_* = -\partial_i(sM_{T,i} + nN_{T,i}) \tag{4.22}$$

where

$$\mathbf{N}_T = \mathbf{N}_R + p\Lambda e\mathbf{n}. \tag{4.23}$$

Thus, in the system (4.18)–(4.20), $\mathbf{M}_T$ replaces $\mathbf{M}_R$ as the effective flux of upstream momentum, and $\nabla \cdot \mathbf{M}_T$ replaces $\nabla \cdot \mathbf{M}_R$ as the measure of the downgradient flux of potential vorticity. This appears to be conceptually beneficial not only because $\mathbf{M}_T$ is a more revealing measure of eddy propagation than $\mathbf{M}_R$, but also because $\nabla \cdot \mathbf{M}_T$ is related only to nonconservative eddy generation[3] and dissipation processes, whereas

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[3] As in the zonal-mean problem, the flux of eddy activity out of the lower boundary is a very important contribution to the total eddy
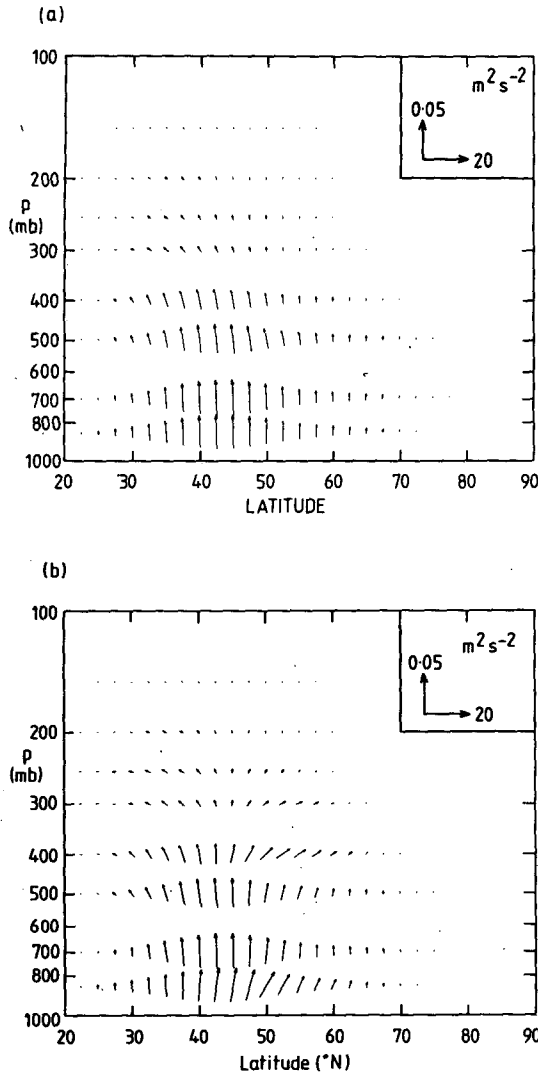
FIG. 9. Zonal means (a) $\langle M_R \rangle$ and (b) $\langle M_T \rangle$ of radiative and total fluxes for the BP eddies. Vectors are mapped according to (3.2). Scales at upper right are correct for $\langle M_R \rangle$, $\langle M_T \rangle$ at 45°N.

$\nabla \cdot M_R$ includes the additional effects of the advection of eddy activity. Thus, a direct link is established in this framework of a conservative basic state between nonconservative eddy processes and the s component of the effective eddy-induced force acting on the mean state; the n-component, however, is apparently unrelated to such effects.

transport; while this does not appears explicitly in (4.20), it enters the problem via the lower boundary condition on $\tilde{w}_{a^*}$. Again, as in the zonal-mean problem, this can be shown to be equivalent to a "delta-function" sheet of divergence of M_T on the lower boundary and as such can be incorporated into this discussion. It should be emphasized that this contribution is a very important one, whose effects are felt (via the ageostrophic circulation) throughout the troposphere.

As noted earlier, this effective cross-stream force also appears in the steady, zonal-mean case but is of little importance for the interaction of eddies with the zonal mean flow. The sustained acceleration of the mean flow thus depends entirely on nonconservative effects. If the waves are nonsteady, however, it is well known that the interaction is also related to eddy transience. In the steady, time-mean problem of interest here, eddy transience is also relevant; in this case, however, it is down-stream transience, associated with spatial variations of eddy amplitudes along a mean streamline. It is these effects that are manifested in the cross-stream momentum flux N_T and the corresponding cross-gradient flux of potential vorticity and, as in the time-dependent, zonally averaged counterpart, the influence of these effects on the mean flow is temporary. These statements are most readily appreciated in the context of the potential vorticity budget, since (4.18) may be written

$$\frac{D\bar{q}}{Dt} = -\nabla \cdot \{s[s \cdot (\overline{u'q'})_*] + n[n \cdot (\overline{u'q'})_*]\} + \bar{S}$$

or, since $s = -\gamma \bar{u}/|\bar{u}|$ where, as before, $\gamma = +1(-1)$ if the mean flow is pseudoeastward (pseudowestward),

$$\frac{D\bar{q}}{Dt} = -\bar{u} \cdot \nabla \left\{ \bar{u} \cdot \frac{(\overline{u'q'})_*}{|\bar{u}|^2} \right\} - \nabla \cdot \{n[n \cdot (\overline{u'q'})_*]\} + \bar{S}.$$

$$(4.24)$$

The term associated with the cross-gradient component of $(\overline{u'q'})_*$—the first term on the rhs of (4.24)—is a transience term, since $\bar{u} \cdot \nabla \equiv D/Dt$. Further, rewriting (4.24) as

$$\frac{D\tilde{q}}{Dt} = -\nabla \cdot \{n[n \cdot (\overline{u'q'})_*]\} + \bar{S} \qquad (4.25)$$

where

$$\tilde{q} = \bar{q} + \frac{\bar{u} \cdot (\overline{u'q'})_*}{|\bar{u}|^2}, \qquad (4.26)$$

it is clear that the effect of the transience term on $\bar{q}$ is a Stokes correction-like term which vanishes where the eddy amplitudes vanish. For example, consider a conservative mean flow ($\bar{S} = 0$) through a region E of conservative eddies (for which $n \cdot (\overline{u'q'})_* = 0$), depicted in Fig. 12. Since $\tilde{q}$ is constant along a mean streamline and since $\bar{q} = \tilde{q}$ outside of E, it follows that the value of $\bar{q}$, while in general not constant along a streamline within E, must revert to its upstream value downstream of E. Therefore only nonconservative effects within E can generate net transport of potential vorticity across mean streamlines. However, because of the transience effect, this transport need not occur in the same location as the nonconservative eddy processes; indeed it is not difficult to conceive of circumstances in which the nonconservative terms serve only to make permanent those changes in the mean state which have been generated upstream by eddy transience (there are a number of zonal-mean analogues for this).
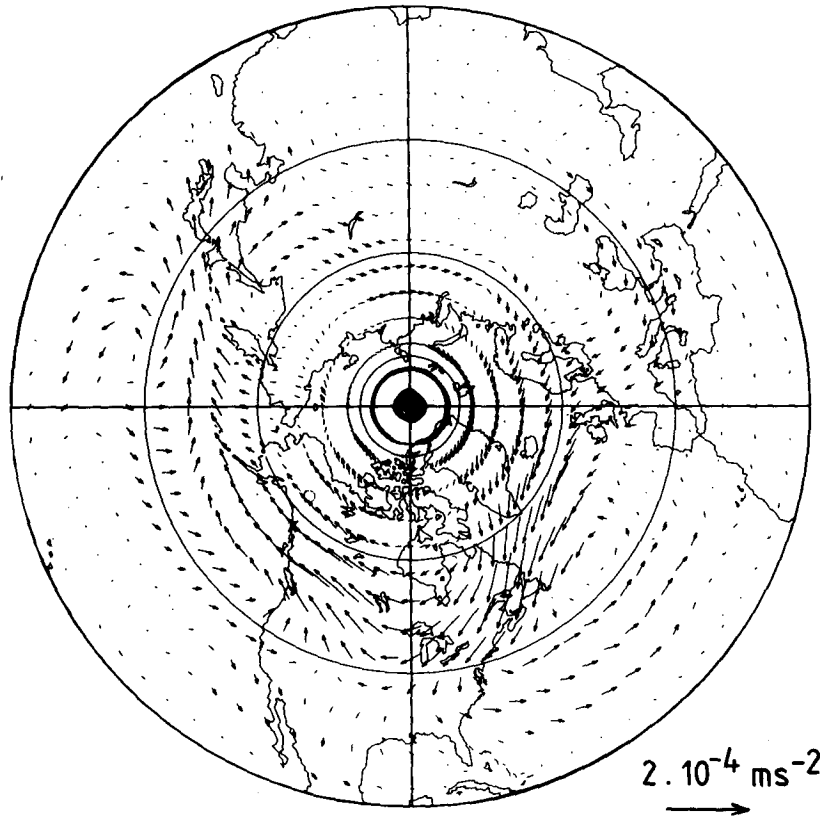
FIG. 10. Potential vorticity flux $\overline{u'q'}$ associated with the BP eddies. Scale at lower right.

5. Conclusions

The properties of the flux $\mathbf{M}_T$ discussed in this paper may be summarized as follows:

(i) For small-amplitude/quasi-geostrophic transient eddies on a spatially slowly varying time-mean flow, $\mathbf{M}_T$ appears as the flux of eddy activity in a conservation relation relating $\nabla \cdot \mathbf{M}_T$ to nonconservative effects.

(ii) For an almost-plane wave, $\mathbf{M}_T$ is parallel to the group velocity.

(iii) If the mean flow is almost-conservative and eddy transport is weak, $\nabla \cdot \mathbf{M}_T$ is proportional to the downgradient component of the residual potential vorticity flux.

(iv) The quasi-geostrophic momentum and thermodynamic equations may be transformed in such a way that the only term describing eddy, mean flow interaction is an effective force, whose upstream component is proportional to $\nabla \cdot \mathbf{M}_T$. As in the zonally averaged problem, the additional effective force associated with lower boundary effects may be incorporated
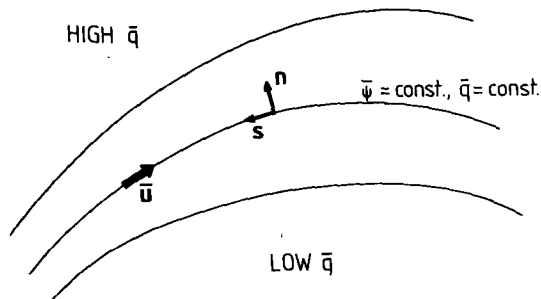

FIG. 11. The unit vectors $\mathbf{n}$ and $\mathbf{s} = \mathbf{k} \times \mathbf{n}$ for a conservative basic state with weak eddy transport. Curves are mean streamlines and isopleths of $\bar{q}$.
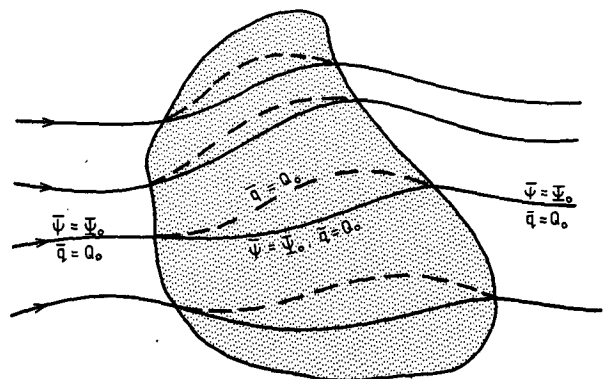

FIG. 12. Schematic illustration of the temporary nature of the effects of transport by a patch of locally conservative eddies. Solid curves, $\bar{\psi}$, $\bar{q}$; Dashed, $\bar{q}$. Stippled region is the region E discussed in the text.

in this formulation by incorporating a delta-function contribution to $\nabla \cdot \mathbf{M}_T$ there; indeed, for tropospheric problems, this term is a very important one, whose effects may do much to counteract the impact of the in situ $\nabla \cdot \mathbf{M}_T$. With this caveat, $\mathbf{M}_T$ may be regarded as an eddy flux of pseudowestward momentum; however the equally important flux of cross-stream momentum is apparently unrelated to $\mathbf{M}_T$.

These constitute generalizations of the properties (i)–(iv) of the EP flux outlined in the Introduction, in the sense that (i)–(iv) reduce to (i)–(iv) for the special case of a purely zonal mean flow if zonal averages are taken. However, the importance of the cross-stream momentum flux in the three-dimensional problem implies that neither the properties of $\mathbf{M}_T$ nor those of the radiative flux $\mathbf{M}_R$ completely summarize the eddy forcing of the mean flow.

Interpretation of $\mathbf{M}_T$ as a measure of the flux of transient eddy activity, via (i) and (ii), rests on the validity of the assumption of small-amplitude eddies on a slowly varying mean state. The small-amplitude assumption is difficult to justify in general but experience with the EP flux in the zonally averaged case and other evidence suggests that the propagation of synoptic eddies away from their region of generation at low levels in middle latitudes may be understood on the basis of quasi-linear dynamics. The validity of the assumption that the mean flow is slowly varying, compared with the length of scales of the eddy statistics, has been addressed and appears qualitatively reasonable for the climatology used here, especially for the band-pass eddies. That is not to say, however, that the same conclusion would be reached in other situations; over shorter time periods, for example, the mean flow could have more structure (e.g., over a blocking event) in which case the slowly varying approximation may be more suspect.

Results for the BP eddies, as determined from a 10-year Northern Hemisphere winter climatology, have revealed the propagation of transient eddy activity in the major storm tracks, with the eddies propagating into the upper troposphere from low levels at the upstream end of the tracks and thence downstream (primarily via simple advection), spreading laterally and predominantly equatorward of the jet. An additional apparent source of eddy activity is located in the upper troposphere, upstream of the Pacific storm track, but this aspect of the results, which clearly needs further attention, may be a consequence of problems of accuracy or interpretation of the eddy enstrophy statistics. While the LP eddy fluxes are a little more barotropic than the BP eddies, their propagation characteristics are much the same, which suggests that the eddy statistics for periods of 10 days or more are still dominated by baroclinic, storm-track disturbances. This contrasts with the results of Hoskins et al. (1983) who found their "E-vector" to be directed eastward for the high-

pass eddies and westward for the LP eddies and thereby deduced (among other things) that the high-pass and LP eddies propagate faster and slower, respectively, than the mean flow. The differences between these and the present conclusions based on the properties of the radiative flux $\mathbf{M}_R$ result primarily from the neglect of the baroclinic term in the zonal component (i.e., the simple relationship between the direction of the E-vector and of the intrinsic group velocity is valid only when this term is neglected). Indeed, those differences that are apparent here in $\mathbf{M}_R$ between BP and LP eddies may result simply from the frequency selection, rather than indicating any more fundamental differences in eddy propagation characteristics, since one would generally expect any low-frequency disturbance in the region of a jet to propagate rapidly upstream *relative to the jet* (Wallace and Lau, 1985). Thgerefore, in order to isolate the lower-frequency, nonbaroclinic systems in the circulation it may be necessary to use other time-filters (cf. Blackmon et al., 1984a,b) or to use other approaches.

The incorporation of advective processes into the flux $\mathbf{M}_T$, despite the unsatisfactory aspects of the results which have been attributed to data problems here, has emphasized that for both BP and LP eddies the flux of eddy activity is dominated by simple advection. Therefore, it is essential to incorporate advective effects (and, as seen here, this is difficult to do, given the demands one must make on data quality) in order to use flux diagnostics such as these to help locate the sources and sinks of eddy activity.

On zonal averaging, it is $\mathbf{M}_R$, and not $\mathbf{M}_T$, which reduces to the EP flux $\mathbf{F}$, and in fact, $\langle \mathbf{M}_T \rangle$ shows rather more poleward propagation than does $\mathbf{F}$, because $\langle \mathbf{M}_T \rangle$ includes additional information on the advection of eddy activity, which is predominantly poleward in the storm tracks. This result serves to highlight the fact that in the presence of a nonzonal mean flow, $\mathbf{F}$ describes the zonally averaged propagation of eddies *relative to the mean flow* rather than relative to fixed coordinates.

The fact that $\mathbf{M}_T$ is an incomplete description of the eddy transport effects on the mean circulation limits the usefulness of the flux as a diagnostic of the interaction, although it may give useful insights (e.g., Illari and Marshall, 1983, and Shutts, 1983, have discussed the possible importance of the downgradient component of the residual potential vorticity flux in some circumstances). In general, however, one may need to seek other avenues, for example via analysis of the potential vorticity flux itself. The most complete approach in this direction was made by Holopainen et al. (1982), who evaluated the irrotational part of the flux, and in the context of the discussion in section 4 it is particularly intriguing that the effective force thus defined as describing the eddy-mean flow interaction was found to be directed approximately upstream.

In general there seems to be no straightforward way

of simplifying the interaction beyond the discussion of section 4 without losing the link with the fundamental properties of eddy propagation. However, Hoskins et al. (1983) showed that useful progress can be made under some further assumptions. If the storm tracks are assumed to be long, thin and zonally aligned, the effective cross-stream momentum flux divergence in (4.9) is dominated by the y-component, in which case Hoskins et al. showed that, given a redefinition of the ageostrophic circulation, the eddy transport is entirely described by a flux **E** of upstream momentum, where **E** is their three-dimensional "E-vector." While this flux is not a flux of conservable eddy activity in the sense that M_R and M_T are, in the almost-plane-wave limit it does (for barotropic eddies though not, as seen in section 3, for the synoptic, baroclinic eddies) have a simple relationship with the eddy group velocity relative to the mean flow.

APPENDIX

Derivation of (3.5)

From (3.4) the perturbation quantity P' is

$$P' = -(f + \bar{\zeta})\frac{\partial \theta'}{\partial p} - \zeta' \frac{\partial \bar{\theta}}{\partial p}$$

$$\approx -f\frac{\partial \theta'}{\partial p} - \zeta' \frac{\partial \bar{\theta}}{\partial p}, \quad (A1)$$

under the quasi-geostrophic assumption that $|\bar{\zeta}| \ll |f|$. However, from (2.1)

$$q' = \zeta' + f\frac{\partial}{\partial p}\left(\frac{\theta'}{\partial \bar{\theta}/\partial p}\right), \quad (A2)$$

and therefore, from (A1)

$$P' = f\theta'\Gamma - q' \frac{\partial \bar{\theta}}{\partial p} \quad (A3)$$

where

$$\Gamma = -\frac{\partial^2 \bar{\theta}/\partial p^2}{\partial \bar{\theta}/\partial p}. \quad (A4)$$

Therefore,

$$\overline{P'^2} = \overline{q'^2}\left(\frac{\partial \bar{\theta}}{\partial p}\right)^2 - 2f\Gamma \frac{\partial \bar{\theta}}{\partial p}\overline{q'\theta'} + f^2\Gamma^2\overline{\theta'^2}. \quad (A5)$$

But, from (A2),

$$\overline{q'\theta'} = \overline{\zeta'\theta'} + \frac{1}{2}f\frac{\partial \bar{\theta}}{\partial p}\frac{\partial}{\partial p}\left(\frac{\overline{\theta'^2}}{(\partial \bar{\theta}/\partial p)^2}\right). \quad (A6)$$

Therefore, with a little manipulation,

$$\overline{P'^2} = \overline{q'^2}\left(\frac{\partial \bar{\theta}}{\partial p}\right)^2 - 2f\Gamma \frac{\partial \bar{\theta}}{\partial p}\overline{\zeta'\theta'} + f^2\Gamma \frac{\partial \bar{\theta}}{\partial p}\frac{\partial}{\partial p}\left(\frac{\overline{\theta'^2}}{\partial \bar{\theta}/\partial p}\right), \quad (A7)$$

and, since

$$\zeta' = \frac{1}{a \cos\phi}\frac{\partial v'}{\partial \lambda} - \frac{1}{a \cos\phi}\frac{\partial}{\partial \phi}(u'\cos\phi),$$

a little manipulation shows that

$$\overline{\zeta'\theta'} = \frac{1}{a \cos\phi}\frac{\partial}{\partial \lambda}(\overline{v'\theta'}) - \frac{1}{a \cos\phi}\frac{\partial}{\partial \phi}(\overline{u'\theta'}\cos\phi)$$

$$- \frac{1}{a \cos\phi}\overline{v'\frac{\partial \theta'}{\partial \lambda}} + \frac{1}{a}\overline{u'\frac{\partial \theta'}{\partial \phi}}. \quad (A8)$$

Using $\theta' = T'p^{-\kappa}$ and the thermal wind relations

$$\frac{1}{a}\frac{\partial T'}{\partial \phi} = -\frac{Hf}{R}\frac{\partial u'}{\partial z}$$

$$\frac{1}{a \cos\phi}\frac{\partial T'}{\partial \lambda} = \frac{Hf}{R}\frac{\partial v'}{\partial z},$$

(A8) becomes

$$p^\kappa\overline{\zeta'\theta'} = \nabla \cdot \mathbf{b} \quad (A9)$$

where **b** is the vector

$$\mathbf{b} = \left(\overline{v'T'}, -\overline{u'T'}, -\frac{fH}{2R}(\overline{u'^2} + \overline{v'^2})\right). \quad (A10)$$

Substitution of (A9) and (A10) into (A7) yields the result (3.5).

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