Using virtual particles for water pathways

Alice Ren

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Theory Seminar 2019

Paola Cessi

Readings:

- Döös, K., 1995. Inter-ocean exchange of water masses. J. Geophys. Res. 100, 13499–13514.
- Blanke, B., S. Raynaud, 1997. Kinematics of the Pacific Equatorial Undercurrent: An Eulerian and Lagrangian Approach from GCM Results. J. Phys. Oceanogr. 27, 1038–1053.
- Döös, K., Jönsson, B., Kjellsson, J., 2017. Evaluation of oceanic and atmospheric trajectory schemes in the TRACMASS trajectory model v6.0. Geosci. Model Dev. 10, 1733–1749.

Döös (1995)

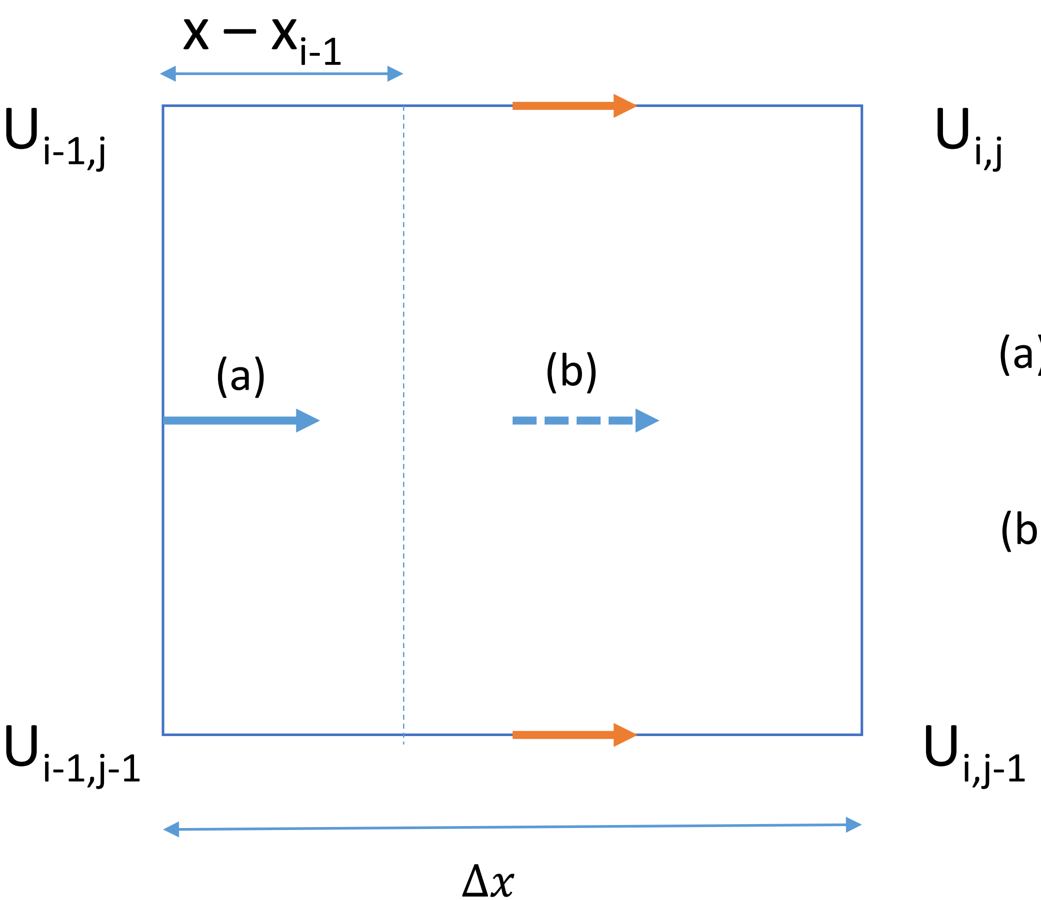
- Lagrangian method to calculate water mass transport between different ocean basins
- Calculate ventilation, mixing, t-s properties and residence time of water mass
- Finds that there is a direct exchange, unventilated, between ocean basins via the Southern Ocean (53%) and an indirect exchange involving circling Antarctica, ventilation, and export north in the Ekman layer (33%)

Numerical Model

- Fine resolution Antarctic model (FRAM)
- 24°S to Antarctica
- 0.25° resolution N-S, 0.5° resolution east-west
- Contains relaxation conditions for temperature and salinity in the surface layer
- Includes mean annual wind stress
- 16 year model run
- For years 10-15, data archived in monthly intervals; results time-averaged to steady velocity field

1. Exact solutions of trajectories to the time-averaged field

Zonal Velocity, u(x)

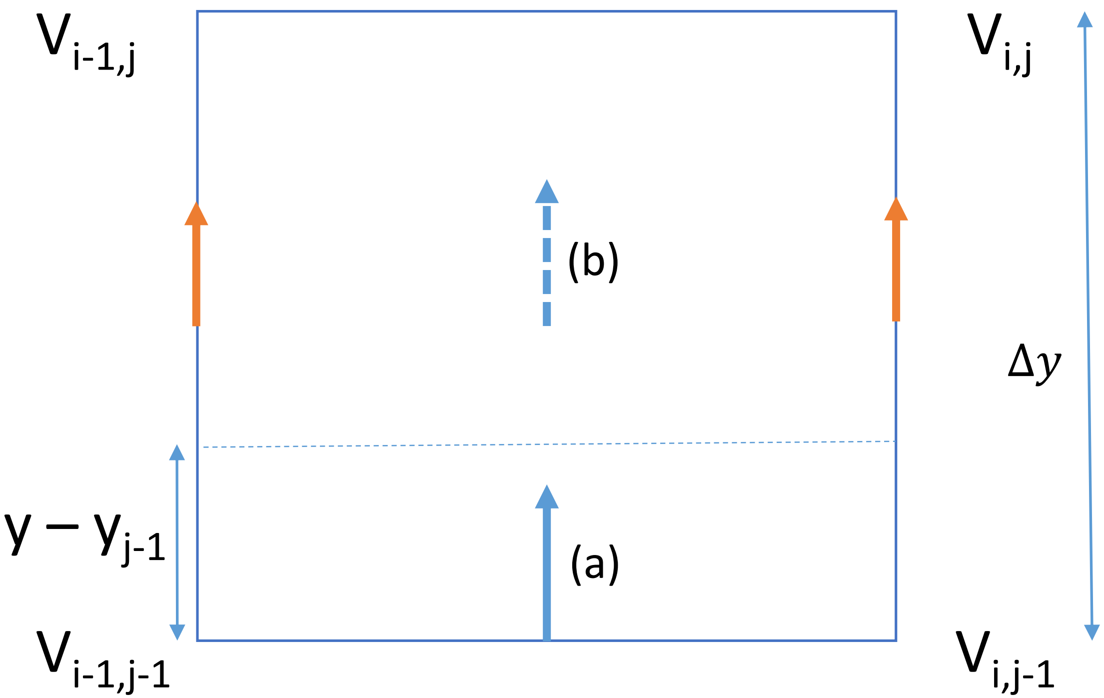


$$u(x) =$$

(a) $$\frac{1}{2}\left(u_{i-1,j} + u_{i-1,j-1}\right)$$

(b) $$+\ \frac{x - x_{i-1}}{2\Delta x}\left(u_{i,j} + u_{i,j-1} - u_{i-1,j} - u_{i-1,j-1}\right)$$

Meridional Velocity, v(y)



$$\mathrm{v}(y) =$$

$$\text{(a)} \quad \frac{1}{2}\left(v_{i-1,j-1} + v_{i,j-1}\right)$$

$$+ \frac{y - y_{j-1}}{2\Delta y}\left(v_{i,j} + v_{i-1,j} - v_{i,j-1} - v_{i-1,j-1}\right) \quad \text{(b)}$$

Back to zonal velocity as example:

$$\frac{dx}{dt} = u(x)$$

$$u(x) = \frac{1}{2}\left(u_{i-1,j} + u_{i-1,j-1}\right) + \frac{x - x_{i-1}}{2\Delta x}\left(u_{i,j} + u_{i,j-1} - u_{i-1,j} - u_{i-1,j-1}\right) \tag{A1}$$

$$\frac{dx}{dt} + \alpha x + \beta = 0 \qquad \alpha = \frac{u_{i-1,j} + u_{i-1,j-1} - u_{i,j} - u_{i,j-1}}{2\Delta x}$$

$$\beta = \frac{x_{i-1}\left(u_{i,j} + u_{i,j-1} - u_{i-1,j} - u_{i-1,j-1}\right)}{2\Delta x} - \frac{1}{2}\left(u_{i-1,j} + u_{i-1,j-1}\right) \tag{A2}$$

Solve the differential equation with boundary conditions $x(t_a)=x_a$ and $x(t_b)=x_b$.

Answer is

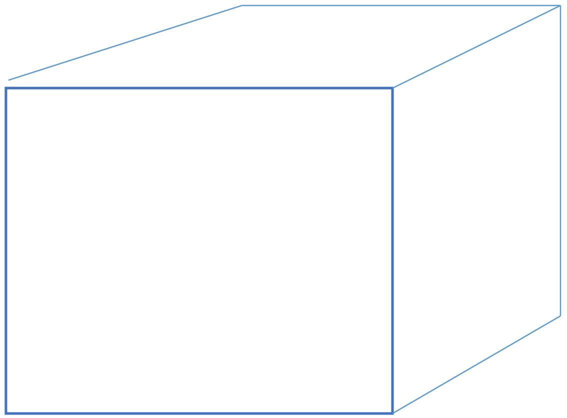
$$x_b + \frac{\beta}{\alpha} = \left(x_a + \frac{\beta}{\alpha}\right) \exp[-\alpha(t_b - t_a)]$$

or

$$t_b = t_a - \frac{1}{\alpha} \log[\frac{x_b + \frac{\beta}{\alpha}}{x_a + \frac{\beta}{\alpha}}]$$

The calculation is repeated for v(y), w(z).

There is a time, t_b , for each of the calculated velocities. The lowest t_b for a grid box determines which side the particle exits.



- Convection parameterized using steady model density field;
    - particle takes random depth within a convectively unstable water column.

Non-divergent particle tracking (incompressible) velocities since

- U is a linear function of x where $\alpha \cong u_x$
- V is a linear function of y where $\alpha_2 \cong v_y$
- W is a linear function of z where $\alpha_3 \cong w_z$

2. Seeding model to calculate volume transport

- Water particles are introduced at the open boundary (25°S) and followed until they leave the model.
- Calculate trajectories for all zonal-vertical grid boxes at open boundary with southward velocities.
- Sub-boxes, N=441 at 20S, 720 zonal, 32 vertical levels, excluding land = 5,569,141 trajectories.
- About 2,400 m horizontal and 1-10 m vertical distance between trajectories.

Can check that seeded water parcels conserve volume:

- Method works for forwards and backwards integration.

- Calculate the particle exits and then integrate backwards and compare the final velocity with the original southward velocity from FRAM.

- In paper, the Pacific receives 1 Sv (10^6 m^3/s) too much, Indian Ocean exports 1 Sv too much.

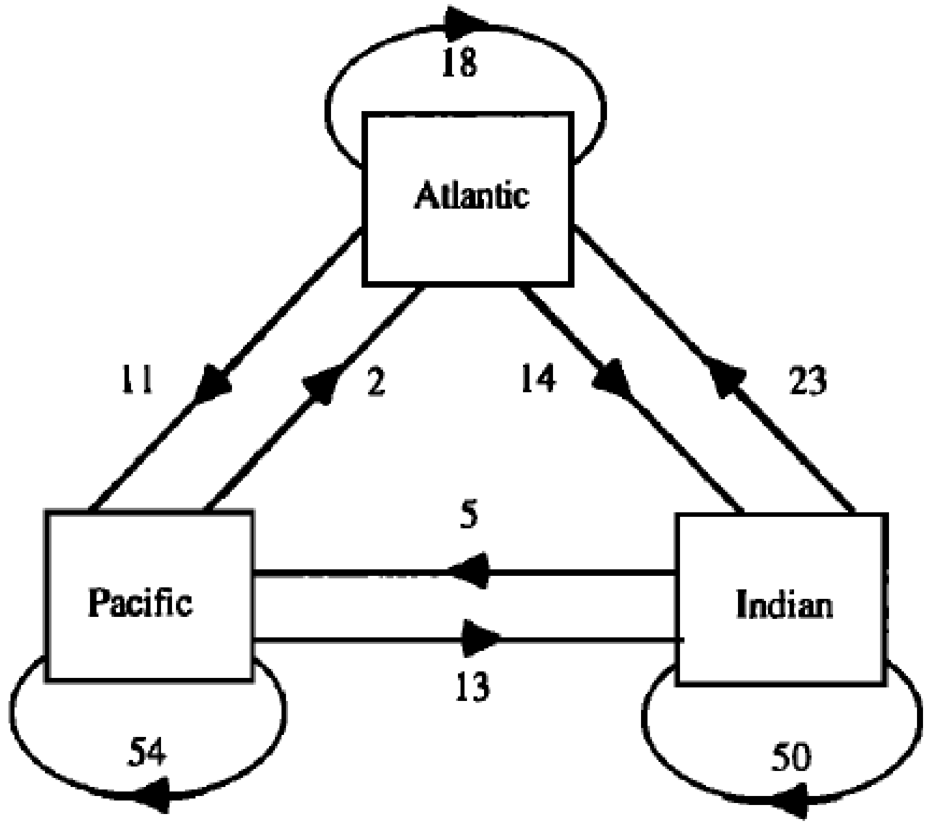


Figure 3. Transports of water between the three world oceans through the Southern Ocean. The transport here is independent of the path. Note that the ~~Atlantic~~ imports 1 Sv too much and the Indian Ocean exports 1 Sv too much. The units are in sverdrups.

Pacific

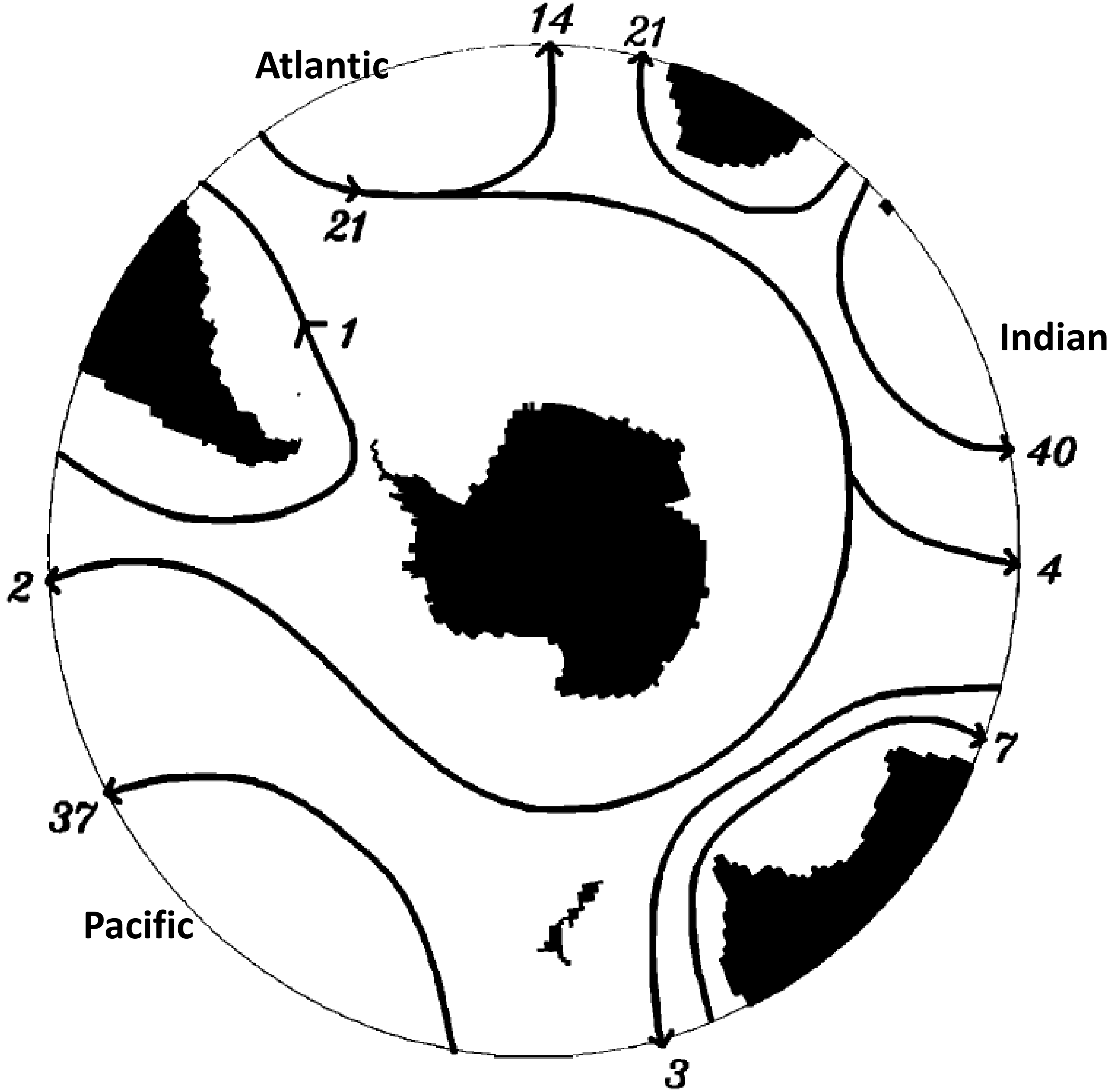


Figure 4. Schematic illustration of the "unventilated direct" ocean routes, with at least 1 Sv that is neither ventilated nor goes around Antarctica. Units are in sverdrups.

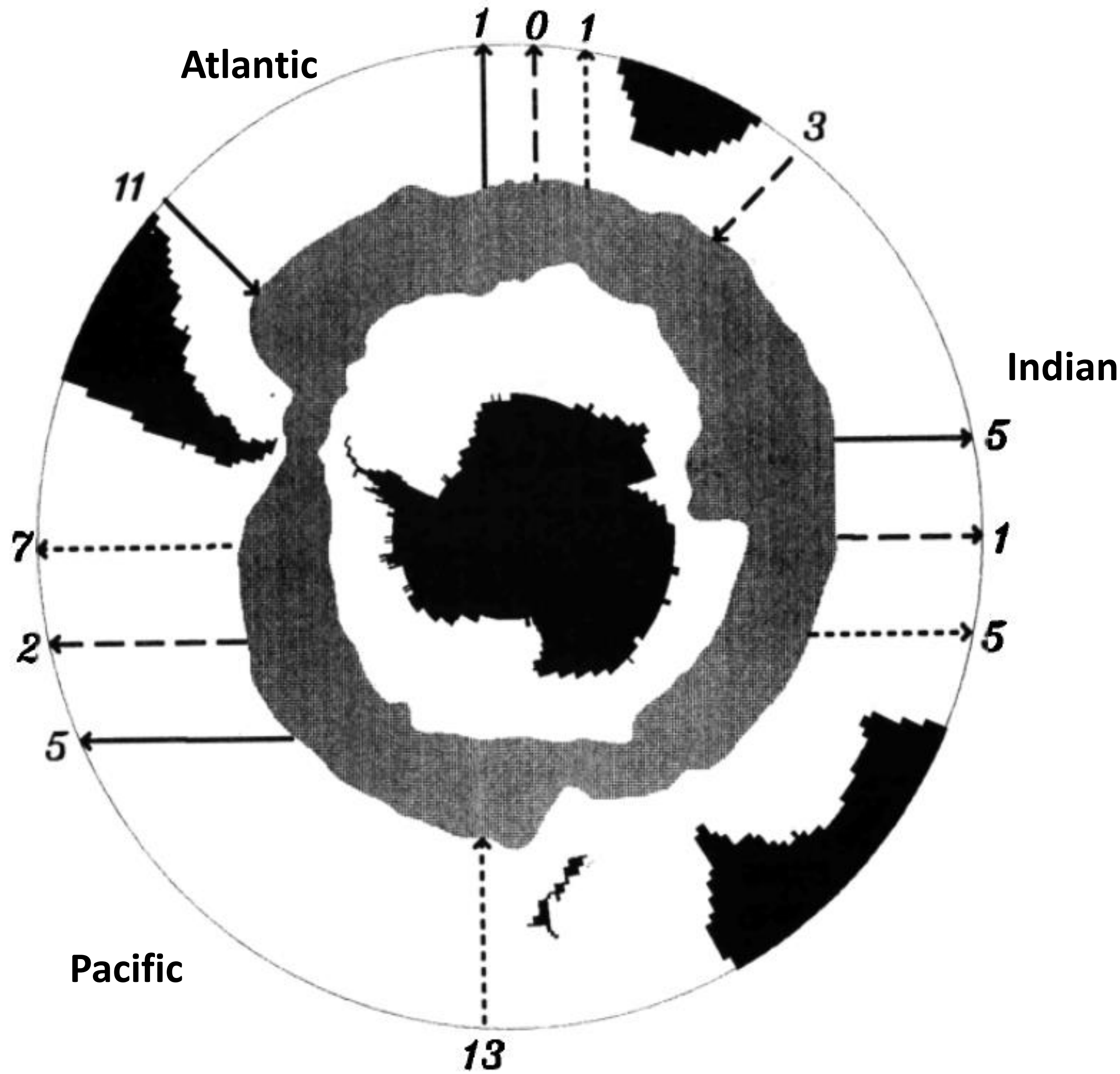


Figure 6. Schmetic illustration of the ventilated routes (numbered) which go around Antarctica at least once and, on average, five times before they are ventilated and then driven rapidly north by the Ekman transport. The solid lines represent water originating from the Atlantic, the long-dashed lines from the Indian Ocean, and the short-dashed lines from the Pacific. Units are in sverdrups.

Other Major Results

- Water makes, on average 4.6 ± 2.3 circuits until it is ventilated, 6.5 ± 3.8 circuits before exiting

- Heat transport into the Atlantic comes 85% from the Indian Ocean and the rest from Drake Passage.

- The time spent in the Southern Ocean varies with a standard deviation of the same order as the mean.

Interesting Points

- “The use of traditional water masses seems to be a doubtful method to trace the interocean exchange of water masses. Only 10 of the 29 Sv of NADW that enter at 24°S in the Atlantic will remain as such.”

- In the ventilated route, “the water rises to the surface in a gradual spiral around Antarctica.”

Weaknesses

- The model is for a steady velocity field; fields do not evolve in time.
- Method is “offline.” Integrating the model with passive tracers would be another method.

Blanke and Raynaud (1997)

- Lagrangian calculations used to evaluate accurately the mass transfers between various sections of the EUC and between the EUC and the tropics

- Use a time-varying set of velocities, able to resolve the annual cycle using monthly fields of velocity, assuming velocity is constant in-between snapshots of time.

- Further take advantage of forward and backwards iteration

Numerical Model

- Laboratoire d'Oceanographie Dynamique et de Climatologie (LODYC) GCM model, in the paper referred to as OPA model
- 47 °N to 65 °S domain
- Zonal resolution 0.33°-0.75°, meridional resolution 0.33°-1.5°, 30 vertical levels
- Climatological wind stress, climatological surface heat flux, salinity relaxed to seasonal climatology
- Annual mean and monthly averages from last year the 10-year experiment

1. Computing Lagrangian Trajectories

Write the divergence of a velocity field as:

$$\Delta \mathbf{V} = \frac{1}{b}[\partial_i(e_2 e_3 U) + \partial_j(e_1 e_3 V) + \partial_k(e_1 e_2 W)], \qquad (A.1)$$

The scale factors in each direction are $e_{1,2,3}$ and represent dx, dy, and dz calculated at **v** gridpoints. The product $e_1e_2e_3=b$ is the volume of a water parcel calculated from the center of a grid cell.

The transport in the x-direction, for example, is $F = e_2 e_3 U$, such that for a non-divergent flow:

$$\partial_i F + \partial_j G + \partial_k H = 0, \qquad (A.2)$$

Transport varies linearly from opposite faces of individual cell. For x-direction

$$F(r) = F_0 + r\Delta F, \qquad (A.3)$$

Where $F(i) = F_i,\ F(i = 0) = F_0,\ \Delta F = F_1 - F_0.$

Next, rewrite the relationship $U = \frac{dx}{dt}$ in terms of transport by using a new set of variables:

$s = (e_1 e_2 e_3)^{-1} t$ and $x = e_1 r.$ Since $dx = e_1 dr,$

$$dt = e_1 e_2 e_3 ds,$$

$$F = e_2 e_3 U,$$

$$\frac{dr}{ds} = F, \qquad (A.4)$$

Solving the differential equation (r=0 for s=0) for r gives:

$$r = \frac{F_0}{\Delta F}[\exp(\Delta F s) - 1]. \qquad \text{(A.5a)}$$

Where if there is no transport difference:

$$\Delta F \to 0$$

$$r = F_0 s. \qquad \text{(A.5b)}$$

Need to compute time to reach the other side, or when r=1, for example

Since

$$ds = \frac{dr}{F}. \tag{A.6}$$

It follows that

$$ds = \frac{dF}{F\Delta F}. \tag{A.7}$$

The pseudo-time, s, is then

$$s = \frac{1}{\Delta F} \ln\left(\frac{F}{F_0}\right). \tag{A.8}$$

To get to the other side, F_1,

$$\Delta s = \frac{1}{\Delta F} \ln\left(\frac{F_1}{F_0}\right), \tag{A.9a}$$

or, if $\Delta F = 0$, its limit when $\Delta F \to 0$:

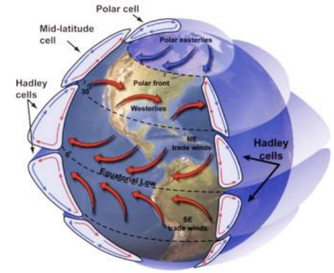
$$\Delta s = \frac{1}{F_0}. \tag{A.9b}$$

- The shortest time defines the travel direction in the cell.
- Due to incompressibility, each given particle conserves its volume (mass).
- Transport of a given water mass is calculated from its own particles and their associated infinitesimal transports.

- Update from Döös (1995) is writing equations to explicitly associate volume transport in calculating trajectories.

- Still need to assume velocity is constant in-between two snapshots of the velocity field (for example, monthly average of June and of July)

Major Results



- All over the Pacific Ocean, the EUC loses water through equatorial upwelling and Ekman divergence.
- West of 150 °W, geostrophic convergence of shallow water at the Equator mixes downward from the overlying SEC.
- East of 150 ° W, the EUC core approaches the surface and becomes more affected by Ekman divergence. The balance is from the west. Two thirds of the EUC transport in the eastern Pacific (120 °W) originates from the western Pacific (150 °E).
- Asymmetric meridional wind causes greater southward export.

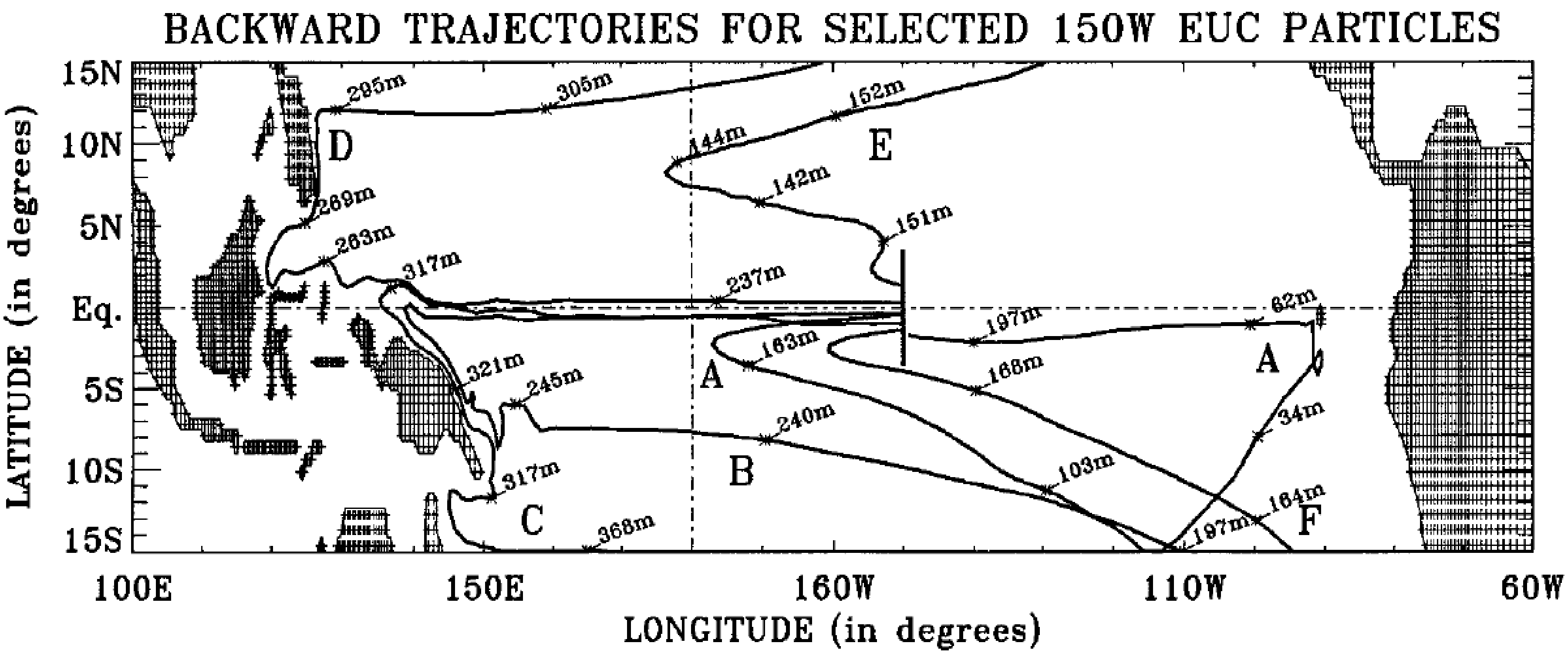


FIG. 10. Six selected backward trajectories ("A" to "F") from the EUC at 150°W. Paths are typical of possible supplies of water to the EUC and are discussed in the text. Depths are given on trajectories.

A=recirculation, **B**=south subtropical gyre, **C**=deeper levels, from East Australia, **D**=North Pacific origin, **E** and **F**= particles entering east of 150°W at 15°N or 15°S

Döös, Jönsson, and Kjellsson (2017)

Considers time dependence to Lagrangian method.

- Two methods:
  - 1) **step-wise stationary -** assumes velocity fields are stationary for a limited time and solves the trajectory path from a differential equation only as a function of space.
  - 2) **time dependent –** uses continuous linear interpolation of the fields in time and solves differential equation as a function of both space and time
- **Time-dependent** scheme greatly improves accuracy with only a small increase in computational time compared to step-wise stationary scheme.

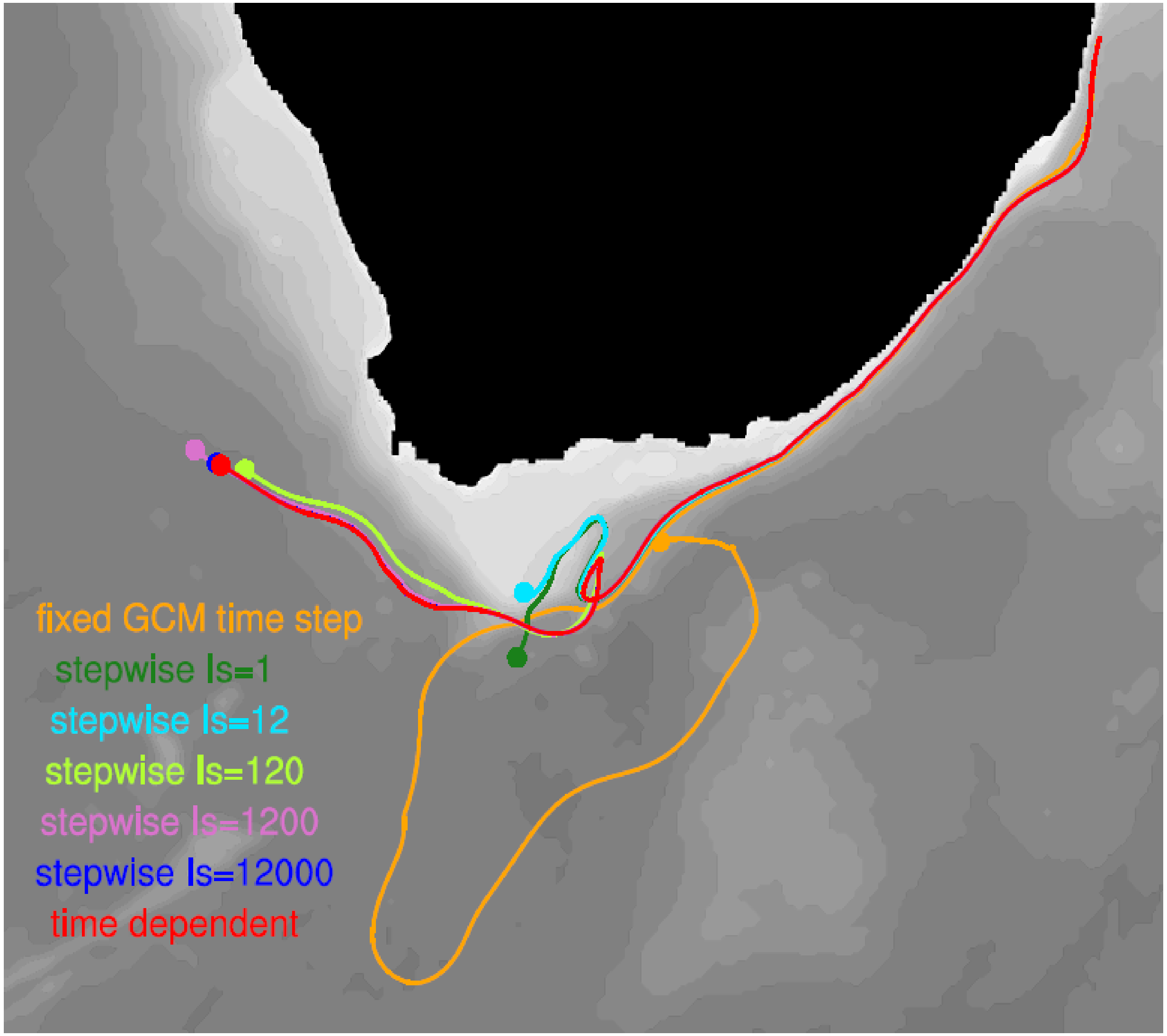


Figure 5. Example of ocean trajectory paths due to different trajectory schemes and number of intermediate time steps. The "time-dependent" method results in red and those obtained with the "stepwise-stationary" method with $I_S = 1, 12, 120, 1200$ and 12000 as well as "fixed GCM time steps". Note that these homologous trajectories were selected to illustrate that "stepwise-stationary" trajectories are closer to "time-dependent" trajectories when the number of intermediate time steps (I_S) is increased.

GCM output is 5-day average

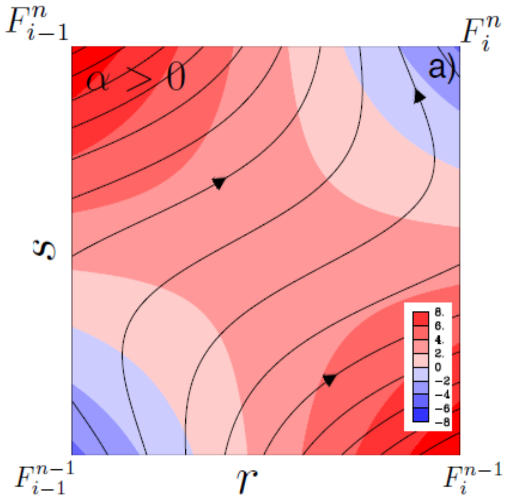
Time dependent Scheme

General idea:

$$Assume\ \ \frac{dx}{dt} + \alpha(t)x = 0, \qquad\qquad ignore\ \beta$$

$$Then\quad x = c_0 e^{\int -\alpha(t)dt}$$

Bilinear interpolation in space and time:



$$
\begin{aligned}
F(r,s) &\\
= \ & F_{i-1}^{n-1} + (r - r_{i-1})\left(F_{i}^{n-1} - F_{i-1}^{n-1}\right) \\
&+ \frac{s - s^{n-1}}{\Delta s}\left[F_{i-1}^{n} - F_{i-1}^{n-1}\right. \\
&\left.+ (r - r_{i-1})\left(F_{i}^{n} - F_{i-1}^{n} + F_{i}^{n-1} - F_{i-1}^{n-1}\right)\right]
\end{aligned}
$$

The transport is F

and as before $r = (x/\Delta x, y/\Delta y, z/\Delta z)$, $s \equiv t/(\Delta x \Delta y \Delta z)$

and $\Delta s = s^{n} - s^{n-1} = (t^{n} - t^{n-1})/(\Delta x \Delta y \Delta z)$

The differential equation is:

$$\frac{dr}{ds} + \alpha\, r\, s + \beta\, r + \gamma\, s + \delta = 0$$

$$\begin{aligned}
\alpha &\equiv -\frac{1}{\Delta s}\left(F_i^n - F_{i-1}^n - F_i^{n-1} + F_{i-1}^{n-1}\right), \\
\beta &\equiv F_{i-1}^{n-1} - F_i^{n-1} - \alpha\, s^{n-1}, \\
\gamma &\equiv -\frac{1}{\Delta s}\left(F_{i-1}^n - F_{i-1}^{n-1}\right) - \alpha\, r_{i-1}, \\
\delta &\equiv -F_{i-1}^{n-1} + r_{i-1}\left(F_i^{n-1} - F_{i-1}^{n-1}\right) - \gamma\, s^{n-1}.
\end{aligned}$$

three cases: $\alpha > 0$, $\alpha < 0$ and $\alpha = 0$

“We strongly recommend the use of the ‘time-dependent’ scheme based on Vries and Döös (2001) in favour of the ‘stepwise-stationary’ scheme. We would also like to dissuade the use of the more primitive ‘fixed GCM time step’ scheme, which is used in other trajectory codes since the velocity fields remain stationary for longer periods creating abrupt discontinuities in the velocity fields, and yielding inaccurate solutions.”