Red noise and regime shifts

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Abstract

The analysis of interdecadal physical and biological variability is made challenging by the relative shortness of available time series. It has been suggested that rapid temporal changes of the most energetic empirical orthogonal function of North Pacific sea surface temperature (sometimes called the Pacific Decadal Oscillation or PDO) represents a “regime shift” between states with otherwise stable statistics. Using random independent time series generated to have the same frequency content as the PDO, we show that a composite analysis of climatic records recently used to identify regime shifts is likely to find them in Gaussian, red noise with stationary statistics. Detection of a shift by this procedure is not evidence of nonlinear processes leading to bi-stable behavior or any other meaningful regime shift.

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1. Introduction

The study of climatic phenomena on interdecadal time scales requires examination of time series that contain significant variability on periods comparable to the record length. Often the energy in relevant time series, such as basin-wide sea surface temperature (SST), grows with increasing period (in analogy to the spectrum of light, such time series are said to have a “red” spectrum). After careful scrutiny of such time series, some have suggested that “regime shifts” occur that separate periods of stable but differing character. Of particular interest in this regard are the large-scale changes of various physical and ecological properties in the North Pacific that occurred near 1976. The question of whether climatic and ecological variability is better thought of as an approximately Gaussian, red-spectrum process with stationary statistics or as a sequence of “regimes” with different statistics is fundamental to understanding and predicting this variability. As Scheffer et al. (2001) make clear, many nonlinear mechanisms exist that could lead to climatic regime shifts, particularly in biological processes. Periods of apparently stationary conditions separated by abrupt shifts of these conditions are likely to result from such nonlinear processes. Nonlinear regime shifts will not, on the other hand, result from the filtered white-noise mechanism for oceanic climate variability discussed by Hasselmann (1976), in which white noise atmospheric forcing with stationary

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statistical characteristics is integrated by an ocean with constant properties to generate red-noise ocean-climate variability.

We address here the detection of regime shifts. It is well known (Kendall and Stuart, 1966, p. 378; Wunsch, 1999) that stationary zero-mean Gaussian red-noise time series have long runs without a zero crossing, and these might easily be mistaken as regimes separated by shifts. On the other hand, Ebbesmeyer et al. (1991) and Hare and Mantua (2000) have developed a methodology that appears to show that variability of physical and ecological variables in the North Pacific is characterized by “conspicuous jumps from one rather stable condition to another”, to quote the discussion of these changes by Scheffer et al. (2001). Using synthetic time series colored to match climatic records we find that such changes are natural features of stationary red-noise time series, and that the regime-shift detecting analyses used in the North Pacific are likely to find regime shifts in stationary Gaussian random red noise.

There is perhaps no better variable than SST in the North Pacific to address issues of interdecadal oceanic change. For decades Namias and collaborators (Namias and Cayan, 1981) examined coupling between SST and atmospheric variables, such as sea level pressure, on climate time scales. While testing Namias’ ideas, Davis (1976) calculated the empirical orthogonal functions (EOFs) of SST north of 20°N in the North Pacific. The most energetic EOF describes a vast region of the central and western Pacific heating and cooling in phase while the eastern Pacific is out of phase, a pattern frequently seen in Namias’ case studies. The temporal amplitude of this dominant EOF (also known as the principal component) has proven such a useful index of oceanographic variability that it has been named the Pacific Decadal Oscillation (PDO) (Mantua et al., 1997).

A major change in the polarity of the PDO occurred in 1976–1977, with the central Pacific cooling and the eastern Pacific warming (Miller et al., 1994), and many atmospheric variables changing concurrently. Several measures of North Pacific biota, such as zooplankton biomass and salmon catch, were also observed to change in concert with the PDO (McGowan et al., 1998). Following Ebbesmeyer et al. (1991), Hare and Mantua (2000) used 100 physical and biological time series in a composite analysis to argue that regime shifts occurred in the North Pacific in 1976 and 1988. Here, we examine the composite analysis and show it is likely to find ostensibly significant steps even if the input time series are red noise.

2. Data and noise

Monthly data for the period 1950–1993 from the Comprehensive Ocean-Atmosphere Data Set (COADS) were used to calculate the first EOF of North Pacific SST on a 2° × 2° grid (Fig. 1). Before calculating EOFs, the average of each calendar month over the 44-year record was subtracted from that month’s values to remove the seasonal cycle. The first mode is essentially the

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**Fig. 1.** The most energetic empirical orthogonal function of North Pacific SST. Positive contours are solid, negative contours are dashed, the zero contour is heavy, and units are arbitrary. The central and western Pacific are out of phase with the eastern boundary.
same as that found by Davis (1976) before the first 1976 putative regime shift and as that employed by Mantua et al. (1997) to define the PDO. The sign of an EOF is arbitrary, as are the units and magnitude; we use the sign of Davis (1976) and normalize the EOF to have unit Euclidean norm (Noble and Daniel, 1977). This sign convention is opposite to that in some recent literature on the PDO (Zhang et al., 1997). The PDO is often defined as the temporal amplitude of the first EOF of North Pacific SST since 1900. By considering data only since 1950, we avoid sparse coverage prior to and during World War II. Hereafter, the term PDO is shorthand for the temporal amplitude of our first EOF.

To examine the composite analysis of jumps, independent random time series were generated to have identical frequency content as the PDO. The periodogram (the squared magnitude of Fourier coefficients vs. frequency) of the PDO (Fig. 2) shows that low frequencies are most energetic, so the PDO can be said to be red. Synthetic noise time series were constructed with the same periodogram as the PDO, but with Fourier coefficient phases that were random, independent, and uniformly distributed. Because noise time series are linear combinations of hundreds of the Fourier coefficients, the noise is nearly Gaussian by the central limit theorem. For comparison, the PDO amplitude and three noise time series are shown in Fig. 3. All series show the long periods of constant sign that characterize red noise (Wunsch, 1999) and have been termed regimes in the PDO.

3. A questionable analysis for regime shifts

The composite analysis that Ebbesmeyer et al. (1991) and Hare and Mantua (2000) have used to characterize the magnitude of regime shifts compares the first and second halves of a collection of $N$ records of $M$ years length. The analysis...
proceeds as follows:

1. Subtract from each of the $N$ records its mean over $M$ years.
2. Separately normalize each half of each record with the standard deviation of that half.
3. The means of the two halves of each record will now necessarily be of opposite sign. Multiply each $M$-year record by the sign of the mean of its first $M/2$ years.
4. Calculate an average for each year across all $N$ time series.
5. Calculate a standard error for each year as $\sigma/\sqrt{N}$ where $\sigma$ is the standard deviation calculated across all $N$ time series.
6. Calculate averages over all $N$ time series and each half of the $M$-year record.

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**Fig. 3.** The PDO and noise generated to have the same frequency content but random phase. All time series are shown as annual averages. The PDO is third from the top. The noise time series are used as input for the composite analysis.
7. Consider a shift as significant if all the annual means in each half-record have the same sign and none is within a standard error of zero.

Hare and Mantua applied this procedure to 100 time series, 61 of them biological, using record lengths of approximately 20 years. We repeated this calculation using \( N = 100 \) random, independent \( M = 20 \)-year synthetic time series with identical frequency content as the PDO. The result (Fig. 4) clearly passes the significance test 7 above. The standard errors are nearly identical (close to 0.1) in all years because the 100 time series are normalized to unit variance. The similarities between Fig. 4 and Hare and Mantua’s Fig. 7 are obvious.

It is fair to ask how often a step as in Fig. 4 is obtained using noise as input. Given 44-year records matched to the PDO series from 1950 through 1993, 25 different 20-year chunks (1950–1969, 1951–1970, ..., 1984–1993) can be analyzed. We performed the composite analysis for each of these 25 possibilities, and tabulated the number of chunks where none of the annual means was within one standard error of zero. This procedure was repeated 10,000 times with different random time series, so 250,000 analyses were performed. In this Monte Carlo calculation, 53% of the 20-year chunks have regime shifts (Table 1) and the average step change is 0.85, only slightly less than the values reported by Hare and Mantua. Thus, given 100 independent and stationary time series of identical frequency content as the PDO, the composite analysis applied to a randomly selected 20-year span will detect a regime shift comparable to that found by Hare and Mantua about half the time.

The composite analysis is designed specifically to make a step. Subtracting the mean from a time series causes it to cross zero. The essential element in the analysis is the multiplication by ±1 so that each input time series contributes positively to the

![Fig. 4. An example of the composite analysis (explained in the text) applied to random, independent noise (as in Fig. 3) of identical frequency content to the PDO. 100 time series of length 20 years went into the analysis. Circles are averages for each year across all 100 time series (step 4 in the composite analysis). Plus and minus one standard error (approximately 0.1) is indicated by the bars on the data (step 5). The solid line indicates the means of the two halves of the record (step 6).](image-url)
step change. Also conducive (though not essential) to the step is the separate normalization of the first and second halves of the records. As the number $N$ of time series in the composite grows, the step in the mean stays of the same magnitude while the standard error decreases like $1/\sqrt{N}$. With, for example, 10,000 time series as input, the standard error becomes 0.01, a limiting shape is approached (Fig. 5), and the probability that any of the annual values is within one standard error of zero vanishes.

The nature of the composite analysis, particularly removal of the $M$-year mean and multiplication by $\pm 1$, ensures that a step will result regardless of the frequency content of the component time series. The likelihood, size, and shape of the resultant step depend in a straightforward way on the frequency spectrum of the input time series. This is shown by applying the Monte Carlo calculation to noise time series constructed using Fourier coefficients chosen from a Gaussian, zero-mean distribution with variance proportional to $f^k$ where $f$ is frequency and $k = -2, -1, 0$ (Fig. 2). The probability of producing a step, and the size of the step, grow with increasing spectral slope (Table 1). Probability and step size are related because as the step grows the larger annual values are less likely to be within one standard error of zero; the standard error is roughly independent of frequency content. Composite analyses that sum 10,000 time series (Fig. 5) show how the underlying spectrum affects step shape. A white spectrum produces a sharper, but larger, step. Ebbesmeyer et al. considered the effect of white noise in the original composite analysis of 40 16-year records, but did not appreciate the importance of red noise in creating a large step.

Red noise time series may be created as first-order autoregressive processes (Box and Jenkins, 1970):

$$x_t = ax_{t-1} + \varepsilon_t,$$

where $a$ is a positive coefficient less than one, $\varepsilon$ is white noise, and the subscripts indicate time. The serial correlation, and frequency content, of the autoregressive process is set by the coefficient $a$. Monte Carlo calculations with $a = 0.99, 0.95, 0.9$ confirm the results of the power law spectra (Table 1). The likelihood of finding a step grows with increasing regressive coefficient. No doubt, other classes of red noise could be considered, all of which would yield steps in the composite analysis.

The composite analysis makes a step because an individual annual value in a time series covaries with the mean of the surrounding ten years. This is true for white noise, and especially so for serially correlated red noise. The covariance between a value and mean decreases as the averaging length increases, so applying the composite analysis to a longer (say 40-year) chunk produces a smaller step. A clear path to creating large steps is to use, as Hare and Mantua do, many short records in the composite analysis.

### Table 1

<table>
<thead>
<tr>
<th>Noise type</th>
<th>Fraction regime shifts</th>
<th>Step magnitude</th>
<th>Step magnitude (shifts only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDO</td>
<td>0.53</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>$f^{-2}$</td>
<td>0.91</td>
<td>1.72</td>
<td>1.73</td>
</tr>
<tr>
<td>$f^{-1}$</td>
<td>0.57</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>$f^0$</td>
<td>0.02</td>
<td>0.39</td>
<td>0.43</td>
</tr>
<tr>
<td>AR, 0.99</td>
<td>0.86</td>
<td>1.53</td>
<td>1.54</td>
</tr>
<tr>
<td>AR, 0.95</td>
<td>0.53</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>AR, 0.90</td>
<td>0.41</td>
<td>0.61</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Noise types have the frequency content of the PDO, are power-law spectra, or are first-order autoregressive with the listed coefficients. The composite analysis is repeated 250,000 times to produce these statistics. A regime shift is defined as having none of the annual means within one standard error of zero. The mean step magnitude is given where the average is over all realizations, and over only those defined to be regime shifts.
4. Conclusion

The narrowest interpretation of these results is that the composite analysis frequently finds steps in random, stationary, Gaussian, independent noise time series. It does this because each time series is multiplied by $\pm 1$ to ensure that it reinforces the step. Compositing many short time series in this way is a recipe for creating a step, and the more series that are composited the greater the certainty of finding a step. If, for example, the analyst were to multiply $N$ random, independent $N$-year records by $N$ freely chosen real parameters (rather than being limited to $\pm 1$) before summing,
A perfect step could always be created. A prior limit on the number of input time series is necessary for meaningful results just as it is in regression analyses (Davis, 1977). Further, if the data were examined before applying the composite analysis, the probability of finding a step would increase by careful choice of time intervals to analyze.

Statistical regime shifts in time series require particular phase relations between Fourier amplitudes for different frequencies. We have constructed synthetic time series with independent Fourier coefficients that cannot have genuine regimes and shown that the composite analysis frequently finds step-like changes in them. In general, a rapid change found using the composite analysis cannot be said to separate two distinct regimes, or to be particularly unusual.

An especially simple model of climate variability is the random walk (Hasselmann, 1976). Such a model integrates stationary white-noise forcing to produce a climate spectrum of $-2$ slope. A true random walk is unrealistic as it has infinite energy at zero frequency, leading Hasselmann to suggest a low-frequency cutoff. The lowest frequency in our simulations was 1 cycle per 44 years, and a spectral slope of $-2$ was considered. This gave a 91% probability of the composite analysis finding a step in 100 randomly selected 20-year records. If the $-2$ slope were extended to 1 cycle per 500 years the probability would rise to 95%, and if a true random walk were used the composite analysis would be even more likely to find a step.

We do not suggest that rapid climate change has not occurred or that nonlinear processes causing genuine regime shifts are not important. The climate record is replete with changes not easily explained as the nearly Gaussian behavior of a linear process. Neither do we claim that the rapid changes in the North Pacific are unimportant or that the physical and biological records are not strongly related. Much evidence exists for the interdependence of the atmosphere, ocean, and biota on a variety of time scales. We do, however, believe that our results show that the existence of changes deemed significant by the composite analysis is not evidence for anything more than Gaussian red noise with stationary statistics.

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References

