Problem Set 3: MAE 127 (Solutions)

1. In problem set 2, you computed wind velocities from wind speed S and direction θ, (hopefully) using the equations:

$$\begin{aligned} u &= -S\sin(\theta * \pi/180) \\ v &= -S\cos(\theta * \pi/180). \end{aligned}$$

where the minus sign appears, because the angle θ reported in the data set indicates the direction from which the wind comes. Use error propagation to estimate the uncertainties in u and v as a function of uncertainties in S and θ.

Assume that uncertainties in S and θ are δ_S and δ_θ respectively. The uncertainty in u is

$$\begin{aligned} \delta_u &= \sqrt{\left(\frac{\partial u}{\partial S}\right)^2 \delta_S^2 + + \left(\frac{\partial u}{\partial \theta}\right)^2 \delta_\theta^2} \\ &= \sqrt{\left(-\sin(\theta\pi/180)\right)^2 \delta_S^2 + \left(-S\pi/180\cos(\theta\pi/180)\right)^2 \delta_\theta^2} \\ &= \sqrt{\sin^2(\theta\pi/180)\delta_S^2 + S^2\pi^2/180^2\cos^2(\theta\pi/180)\delta_\theta^2} \\ \delta_v &= \sqrt{\left(\frac{\partial v}{\partial S}\right)^2 \delta_S^2 + + \left(\frac{\partial v}{\partial \theta}\right)^2 \delta_\theta^2} \\ &= \sqrt{\left(-\cos(\theta\pi/180)\right)^2 \delta_S^2 + \left(S\pi/180\sin(\theta\pi/180)\right)^2 \delta_\theta^2} \\ &= \sqrt{\cos^2(\theta\pi/180)\delta_S^2 + S^2\pi^2/180^2\sin^2(\theta\pi/180)\delta_\theta^2}. \end{aligned}$$

Without further information, we can't simplify this any further.

2. How much CO_2 can be stored in the ocean? Burning fossil fuels releases CO_2 into the atmosphere, where it is responsible for greenhouse warming, but not all of the fossil fuel remains in the atmosphere. Some is taken up by plants, and some is stored in the ocean. Problems 2 and 3 look at the problem of estimating air-sea CO_2 gas exchange.

The gas transfer velocity defining air-sea CO_2 exchange is a function of wind speed:

$$k = 0.0283u^3(Sc/660)^{-1/2},$$

where u represents the instantaneous wind speed (officially at 10-m elevation, but we won't worry about that for this problem), and Sc is the Schmidt number.

$$Sc = 335.6(MW)^{1/2}(1 - 0.066T + 0.002043T^2 - 0.000026T^3)$$

where MW is molecular weight and is equal to 44 grams/mole for CO_2, and T is sea surface temperature in °C. Use the wind data and sea surface temperature data from buoy.mat, the data set that we began using in problem set 1, to estimate the gas transfer velocity as a function of time. Plot the pdf of the gas transfer velocity. What are its mean and standard deviation?

Here is code to compute the gas transfer velocity as a function of time and plot its pdf, mean, and standard deviation.

```
% load data and reflag bad values
load buoy.mat
wind_speed(find(wind_speed==99))=NaN;
water_temp(find(water_temp==999))=NaN;

% compute gas transfer velocity
MW=44;
Sc=335.6*(MW)^.5*(1-0.066*water_temp + 0.002043*water_temp.^2  ...
          - 0.000026*water_temp.^3);
k=0.0283 * wind_speed.^3 .* (Sc/660).^(-.5);

% plot time series and pdf
subplot(1,2,1); plot(datenum(year,month,day,hour,0,0),k); datetick;
xlabel('time (years)'); ylabel('gas transfer velocity (m/s)')
title('Time series of gas transfer velocity estimated for Santa Monica Basin')

dk=1;
range=0:dk:50;
[a,b]=hist(k,range);
subplot(1,2,2); plot(b,a/sum(a)/dk);
xlabel('gas transfer velocity (m/s)'); ylabel('pdf')
title('PDF of gas transfer velocity from Santa Monica Basin')

% output statistics using nanmean, nanstd
[nanmean(k) nanstd(k)]

% if stats toolbox is not available:
[mean(k(find(~isnan(k)))) std(k(find(~isnan(k))))]
```

The resulting time series and pdf are shown in Figure 1. In the time series plot (left) gas transfer velocities are highly variable. The pdf (right) shows that they are always positive and commonly near zero with some outliers.

The mean k is 3.06 m s^{-1} and the standard deviation is 7.18 m s^{-1} with $N = 43381$.

3. Assuming that temperatures are accurate to $\pm 0.2°$C and velocities to 0.5 m/s, propagate errors through the equation for k to estimate its uncertainty. Show your algebra. Are your uncertainty estimates consistent with the standard deviation computed in problem 2?

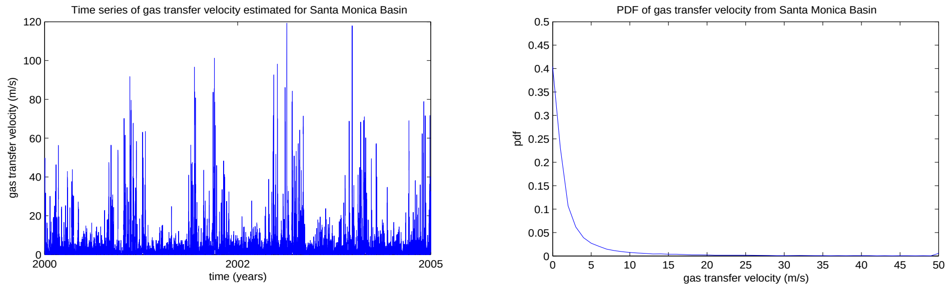

Figure 1: *(left) Time series of gas transfer velocity k estimated from observations collected at a buoy in the Santa Monica Basin. (right) Probability density function of gas transfer velocities.*

We can express k as:

$$k = 0.0283\sqrt{660}\frac{u^3}{\sqrt{Sc}} = 0.7270\frac{u^3}{\sqrt{Sc}}$$

with

$$Sc = 2226.1(1 - 0.066T + 0.002043T^2 - 0.000026T^3)$$

Its uncertainty is:

$$\begin{aligned}
\delta_k &= \sqrt{\left(\frac{\partial k}{\partial u}\right)^2 \delta_u^2 + + \left(\frac{\partial k}{\partial T}\right)^2 \delta_T^2} \\
&= \sqrt{\left(\frac{3 \times 0.7270 u^2 \delta_u}{\sqrt{Sc}}\right)^2 + \left(-\frac{1}{2}\frac{0.7270 u^3 \delta_T}{Sc^{3/2}}\frac{\partial Sc}{\partial T}\right)^2} \\
&= \sqrt{\left(3k\frac{\delta_u}{u}\right)^2 + \left(\frac{k\delta_T}{Sc}\left[1113.0(-0.066 + 2 \times 0.002043T - 3 \times 0.000026T^2)\right]\right)^2} \\
&= \sqrt{\left(3k\frac{\delta_u}{u}\right)^2 + \left(\frac{k(-73.46 + 4.548T - 0.087T^2)\delta_T}{Sc}\right)^2}.
\end{aligned}$$

We can test this out with our data, using the values of Sc and k computed above:

```
delta_T=0.2;
delta_u=0.5;
delta_k=sqrt((3 * k * delta_u ./wind_speed).^2 ...
  +(k .*(-73.46*ones(size(water_temp))+4.548*water_temp...
  -0.087*water_temp.^2)*delta_T ./Sc).^2);

% look at results:
[nanmean(delta_k) nanmedian(delta_k)]
```

The mean uncertainty $\overline{\delta}_k$ is 0.70 m s^{-1}, which is smaller than the standard deviation of the raw data that you computed in question 2. This is not surprising since wind speed and *temperature are highly variable. An inconsistency would occur if the expected error were much larger than the observed standard deviation.*