Problem Set 6: MAE 127

due Friday, May 20, 2005

1. Carbon dioxide levels in the atmosphere have risen steadily since measurements began in the 1950s. Atmospheric CO_2 concentrations seem to be growing exponentially. The annually-averaged Keeling data are available from the UCSD data server or from http://www-pord.ucsd.edu/~sgille/mae127/ps6.html. Fit the data with an exponential curve of the form:

$$C = x_0 + x_1 \exp\left(\frac{t-1900}{x_2}\right).$$

You won't be able to solve this using a simple linear least squares fit, but you can try two different possibilities. For option 1, assume that x_0 is 290, take the natural log of both sides of the equation, and solve it using a least-squares fit. What do you predict atmospheric CO_2 levels to be in 2100?

To solve for the exponential function, we compute

$$\log(C - x_0) = \log(x_1) + \frac{t-1900}{x_2}.$$

Thus we fit using a matrix A containing a column of ones and a column of times (relative to 1900). The algorithm is as follows:

```
load keeling.mat
A=[ones(size(keeling_year)) keeling_year-1900];
x=inv(A'*A)*A'*log(keeling_co2-290);
plot(keeling_year,290+exp(A*x),keeling_year,keeling_co2)
xlabel('time'); ylabel('atmospheric CO_2 (ppmv)')
```

This produces $x(1) = 1.6383$ *and* $x(2) = 0.0277$, *corresponding to the values* $x_1 = \exp(x(1)) = 5.1464$ *and* $x_2 = 1/x(2) = 36.1081$, *implying a 36 year e-folding scale for* CO_2 *growth in the atmosphere.*

If we plug in the year 2100 to these results:

```
290+exp(x(1)+(2100-1900)*x(2))
```

we predict that CO_2 *concentrations in the atmosphere should reach 1599.3 ppmv in 2100, a rather sobering level that is substantially greater than double present concentrations.*

For comparison it's interesting to look at the linear fit:

```
x_lin = inv(A'*A)*A'*(keeling_co2-290);
hold on; plot(keeling_year,290+A*x_lin,'r')
```

This produces a constant of -59.4079 and a slope of 1.3712, implying CO_2 *concentrations of 505 ppmv in 2100, but it's not a very good fit to the data, as indicated in Figure 1.*

2. Now consider the same least-squares fit as a nonlinear fitting problem. Solve for x_0, x_1, and x_2 using "fminsearch". The Matlab function "fminsearch" requires an input cost function

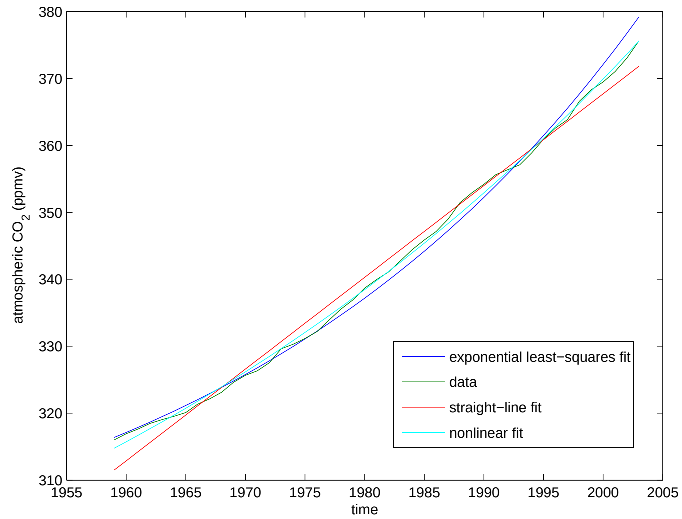

Figure 1: Data and fits for Keeling CO_2 curve.

to be minimized. You may use my example, "expfit.m", or write your own. Based on this fit, what do you predict atmospheric CO_2 levels to be in 2100? Plot your output. Comment on your results. Are the results from question 1 preferable to the results from the nonlinear fit? Assuming that data are accurate to ±0.5 ppmv, what is χ^2 for the two cases?

In this case, we minimize a cost function defined as:

$$\epsilon = \sum_{i=1}^{N} \left(x_0 + x_1 \exp\left(\frac{t_i - 1900}{x_2} \right) - C_i \right)^2$$

by tuning the three variables x_0, x_1, and x_2. To do this we have to start by guessing values for our unknowns. I tried $x_0 = 290, x_1 = 50, x_2 = 40$.

```
x3=fminsearch(@(x) expfit(x,keeling_co2,keeling_year),[290; 50; 40]);
plot(keeling_year,x3(1)+x3(2)*exp((keeling_year-1900)/x3(3)),'c')
```

This produces $x_0 = 256.45, x_1 = 22.37, x_2 = 61.58$.

In the plot, this provides good agreement with the data. To quantify the fit, we can use χ^2, which is here equal to the output of "expfit" divided by the uncertainty squared:

```
chi2=expfit(x3,keeling_co2,keeling_year)/0.5^2.
```

Thus $\chi^2 = 93.4$ with $N - M = 45 - 3 = 42$.

For comparison the exponential least-squares fit produced a χ^2 value of

```
chi2=sum((exp(A*x)-keeling_co2+290).^2)/0.5^2
```

which is 391.6. Thus the nonlinear fit appears to produce better results.

The nonlinear fit suggests that CO_2 concentrations in 2100 should be

```
x3(1)+x3(2)*exp((2100-1900)/x3(3))
```

or about 832 ppmv.

3. In problem set 4, you least-squares fit an annual cycle to one year of temperature data. Using the same data that you used in problem set 4, now find the Fourier transform of the data. Show that the Fourier transform produces results that are consistent with results from your least-squares fit.

In problem set 4 we found, We least squares fitted a constant and a low-frequency cycle corresponding to one cycle per year. Here we compute the fft for the same data:

```
X =fft(alpine_min);
```

The constant is at zero frequency, which is the first value of X, $X(1) = 17973$. *To find the amplitude we divide by* $N = 366$ *to find a mean of* $X(1)/366 = 49.1066$, *exactly as we found in problem set 4.*

The cosine and sine amplitudes will correspond to the real and imaginary parts of $X(2)$. *Here* $X(2) = -1548.7+639.91i$. *We divide by* $N/2$ *to get* $2X(2)/N = X(2)/183 = -8.4626+3.4968i$. *Thus the cosine amplitude is* -8.462 *and the sine amplitude is* 3.4968. *These values are similar to, but not identical to the problem set 4 solutions. The reason they are not identical is that in problem set 4, the first data point was on day 1, while the Fourier transform assumes that the first data point is at time 0. We can actually duplicate the shift by multiplying by* $\exp(-2*\pi*i/366)$. *Then* $2X(2)/N\exp(-2*\pi*i/366) = -8.4013+3.6415i$ *which exactly matches the problem set 4 solution.*

Another way to assure yourself that the results are correct is to look at the amplitude. The total amplitude of the annual cycle is $\sqrt{\Re X(2)^2 + \Im X(2)^2} = |X(2)|^2 = 9.16$, *as we found in problem set 4.*