Problem Set 8: MAE 127

due Friday, June 3, 2005

The problem set 8 page on the course web site and the UCSD ieng9 data server both contain a file called ncep_sample.mat containing East-West wind (in m s^{-1}), longitude (in degrees), and time (in hours, starting from January 1, 0001).

1. Compute a two-dimensional spectrum for the data. Remove the time mean from each data point before you start, so that your signal isn't dominated by the mean. Split the data into 10 equally long time segments, so that error bars can be computed. What frequency/wavenumber combination is most energetic? What nonzero freqeuncy/wavenumber combination is most energetic? How much of the variability in the data do these frequency/wavenumber combinations represent?

For bonus points, estimate the error bars.

Here's the basic strategy:

```
% compute spectrum
Um=U-ones(1460,1)*mean(U);
seg=146;
for i=1:10
 f1(:,:,i)=fft2(Um((i-1)*seg+1:i*seg,:));
end
s1=sum(abs(f1).^2,3)/(146*192);

% reshape and plot
s2=[s1(1:74,97:192) s1(1:74,1:96)];
s2(2:end-1,:)=2*s2(2:end-1,:);
imagesc(-96:95,0:72,log10(s2)); axis xy; colorbar
xlabel('wavenumber'); ylabel('frequency')

% find the maximum
[y,i]=max(s2(:));
[l,m]=ind2sub(size(s2),i)  % this occurs at l=1, m=97, corresponding
                           % to frequency/wavenumber 0/0.
y/sum(s2(:))  % = 0.04

% find the maximum not at zero
s1test=s1(2:end,2:end);
[y,i]=max(s1test(:));
2*y/sum(s1test(:)) % gives the fraction of variance explained by this peak.
                   % = 0.0295, about 3% of the variance
[l,m]=ind2sub(size(s1test),i) % locates maximum variance at l=145, m=2
                           % this corresponds to frequency -1, wavenumber 2
                           % or alternatively, wavenumber -2, frequency 1
```

Figure 1 shows the resulting two dimensional spectrum.

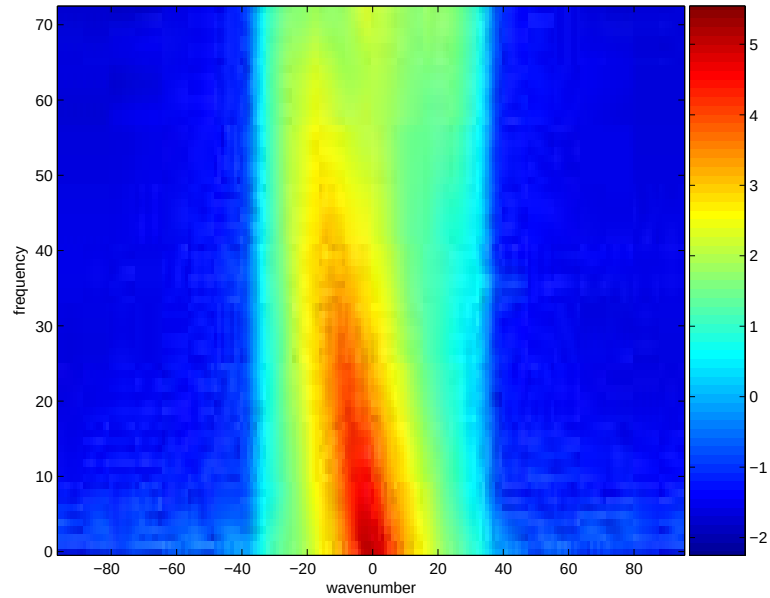

Figure 1: Two-dimensional frequency-wavenumber spectrum of zonal wind observations from the Southern Ocean. Colorscale corresponds to $\log_{10}$ of spectrum.

To estimate error bars, we use the same method that we used for one-dimensional spectra. Since the data have been broken into 10 segments, we compute:

```
nu=2*10;
err_low = nu/chi2inv(1-.05/2,nu);  % 0.5853
err_high = nu/chi2inv(.05/2,nu);  % 2.0853
```

This indicates that on a log scale, if the $\log_{10}$ of the spectrum for a particular frequency–wavenumber combination is 1, then the uncertainty of this estimate ranges from 0.59 to 2.09. This tells us that peaks that exceed the background spectrum by about 1 unit in log space are statistically significant.

2. Compute EOFs for the same demeaned data. What fraction of the variance in the data is explained by the 1st, 2nd, and 3rd mode EOFs?

EOFs are a quick one-line computation.

```
[u,s,v]=svd(Um);
diag(s(1:3,1:3).^2)/sum(diag(s.^2))
plot(diag(s.^2)/sum(diag(s.^2)))
```

The fraction of variance is plotted in Figure 2 and the corresponding numerical values indicate that the first 3 EOFs explain 12.7%, 11.0%, and 9.2% of the variance respectively.

To plot the first mode EOFs in time and space:

```
subplot(1,2,1);
plot(time-time(1),u(:,1)); xlabel('time (hours from start of year)');
```

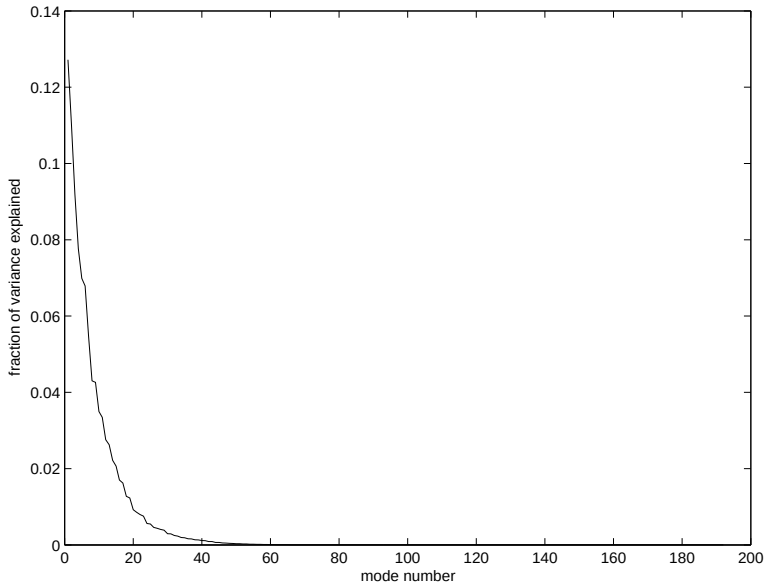

Figure 2: Fraction of variance explained by EOFs of Southern Ocean wind data.

```
ylabel('mode 1 structure');

subplot(1,2,2);
plot(lon,v(:,1)); xlabel('longitude'); ylabel('mode 1 structure');
```

The resulting structures in Figure 3 indicate that the leading order response is stronger in the Pacific Ocean than the Atlantic or Indian Oceans, and that the pattern varies rapidly.

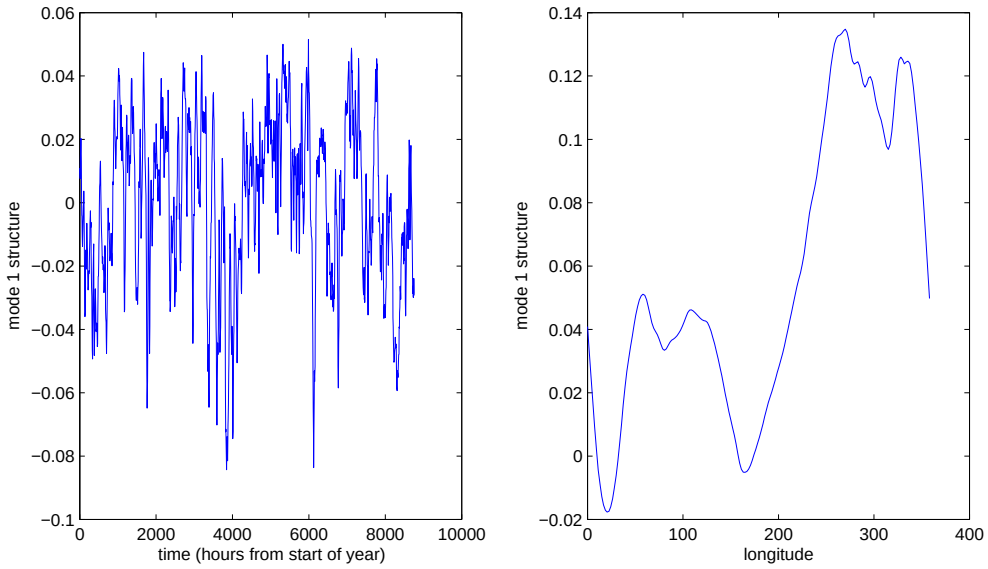

Figure 3: Temporal (left) and spatial (right) structure of mode 1 EOF.