

How nonlinearities in the equation of state of seawater can confound estimates of steric sea level change

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[1] The process of mixing water masses that have differing temperatures, salinities, and pressures creates water that is typically denser than the average density of the source waters. Thus mixing the ocean, either along isopycnals or vertically, tends to decrease global sea level, even when no heat or salt is added to the ocean. This effect is small in the global ocean, but regional effects can be substantial. Two recommendations emerge from this study. First, numerical predictions of steric sea level rise should be designed to minimize sensitivity to erroneous isopycnal or vertical/diapycnal mixing rates that might over or under mix water masses. Second, retrospective analyses of steric sea level rise in historic data should avoid using gridded temperature and salinity fields to compute steric sea level change, since mapping smooths data in the same way that mixing does. **INDEX TERMS:** 4556 Oceanography: Physical: Sea level variations; 4271 Oceanography: General: Physical and chemical properties of seawater; 4203 Oceanography: General: Analytical modeling; **KEYWORDS:** sea level, equation of state

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1. Introduction

[2] Cabbeling results from the nonlinearities in the equation of state of seawater: at constant pressure, if you mix two volumes of seawater of different temperatures but equal density, the resulting water will turn out to be denser than the original water masses [e.g., Witte, 1902; Foster, 1972]. In addition, if pressure and temperature both differ, then because of thermobaricity, the mixture of two volumes of water can be either denser or lighter than the source water [e.g., McDougall, 1987]. Cabbeling and thermobaricity are often neglected in climate studies, because they are assumed to play only a minor role in large-scale ocean processes. However, situations exist in which cabbeling and thermobaricity matter. This paper examines the impact of nonlinearities in the equation of state on global sea level estimates.

[3] Over the past 100 years, global sea level has risen at an estimated rate of 1.0 to 2.4 mm/y [Church *et al.*, 2001; Douglas and Peltier, 2002]. This rise is attributed to a combination of factors including steric warming of the upper ocean, surface groundwater changes, and meltwater from glaciers, the Greenland ice sheet, and the Antarctic ice sheet [Church *et al.*, 2001]. Although the 20th century sea level rise budget is not fully understood [Douglas and Peltier, 2002; Munk, 2002; Meier and Wahr, 2002], steric sea level rise is thought to be a major contributor to the observed rise. The 2001 Intergovernmental Panel on Cli-

mate Change report attributed about a third of sea level rise to steric warming of the upper ocean [Church *et al.*, 2001], while [Cabanes *et al.*, 2001] concluded that sea level rise during the second half of the 20th century could be entirely explained as a steric warming effect estimated at $0.50 \pm 0.05 \text{ mm yr}^{-1}$. The analysis in this paper will look at a possible bias in steric sea level estimates that may provide a modest contribution to the uncertainties in global sea level rise.

[4] Steric sea level rise results predominantly from heating the upper ocean. Hydrographic data collected since 1950 indicate that the upper 3000 m of the ocean has warmed, with the upper 1000 m heating by about 0.1°C [Levitus *et al.*, 2000]. (Salt has been shown to be a smaller contributor to steric sea level change [Antonov *et al.*, 2002].) Studies of steric sea level rise have considered two mechanisms by which heat can enter the ocean and decrease ocean density. Vertical diffusion carries heat straight downward through the water column [Wigley and Raper, 1987; Sokolov and Stone, 1998], while subduction models allow heat to penetrate into the ocean along isopycnal surfaces [Church *et al.*, 1991; Jackett *et al.*, 2000].

[5] The present study shows how sea level can change even if the total heat content of the ocean remains constant. Owing to the nonlinearities in the equation of state, mixing processes in the ocean increase density and therefore decrease sea level. Section 2 considers the impact of isopycnal mixing, while section 3 examines how vertical mixing alters sea level. Mixing phenomena have implications both for predicting future sea level change and for evaluating the sea level rise during the

past century, and these implications are explored in section 4. Results are summarized in section 5.

2. Isopycnal Mixing

[6] Mixing in the ocean is normally imagined to occur predominantly along constant density surfaces, often formally described as neutral density surfaces. What if along-isopycnal mixing were so rapid that temperature and salinity became completely homogenized on neutral density surfaces?

[7] For this analysis, gridded data from the upper 3000 meters of the World Ocean Atlas 2001 [Stephens *et al.*, 2002; Boyer *et al.*, 2002, henceforth WOA01] were used to evaluate how spatial variations of temperature and salinity control global mean sea level. The atlas data merge more than 50 years of shipboard observations, which have been vertically interpolated onto 28 unevenly spaced standard levels between 0 and 3000 m depth and horizontally averaged onto a 1° by 1° grid.

[8] Neutral density was computed for every location in the atlas data, using the database developed by [Jackett and McDougall, 1997]. The neutral density database covers the latitude range between 80°S and 64°N , but excludes the Arctic Ocean. For most of the world's ocean, neutral density ranges between 20 and 30 neutral density units, although values as small as 10 occur in isolated locations where salinity is particularly low.

[9] On the basis of neutral density, seawater from the entire latitude range included in the neutral density database was sorted into bins, spaced at increments of 0.1 neutral density units. In each of these bins, mean salinity and mean potential temperature were computed. Potential temperature was used rather than in situ temperature in this calculation, because under conditions where pressure can change, potential temperature is closer to being a conservative variable than is in situ temperature. The averaging procedure also required scaling each gridded potential temperature and salinity value appropriately to account for variations in horizontal area with latitude and vertical thickness with depth. Finally at each location, potential temperature was converted back into temperature so that density and specific volume anomaly $\delta = 1/\rho(S, T, p) - 1/\rho(35, 0, p)$ could be computed using the UNESCO equation of state [Millero *et al.*, 1980; Millero and Poisson, 1981]. Dynamic height is then $D = \int_{p_{bot}}^0 \delta(S, T, p) dp/g$, where g is gravity, S is salinity, T is temperature, and p_{bot} is the pressure at the reference level, where dynamic topography is assumed to be constant [e.g., Pond and Pickard, 1983]. The difference between the dynamic height of mixed and unmixed water indicates the impact of mixing on sea level. Since data are archived at standard depths, rather than pressure surfaces, following [Levitus, 1982] dp is approximated as $\rho_o g dz$ in dynamic height calculations, with $\rho_o = 1.03 \text{ kg m}^{-3}$. Errors resulting from this approximation are estimated to be less than 1%.

[10] Results show that the process of mixing temperature and salinity along isopycnal surfaces decreases global mean sea level by 29 mm. This exceeds the estimated 20 mm sea level rise due to steric warming that is believed to have occurred between 1955 and 1996 [Cabanes *et al.*, 2001]. Note that the 29 mm decrease reported here is entirely due to the equation of state; the process of mixing waters with

different temperatures and salinities increases the average in situ density of the water. There is no physically conceivable way to mix in situ density along constant neutral density surfaces so that global sea level remains unchanged. Temperature, salinity, and pressure changes all contribute to the observed sea level change, but the largest contributions come as a result of homogenizing potential temperature. If salinity were unchanged, while temperature was averaged along neutral density surfaces, global sea level would still drop by 28 mm. In contrast if temperature were fixed, while salinity was averaged along neutral density surfaces, sea level would drop by just 2.7 mm. Changes in pressure appear to play only a minor role; if temperature and salinity were homogenized along current neutral density surfaces, but the entire ocean was assumed to have a pressure of 1000 mbars, regardless of depth, then sea level would drop 28 mm.

[11] Figure 1a shows the spatial distribution of sea level change resulting from homogenizing salinity and potential temperature along neutral density surfaces. Sea level decreases slightly over most of the world's oceans, with the largest changes in regions of strong geostrophic currents, where density surfaces tend to outcrop to the surface. In these places, isopycnal homogenization substantially increases upper ocean density and therefore decreases sea level. This process of along-isopycnal mixing illustrates how nonlinearities in the equation of state of seawater influence global ocean processes.

3. Vertical Mixing

[12] Ocean climatology is influenced not only by along-isopycnal mixing, but also by diapycnal mixing, which is nearly approximated as vertical mixing. Vertical eddy diffusivities are estimated to vary between 10^{-5} and $10^{-3} \text{ m}^2 \text{ s}^{-1}$ (0.1 to $10 \text{ cm}^2 \text{ s}^{-1}$) [Munk and Wunsch, 1998]. Regional variations in diapycnal mixing are thought to depend on a number of factors, including regional variations in wind-forcing and the conversion of tidal energy into internal wave energy, which is more rapid over rough bottom topography than smooth bottom topography [Polzin *et al.*, 1997; Ledwell *et al.*, 2000], but no precise map of vertical dissipation exists at present. For this study, an evaluation of the importance of knowing vertical diffusivities was carried out by turning off horizontal advection and allowing the entire ocean to diffuse vertically, with a constant diffusivity.

[13] This calculation again used WOA01 gridded atlas data. Potential temperature and salinity were allowed to diffuse vertically through the upper 3000 m of the water column, but all horizontal advection and heat input were shut off. The time evolution of potential temperature was described by:

$$\frac{\partial \theta}{\partial t} = \kappa \frac{\partial^2 \theta}{\partial z^2}, \quad (1)$$

and salinity was diffused in the same way. This equation was solved subject to the boundary conditions

$$\left. \frac{\partial \theta}{\partial z} \right|_{z=0} = \left. \frac{\partial \theta}{\partial z} \right|_{z=3000\text{m}} = 0, \quad (2)$$

meaning that no heat or salt could enter or leave the vertical water column. Data were considered only from locations

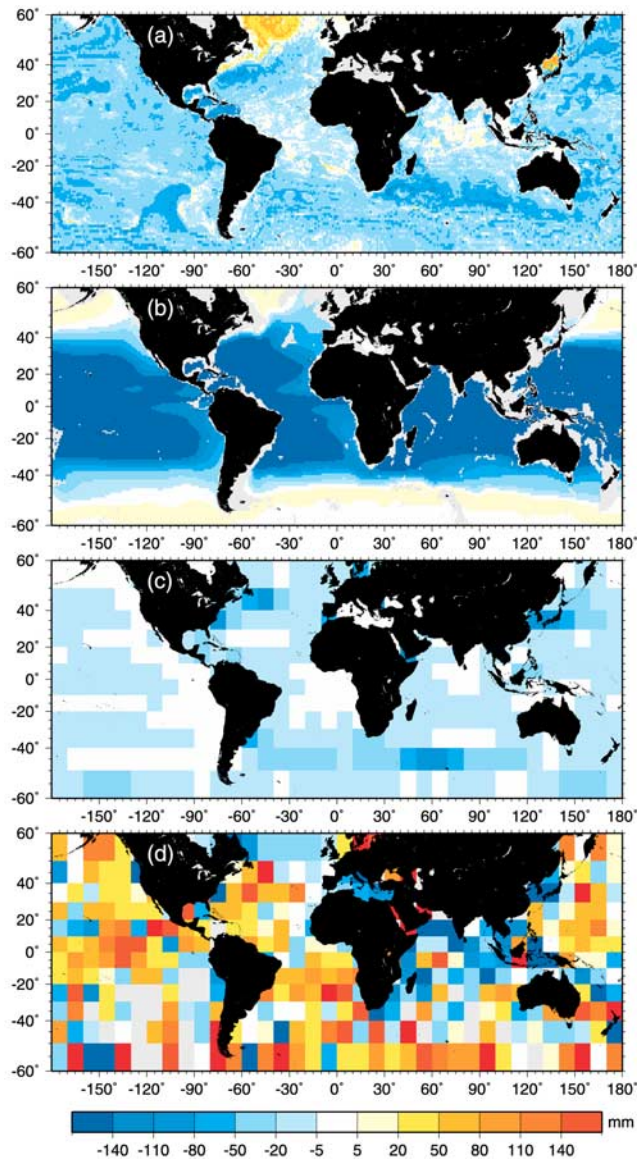


Figure 1. Sea level change associated with (a) homogenizing potential temperature and salinity along neutral density surfaces, (b) vertically diffusing the ocean for 10 years with a vertical diffusivity of $\kappa = 10^{-3} \text{ m}^2 \text{ s}^{-1}$, (c) difference between biased and unbiased estimates of sea surface height, $\bar{D} - \bar{D}$, relative to 3000 m depth, averaged in 10° by 10° bins, and (d) steric sea level change between the 1950s and the 1990s, computed from 10° latitude by 10° longitude averages of specific volume anomaly. Gray areas indicate regions with no data at any depth in either the 1950s or the 1990s. All panels use the same color scale.

where the ocean was at least 3000 m deep. Equation (1) can be expressed using matrix notation: $\partial\theta/\partial t = \kappa A\theta$, where A is a matrix representing the vertical diffusion operator. The solution to (1) is then of the form $\theta(z, t) = \exp(\kappa A t) \theta_o(z, t = 0)$, and an implicit solution to the equation is obtained using a matrix exponential package. As in section 2, diffused θ fields were converted to temperature, density was computed from the mixed and unmixed temperatures and salinities, and dynamic height differences were calculated.

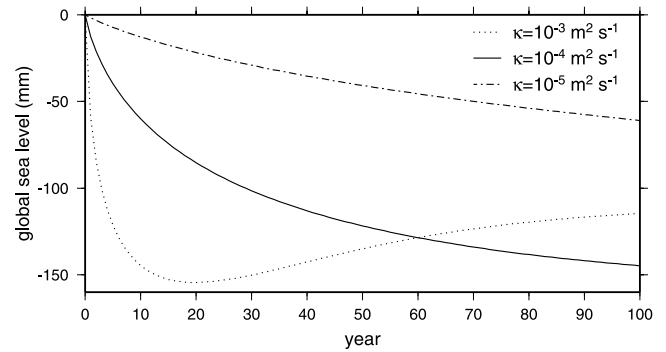


Figure 2. Globally-averaged sea level drop resulting from vertically diffusing the ocean with diffusion coefficients ranging from $\kappa = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (dotted line) to $\kappa = 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (dash-dotted line). The curves are self-similar: 10 years of diffusion with $\kappa = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ are equivalent to 100 years of diffusion with $\kappa = 10^{-4} \text{ m}^2 \text{ s}^{-1}$. For long time periods, sea level drop asymptotes to 109 mm.

[14] All values of κ decrease global sea level over time. Figure 2 and Table 1 summarize the results for a range of κ 's. When κ is large, sea level can decrease by 10 to 15 cm in just a few decades, because the vertical diffusion of temperature and salinity mixes surface properties rapidly through the upper ocean.

[15] Ultimately, after 200 or more years, when water is uniformly mixed throughout the upper 3000 m of the ocean, global sea level is lower by 109 mm. In the extreme case in which the entire ocean is fully homogenized as a result of complete horizontal and vertical mixing, sea level is reduced by 165 mm relative to the climatology.

[16] Figure 1b shows the spatial distribution of sea level after 10 years of vertical diffusion with a diffusivity $\kappa = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (or equivalently 100 years of diffusion with $\kappa = 10^{-4} \text{ m}^2 \text{ s}^{-1}$). Diffusion decreases sea level nearly everywhere, but changes are largest in the tropics, where stratification is initially strongest, so that vertical diffusion mixes widely disparate water masses. Sea level can increase slightly at high latitudes, because for colder temperatures, the process of warming deep water increases sea level more than cooling surface waters lowers sea level.

4. Discussion: Implications for Modeling and Data Analysis

[17] Skeptical readers will dismiss the cases discussed in sections 2 and 3 as toy problems with little applicability to the real ocean. Indeed the real ocean is not observed to mix

Table 1. Change in Sea Level That Would Result From Vertically Diffusing the Global Ocean for 10 or 20 Years (While Shutting Off Horizontal Advection and Diffusion Processes)

Diffusivity ($\text{m}^2 \text{ s}^{-1}$)	Sea Level Change (mm)	
	10 Years	20 Years
10^{-3}	144	150
10^{-4}	59	84
10^{-5}	12	21

fully along isopycnals or to experience rapid top-to-bottom diapycnal mixing on timescales of decades. However the results from these tests have implications both for numerical modelers and for researchers attempting to interpret sparse historical data sets.

4.1. Numerical Models and Steric Sea Level Change

[18] Forecasts of long-term sea level rise depend on numerical climate models. One challenge in developing these models is to establish realistic values of horizontal and vertical diffusion coefficients. A Fickian diffusion operator is typically adopted to parameterize a broad range of unresolved subgrid-scale eddy motions that are not well-quantified. In order to maintain computational stability, numerical climate models often use diffusivities that exceed estimated realistic values. As a result, temperature and salinity gradients in climate models can diffuse away over time. The results of this study suggest that as a result of the nonlinearities in the equation of state, this long-term diffusion process will also slowly form denser water and decrease global sea level. In more realistic scenarios, in which air-sea heat exchange occurs, oceanic diffusion also strongly influences how much heat penetrates into the ocean. High diffusion results in rapid transfer of heat between the surface and subsurface and comparatively lower sea surface temperatures, which in turn allow more rapid heat exchange between the atmosphere and ocean. For example, [Sokolov and Stone, 1998] and [Sokolov *et al.*, 1998] have shown that sea level rise in a simplified climate model is sensitive to vertical diffusion rates. [Jackett *et al.*, 2000] explored how differing parameterizations for isopycnal mixing influenced sea level rise in coupled climate simulations. The results shown here suggest that both vertical diffusion and along-isopycnal diffusion can influence sea level estimates even in the extreme case in which no heat is added to the global ocean. For this reason, non-Boussinesq numerical projections of global sea level rise should be designed so that results are not strongly sensitive to inaccurate mixing rates. This could be achieved by choosing realistic isopycnal and vertical/diapycnal mixing rates, or as in many recent sea level rise studies, by comparing anthropogenic warming scenarios with constant-climate results derived from the same model.

4.2. Historical Hydrographic Data and Steric Sea Level

[19] Nonlinearities in the equation of state should also be taken into consideration in analyses of sea level rise from hydrographic data. Global estimates of steric sea level are often derived from objectively mapped temperature and salinity data. The process of mapping temperature and salinity on depth surfaces is analogous to mixing water on constant depth surfaces. Thus the mapping process tends to produce sea level estimates that are biased low compared with the true sea level. In regions where measurements are sparse, mapped quantities may represent an average of only one or two values, and sea level will be comparatively unlikely to be biased. In contrast, if many independent observations are available, mapped temperatures and salinities will each represent averages of all measurements and the resulting sea level will tend to be biased low relative to the true sea level.

[20] Hydrographic data at standard depths from the World Ocean Database 2001 [Conkright *et al.*, 2002, henceforth WOD01] were used to evaluate the impact of this effect. For each hydrographic station, specific volume anomaly δ was computed at each depth for which valid temperature and salinity measurements were available (Data were judged valid if the error flag supplied in WOD01 was less than 6, meaning that the observed temperature or salinity value differed strongly from no more than one of three quantities: the annual mean, the seasonal mean, and the monthly mean.). Then T , S , and δ were bin averaged at constant depth in 2.5° by 2.5° squares, 5° by 5° squares, and 10° by 10° squares to obtain \bar{T} , \bar{S} , and $\bar{\delta}$. Mean temperature and salinity were used to compute $\delta(\bar{T}, \bar{S})$, the specific volume anomaly that would be obtained from mapped temperature and salinity. Finally dynamic topography was computed from the mean specific volume anomaly, $\bar{D}(p_b) = \int_{p_b}^0 \delta dp/g$, and from the specific volume anomaly of the mean temperature and salinity $\tilde{D}(p_b) = \int_{p_b}^0 \delta(\bar{T}, \bar{S}) dp/g$. $\bar{D}(p_b)$ represents the unbiased sea level relative to a reference pressure p_b , and $\tilde{D}(p_b)$ will be biased low as a result of mapping T and S before computing δ .

[21] The mapping procedure used here is simple compared with the iterative technique employed for the World Ocean Atlas 2001, in which the radius of influence from which observations are taken decreases with each successive iteration from 888 km, to 666 km, and finally to 444 km [Stephens *et al.*, 2002]. Under the WOA technique, the effective averaging radius varies depending on the number of observations available at a particular location. Results reported here are therefore not directly comparable to atlas results.

[22] Figure 1c maps the dynamic height difference $\bar{D} - \tilde{D}$ relative to 3000 m in 10° by 10° bins. The mean difference is 4.6 mm when 2.5° bins are used, 6.3 mm when 5° bins are used, and 11.1 mm when 10° bins are used. More refined mapping algorithms are expected to yield similar results: when the bin-averaging approach is replaced with an objective mapping algorithm [Bretherton *et al.*, 1976] using a 5° Gaussian decorrelation scale, the bias $\bar{D} - \tilde{D}$ is 5.9 mm, similar to the bias obtained for 5° bins. Separate calculations indicate that the bias would be 1 to 2 mm greater if observed measurements were used rather than standard level data [L. Miller, personal communication, 2003]. Overall, these results indicate that the bias in sea surface height estimates increases when more data (and particularly more disparate data) are available to be averaged. Bias is particularly large in regions of large mesoscale variability, such as the Gulf Stream, where mean temperature and salinity may differ substantially from individual realizations.

[23] What impact does this bias have on estimates of decadal scale sea level rise? Prior to 1970, data were collected at pre-determined sample depths using reversing thermometers and Nansen bottles. Electronic instruments were introduced in the early 1970s to measure conductivity, temperature and pressure continuously through the water column. Because of the time required to gather bottle data, pre-1970 observations were often more sparsely spaced than contemporary measurements. (Numbers of observations collected have also fluctuated with shifts in scientific interests and funding priorities.) Mapped temperatures and

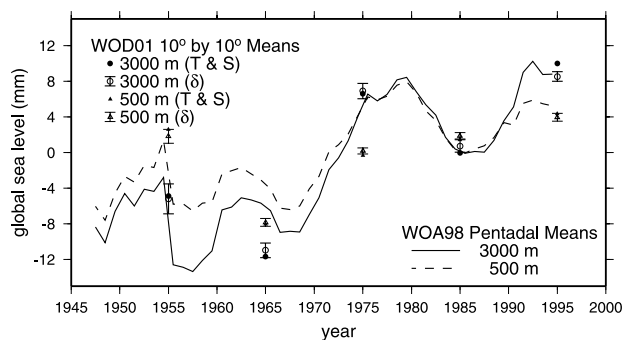


Figure 3. Globally-averaged thermosteric sea level rise, computed from WOA98 pentadal temperature fields, for the upper 500 m and 3000 m. Open circles and open triangles with error bars indicate decadal averages of unbiased dynamic height \bar{D} relative to 3000 m and 500 m, while closed circles and triangles indicate decadal averages of biased dynamic height \tilde{D} . At each depth, data from each 10° by 10° bin were used only if observations were available for each decade from the 1950s through the 1990s. Because comparatively fewer data are available in the 1990s than in the 1960s through 1980s, the biased estimate appears to overestimate total sea level rise.

salinities from the 1950s and 1960s often represent an average of fewer values than mapped temperature and salinity fields from the 1980s. As a result, steric sea level in the 1980s, for example, is expected to be biased low relative to steric sea level in the 1950s.

[24] Figure 3 plots decadal averages of steric sea level rise from the 1950s through the 1990s. The 40-year trends of unbiased sea level \bar{D} (open circles and triangles) differ by 1 to 2 mm from biased sea level \tilde{D} (solid circles and triangles). These differences are small compared with the statistical uncertainties in the observed trend. Because comparatively fewer data are available in the 1990s than in earlier decades, the biased estimate \tilde{D} underestimates total sea level rise. For reference, Figure 3 also shows thermosteric sea level rise computed from 5-year running mean temperature fields available as part of the World Ocean Atlas 1998 [Levitus *et al.*, 1998, henceforth WOA98]. These data indicate a total sea surface elevation rise of 17 mm relative to 3000 m depth between 1947 and 1994, attributable to thermosteric effects. Decadal averages from this analysis are in general agreement with the WOA98 sea level rise estimates. Differences between this analysis and the WOA98 estimates may result from modifications to the hydrographic data base between 1998 and 2001, and they may also reflect the uncertainties inherent in gridding sparse, irregularly spaced data.

[25] Most of the thermosteric sea level rise is concentrated in the upper ocean; 86% is in the upper 1000 m and 66% within the upper 500 m. Thus sea level rise estimates are particularly sensitive to sampling and mapping errors in the upper ocean.

[26] Figure 1d shows the spatial distribution of the sea level change from the 1950s to the 1990s. To compute this change, specific volume anomalies were averaged for each decade in 10° longitude by 10° latitude bins at each standard depth in WOD01. Then the differences between the 1950s

and the 1990s were computed and integrated to determine the change in sea level. For this calculation, temperature and salinity were assumed not to change at depths for which data were unavailable in either the 1950s or the 1990s. Geographic boxes that contained no data at any depth for either the 1950s or the 1990s are indicated in gray. The regions of most rapid sea level rise are in the Southern Ocean; uncertainties are also larger in the Southern Ocean because of the paucity of observations.

[27] The solid line in Figure 4 shows the difference in global sea level trends between the 1950s and the 1980s, $\Delta\bar{D} - \Delta\tilde{D}$, plotted as a function of reference depth. Here $\Delta D = D(1980s) - D(1950s)$. This provides a measure of how steric sea level trends estimated from gridded temperature and salinity data can underestimate total steric sea level rise. Throughout most of the water column, sea level rise estimates differ by 0.5 mm to 1 mm over the 30 year time interval.

[28] Circles in the left panel of Figure 4 indicate $(\Delta\bar{D} - \Delta\tilde{D})/\Delta\tilde{D}$. This measures the fraction by which mapped temperature and salinity underestimate the trend relative to the unbiased estimate obtained from the specific volume anomaly. In this case, for reference levels greater than about 1000 m, gridded data are likely to underestimate sea level rise by about 5%. Fractional errors are larger when sea level rise is referenced to a shallow depth, because total steric sea level rise is small between the 1950s and 1980s in the upper ocean.

[29] The right panel of Figure 4 shows the number of data points available at each depth. Throughout the water column, substantially more data were collected during the

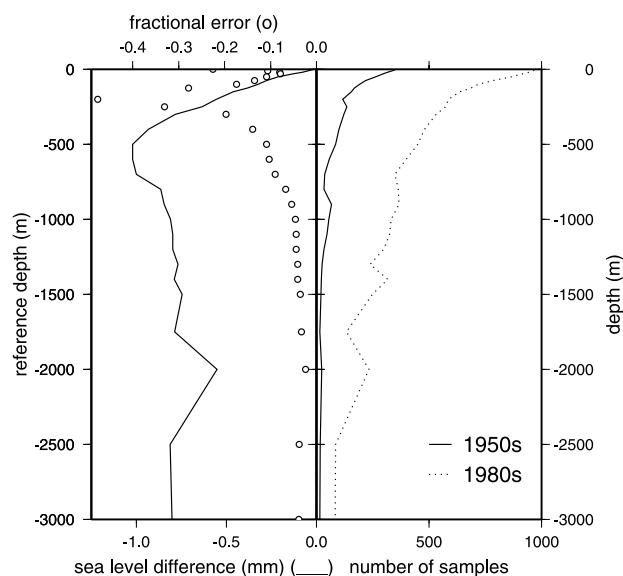


Figure 4. (left) Fractional error of sea level rise from the 1950s to the 1980s (circles), $(\Delta\bar{D} - \Delta\tilde{D})/\Delta\tilde{D}$. (solid line) Difference between biased and unbiased sea level $\Delta\bar{D} - \Delta\tilde{D}$. Values are plotted as a function of reference depth, z_{bot} . At each depth, data from each 10° by 10° bin were used only if observations existed from both the 1950s and from the 1980s. (right) Average number of data points available at each depth in the 1950s and 1980s. Differences in spatial coverage are not reflected in this plot.

1980s than during the 1950s, which accounts for the observed biases. The details of the results differ depending on the decades chosen for comparison. Although the global sea level rose from the 1980s through the 1990s, fewer observations are available from the 1990s, and the bias in sea level rise estimates is therefore smaller or of opposite sign for the 1990s.

[30] The numbers reported here should be viewed as only loose guidelines for the potential bias in steric sea level rise estimates determined from gridded temperature and salinity fields. Since sampling patterns vary substantially as a function of location, depth, and time, the actual impact of this effect will strongly depend on the regions and time intervals under consideration. Regional errors may be larger than the global biases reported here and could lead to erroneous estimates of dynamic topography (which is used for a variety of purposes including geoid estimation) and biased geostrophic transport calculations.

5. Summary and Conclusions

[31] Two idealized situations have been considered. The first showed that mixing along isopycnal surfaces will lower global sea level as a result of cabbeling. The second illustrated that vertical mixing of the global ocean also lowers global sea level, particularly in low-latitude regions where stratification between the surface and deep ocean is significant. These results have implications for studies of global sea level change.

[32] First, forecast models of long-term global sea level change should be designed so that results are not biased by inaccurate isopycnal and vertical diffusion parameters. High diffusivities that are sometimes included for numerical stability may over-mix the ocean and could lead to underestimates of long-term sea level change (at least if climate-change model scenarios were not bench-marked against constant-climate scenarios using the same model). In extreme cases, excess isopycnal mixing rates will result in a maximum underestimate of global sea level of 28 mm for the upper 3000 m of the ocean. Excess vertical diffusivities can depress global sea level by as much as 150 mm relative to 3000 m depth.

[33] Second, if temperature and salinity are smoothed, in either the horizontal or vertical direction, prior to computing sea level, then sea level estimates will be biased low. The effect of gridding sparse data, is likely to be no more than a few millimeters per century, but these differences can magnify errors in long-term global sea level estimates. Retrospective analyses of historic data should avoid mapping temperature and salinity prior to computing sea surface height, since smoothing is equivalent to mixing and will tend to decrease global sea level. Instead, sea level should be estimated from each independent temperature-salinity profile before any mapping procedure is carried out.

[34] Other studies have also pointed to the importance of choosing appropriate methods to map hydrographic data. [Lozier *et al.*, 1994] noted that mapping or averaging temperature and salinity horizontally created water with anomalous properties. The surprise in this study is that even if temperature and salinity are mapped (or mixed) on isopycnal surfaces, the resulting water can appear anomalously dense.

[35] In the examples explored in this paper, nonlinearities in the equation of state are unlikely to explain more than 1 mm of total sea level rise over the 30-year period from the 1950s to the 1980s or about 5% of the total sea level rise. This represents much less than 1 cm per century and appears likely to resolve only a small fraction of the uncertainties in the global sea level rise budget, although the effect may be much larger in regions or depth ranges where the sampling patterns have changed dramatically over time. Since the bias associated with nonlinearities in the equation of state is small compared with total uncertainties in the sea level rise budget, the calculations here will not provide much assistance in closing the global sea level rise budget. Other mechanisms, such as freshwater input [e.g., Munk, 2003], will need to be invoked to explain fully the observed sea level changes discussed in the introduction.

[36] **Acknowledgments.** C. K. Shum initially suggested examining the impact of mixing on global sea level, and L. Miller shared his own results and insights through the course of this work. S. G. Llewellyn Smith helped find an efficient solution to the vertical diffusion problem. Comments from J. M. Toole, T. J. McDougall and an anonymous reviewer improved the presentation of this work. This research was supported by the National Science Foundation under grant OCE-9985203/OCE-0049066 and by the National Aeronautics and Space Administration under JPL contract 1224031.

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