Problem Set 2: SIO 221B, Data Analysis

due Friday, October 18, 2002

1. A random variable x has probability density $F_x = \frac{1}{2}(\delta(x-\alpha)+\delta(x+\alpha))$. A new random variable is defined as $y = \frac{1}{3}(x_1 + x_2 + x_3)$ where the x_i may be assumed to be independent realizations of x. What is the probability density for y? What are its first three moments?

2. Using the same data set that you used for problem set 1 ('rcm00001.str' from the course website), map the joint probability density function for u and v velocities and the covariance of u and v. Are u and v uncorrelated? How would you rotate the pdf to make u and v be uncorrelated? Is there any relationship between the dominant variability and the direction of mean flow?

3. (Problem 6 from the notes on fundamentals of statistics). The sum of two normally distributed variables is also normally distributed. Thus the low moments of the sum should be related as in equation (30) of the notes. Let $Z = X + Y$ where X and Y are independent normally distributed variables with $\langle X\rangle = \langle Y\rangle = 0$. What is the third moment of Z in terms of the statistics of X and Y? Show that the fourth moment of Z obeys equation (30).

4. Standard random number generating packages on most computer systems will produce random numbers with a uniform distribution between 0 and 1. That is:

$$F_1(x)dx = \begin{cases} dx & \text{for } 0 < x < 1 \\ 0. & \text{otherwise} \end{cases} \tag{1}$$

(In Matlab and Fortran, uniform distributions are generated by the "rand" function.)

In contrast geophysical noise rarely has a uniform distribution. In order to carry out a Monte Carlo simulation, suppose that you require noise with the symmetrical exponential distribution that you considered in Problem set 1:

$$F_2(x)dx = \frac{1}{\sigma\sqrt{2}} \exp\left[\frac{-|x|\sqrt{2}}{\sigma}\right] \tag{2}$$

with $\sigma = 1$.

a. Verify that the pdf $F_2(x)$ is normalized correctly.

b. Derive the appropriate algorithm to generate noise that is distributed as specified by $F_2(x)$.

c. Use your algorithm to convert uniform random numbers from $F_1(x)$ so that they have the distribution $F_2(x)$. Plot the pdf of the original uniform distribution and your record with the new distribution.