Problem Set 7: SIO 221B, Data Analysis

due Friday, December 6, 2002

1. The covariance of the variable x is of the form:

$$\langle x(t)x(t+\delta t)\rangle = \begin{cases} (1-|t|)\langle x(t)x(t)\rangle & \text{for } |t| \le 1 \text{ day} \\ 0 & \text{otherwise} \end{cases}$$

Assume time is measured in days. If data are sampled continuously for 365 days, how many independent samples do you have?

2. Download the Amsterdam Island bottom pressure recorder record (amsterdam_bpr.dat) from http://www-mae.ucsd.edu/~sgille/sio221b/. In this data record, data are archived hourly. Column 6 contains bottom pressure, and column 8 contains detided bottom pressure.

a. Compute and plot the discrete autocovariance for columns 6 and 8. (You can do this using the "xcov" function in matlab, but be sure to read the help screen for this function, and consider whether you should use the biased or unbiased estimator.)

b. How many independent samples are in the two data records?

c. How does the time interval between independent samples compare with the time required for the autocovariance function to drop by $1/e$ from its maximum? What about the time lag required to reach the first zero crossing of the autocovariance function?

3. (Notes on statistically optimal linear estimators), problem 5). A numerical model of tides involves the large-scale flow field U, but not the small-scale components u. In this model, the bottom drag $\tau = C_D(U+u)|U+u|$ must be parameterized in terms of U. Consider the approximation $\hat{\tau} = \gamma(U)\cdot U$ in one dimensional flow, with $\langle u\rangle = 0$.

a. Find equations determining the optimal function $\gamma(U)$ for the "beauty principle" of minimizing the mean-square error in tidal dissipation. You may wish to assume that there is a scale-separation between large-scale and small-scale motions, so that $\langle Uu\rangle = 0$.

b. Find γ if u is normally distributed with known variance. In other words, $\langle u^2\rangle = \mu_2$.

c. Compare the mean-square dissipation error using the answer from (b) with that from the naive modeler's choice $\hat{\tau} = C_D U|U|$.