Why use variance preserving spectra

Usually when we plot spectra we start with log-log axes. These are useful for observing that spectra are red and for making grand statements about k^{-2} slopes. But they also have some disadvantages. Notably, on a log-log plot, you can't really say anything useful about the relative fraction of variance within one band of frequencies relative to another band of frequencies. Variance-preserving spectra are designed to provide a useful measure of the signal variance. Emery and Thomson provide a succinct derivation of this. If you plot the spectrum S_{xx} times frequency f, as a function of log(f), then area under the spectral curve between frequencies f_0 and f_n will be:

$$\sigma^2 = \int_{f_0}^{f_n} f S_{xx}(f) d(\log(f)) = \int_{f_0}^{f_n} S_{xx}(f) df. \tag{1}$$

Here note that $d(\log(f)) = df/f$. Thus area under the curve between two frequencies gives a measure of spectral signal variance in that frequency band.