Problems Week 5

Due Thursday, October 27, 2016

1. **Compute a spectrum with error bars.** For this problem set, please once again generate a data set with Gaussian white noise. To start, you can consider a 500×10 matrix of Gaussian white noise, which will give you 10 segments, each 500 points long. Compute and plot the spectrum for this record. (You should have 10 Fourier transforms, each with 500 Fourier coefficients.) Use the χ^2 formulation to compute an uncertainty for your spectrum, and show this on your plot.

2. **Use Monte Carlo simulation to verify the χ^2 error bar.** To check the error bar, you'll want to generate multiple ensembles of data and compute spectra for each of them. I would suggest using 200 matrices that are 500×10 elements each. Compute spectra for each of them. Examine the pdf of your values. (Because we're looking at Gaussian white noise, you can merge all the frequencies for this.) Is the pdf consistent with your expectations for a variable with a χ^2 distribution? //

   Now, for each frequency, sort the 200 realizations of the pdf by size (e.g. using the "sort" command in Matlab. Since you are looking for the 95% confidence range, you'll want to find the limits that exclude the lowest 2.5% and the highest 2.5% of your data—presumably the 6th and 195th points in each sorted set of pdfs.) What is the ratio of the upper limit to the lower limit? Are the error bars derived using the Monte Carlo process consistent with those from question 1?

3. **Evaluate whether windowing alters degrees of freedom.** Now apply a Hanning window to your data and repeat the analysis in question 2. Does the use of a Hanning window change the estimated uncertainties?

4. **Evaluate whether using a 50% overlap modifies the degrees of freedom.** Now modify your artificial data set to have white noise data segments that overlap by 50%. Compute spectra (again using a Hanning window). Do the empirical error bars from the Monte Carlo process match your expectations based on the number of segments that you have available? Does the use of overlapping segments reduce your effective degrees of freedom? If you don't window, does the use of overlapping segments reduce your effective degrees of freedom? The Lecture 9 notes replicate 2 tables from a couple of textbooks that don't appear to agree. Which, if any, of these tables appears to be more useful for the Hanning window?