Standing internal tides in the Tasman Sea observed by gliders

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ABSTRACT

Low-mode internal tides are generated at tall submarine ridges, propagate across the open ocean with little attenuation, and reach distant continental slopes. A semidiurnal internal tide beam is identified in previous altimetric observations and modelling, emanates from the Macquarie Ridge, crosses the Tasman Sea, and impinges on the Tasmanian slope.

Spatial surveys covering within 150 km of the slope by two autonomous underwater gliders with maximum profile depths of 500 and 1000 m show the steepest slope near 43°S reflects almost all of the incident energy flux to form a standing wave. Starting from the slope and moving offshore by one wavelength (~150 km), potential energy density displays an antinode-node-antinode-node structure, while kinetic energy density shows the opposite.

Mission-mean mode-1 incident and reflected flux magnitudes are distinguished by treating each glider’s survey as an internal wave antenna. Incident fluxes are 1.4 and 2.3 kW m$^{-1}$ from the two missions, while reflected fluxes are 1.2 and 1.8 kW m$^{-1}$. From one glider surveying the region of highest energy at the steepest slope, the reflectivity estimates are 0.8 and 1, if one considers the kinetic and potential energy densities separately. These results are in agreement with mode-1 reflectivity of 0.7–1 from a model in one horizontal dimension with realistic topography and stratification. The direction of the incident internal tides are consistent with altimetry and modelling, while the reflected tide is consistent with specular reflection from a straight coastline.
1. Introduction

Long wavelength (i.e., vertical mode 1) internal tides are generated at tall, steep, submarine ridges (Holloway and Merrifield 1999; Merrifield et al. 2001; Rudnick et al. 2003; Garrett and Kunze 2007) and propagate thousands of kilometers across ocean basins (Ray and Mitchum 1996, 1997; Simmons et al. 2004; Zhao and Alford 2009). Their decay may occur through either non-linear wave-wave interactions (MacKinnon et al. 2013a,b), scattering from midocean topography (Müller and Liu 2000; Johnston et al. 2003; Bühler and Holmes-Cerfon 2011; Mathur et al. 2014), or shoaling on the continental slope (Nash et al. 2004; Kelly et al. 2013a). For the latter situation, a particularly good geometry for observations occurs in the Tasman Sea. A well-defined principal, lunar, semidiurnal ($M_2$) internal tide is generated at the Macquarie Ridge south of New Zealand, propagates west-northwestward across the Tasman Sea, impacts the continental slope of Tasmania according to modelling (Simmons et al. 2004) and altimetry (Z. Zhao and M. Alford, pers. comm.), and then reflects back into the Tasman Sea setting up a standing internal tide (Simmons et al. 2004, see their Figure 7 for the incident tide and Figure 8 for the standing wave pattern in a global model). Our contribution is to make extensive subsurface observations using Spray autonomous underwater gliders, to calculate incident and reflected energies, and to estimate the reflectivity of the slope.

Internal wave transmission and reflection at topography is determined both by the height and steepness of the topography (Müller and Liu 2000; Johnston and Merrifield 2003; Kelly et al. 2013b; Mathur et al. 2014). The ratio of the topographic slope ($\gamma$) to the slope of an internal wave characteristic ($s$) defines supercritical ($\gamma/s > 1$), critical ($\gamma/s = 1$), or subcritical ($\gamma/s < 1$) topographies, where

$$s(z) = \sqrt{[\omega^2 - f^2]/[N(z)^2 - \omega^2]}$$

$\omega = 1.405 \times 10^{-4}$ s$^{-1}$ is the $M_2$ radial frequency, $f$ is the Coriolis frequency, and $N(z) = \sqrt{-g \partial_z \sigma_{\theta_o}(z)/\sigma_{\theta_o}(z)}$ is the buoyancy frequency,
$g$ is the gravitational acceleration, $z$ is the vertical coordinate with positive upward, $\sigma_\theta$ is the potential density, $\sigma_{\theta_0}(z)$ is the unperturbed potential density, and $\partial_z$ denotes the vertical gradient. In general, wave transmission is found for subcritical topography and wave reflection at supercritical topography. Wave scattering to shorter wavelength (i.e., higher vertical modes) is expected at near-critical to supercritical slopes. Wave dissipation is expected at near-critical slopes, where energy is focussed along internal wave characteristics leading to elevated shear (Johnston and Merrifield 2003; Nash et al. 2004). Numerous details complicate this summary of wave transmission past sub-, near-, and supercritical slopes including stratification, phasing between different incident waves, phasing between the internal and surface tides, shelf/slope geometry, and angle of wave incidence (Chapman and Hendershott 1981; Kelly and Nash 2010; Martini et al. 2011; Hall et al. 2013).

In much of the ocean, propagating internal tides are found, but standing or partially-standing waves appear rare (Alford and Zhao 2007a,b). This result implies that a considerable portion of internal tide energy is either dissipated at midocean topography or continental slopes, transmitted past the topographic obstacles, or scattered into higher modes (Kelly et al. 2013b).

Several observations of standing semidiurnal internal tides are noted. Two wave sources produce standing or partially-standing internal tides around Oahu, Hawaii (Eich et al. 2004; Alford et al. 2006; Nash et al. 2006; Martini et al. 2007). The narrow shelf off California also allows for reflection of an incident wave to produce a standing wave (Winant and Bratkovich 1981; Petruncio et al. 1998). With multiple submarine ridges acting as wave sources on the Hawaiian Ridge, still more complex interference patterns are found (Rainville et al. 2010). Separating the incident and reflected waves, for example, out of the total velocity and density in either models or observations requires resolution of the waves in both space and time.
The portion of the Tasmanian slope under consideration is supercritical to the semidiurnal internal tide, which suggests much of the energy is reflected and standing waves are expected (See Figures 7–8 in Simmons et al. 2004). To motivate results described later in Sections 4–5, a snapshot of the solution to Poincaré wave reflection from a vertical wall is superimposed on a map of Tasmania (Figure 1). When a mode-1 internal tide (a rotating, shallow water wave or Poincaré wave with wavelength: \( \lambda = 150 \text{ km} \), Appendix A), reaches a supercritical continental slope (or a vertical wall in Figure 1), reflection occurs. Under the condition of no normal flow through the wall, the incident wave reflects completely and combines with the reflected wave to produce a standing wave (LeBlond and Mysak 1978). Thus the standing wave structure from the slope in the offshore direction is node-antinode-node for cross-shore velocity with distances between antinodes (blue/red) and nodes (white) equal to \( \lambda/2 \) (Figure 1). A displacement antinode is at the coast and at other velocity nodes. Adjacent displacement and velocity antinodes are separated by \( \lambda/4 \).

Using similar modal solutions of Laplace’s tidal equations in a one horizontal (cross-shore) dimensional global model, a mode-1 internal tide incident on realistic slope topography with realistic stratification transmits 20% of its energy onto the shelf, scatters 40% to higher modes, and reflects 40% back into the deep ocean (Kelly et al. 2013b). Larger values of reflection (50–70%) are found for the Tasmanian slope, while a higher-resolution calculation presented in Section 6 shows up to 100% reflection of an incident mode-1 internal tide along the steepest portions of the Tasmanian slope where we find the highest energy density. Interestingly, in the study region where the largest semidiurnal waves are located, the continental slope is steepest and has exposed bedrock (Hill and Exon 2004), which supports the idea that internal tides may scour and shape the continental slope (Cacchione et al. 2002).
To observe incident and reflected waves, determine their energies, and assess the reflectivity of the slope, two Spray gliders were deployed off the continental slope for this first phase of Tasmanian Tidal Dissipation Experiment (TTIDE, see http://ttide.ucsd.edu) in 2012 and 2013. Gliders are versatile tools (Sherman et al. 2001; Rudnick et al. 2004), which can move around for spatial surveying, hold station for time series, and do both during a single mission as was done here (Figure 2). While such measurements are made from other platforms, gliders have observed internal tides (Johnston et al. 2013; Rainville et al. 2013), high-frequency internal waves (Rudnick et al. 2013), and related mixing via finescale parameterizations (Johnston and Rudnick 2015).

This paper describes gliders and the data analysis methods (Section 2). Internal tide variability is presented in Section 3, which Section 4 reveals as cross-shore standing wave patterns. To identify wave propagation directions, we treat the glider surveys as antennae to estimate the amplitude of incident and reflected internal tides along with their wavenumber magnitudes and directions (Section 5). Estimates of mode-1 energy densities and fluxes are made in Sections 4–5. A discussion and conclusions follow in Sections 6–7.

Several appendices cover our methods in more detail. Appendix A provides equations to describe the simplest theory suitable to the observed geometry—reflection of a mode-1 Poincaré wave incident at an angle to the coast (i.e., a vertical wall) to produce a standing wave with no energy loss. Appendix B describes the wavelet transform method used to bandpass and reconstruct the data around the diurnal and semidiurnal frequencies. Appendix C details how mode-1 estimates of energy are made from depth-limited observations. Appendix D describes the internal-wave antennae.
2. Methods

a. Data

Two Spray glider missions covered the region within one mode-1 $M_2$ $\lambda \sim 150$ km of the Tasmanian slope (Figure 2), where the incident internal tides are expected to be strongest according to models (Simmons et al. 2004) and altimetry (Z. Zhao and M. Alford, pers. comm.). The two missions took place from (a) 2 November 2012 to 14 March 2013 (mission 12B05501, for which the first two digits denote the year as 2012, hexadecimal month B denotes November, 055 denotes Spray serial number 55- hereafter Spray 55, and the final two digits denote mission 1 in this month) and (b) from 26 August to 31 October 2013 (mission 13805601, hereafter Spray 56). Data plots are available at http://spray.ucsd.edu.

The payload comprises: (a) a pumped Sea-Bird Electronics (SBE) 41CP conductivity-temperature-depth instrument (CTD) from which potential temperature ($\theta$), salinity ($S$), in situ density ($\rho$), and $\sigma_{\theta}$ are obtained; (b) a Seapoint chlorophyll $a$ (chl $a$) fluorometer; and (c) a Son-tek 750 kHz acoustic Doppler profiler (ADP) aligned to measure horizontal velocities in five 4-m vertical range bins (Davis 2010; Todd et al. 2011). Data are obtained on ascents, when the CTD experiences clean flow (Figure 3). Data from Spray 56 are averaged in 10-m bins from 10–500 m, while data from Spray 55 are averaged in 20-m vertical bins from 20–1000 m due to a noisier conductivity cell.

A glider moves up-/downwards through the ocean by moving mineral oil out of/into the pressure case and inflating/deflating an external bladder (Sherman et al. 2001). Wings create lift and move the glider forward. Return dive cycles to 500/1000 m were $\sim$1.5/5 hours for Spray 56/55, which give a Nyquist frequency sufficient to sample $D_2$ signals. During a dive cycle to 500/1000 m, a glider moves laterally about 1.6/5.4 km through the water and produces a sawtooth path in depth.
and horizontal distance (or time). The dive angle is $17^\circ$ for maximum endurance for Spray 55 and $30^\circ$ for maximum profiling speed for Spray 56 (Sherman et al. 2001).

Spray 55 completed a broad survey, while Spray 56 concentrated in the area which Spray 55 showed to be the most energetic. Spray 56 also obtained a two-week time series near the coast (27 September–10 October 2013 near 48° 23′S). Spray 55 completed 645 CTD profiles with most to 1000 m and was configured to provide maximum endurance and spatial coverage, while Spray 56 completed 968 CTD/ADP profiles with most to 500 m to provide better temporal resolution of the semidiurnal signal. With two-way Iridium satellite communication, waypoints can be changed based on analysis of subsampled data, which are transmitted after every dive. These missions demonstrate the versatile capabilities of the gliders.

In both cases, the relatively slow moving gliders ($\sim$20 km/day) confuse spatial and temporal variability because high frequency signals (i.e., internal waves with frequencies from $f$ to $N$) are projected into spatial variations (Rudnick and Cole 2011). The projection arises from two distinct effects in the sampling (smearing from the slow motion and aliasing due to a sampling frequency in the internal wave band), which cannot be disentangled subsequently. For analysis of internal tides, the gliders slow motion is an advantage for spatial surveys— they can be considered slowly-moving time series, from which we estimate tidal signals similar to Johnston et al. (2013). For semidiurnal and diurnal internal waves, smearing projects into $\lambda \sim 10$ and 20 km (equation 5 in Rudnick and Cole 2011), which is within our averaging window (Section 2b).

Velocity profiles are processed much like lowered acoustic Doppler current profiles obtained from ships (Visbeck 2002). Depth-mean currents are calculated from the difference over a dive cycle in positions from Global Positioning System (GPS) fixes and the glider’s dead reckoning via measured attitude (Todd et al. 2009) (Figure 3e). Velocities (both depth-means and those measured by the ADP relative to the glider) are combined in a linear system of equations, from which
water velocities are extracted by a least squares procedure yielding depth-dependent eastward \((u)\) and northward \((v)\) velocities (Todd et al. 2011) (Figure 3c–d). Similar results are obtained by objectively-mapping and vertically-integrating ADP-measured shear and then referencing to the depth-mean current (Davis 2010).

\[b. \text{Tidal analysis}\]

In Section 3, tidal amplitudes \((A)\) and phases \((\phi)\), which vary in \(z\) and time \((t)\), are obtained from a wavelet transform (Appendix B). The Morlet wavelet used here is a complex exponential within a Gaussian envelope and results are virtually identical to the method described in Johnston et al. (2013), which used a moving square window of several days’ length to calculate harmonic fits. Due to the limited window lengths in both cases, frequency resolution is insufficient to distinguish tidal constituents within each of the diurnal \((D_1)\) or semidiurnal \((D_2)\) pass bands, which are \(\pm 25\%\) of the \(M_2\) and \(K_1\) \((7.292 \times 10^{-5} \text{ s}^{-1})\) radial frequencies. Bandpassed data are reconstructed using these amplitudes and phases (Figure 4). Phases are referenced to 0000 UTC on 1 May 2013, a point between the two glider missions. In several instances in the paper, depth-mean phases are calculated from the real and imaginary components of depth-varying phases. Profiles are irregularly spaced in time and are used without interpolating to a regular time grid. The mean time interval between profiles is \(5.1 \pm 0.2\) hours (1 standard deviation) for Spray 55 and \(1.6 \pm 0.1\) hours Spray 56. The corresponding range in frequency is \(\pm 4/6\%\) for Spray 55/56, which is small compared to the width of the pass bands.

Vertical displacements are calculated as: \(\eta'(z,t) = -\sigma'_y/\partial_z\langle \sigma_y \rangle\) where primes denote tidally band-passed data and \(\langle \cdot \rangle\) denotes 3-day lowpassed data also obtained using wavelets.

Unlike Johnston et al. (2013) which sampled the mode-1 structure sufficiently, we avoid mode fits here, since Spray 55/56 reach 1000/500 m, which, over much of the survey region, are above
the mode-1 $\eta$ maximum at depths of $\sim 1300$ m in 3000 m of water. However, relatively uniform
values of $u$, $v$, and $\eta$ are found in the data from Spray 55/56 in the depth ranges 500–1000 m/300–
500 m (Figure 4). For Spray 55, the mesoscale variability seen in this western boundary area is
roughly equal to the tidal variability. Using deeper data minimizes the mesoscale and emphasizes
the internal tide displacements. For Spray 56, we avoid using data in the weakly-stratified surface
layer, which extends down to 200–300 m (the mean mixed layer depth is 259 m based on $\Delta \sigma_\theta =$
0.1 kg m$^{-3}$ from the near-surface value, Figure 3).

In Section 4, depth-mean tidal amplitudes in these limited depth ranges (500–1000 m for
Spray 55 and 300–500 m for Spray 56) are used to calculate energy and flux. Total energy is:
$$E_t = \langle \sigma_\theta \rangle (A_u^2 + A_v^2 + \langle N^2 \rangle A_\eta^2)/4,$$
where kinetic energy ($E_k$) is the sum of terms 1 and 2, while
term 3 is potential energy ($E_p$). Flux magnitude is $F = E_t c_g$, where $c_g$ is the group velocity.
Mode-1 energy estimates from depth-limited measurements are obtained by a scaling of the band-
passed tidal variances (Appendix C). A brief description of the process follows: the variances
are multiplied by factors accounting for the limited-depth measurement of the mode structure ($b$)
and the depth-integrated mode structure ($I$). Mission means for $\sigma_\theta$ and $N^2$ are used. No attempt
is made to remove the barotropic tide, which has $D_1$ and $D_2$ currents of about 0.03 m s$^{-1}$ each
(Egbert et al. 1994).

To demonstrate how a glider observes a standing wave pattern under ideal conditions, synthetic
data are produced from the sum of incident and reflected Poincaré waves with $\lambda = 150$ km (Fig-
ure 1, equation A5). The synthetic glider moves at 0.25 m s$^{-1}$, produces a profile every 5 hours
over a 125-day mission, and surveys in a zig zag pattern extending one $\lambda$ in both the cross- and
alongshore directions (Figure 5). The time dependent velocity seen by the glider is shown in Fig-
ure 5a and then bandpassed as described above to produce amplitudes at the site of each profile
(Figure 5b). Due to the width of the averaging window, the glider under-/overestimates the antin-
odes/nodes. Velocities are scaled by the incident wave amplitude \((A_{uo} = ABC; \text{Appendix A})\) and similarly for \(p\) \((A_{po} = A)\). Changing the time interval between profiles, survey area, the survey leg spacing, or mission duration by a factor of two has some impact on these results, but these results are representative of what can be achieved with a single typical glider mission.

By averaging the analytic and synthetic glider data to produce a mean cross-shore section, \(u\) nodes are found at distances of 0, 75, and 150 km offshore with antinodes between (Figure 6b). \(v\) antinodes are slightly offset from \(u\) antinodes (Figure 6c; Appendix A), while \(p\) antinodes are found at distances of 0, 75, and 150 km offshore. This cross-shore structure of the standing wave is identified by the glider with a \(\sim 20\%\) reduction in amplitude compared to the analytic solution.

c. Identifying incident and reflected waves

In Section 5, each glider sampling pattern is used as an antenna to identify amplitudes, wavelengths, and propagation directions of the incident and reflected waves similar to previous altimetric examinations of internal tides (Zhao and Alford 2009; Zhao 2014). The result is also similar to a directional wave spectrum for surface gravity waves. We use a harmonic fit of the form 

\[ A \exp[i(k \cdot x - \omega t - \phi)], \]

where the wavenumber, \(k = (k, l)\), and position, \(x = (x, y)\), are vectors and are described by eastward and northward components. Wavelengths are obtained from the wavenumber magnitude as: \(\lambda = 2\pi/|k|\). Propagation directions \((\alpha)\) are obtained as \(\tan \alpha = l/k\) and so east, north, west, and south correspond to \(\alpha = 0^\circ, 90^\circ, 180^\circ, \) and \(-90^\circ\). \(\phi\) is the phase with respect to 1 May 2013. Rather than use a relatively short time window, as with the wavelet analysis in Sections 3–4, we make use of the entire dataset for each glider to obtain \(A\) and \(\phi\), which are coherent over the record. \(k\) are specified for each fit in the ranges of \(\lambda = 60–180 \text{ km}\) and \(\alpha = 0–360^\circ\). This process is repeated at each depth. Finally, depth-mean \(A\) is calculated for each \(k\), which is the focus of Section 5. Maxima in \(A\) are used to identify wavenumbers of the incident and
reflected waves at either the expected mean \( \lambda \) of 150 km in the study area or any \( \lambda \). The incident and reflected waves are now specified from this analysis. Reflectivity is then estimated by scaling the amplitudes (Appendix C) to produce incident \( (F_i) \) and reflected \( (F_r) \) fluxes: \( R = F_r / F_i \).

Again to understand how well incident and reflected waves can be discerned, we examine results from the synthetic data (Figure 5) and apply the directional wave analysis (Figure 7). Maxima in \( A \) at any \( \lambda \) (blue vector) and at the actual \( \lambda \) of 150 km (purple vector), identify the specified synthetic incident and reflected wavenumber directions at 10\(^\circ\) angles from west and east (white vector), as expected for reflection from a meridional wall. The fits underestimate \( A \) by 10–20\%. The angular resolution of this survey pattern is about ±15\(^\circ\). These results are obtained with reasonable choices for glider sampling, but depend on the survey area, survey spacing, mission duration, and sampling interval. However, these results are representative to changes by a factor of two. These results from sampling synthetic data as a glider under ideal conditions are comparable to the actual data obtained under less than ideal conditions (Section 5).

The sensitivity of each antenna from the two actual glider missions is summarized below and described in more detail in Appendix D. Tests with synthetic data using the tracks of Spray 55 and 56 show how well each wave antenna measures \( A \) and \( \alpha \) for varying (a) phase differences between westward and eastward waves \( (\Delta \phi = \phi_w - \phi_e) \) and (b) propagation directions of the eastward wave \( (\alpha_e) \). \( \alpha_w \) is fixed at -150\(^\circ\) (roughly west-northwestward). A 20\% change at most in \( A \) is obtained by varying \( \alpha_e \) by ±15\(^\circ\) (which is roughly our antenna’s angular sensitivity). Since amplitudes are associated with an uncertainty of ±20\%, those in energy and \( R \) can be about ±40\%. When considering \( A_w \) and \( A_e \) both are similarly over- or underestimated by the antenna and so the effect on the reflectivity calculated from one quantity (i.e., \( \eta \)) is minimal. However, this is for a given \( \Delta \phi \), but \( \Delta \phi \) is different for \( \eta, u, \) and \( v \) upon reflection (Appendix A). Since \( E_k \) is often much larger than our measured \( E_p \), this last issue is of minor concern in our case.
d. Modelling incident and reflected waves

The simple picture of a Poincaré wave reflecting from a coastal wall can be extended to depth-variable internal tides encountering realistic topography with realistic stratification via the coupling equations for linear tides model (CELT). CELT is forced with an incident mode-1 $M_2$ internal tide and determines the modal solutions to Laplace’s Tidal Equations by matching horizontal velocity and pressure at a series of topographic steps in one horizontal dimension. The solutions, which simultaneously satisfy the matching conditions at all of the topographic steps, are determined from a single matrix inversion (Kelly et al. 2013a). CELT has the advantages of (a) incorporating arbitrary realistic stratification and one-dimensional topography without requiring numerically-expensive simulations and (b) intrinsically separating the incident and reflected components of the solution, making it easy to compute $R$.

CELT has been applied globally to the continental margins (including the Tasmanian slope at low alongshore resolution) with normally-incident $M_2$ internal tides (Kelly et al. 2013b). Here, the model is refined for comparison with glider observations by (a) quadrupling the alongshore resolution to one topographic profile every 25 km and (b) forcing with obliquely-incident waves ($\alpha = 150^\circ$), which are accommodated by conserving alongshore wavenumber (Chapman and Hendershott 1981). However, differences in reflectivity for waves at normal incidence and $\alpha = 150^\circ$ are minor. The model is configured with 500-m horizontal resolution, 32 vertical modes, a vertical viscosity of $10^{-5} \text{ m}^2 \text{ s}^{-1}$, bathymetry from the General Bathymetric Chart of the Oceans (GEBCO), and stratification from the World Ocean Atlas (Antonov et al. 2010; Locarnini et al. 2010).

3. Internal tides

Tidal variability is visible in the vertically binned $\theta$, $S$, $\sigma_\theta$, $u$, and $v$ data from Spray 56 (Figure 3). Several features are apparent, which affect our subsequent analysis of Spray 56 data: (a) the
mixed layer extends to 200–300 m as seen in the uniform $\theta$ and $S$; (b) velocities display no spring-neap signature even during the two-week time series from 27 September–10 October 2013 near the coast; (c) displacements of isotherms, isohalines, and isopycnals appear vertically coherent at least over depths of 300–500 m; and (d) tidal and mesoscale velocities are comparable in magnitude.

For Spray 56, the $D_2$ bandpassed data in Figure 4a–d show: (a) vertically-coherent displacements from 300-500 m; (b) larger displacements from 0–200 m, which are not always correlated with those below 300 m; and (c) no clear spring-neap temporal variability with the possible exception of $\eta$ during the time series from 27 September–10 October 2013. For Spray 55, similar vertical coherence for $\eta$ is found from 500–1000 m (Figure not shown.). Also similar to Spray 56, Spray 55’s depth-dependent $D_2 \eta$ and depth-mean $D_2 u$ and $v$ data are not large at the same times. In the following section, depth means will be taken over these limited depth ranges to emphasize the $D_2$ signal.

$D_1 \eta$ and velocities are strongest at the same time and near topography consistent with a forced, subinertial, topographically-trapped wave (e.g., Johnston and Rudnick 2015) (Figure 4e–h). For Spray 56, $D_1$ is most prominent both at the slope and the offshore seamount (East Tasman Rise) around 3 September, 15 September, and 7 October 2013. Also, Spray 55 shows $D_1 \eta$ and depth-mean $u$ and $v$ are correlated in time (Figure not shown.). While both Spray 55 and Spray 56 find $D_1$ amplitudes are comparable to $D_2$, the remainder of this paper focuses on the $D_2$ component and the standing waves.

4. Standing waves

To display the standing wave structure of the $D_2$ internal tides, depth-mean $A$ and $\phi$ are binned in $1/8^\circ \times 1/8^\circ$ latitude-longitude bins for each of Spray 55 and 56 (Figures 8-9). Due to both the different profiling and stratification encountered by each glider, binning is done separately
for each glider. However, once these factors are accounted for by estimating mode-1 $E_k$ and $E_p$, these quantities from both gliders are averaged together with a weighting based on the number of profiles per bin (Figure 10).

Maxima/minima in $A$ or $E$ are antinodes/nodes of a standing wave. In the synthetic data, the standing wave pattern expected moving $\lambda/2 = 75$ km from the slope in the offshore direction is node-antinode-node for $E_k$ or $A_{u,v}$ (Figures 6b–c) and antinode-node-antinode for $E_p$ or $A_\eta$ (Figure 6a). From the glider observations, $A_\eta$ is maximum parallel to the slope and about 75 km offshore (Figures 8a and 9a; note 75 km scale in latter). $A_u$ and $A_v$ have (a) maxima in between the $A_\eta$ maxima and (b) minima at the shelf and offshore (Figures 8b–c and 9b–c). Further offshore, Spray 56 shows another possible $A_\eta$ node and $A_v$ antinode near Cascade Seamount (Figure 9a and c).

For Spray 55, $u$ and $v$ are depth-means over the entire 1000-m profiling range obtained from the differences between dead-reckoning and GPS positions, while, for Spray 56, $u$ and $v$ are obtained from the depth-varying velocities measured by the ADP and then averaged from 300-500 m.

Overall, the phase structure is slowly-varying over the central area of the survey, where amplitudes are generally largest and sampling is densest (Figures 8d–f and 9d–f). This result indicates the $D_2$ internal tide is narrow banded during each deployment. In the region of large $A_\eta$ near $42^\circ 30\prime$S, the $180^\circ$ change in $\phi_\eta$ from the inshore to offshore area also indicates standing waves or, in other words, the inshore antinode is out of phase with the offshore antinode. In particular, (a) for Spray 55, $\phi_\eta$ goes from $90^\circ$ (orange color) to $-90^\circ$ (light blue color; Figure 8d) and, (b) for Spray 56, $\phi_\eta$ goes from $180^\circ$ (red color) to $0^\circ$ (green color; Figure 9d). For Spray 56, $\phi_u$ is near $0^\circ$ (yellow/green color in Figure 9e) at the $u$ antinode and $90^\circ$ out of phase with $\phi_\eta$ at the $\eta$ antinodes. The measurements of $\eta$ and $u$ are independent and both data types identify a standing wave.
To compare $D_2$ waves from the different background $N^2$ and depths sampled by Spray 55 and 56, mode-1 energy estimates are made by scaling the binned depth-mean $D_2 \eta$ and velocity variances, i.e., $A_\eta^2$ and $A_{u,v}^2$ (Appendix C). The maps of $E_p$ and $E_k$ from Spray 55 (Figures 10a–c) and Spray 56 (Figures 10d–f) show the same standing wave patterns described above for $A_\eta$ and $A_{u,v}$ (Figures 8a–c and 9a–c). However, now the data from both gliders can be combined in a consistent way (Figure 10g–i). The node for $E_p$ and the antinode for $E_k$ are further emphasized. $E_p$ estimates are largely insensitive to the different glider sampling strategies. Spray 56’s depth-dependent $u$ and $v$ measurements and subsequent estimate of mode-1 $E_k$ are within a factor of 1–2 of Spray 55’s estimates from depth-independent $u$ and $v$. Small changes in either the incident or reflected wave amplitude, direction, or phase could produce large variations in the interference pattern. However, the similarity of $E_p$ and $E_k$ over missions that span 10 months demonstrate a consistent incident and reflected internal tide, which combine to form a standing wave.

The standing wave pattern is emphasized by further averaging the spatial pattern of $E_p$ and $E_k$ (Figures 10g–h) in the alongshore direction to produce the mean cross-shore structure of $E_p$ and $E_k$ (Figure 11). Coordinates are rotated 18° about 42.5°S, 148.5°E (dashed line in Figure 1 shows alongshore direction). Two antinodes in $E_p$ of $\sim$1 kJ m$^{-2}$ are found at distances of 15 and 90 km offshore (orange line) and are separated by $\lambda/2$ as expected for a mode-1 standing wave (black line). Two antinodes in $E_k$ of $\sim$3 and 2 kJ m$^{-2}$ are found at distances of 60 and 135 km offshore, a spacing of $\lambda/2$. $E_k$ nodes are found near $E_p$ antinodes as expected from the analytic data (Figure 6).

To match the observations, the analytic $E_p$ is chosen to be 1 kJ m$^{-2}$ and is shifted by 15 km offshore. The analytic $E_k$ is fully prescribed by these choices for $E_p$ and consequently has antinodes of 2.8 kJ m$^{-2}$, which lie in between the two observed maxima and are consistent with the expected $E_k/E_p$ ratio at 42.5°S. A number of discrepancies are noted, likely due to the presence of
the shelf: (a) the analytic results are shifted offshore to match the observations; (b) the $E_p$ antinode closest to the slope decreases shoreward to the westernmost extent of our survey; and (c) the $E_k$ node closest to the slope does not reach zero.

Overall, the resemblance of analytic cross-shore structure with the mean cross-shore structure in $E_k$ and $E_p$ indicates a mode-1 $D_2$ standing or partially-standing internal tide. If the minima/maxima indeed reflect the difference/sum of incident and reflected waves, rather than other $D_2$ signals in the area, then $E_p$ suggests an incident and reflected wave of 0.6 and 0.4 kJ m$^{-2}$, while $E_k$ suggests 2.2 and 1.2 kJ m$^{-2}$. The reflectivity is 0.7 and 0.6 in these two cases. This issue is investigated further in the following section.

For a single plane wave, $F = E_t c_g$ and so $F$ and $E_t$ have the same patterns. Since both an incident and reflected wave are present, amplitudes are a factor of 2 higher for perfect reflection. Energies will then be a factor of 4 higher. $c_g$ is about 1.5 m s$^{-1}$ during both missions. Thus an estimate of the incident flux is obtained by dividing the maximum $E_t$ in Figure 10i of 5 kJ m$^{-2}$ by 4 to account for the standing wave and then multiplying by $c_g$ yielding a maximum $F$ of about 2 kW m$^{-1}$ averaged over spring-neap cycles.

5. Incident and reflected waves from directional wave fits

In this section, the incident and reflected waves in the standing wave pattern are isolated by using the entire record and fitting both in time and space to produce directional wave fits. The waves’ amplitudes ($A$), wavelengths ($\lambda$), and propagation directions ($\alpha$) are identified, which cannot be done using the localized methods in the previous section. Details of the procedure are in Section 2c and Appendix D. The synthetic glider directional wave fit shows how the synthetic glider sampling pattern captures incident and reflected waves under ideal conditions (Figure 7). The synthetic results are quite similar to the directional wave fits to the observations (Figures 12–13). For Spray
$A_u$ and $A_v$ are obtained from fits to the depth-mean velocity of the glider over its profiling range of 0–1000 m (Figures 12b–c). Depth-varying $A_\eta$ is computed as a function of $\alpha$ and $\lambda$ and then averaged from 500–1000 m (Figure 12a). Similarly, for Spray 56, depth-varying $A_\eta$, $A_u$, and $A_v$ are calculated at each $\alpha$ and $\lambda$ and then averaged from 300–500 m (Figure 13). For Spray 56, the patterns of the directional wave fits are similar for both depth-mean and depth-varying $u$ and $v$ (figure not shown) lending confidence to the directional wave fits of depth-mean $u$ and $v$ for Spray 55.

For Spray 55, the incident wave propagates at $\alpha = 150^\circ$ or roughly west-northwestward as identified by the maximum amplitude at any $\lambda$ (blue vector) or the mean $\lambda$ (purple vector, Figure 12). This angle is consistent with the incident wave in models and altimetry and the synthetic Poincaré wave in Figure 1a. The reflected wave is at $\alpha \sim 0^\circ$ or eastward for maximum amplitudes at any $\lambda$ (blue vector) or the mean $\lambda$ (purple vector). This direction is consistent with specular reflection from the continental slope aligned at 15°.

For Spray 55, the incident and reflected $A_u$ and $A_v$ are similar, but $A_\eta$ weakens by about 25% upon reflection. Roughly speaking the amplitudes obtained here are about half of those seen in the localized fits (Figure 8), which measure the total amplitude due to both incident and reflected waves. This result is consistent with a standing wave. Depth-mean amplitude variances are scaled again (Section 4 and Appendix C) with results summarized in Table 2. This scaling allows us to combine the displacement and velocity data and also shows that our estimates of $E$ are dominated by $E_k$ for reasons that are unclear. Reflectivity is defined by the flux ($R = F_e/F_w = 0.9$, where subscripts e/w denote east-/westward or reflected/incident waves), but can also be considered individually for $E_p$ and $E_k$ with $R = 0.6$ and 0.9.

Next we consider similar results for Spray 56. Incident wave direction is closer to west ($\alpha_w = 180^\circ$) than west-northwestward as with Spray 55, while the reflected wave is eastward ($\alpha_e = 0^\circ$, \ldots
Figure 13). These incident directions are not within the antenna’s sensitivity (±15°, Appendix D) of the directions for Spray 55, but are within ±30°. The direction obtained by identifying the maximum $A$ at mean $\lambda$ (purple vectors) is consistent between displacements and velocities. The antenna has also identified a northward wave with maximum $A_\eta$ at $\lambda = 100$ km and a weaker signal in maximum $A_v$ at 180 km. There are two possible explanations for this wave with a mode-1 $\lambda = 100$ km suitable for the shallower water depth along the slope: it is either a coastal trapped wave or a superposition of incident and reflected waves, both of which have northward propagation. It is unclear why maximum northward $A_v$ is associated with a larger $\lambda$. Nearly-trapped, mode-1, superinertial internal edge waves are possible over the shelf and decay into the deep sea at step topography (Chapman 1982). $E_p$ for the northward wave is 0.06 kJ m$^{-2}$, which is the same as the incident/reflected waves and is obtained from a suitable scaling of $A_\eta$ for the shallower water (Appendix C). This northward wave is not considered in the reflectivity calculation. For Spray 56, $R = 1, 0.8,$ and 0.8 based on $E_p$, $E_k$, and $F$ associated with the westward incident wave and the eastward reflected wave (Table 2). Since Spray 56 sampled mostly within the beam and measured currents directly, these $R$ are our best estimates in our opinion.

6. Discussion

Incident and reflected $D_2$ internal tides interfere constructively and destructively to produce a standing wave pattern within 150 km of the Tasmanian slope. Two approaches are used: (a) localized $D_2$ bandpassing using wavelets identifies the tidal signal over a ~1.5-day window and (b) directional harmonic fits to the entire record reveal the signal that remains coherent over areas inside/outside the main beam of energy and over times of spring/neap tides. Localized bandpassing identifies the spatial pattern of the standing wave but offers little time coverage at a given point. Directional wave fits isolate the incident and reflected waves, but offer neither spatial discrimi-
nation nor measurements of the incoherent signal. Comparisons of the spatial structure (Figures 10–11) and directional wave fits (Figures 12–13) from the observations with the synthetic data sampled under ideal conditions (Figures 6–7) show reasonable agreement indicating our methods can detect the standing wave signal.

Comparison of $A_\eta$ from the two methods can be crudely attempted as follows. First, assume a standing wave composed of equal incident and reflected waves and then localized bandpassing overestimates the true amplitude by a factor of 2. Next, assume spatial coverage in/out of the beam is equal (Figure 10i), then directional wave fits may underestimate the true signal by a factor of 2. From Figures 8a and 9a, $A_\eta$ reduced by a factor of 2 is about 8 m, while from Figures 12a and 13a, $A_\eta$ increased by a factor of 2 is 6–13 m. This rough scaling suggests that (a) the coherent mode-1 signal is dominant during the two glider missions and (b) the glider sampling distinguishes the incident and reflected waves.

The main limitation of these measurements is the maximum profiling depth. Our mode-1 estimates are not based on fits to vertical structure because the maximum depth of the profiles does not reach the mode-1 $\eta$ maximum. However, we scale depth-mean values to produce estimates of mode-1 amplitudes. Since the gliders profile to different maximum depths and since the stratification differed between the two missions, data are combined as estimated depth-integrated mode-1 energy densities.

The main advantage of our measurements is their spatial coverage, which show the $\eta$ structure of antinode-node-antinode-node going from the slope to one $\lambda$ offshore. Velocity antinodes are found at the $\eta$ nodes. Despite the large stratification changes, strong mesoscale variability in a western boundary, and the different sampling depths and patterns, the standing wave’s spatial structure appears stable over missions spanning 10 months, although its magnitude fluctuates.
For a standing wave, the ratio of $E_k/E_p$ can vary tremendously depending on the location within the interference pattern. Comparing maxima of $E_k \sim 5$ kJ m$^{-2}$ and $E_p \sim 1.5$ kJ m$^{-2}$ yields the expected ratio for this latitude of $E_k/E_p = (\omega^2 + f^2)/(\omega^2 - f^2) \sim 3$ (Figures 10g–h) and similar agreement is found for the cross-shore means (Figure 11). However, separating into incident and reflected waves produces $E_k/E_p$ ratios that range from 4–25, although values near 3 are expected for plane waves (Table 2).

Uncertainties in modal energy calculations with measurements over a limited depth range, as in this case, certainly exceed 50% (Nash et al. 2005), which must be further combined with uncertainties from the antenna of an additional 40% to yield a total uncertainty of at least ±90%. Uncertainties in energy derived from the antenna are based on tests with synthetic data with sampling in space and time like Spray 55 and Spray 56. However, despite the large uncertainties, our estimates of the incident mode-1 $F$ of 2 kW m$^{-1}$ from both velocity and displacement measurements from Spray 56 are in agreement with the lower bound from altimetry of 2–3 kW m$^{-1}$ (Z. Zhao and M. Alford, pers. comm.).

The other advantage of a wide-ranging glider survey is that this coverage over space and time can be used as an internal-wave antenna. The incident and reflected waves are separated and show most of the incident energy is reflected (Table 2 and Figures 12–13). The coastal reflectivity as measured by the incident and reflected waves’ estimated mode-1 flux magnitude is $R = 0.6–1$. Since $E_k$ is much larger than $E_p$, $R$ based on $F$ is the same as that based on $E_k$ with $R = 0.9$ and 0.8 for Spray 55 and 56. For Spray 55, which ranged over a wider area, $R$ based on $E_p$ is 0.6, but for Spray 56 is 1. Data from Spray 56 are in the central portion of the incident beam, which was identified by Spray 55. However, we use a more limited depth range of 300–500 m for estimating mode 1 from Spray 56 compared to 500–1000 m for Spray 55. Despite the large stratification
changes and the different sampling ranges considered, $R$ based on either $E_k$ or $E_t$ is consistent
between missions spanning 10 months.

Based on *Spray 56* data, which is in the most energetic region with steepest slopes, our best
estimate is $R = 0.8$ or 1 from either $E_k$ or $E_p$, which is consistent with $R = 0.7–1$ from CELT for
mode 1 in this region (Figure 14). Estimates of $E_p$ from *Spray 55*, which covered both the less
energetic regions with shallower slopes and the higher energy region at steeper slopes (Figure 10),
produce lower $R = 0.6$, consistent with the mean value of $R = 0.6$ from CELT over this wider area
from 41–45°S. Furthermore, a northward $D_2$ wave was found with a 100-km wavelength in $\eta$ and
an $E_p$ equal to the reflected/incident waves. This result indicates *Spray 56* was in the reflection
region, where the incident and reflected waves combine to produce a northward wave in $\eta$ at the
coastal antinode.

The mean cross-shore structure of $E_p$ and $E_k$ from all the data (Figure 11) can also be used to
estimate $R \sim 0.7$ and 0.6. This result does not isolate the incident and reflected waves, includes
$D_2$ variability from other sources, samples regions both in- and outside the main beam of energy,
and thus should be considered a lower bound.

It appears the East Tasman Plateau refracts the incident waves and focuses the energy on the
region sampled by the gliders as noted for idealized submarine ridges by Johnston and Merrifield
(2003). Coarser coverage was obtained by the gliders in the north, but energy appears to be low
there. Scouring of the slope by the focused incident waves may explain why this portion of the
slope is steeper than the adjacent slope topography to the north or south (Cacchione et al. 2002).

**7. Conclusions**

Incident semidiurnal internal tides with an estimated mode-1 flux of 1–2 kW m$^{-1}$ are consistent
with (a) the direction of internal tides emanating from the Macquarie Ridge, south of New Zealand
(Simmons et al. 2004) and (b) an unpublished lower bound on the magnitude from altimetry using methods similar to Zhao and Alford (2009). Our mode-1 calculations are not mode fits but rather estimates based on depth means over a subset of the gliders’ profiling range. Our best estimate of mode-1 reflectivity is 0.8 and 1 from $E_k$ and $E_p$ in agreement with results from CELT in the most energetic region at the steepest slopes near 43°S. From 41–45°S, observed and modelled $R = 0.6$, when shallower slopes are included. The incident waves reflect almost entirely form the steepest portion of the Tasmanian continental slope, where the estimated mode-1 energy density is highest and reaches 5 kJ m$^{-2}$. As a result, standing internal tides are found within 150 km of the slope. Displacement antinodes are located at the shelf and 75 and 150 km further offshore, which is consistent with the 150-km wavelength of the waves. The velocity antinodes are found at the displacement nodes. The main advantage of our glider-based measurements is extensive coverage in space and time, but the main limitation is the limited profiling depth, which leads to large uncertainties.

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APPENDIX A

Reflection of a rotating, shallow-water gravity wave obliquely incident on a coast

We examine a rotating shallow-water gravity wave (or Poincaré wave) obliquely incident, (i.e., with a westward component to its propagation direction) onto a straight coast along \(x = 0\), which then fully reflects (i.e., with an eastward component to its propagation direction) as described by LeBlond and Mysak (1978). To identify these waves in our data, we examine the amplitudes and phases of the waves. The incident and reflected waves (subscripts \(w, e\)) have pressure perturbation \(p\) and cross-/alongshore velocities \((u/v)\) given by:

\[
\begin{bmatrix}
    p_{w,e} \\
    u_{w,e} \\
    v_{w,e}
\end{bmatrix} = A_{w,e} \begin{bmatrix} 1 \\
    B(\mp \omega k + i f l) \\
    B(\mp i f k + \omega l)
\end{bmatrix} e^{i(\mp kx + ly - \omega t)} \tag{A1}
\]

where \(A\) denotes the amplitude of \(p\) and \(B = \sigma^{-1}_{\theta o} (\omega^2 - f^2)^{-1}\). With no change in wavenumber magnitudes or frequency upon reflection and no net cross-shore flow at the coast \((u_t = u_w + u_e = 0\) at \(x = 0\), where subscript \(t\) denotes total), we obtain the reflection coefficient:

\[
R = (\omega k - if l)/(\omega k + if l) = e^{2i\beta} \tag{A2}
\]

where \(\beta = \tan^{-1}(-fl/\omega k)\). Then, the wave amplitudes are related by: \(A_e = RA_w\). Since \(|R| = 1\), \(|A_e| = |A_w| = A\).

The total wave field is the sum of the incident and reflected waves (i.e., \(p_t = p_w + p_e\)). An intermediate step in the calculation demonstrates the phase difference between the reflected and
incident waves. On the right hand side, term 1 inside the square brackets is the incident wave and
term 2 is the reflected wave:

\[
\begin{bmatrix}
 p_t \\
 u_t \\
 v_t \\
\end{bmatrix}
= A \begin{bmatrix}
 e^{-ikx} & + & e^{i(kx+2\beta)} \\
 BC[e^{i(-kx+\pi+\beta)} & + & e^{i(kx+\beta)}] \\
 BC[e^{i(-kx-\alpha)} & + & e^{i(kx+2\beta+\alpha)}] \\
\end{bmatrix} e^{i(ly-\omega t)} \tag{A3}
\]

where \( \alpha = \tan^{-1}(f/k/\omega l) \) and \( C = \sqrt{\omega^2k^2 + f^2l^2} \). The phase differences between the reflected and the incident waves at the coast are:

\[
\begin{bmatrix}
 \Delta \phi_p \\
 \Delta \phi_u \\
 \Delta \phi_v \\
\end{bmatrix}
= \begin{bmatrix}
 2\beta \\
 \pi \\
 2(\beta + \alpha) \\
\end{bmatrix} \tag{A4}
\]

The total wave field is:

\[
\begin{bmatrix}
 p_t \\
 u_t \\
 v_t \\
\end{bmatrix}
= 2A \begin{bmatrix}
 \cos(kx + \beta) \\
 iBC \sin(kx) \\
 BC \cos(kx + \beta + \alpha) \\
\end{bmatrix} e^{i(ly-\omega t+\beta)} \tag{A5}
\]

For the Tasmanian slope at 42.5°S, the coast is oriented at a bearing of \( \sim 18^\circ \) with a roughly west-northwestward incident wave (i.e., roughly 10–20° from the coast normal) and a roughly eastward reflected wave. The cross-/alongslope \( \lambda \) are 150 and 900 km, \( \alpha \) is -76 to -63°, and \( \beta \) is 7–14°. The alongslope \( \lambda \) far exceeds the actual width of incident wave packets anticipated (Simmons et al. 2004).

APPENDIX B

Bandpassing and reconstructing tidal signals using wavelet transforms
Like Fourier transforms, wavelet transforms are used to analyze the frequency content of a time series, but have the advantage of being selective in both time and frequency (Kumar and Foufoula-Georgiou 1997; Torrence and Compo 1998). This quality is desired for the analysis of internal tides because of their intermittency: (a) various constituents are present at different locations as the glider moves and/or (b) the amplitude and phase of individual constituents vary over position and time with either the spring-neap cycle or as the result of changing environmental conditions. Johnston et al. (2013) use harmonic fits applied to data with a sliding 3-day window for identical purposes and with almost identical results. Additional advantages of wavelets will be noted below.

To obtain localization in time, the complex Morlet wavelet is frequently used and combines sinusoidal oscillations within a Gaussian envelope:

$$\psi(t) = e^{i2\pi ht} e^{-t^2/2}$$  \hspace{1cm} (B1)

The family of wavelets at different scales ($s$) and different central times ($t_o$) are obtained from the mother wavelet ($\psi$) as:

$$\psi_{s,t_o}(t) = |s|^{-1/2} \psi \left( \frac{t-t_o}{s} \right)$$ \hspace{1cm} (B2)

where $s$ dilates and contracts the mother wavelet, while $t_o$ shifts its location in time. Here we always use $h = 1$, which means the central/mean (denoted by an overbar) frequency of the wavelet is the inverse of its scale: $\bar{f}_s = h/s$. The $e$-folding time of the Morlet wavelet is $\sqrt{2}s$ and its full width at this level is $2\sqrt{2}s$, which is similar to the 3-day window used in Johnston et al. (2013) for diurnal waves (i.e., $s = 1$ day). Minor differences from the harmonic fits in Johnston et al. (2013) arise due to the shape and length in time of the windowing function. Edge effects are avoided by considering data more than $1.5s$ days from the edges (i.e., the amplitude of the Gaussian envelope is less than $1/e$).
Two additional advantages of wavelets are noted. (a) The Morlet wavelet is optimal in that it has the smallest possible product of frequency spread and spread in time. In contrast, the Fourier transform can have a line spectrum for a single sinusoid, but in the time domain the sinusoid extends to all times and conversely for an impulse function in time. (b) The quality factor, $Q$, of a wavelet is the ratio of $\bar{f}_s$ to its frequency spread, which is constant for a wavelet family and for the Morlet wavelet it is: $Q = 2\sqrt{2\pi h}$ or about 3 oscillations across the full width of the envelope at the $1/e$ level for $h = 1$. Within the Gaussian envelope the number of oscillations is the same even as scale increases.

The wavelet transform and the wavelet coefficients are defined as:

$$\tilde{g}(s,t_o) = \int_{-\infty}^{\infty} g(t) \psi^*_{s,t_o}(t) \, dt \quad (B3)$$

where $\tilde{g}(s,t_o)$ are the wavelet coefficients, the asterisk denotes the complex conjugate, and $g(t) = u + iv$ is a complex time series of velocity, for example. Via Parseval’s theorem, this convolution is most efficiently calculated as a product in Fourier space and followed by an inverse Fourier transform:

$$\tilde{g}(s,t_o) = |s|^{1/2} \int_{-\infty}^{\infty} \hat{g}(f) \hat{\psi}^*(fs)e^{i2\pi ft_o} \, df \quad (B4)$$

where the Fourier transform (denoted by a circumflex) of the times series is $\hat{g}$ and that of the Morlet wavelet is known analytically:

$$\hat{\psi}_{s,t_o}(f) = \int_{-\infty}^{\infty} \psi_{s,t_o}(t)e^{-i2\pi ft_o} \, dt = \sqrt{2\pi}e^{-[2\pi(h-f)]^2/2} \quad (B5)$$

It is convenient to choose scales in powers of 2. To provide better resolution each octave (or doubling of scales) contains an equal number of voices ($n = 16$ in our case) and so $s = 2, 2^{1/16}, 2^{1/8}, 2^{3/16}, 2^{1/4},$ and so on. The reconstructed signal over a range of scales ($s_{\text{min}}$ to $s_{\text{max}}$, i.e., a...
bandpass) is obtained from an inverse wavelet transform and is denoted by an upper case letter as:

$$G(t) = C_{\psi}^{-1} \ln 2 \int_{s_{\min}}^{s_{\max}} \tilde{g}(s, t) s^{-1/2} ds$$  \hspace{1cm} (B6)

where:

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{\hat{\psi}^*(f)}{|f|} df$$  \hspace{1cm} (B7)

which is 1.07 for the Morlet wavelet family with $h = 1$. This reconstruction can be performed over the entire range of $s$ or over a selected frequency range (tidal frequency limits are in Section 2b) to form a bandpass or lowpass filter as we do here. So for a complex time series for velocity as noted above: $G(t) = U(t) + iV(t)$.

Analogous to spectral methods, the energy of $g(t)$ is:

$$\int_{-\infty}^{\infty} |g(t)|^2 dt = C_{\psi}^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\tilde{g}(s, t_0)/s|^2 ds dt_0$$  \hspace{1cm} (B8)

where

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{|\hat{\psi}^*(f)|^2}{|f|} df$$  \hspace{1cm} (B9)

From equation B8, the wavelet energy density in the $s$-$t_0$ plane or scalogram is $C_{\psi}^{-1} |\tilde{g}(s, t_0)/s|^2$. The admissibility condition ($C_{\psi} < \infty$) insures the wavelet is compact in frequency and has zero mean. However, this condition is not exactly satisfied by the Morlet wavelet, which has a mean value of $O(10^{-10})$.

Phase is obtained by first shifting the wavelet coefficients to a fixed reference time instead of the localized $t_0$:

$$\tilde{g}(s, t) = \tilde{g}(s, t_0)e^{-i2\pi \bar{f}_s(t-t_0)} df$$  \hspace{1cm} (B10)

and then as with rotary spectra, we identify the phase from the complex coefficients for both positive and negative $s$ (for which, for example, the real component with $s > 0$ is denoted for
compactness as $\tilde{g}_r^+$):

$$\phi_u = \tan^{-1}\left(\frac{-\tilde{g}_i^+ + \tilde{g}_i^-}{\tilde{g}_r^+ + \tilde{g}_r^-}\right)$$ (B11)

$$\phi_v = \tan^{-1}\left(\frac{\tilde{g}_r^+ - \tilde{g}_r^-}{\tilde{g}_i^+ + \tilde{g}_i^-}\right)$$ (B12)

The phase associated with positive $s$ is obtained by considering only those contributions, i.e.,

$$\phi_u^+ = \tan^{-1}(\tilde{g}_i^+/\tilde{g}_r^+)$$

While phase is obtained for a given single value of $\tilde{f}_s$ from the shifted wavelet coefficients (Equations B10–B12), amplitudes are calculated from the complex tidally-bandpassed reconstructions (Equation B6) as follows:

$$A_u = |(G_r^+ + G_r^-) + i(-G_i^+ + G_i^-)|$$ (B13)

$$A_v = |(G_i^+ + G_i^-) + i(G_r^+ - G_r^-)|$$ (B14)

Similar to the calculation above, the amplitudes for positive $s$ are obtained by considering those contributions only: e.g., $A_u^+ = |G_r^+ - iG_i^+|$.

When using the wavelets for bandpass or lowpass filtering, edge effects must be taken into account as with any filter. The area free of edge effects in wavelet analysis is referred to as the cone of influence, for which (a) we use the $e$-folding level noted above and (b) the extent of good data decreases with increasing $s$. To obtain the lowpassed data, we subtract the highpassed data from the unfiltered data to minimize the amount of data lost to edge effects. Then, one practical consideration involves some noise at the Nyquist frequency. To remove this noise a two point running mean over adjacent time steps is sufficient, but then another two point running mean is applied to move the times back into alignment with the original time steps resulting in a three-point filter with weights of $[0.25 0.5 0.25]$.

In summary there are several advantages to using wavelets for bandpassing and signal reconstruction of intermittent and/or spatially-variable internal tides: (a) wavelets are selective in time and frequency, (b) the Morlet wavelet is optimally compact in both time and frequency, (c)
wavelets have a constant $Q$ across scales, and (d) a consistent method is used to obtain lowpassed variables.

**APPENDIX C**

**Estimating mode-1 structure from means over a limited depth**

Means over limited depth ranges of 500–1000 and 300–500 m for *Spray* 55 and *Spray* 56 are used to estimate the mode-1 structure of the incident, reflected, and total wave fields. The estimated mode-1, depth-integrated energies are obtained from depth-mean amplitudes ($A$) in longitude-latitude bins (Section 4) or depth-mean amplitudes from the directional wave fits (Section 5):

$$E_k = \langle \langle \sigma_{\theta o} \rangle \rangle_H (\langle A_u \rangle_z^2 + \langle A_v \rangle_z^2) b_p I_p / 4 = B_k (\langle A_u \rangle_z^2 + \langle A_v \rangle_z^2)$$

**C1**

$$E_p = \langle \langle \sigma_{\theta o} \rangle \rangle_H (\langle N^2 \rangle_z) \langle A_\eta \rangle_z^2 b_p I_h / 4 = B_p \langle A_\eta \rangle_z^2$$

where $\langle \cdot \rangle$ will denote a mission (i.e., time) mean. Additional averaging in depth from either (a) the surface to the mission-mean bottom depth ($\langle -H \rangle$) is written as $\langle \cdot \rangle_H$ or (b) over the limited depth described above is $\langle \cdot \rangle_z$. Rather than allowing spatial variability, we use $\langle -H \rangle$ and the mission-mean mode structure found from the surface to $\langle -H \rangle$: $\langle \tilde{h}(z) \rangle$ and $\langle \tilde{p}(z) \rangle$ for displacements and pressure. The ratio of the maximum mode amplitude (i.e., 1) to the amplitude over the depth range measured by the glider is: $b_h = 1 / \langle \tilde{h} \rangle_z$. The depth-integrated squared mode structure for displacement is:

$$I_h = \int_{-\langle H \rangle}^{0} \langle \tilde{h}(z) \rangle^2$$

**C2**

The estimated mode-1 amplitude is $\langle A_\eta \rangle_z b_h$. The calculations for mode-1 pressure or velocity are identical. When using depth-mean velocities from *Spray* 55 data, the depth in consideration...
is actually the full profiling range of 0–1000 m. Values for these quantities are noted in Table 1. Finally, the scaling factors for the bandpassed tidal variances are: \[ B_{E_k} = \langle \langle \sigma_{\theta_o} \rangle \rangle H b_p^2 I_p / 4 \] and \[ B_{E_p} = \langle \langle \sigma_{\theta_o} \rangle \rangle H \langle \langle N^2 \rangle \rangle H b_h^2 I_h / 4. \]

APPENDIX D

Antenna sensitivity

The sensitivity of each survey pattern or internal-wave antenna is tested with synthetic data to evaluate uncertainties in the incident (westward, \( w \)) and reflected (eastward, \( e \)) waves’ amplitudes \((A_w, A_e)\), propagation directions \((\alpha_w, \alpha_e)\), and \( \lambda \). Only \( \alpha_e \) and phase difference \((\Delta \phi = \phi_w - \phi_e)\) are varied in these tests. The synthetic data are sampled in space and time along the tracks of Spray 55 and Spray 56. The synthetic waves have \( \lambda = 150 \text{ km} \) and unit magnitude. \( \alpha_w \) is fixed at 150° for these tests similar to the west-northwestward incident wave found in our data. For these sensitivity tests, \(-90^\circ < \alpha_e < 90^\circ\), i.e., from south to east to north. From the coastal geometry and our results, it is most likely \(| \alpha_e | < 30^\circ\). \( \Delta \phi \) goes from -180° to 180°.

To illustrate the method, we consider representative examples for Spray 55 and Spray 56 with \((\alpha_w, \alpha_e) = (150^\circ, 0^\circ)\) and \( \Delta \phi = 0^\circ, 90^\circ, 180^\circ, \) and -90° (Figures 15–16). The amplitudes and directions of the incident and reflected waves in the synthetic data are noted in gray. The blue vector identifies maximum \( A_w \) and \( A_e \) at any \( \lambda \), while the purple vector is the maximum at the wavelength of interest, 150 km. Each antenna can discern the incident and reflected waves, as evidenced by the vector differences. Highest amplitudes are seen in sectors of roughly \( \pm 15^\circ \) and \( \lambda = 150 \pm 30 \text{ km} \). Similar results are found for single test waves but are not shown. Substantial
differences in amplitude are found for differing $\Delta \phi$, but for a given $\Delta \phi$ both $A_w$ and $A_e$ increase similarly. The direction of the waves is relatively unaffected by $\Delta \phi$.

Similar tests are run over the full range of $\Delta \phi$ and $\alpha_e$, which are then summarized in Figures 17–18. Maximum $A_w$ and $A_e$ at 150 km are noted along with the corresponding $\alpha_w$ and $\alpha_e$. The sensitivity of these quantities are examined below. For both antennae and for the most likely reflected wave direction, $|\alpha_e| < 30^\circ$, both $\alpha_w$ and $\alpha_e$ are within about $15^\circ$ of their synthetic values (Figures 17b, 17d, 18b, and 18d).

Amplitude variability is more complex. $A_w$ and $A_e$ vary by roughly $\pm 50\%$ for both Spray 55 and Spray 56 over the full range of parameter space (Figures 17a, 17c, 18a, and 18c). For specified $\alpha_e$ and $\Delta \phi$, for both antennae, $A_w$ and $A_e$ vary similarly and therefore have little effect on estimates of reflectivity based on one variable, i.e, $\eta$ and then $E_p$. The patterns for $A_w$ and $A_e$ are not the same because of imperfect identification of maximum $A$ at a given $\lambda$. However, when combining $E_p$ and $E_k$, the underlying waves will have different $\Delta \phi$ in theory (Appendix A) with $\Delta \phi_\eta \sim 0^\circ$ for a coastal antinode and $\Delta \phi_\eta \sim 180^\circ$ for a coastal node. For a worst case scenario with Spray 56, we again find for $\Delta \phi$ which differ by $180^\circ$, $A_\mu$ could be 1.5 at $(\alpha_e, \Delta \phi) = (30^\circ, -120^\circ)$, while $A_\eta$ could be 0.5 at $(\alpha_e, \Delta \phi) = (30^\circ, 60^\circ)$. Thus, $E_k$ and $E_p$ could have uncertainties of over $100\%$. However, $R$ estimated from a single quantity, such as $E_k$ or $E_p$, has much smaller uncertainties from variations in $\Delta \phi$, since $A_w$ and $A_e$ vary in concert.

Another estimate of $A$ uncertainty comes from the angular sensitivity seen for a test wave of $\pm 15^\circ$ (Figures 15–16), which is similar to what appears in the data. By altering $\alpha_e = 0^\circ$ by $\pm 15^\circ$ in the synthetic data (Figures 17a, 17c, 18a, and 18c), $A$ changes by $\pm 0.2$ for a $20\%$ uncertainty in $A$ or about $40\%$ in energy or reflectivity.
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TABLE 1. Quantities used to estimate mode-1 energy density and flux magnitude (Appendix C) from glider observations over depths from 500–1000 m for Spray 55 and 300-500 m for Spray 56.

<table>
<thead>
<tr>
<th>glider</th>
<th>⟨H⟩</th>
<th>⟨⟨σθ⟩⟩_{H}</th>
<th>⟨⟨N^2⟩⟩_{H}</th>
<th>b_θ</th>
<th>b_p</th>
<th>I_h</th>
<th>I_p</th>
<th>B_{Ep}</th>
<th>B_{Ek}</th>
<th>c_g</th>
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<tbody>
<tr>
<td></td>
<td>[m]</td>
<td>[kg m^{-3}]</td>
<td>[\times 10^{-6} s^{-2}]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[kg m^{-3} s^{-2}]</td>
<td>[\times 10^5 kg m^{-3}]</td>
<td>[m s^{-1}]</td>
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<td>Spray 55</td>
<td>3026</td>
<td>27.4</td>
<td>6.4</td>
<td>1.3</td>
<td>1.4</td>
<td>1428</td>
<td>959</td>
<td>3.9</td>
<td>4.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Spray 56</td>
<td>2641</td>
<td>27.3</td>
<td>5.1</td>
<td>2.0</td>
<td>1.1</td>
<td>1250</td>
<td>1075</td>
<td>6.5</td>
<td>3.3</td>
<td>1.5</td>
</tr>
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</table>
TABLE 2. Amplitudes and estimated mode-1 energy density and flux magnitude for incident (westward, W) and reflected (eastward, E) waves from directional fits (Section 5 and Figures 12–13).

<table>
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<tr>
<th>glider wave</th>
<th>direction</th>
<th>( A_\eta ) [m]</th>
<th>( A_u ) [cm s(^{-1})]</th>
<th>( A_v ) [cm s(^{-1})]</th>
<th>( E_p ) [kJ m(^{-3})]</th>
<th>( E_k ) [kJ m(^{-3})]</th>
<th>( F ) [kW m(^{-1})]</th>
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<tr>
<td>Spray 55 E</td>
<td>4.7</td>
<td>2.7</td>
<td>2.4</td>
<td>0.09</td>
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<tr>
<td>Spray 56 W</td>
<td>3.1</td>
<td>5.6</td>
<td>3.7</td>
<td>0.06</td>
<td>1.5</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>Spray 56 E</td>
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<td>5.1</td>
<td>3.2</td>
<td>0.06</td>
<td>1.2</td>
<td>1.8</td>
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<td>Summary of Spray 56 directional wave fit tests</td>
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FIG. 1. To illustrate the geometry of the internal tides (a) incident upon and (b) reflected from a coastal wall (dashed line), the instantaneous cross-slope velocity component of an idealized mode-1 Poincaré wave with a constant 150-km wavelength is shown superimposed on a map of Tasmania. The incident and reflected wave direction are shown (black arrows). The wall (dashed line along a bearing of 18°) is aligned with the steepest section of the slope. The incident and reflected waves combine to produce a standing wave in (c). The inset in Figure 1a shows the area under consideration (red box) in subsequent figures with AUS, TAS, and NZ denoting Australia, Tasmania, and New Zealand. Topography is contoured at 1000-m intervals. The East Tasman Plateau (ETP) is the broad rise offshore, within which is a steep pinnacle, the Cascade Seamount (CS).
FIG. 2. Glider tracks for (a) Spray 55 and (b) Spray 56 are shown with color indicating time. Increasingly dark dots along the track denote increasing spring-neap phase, which is expressed in terms of days. In both cases, a wide range of phases is seen in the areas with greatest sampling. In Figure 2b, Spray 56 profiles for 2 weeks near 42.4°S (green to yellow colors).
FIG. 3. Data from Spray 56 are shown with visible $D_1$ and $D_2$ oscillations and longer duration mesoscale variations in depth-varying (a) $\theta$, (b) $S$, (c) $u$, and (d) $v$ and also in (e) depth-mean $u$ and $v$. $\sigma_\theta$ is contoured at 0.1 kg m$^{-3}$ intervals (gray lines). A weakly-stratified surface layer extends to 200–300 m (Figures 3a–b).
Fig. 4. Spray 56 data are bandpassed in the $D_2$ band: (a) $\eta$, (b) $u$, (c) $v$, and (d) depth-mean $u$ and $v$. Similarly, data are bandpassed in the $D_1$ band: (e) $\eta$, (f) $u$, (g) $v$, and (h) depth-mean $u$ and $v$. $D_1$ and $D_2$ amplitudes appear roughly equal. $D_1\eta$ and velocities are large at the same time indicative of a forced or trapped wave, while $D_2\eta$ and velocities are large at different times (or places, since the glider is moving) supportive of the idea of a standing wave.
FIG. 5. The synthetic wave field in Figure 1 is plotted in cross- and alongshore coordinates with a wall at cross-shore distance = 0 km. The total amplitude ($A_u$) is in color contours and scaled by the amplitude of the incident wave ($A_{uo}$). (a) The absolute value of the time-dependent $u$ measured by the synthetic glider is scaled and plotted as colored dots and (b) these data are then bandpassed to produce amplitudes, which are then scaled as above. The synthetic glider’s spatial pattern reveals a standing wave.
FIG. 6. The mean cross-shore structure of (a) $A_p$, (b) $A_u$, and (c) $A_v$ are obtained by averaging the analytic (black) and synthetic glider (blue) data (e.g., Figure 5) in the alongshore direction. Amplitudes are scaled by the incident wave amplitude. $p$ antinodes and $u$ nodes are found at distances of 0, 75, and 150 km offshore. $u$ and $v$ antinodes are slightly offset. The coastal wall is at cross-shore distance = 0 km.
FIG. 7. Polar diagrams show directional $D_2$ wave fits to synthetic glider data (e.g., Figure 5): (a) $A_p$, (b) $A_u$, and (c) $A_v$ scaled by the incident wave amplitude (color denotes amplitude) as functions of $\lambda$ (radial axis) and $\alpha$ (propagation direction, angular axis). Directions are labelled north (N), east (E), south (S), and west (W) with grid lines every 30°. Wavelength grid lines are at 50, 100, and 150 km. Maxima in $A$ at the prescribed $\lambda$ (purple vector) and at any $\lambda$ (blue vector) are used to identify the propagation direction of the incident (westward) and reflected (eastward) waves. The specified synthetic incident and reflected waves are denoted by white vectors.
FIG. 8. $D_2$ amplitudes from Spray 55’s bandpassed data are averaged in 1/8° bins to reveal the standing wave patterns for (a) $\eta$ of antinode-node-antinode extending from the slope to offshore and (b) $u$ and (c) $v$ of node-antinode-node extending from the slope to offshore. $A_\eta$ is averaged from 500–1000 m, while $A_u$ and $A_v$ are from the depth-mean velocity from 0–1000 m from differences between GPS and dead-reckoned positions. Phases vary slowly in the regions of strongest amplitudes for (d) $\eta$, (e) $u$, and (f) $v$. 
FIG. 9. $D_2$ amplitudes and phases from Spray 56’s bandpassed $\eta$, $u$, and $v$ data from 300–500 m similar to Figure 8. A 75 km scale bar indicates a half wavelength, the expected distance between either antinodes or nodes.
Fig. 10. Estimated mode-1 $D_2$ energy density is calculated using the amplitudes in Figures 8–9 and the method in Appendix C, yielding for Spray 55: (a) $E_p$, (b) $E_k$, and (c) $E_t$. Similarly for Spray 56, bin means are shown for (d) $E_p$, (e) $E_k$, and (f) $E_t$. Energies from both Spray 55 and Spray 56 (i.e., Figures 10a–f) are combined using a weighted mean based on the number of profiles, again yielding (g) $E_p$, (h) $E_k$, and (i) $E_t$. 
Fig. 11. The mean cross-shore structure of (a) $E_p$ and (b) $E_k$ are obtained from averaging the spatial pattern of $E_p$ and $E_k$ (Figures 10g–h) in the alongshore direction (orange lines). The most shoreward observations over the slope are at a cross-shore distance = 0 km. The analytic $E_p$ (black line) is shifted 15 km offshore to better match the observations and is specified to be 1 kJ m$^{-2}$ using $A_p^2$ from Figure 6. Once $E_p$ is specified this way, the resulting $E_k$ (black line) cannot be adjusted further, reaches maxima of 2.8 kJ m$^{-2}$, and has nodes at $E_p$ antinodes.
FIG. 12. Polar diagrams show directional $D_2$ wave fits to data from Spray 55 yielding (a) mean $A_\eta$ from 500–1000 m and depth-mean (b) $A_u$ and (c) $A_v$ from 0–1000 m (color denotes amplitude) as functions of $\lambda$ (radial axis) and $\alpha$ (angular axis). Similar fits are made at each depth before producing this depth mean. Directions are labelled north (N), east (E), south (S), and west (W) with grid lines every 30°. Wavelength grid lines are at 50, 100, and 150 km. Maxima in $A$ at the mean mode-1 $\lambda$ (purple vector) and at any $\lambda$ (blue vector) are used to identify the propagation direction of the incident (westward) and reflected (eastward) waves. Results are tabulated in Table 2.
FIG. 13. Directional wave fit as in Figure 12, but averaged from 300–500 m for Spray 56. Incident (westward) and reflected (eastward) waves are found. Also a northward wave is identified in $A_\eta$ with $\lambda = 100$ km.
Fig. 14. Simulated mode-1 reflectivity (colored lines) from CELT is 0.7–1 at the steepest topography near 43°S in the most energetic area (Figure 10), while the mean reflectivity over the slope from 41–45°S is 0.6. Line lengths indicate the actual bathymetry used in the solution (avoiding the East Tasman Plateau and Cascade Seamount), while the model bathymetry extends endlessly further on-/offshore at local shelf/abyssal values. The arrow indicates the incident wave direction (α = 150°). Topography is contoured at 1000-m intervals.
Fig. 15. Polar diagrams show directional $D_2$ wave fits to synthetic incident and reflected wave data sampled in space and time along Spray 55’s track. Input values are denoted by the gray vectors with $A_w$ and $A_e = 1$ (colors), $\alpha_w = 150^\circ$, $\alpha_e = 0^\circ$, and $\lambda = 150$ km. Maxima in $A$ at the mode-1 $\lambda$ (purple vector) and at any $\lambda$ (blue vector) are used to identify $\alpha_w$ and $\alpha_e$. Tests are repeated with different phase differences between incident and reflected waves: (a) $\Delta \phi = 0^\circ$, (b) $\Delta \phi = 90^\circ$, (c) $\Delta \phi = 180^\circ$, and (d) $\Delta \phi = -90^\circ$. Directions are labelled north (N), east (E), south (S), and west (W) with grid lines every $30^\circ$. Wavelength grid lines are at 50, 100, and 150 km. Angular spread appears to be $\pm 15^\circ$ for the region $A > 0.5$. 
FIG. 16. Directional wave fit test as in Figure 15, but for Spray 56.
FIG. 17. Synthetic data with fixed $A_w = A_e = 1$, $\alpha_w = 150^\circ$, and $\lambda = 150$ km, but with varying $\Delta \phi$ and $\alpha_e$ are sampled in space and time as Spray 55 did and then directional wave fits are used to identify (a) $A_w$, (b) $\alpha_w$, (c) $A_e$, and (d) $\alpha_e$ at the mean $\lambda$ (purple vectors in Figure 15). Results for realistic $|\alpha_e| < 30^\circ$ suggest maximum uncertainties of about 20% in $A$ and 15° in $\alpha$. 
FIG. 18. As in Figure 17, results from directional wave fit tests for Spray 56 (purple vectors in Figure 16) are summarized. Results for realistic $|\alpha_e| < 30^\circ$ suggest maximum uncertainties of about 40% in $A$ and 15° in $\alpha$. 