Bottom Drag, eddy diffusivity, wind work and the power integrals

Bill Young, Andrew Thompson & Paola Cessi
Scripps Institution of Oceanography

MIT, October 2005
Two-layer baroclinic turbulence

The eddy diffusivity of heat is:

\[ D_\tau \equiv U^{-1} \langle \psi_x \tau \rangle \]

\( above \langle \rangle = x, y, t \) average

Dimensional considerations (HH80) show:

\[ D_\tau = U\lambda \times D^*_\tau \left( \frac{\lambda}{L}, \frac{\kappa\lambda}{U}, \frac{\beta\lambda^2}{U}, \frac{\nu}{UL^7} \right) \]

This is the cleanest example of the eddy diffusivity problem.

\[
\begin{align*}
\nabla^2 \psi_t + J(\psi, \nabla^2 \psi) + J(\tau, \nabla^2 \tau) + U \nabla^2 \tau_x &= -\kappa \nabla^2 (\psi - \sqrt{2} \tau) - \nu \nabla^8 (\nabla^2 \psi), \\
(\nabla^2 - \lambda^2) \tau_t &+ J(\psi, (\nabla^2 - \lambda^2) \tau) + J(\tau, \nabla^2 \psi) + U (\nabla^2 + \lambda^{-2}) \psi_x \\
&= -\kappa \nabla^2 (2\tau - \sqrt{2} \psi) - \nu \nabla^8 (\nabla^2 - \lambda^2) \tau.
\end{align*}
\]
The eddy diffusivity of heat is:

\[ D_\tau \equiv U^{-1} \langle \psi_x \tau \rangle \]

Recent work on parameterizing D (LH95, HL96, LH03) results in:

\[ D_{\tau}^{LH03} = U \lambda \times 1.75 \beta_*^{-2} (1 - \beta_*)^{5/2} \]

These LH theories are based on inertial range arguments (R77, S78, S80), and ignore the power integral:

\[ U^2 \lambda^{-2} D_\tau \approx \kappa \left\langle |\nabla \psi - \sqrt{2} \nabla \tau|^2 \right\rangle \]

\[
\frac{1}{2} \frac{\partial}{\partial t} \left( \langle |\nabla \psi|^2 \rangle + \langle |\nabla \tau|^2 \rangle + \lambda^{-2} \langle \tau^2 \rangle \right) + U \lambda^{-2} \langle \psi_x \tau \rangle = \kappa \left\langle |\nabla \psi - \sqrt{2} \nabla \tau|^2 \right\rangle + \text{ssd}
\]

Time rate of change of energy  
Energy production  
Ekman dissipation  
Hyperm-viscosity
D from 72 simulations with $\beta=0$

\[
D = U\lambda \times D_* \left( \frac{L}{\lambda}, \frac{\kappa\lambda}{U}, 0 \right)
\]

The dashed curve is:
\[
D_* \approx 2.4 \exp \left( \frac{2U}{3\kappa\lambda} \right)
\]

With $\beta=0$, $D$ is very sensitive to bottom drag --- the inverse cascade is halted only by bottom drag.
Now the $\beta$-effect

The inverse cascade of the barotropic mode can be arrested by $\beta$ (without dissipation).

$$\ell_{\text{Rhines}} = \sqrt{\frac{U}{\beta}}$$

Perhaps non-dissipative scaling arguments work?

$$D_{\tau}^{LH03} = U\lambda \times 1.75\beta_*^{-2}(1 - \beta_*)^{5/2}$$

However we still have: $U^2\lambda^{-2}D_{\tau} \approx \kappa \left\langle |\nabla \psi - \sqrt{2}\nabla \tau|^2 \right\rangle$
QG eddy diffusivity, non-zero $\beta$

$$D_{\tau}^{LH03} = U \lambda \times 1.75 \beta_*^{-2} (1 - \beta_*)^{5/2}$$
QG conclusions

- There is no part of the parameter space where bottom drag is not qualitatively important.
- There are two decisive parameters, $\beta$ and $\kappa$.
- The power integral is important e.g., the TY05 parametrization of $D$.
- Is there a useful generalization of the power integral to PE models?

$$U^2 \lambda^{-2} D_\tau \approx \kappa \left\langle |\nabla \psi - \sqrt{2} \nabla \tau|^2 \right\rangle$$
Idealized models of the ACC

How does the thermocline depth, \( h \), depend on external parameters? Using TEM arguments:

\[
\overline{u'b'} \approx -D_{\text{eddy}} \overline{b_y} \approx \frac{\tau_s}{f_0} \overline{b_z} \quad \Rightarrow \quad h \approx \frac{\tau L_y}{f D_{\text{eddy}}}
\]

How to find the eddy-\( D \)? For example, KJM suggest:

\[
D^{\text{KJM}}_{\text{eddy}} = L_y \times \frac{g' h}{f L_y}
\]

\[
\frac{D u}{D t} + \hat{z} \times f_0 u + \nabla p = \nabla \cdot \nu \nabla u + b \hat{z} + \gamma_s(z) \tau_s(z) \hat{x} - r \gamma_b(z) u ,
\]

\[
\frac{D b}{D t} = \nabla \cdot \kappa \nabla b ,
\]

\[
\nabla \cdot u = 0 .
\]

Bottom drag is now \( r \), and the wind stress is \( \tau \).

The zonal mean bottom velocity is determined by \( r \).

\[
\bar{u}_b(y) = \frac{\tau_s(y)}{r H}
\]
The power integral

In this problem the power integral is:

\[ \langle w'b' \rangle \approx - \langle \bar{w}\bar{b} \rangle \approx r \langle |u'_b|^2 \rangle \]

PY05  GGS74

TEM arguments give:

\[ \langle w'b' \rangle \approx - \left\langle v'b' \frac{\bar{b}_y}{b_z} \right\rangle \approx \left\langle D_{\text{eddy}} \frac{\bar{b}_y^2}{b_z} \right\rangle \]

Note: \( \bar{w} \approx w_E = - \frac{1}{f_0} \frac{d\tau_s}{dy} \) and so: \(-\langle \bar{w}\bar{b} \rangle \sim \frac{h}{H} \frac{\tau g'}{f_0 L_y} \sim r u'_b^2 \)

CPY05 “eddy transfer velocity”.
The power integral - derive GGS74

\[ \langle \gamma_s \tau_s u \rangle + \langle wb \rangle = r \langle \gamma_b |u|^2 \rangle + \langle \nu \| \nabla u \|^2 \rangle \]

Wind work \hspace{1cm} \text{PE } \rightarrow \text{ KE } \hspace{1cm} \text{Bottom drag } \hspace{1cm} \text{viscosity }

After some simplifications: \( H^{-1} \langle \tau_s \bar{u}_s \rangle \approx r \langle |u_b|^2 \rangle = r \langle \bar{u}_b^2 \rangle + r \langle |u'_b|^2 \rangle \)

(N.B. true even with topography)

How about PE \( \rightarrow \) KE???

\[ \langle wb \rangle = \langle \kappa b_z \rangle \]

very small

Now estimate the wind work using:

\[ \bar{u}_s \approx \bar{u}_b - \frac{1}{f_0} \int_{-H}^{0} \bar{b}_y(y, z) \, dz \]

Thus:

\[ -H^{-1} \left\langle \int_{-H}^{0} \bar{b}(y, z) \, dz \, w_E(y) \right\rangle \approx r \langle |u'_b|^2 \rangle \]

Note: \( \bar{w} \approx w_E = -\frac{1}{f_0} \frac{d \tau_s}{dy} \) and so:

\[ \frac{h}{H} \frac{\tau g'}{f_0 L_y} \sim r u_b^2 \]