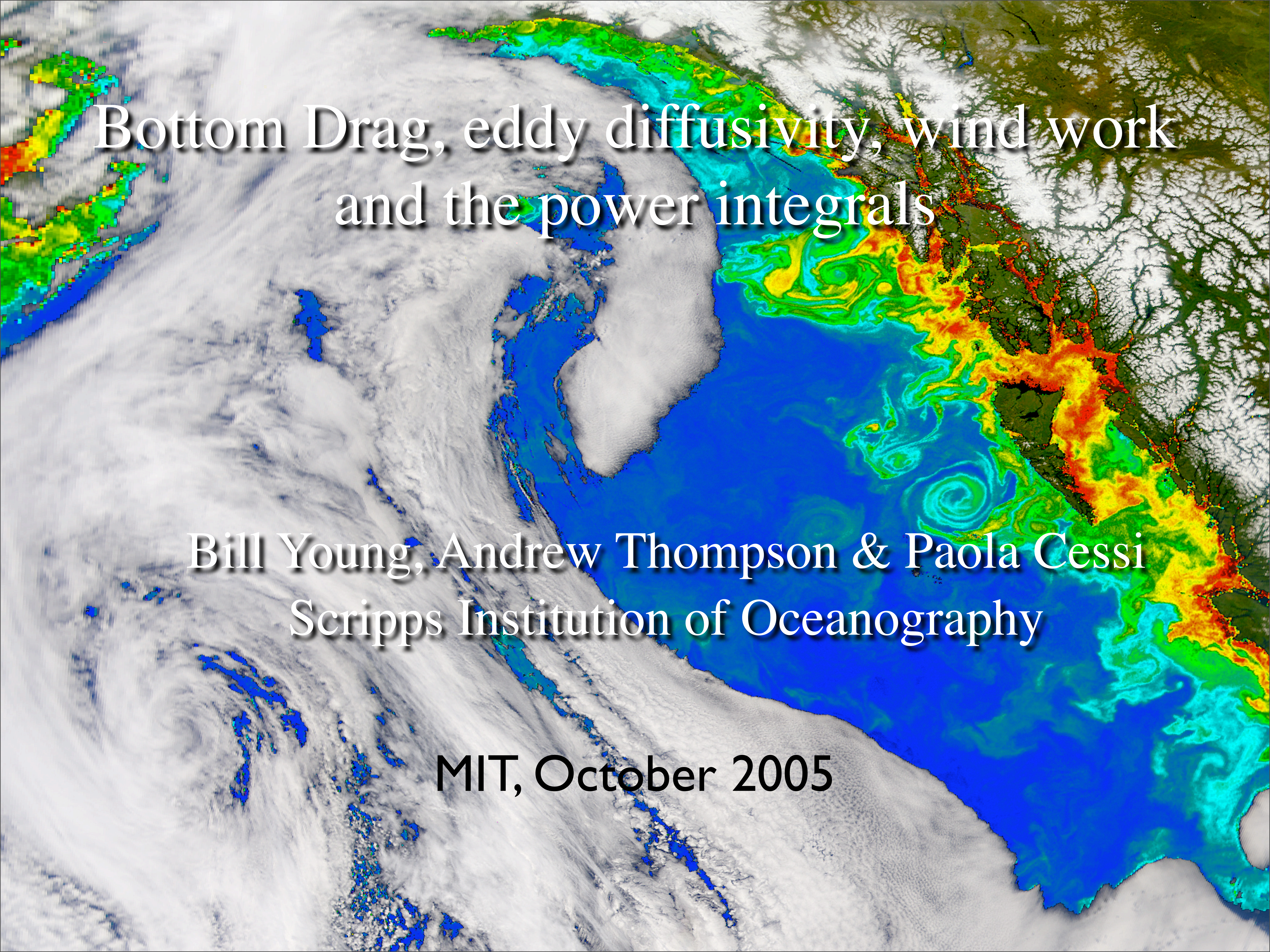
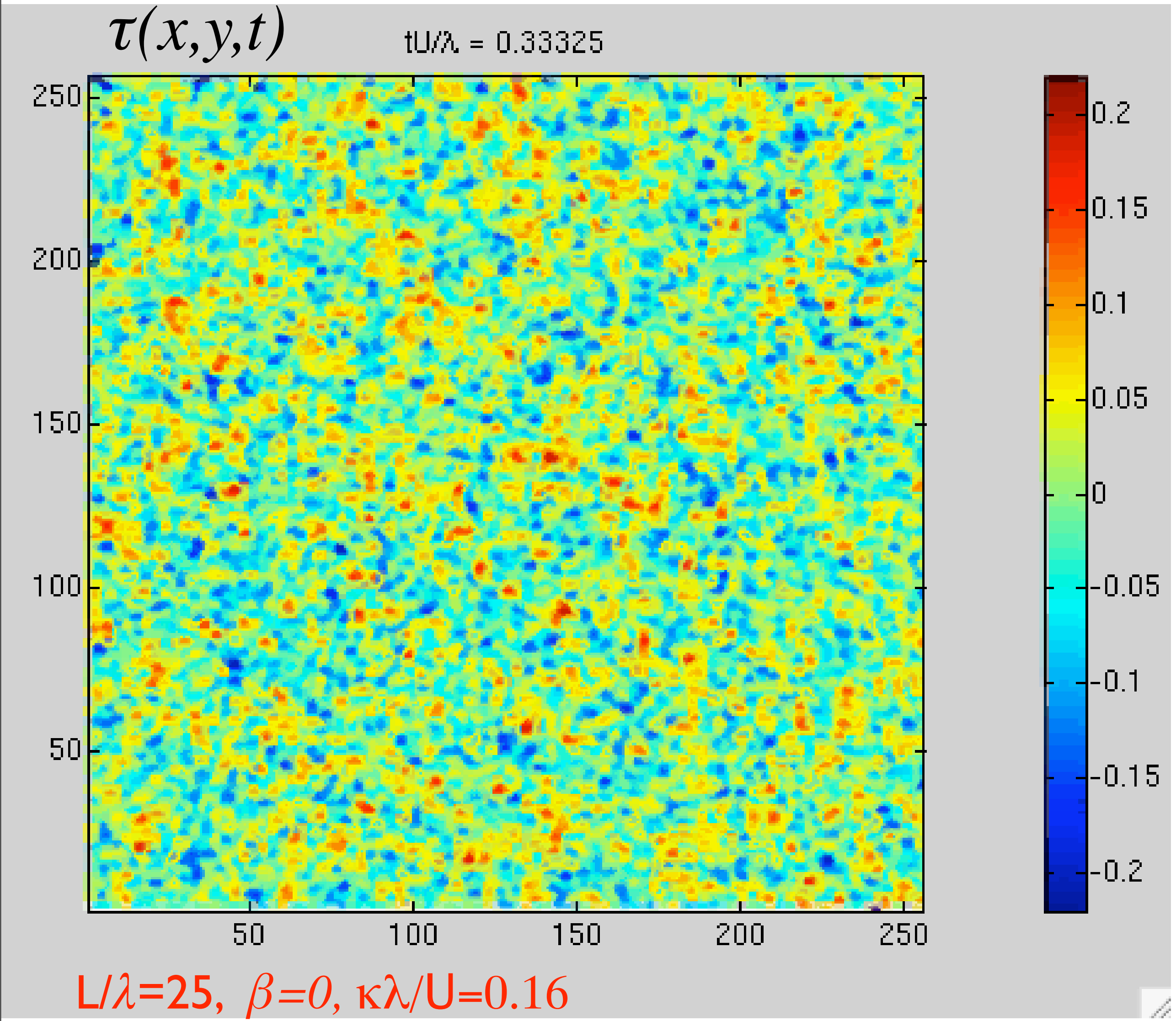


Bottom Drag, eddy diffusivity, wind work and the power integrals

Bill Young, Andrew Thompson & Paola Cessi
Scripps Institution of Oceanography

MIT, October 2005

Two-layer baroclinic turbulence



The eddy diffusivity of heat is:

$$\boxed{D_\tau \equiv U^{-1}\langle \psi_x \tau \rangle}$$

above <> = x, y, t average

Dimensional considerations (HH80) show:

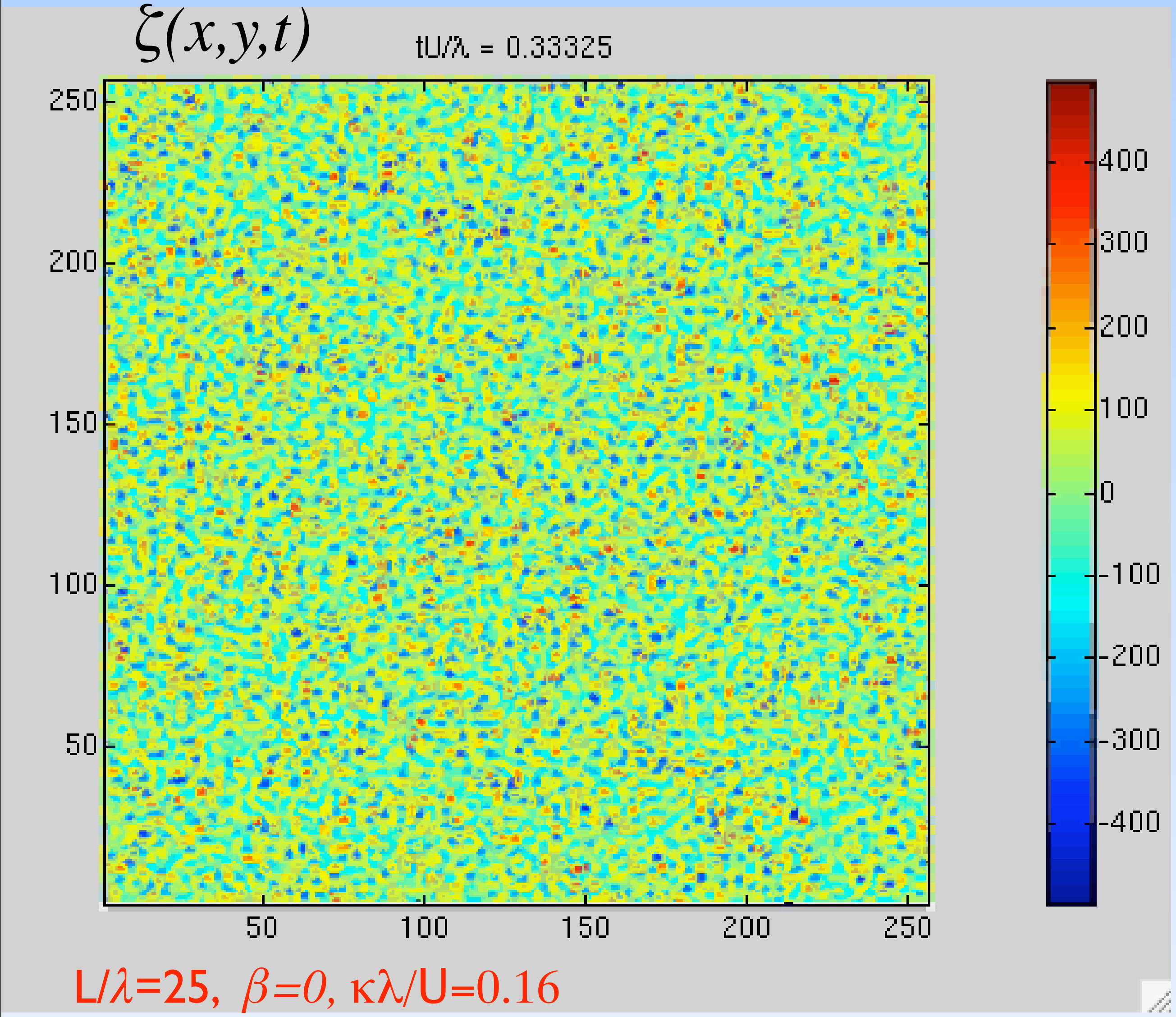
$$D_\tau = U\lambda \times D_\tau^*\left(\frac{\lambda}{L}, \frac{\kappa\lambda}{U}, \frac{\beta\lambda^2}{U}, \frac{\nu}{UL^7}\right)$$

This is the cleanest example of the eddy diffusivity problem.

$$\nabla^2\psi_t + J(\psi, \nabla^2\psi) + J(\tau, \nabla^2\tau) + U\nabla^2\tau_x = -\kappa\nabla^2(\psi - \sqrt{2}\tau) - \nu\nabla^8(\nabla^2\psi)\,,$$

$$\begin{aligned}\left(\nabla^2 - \lambda^{-2}\right)\tau_t \quad &+ \quad J(\psi, (\nabla^2 - \lambda^{-2})\tau) + J(\tau, \nabla^2\psi) + U\left(\nabla^2 + \lambda^{-2}\right)\psi_x \\ &= \quad -\kappa\nabla^2(2\tau - \sqrt{2}\psi) - \nu\nabla^8(\nabla^2 - \lambda^2)\tau\,.\end{aligned}$$

The eddy diffusivity of heat is: $\boxed{D_\tau \equiv U^{-1}\langle \psi_x \tau \rangle}$



Recent work on parameterizing D (LH95, HL96, LH03) results in:

$$D_\tau^{LH03} = U\lambda \times 1.75\beta_*^{-2}(1-\beta_*)^{5/2}$$

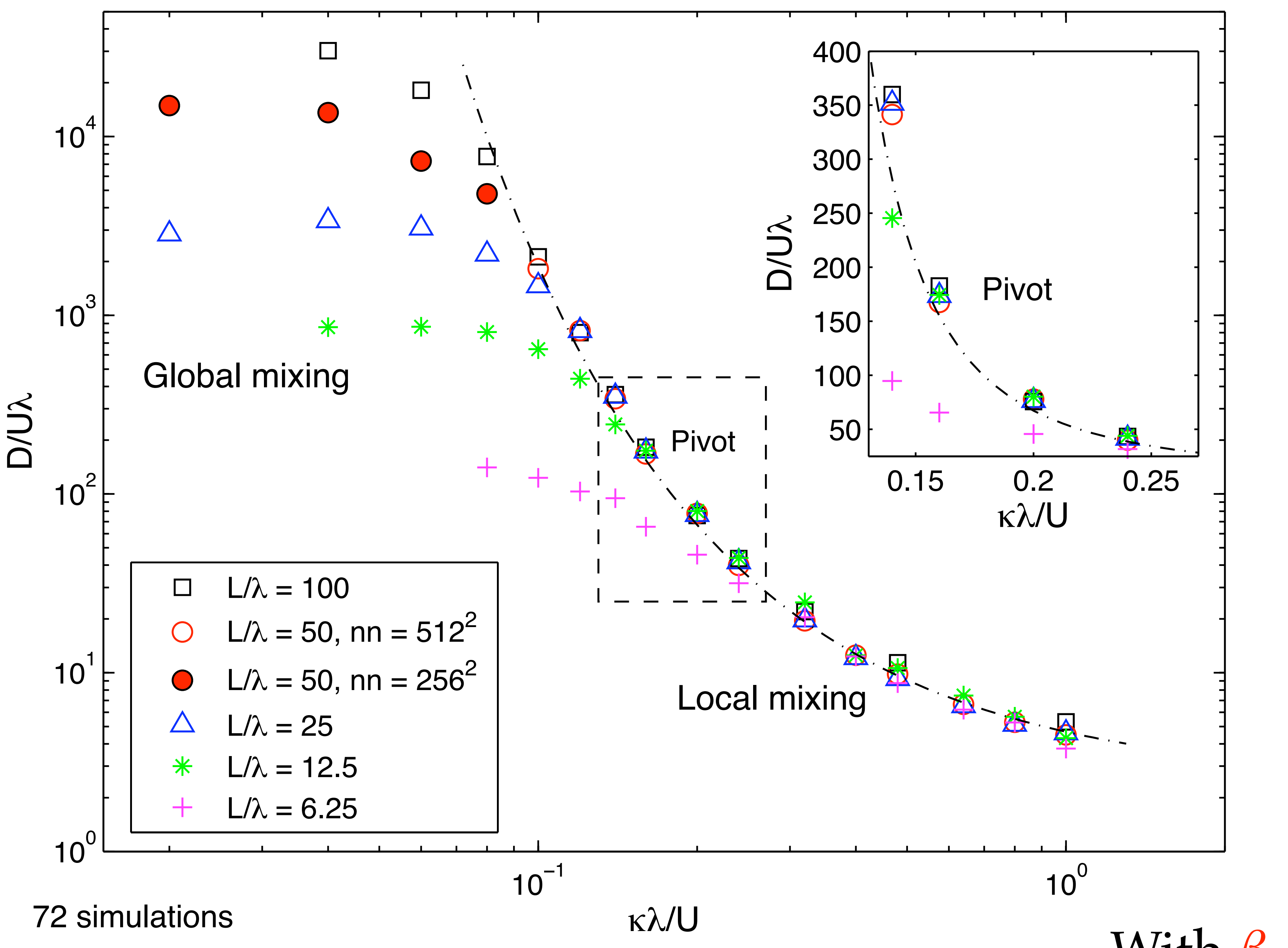
These LH theories are based on inertial range arguments (R77, S78, S80), and ignore the power integral:

$$U^2\lambda^{-2}D_\tau \approx \kappa\left\langle |\boldsymbol{\nabla}\psi - \sqrt{2}\boldsymbol{\nabla}\tau|^2\right\rangle$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\left\langle|\nabla\psi|^2\right\rangle + \left\langle|\nabla\tau|^2\right\rangle + \lambda^{-2}\left\langle\tau^2\right\rangle\right) + U\lambda^{-2}\left\langle\psi_x\tau\right\rangle = \kappa\left\langle|\nabla\psi - \sqrt{2}\nabla\tau|^2\right\rangle + ssd$$

Time rate of change of energy | Energy production | Ekman dissipation | Hyper-viscosity

D from 72 simulations with β=0



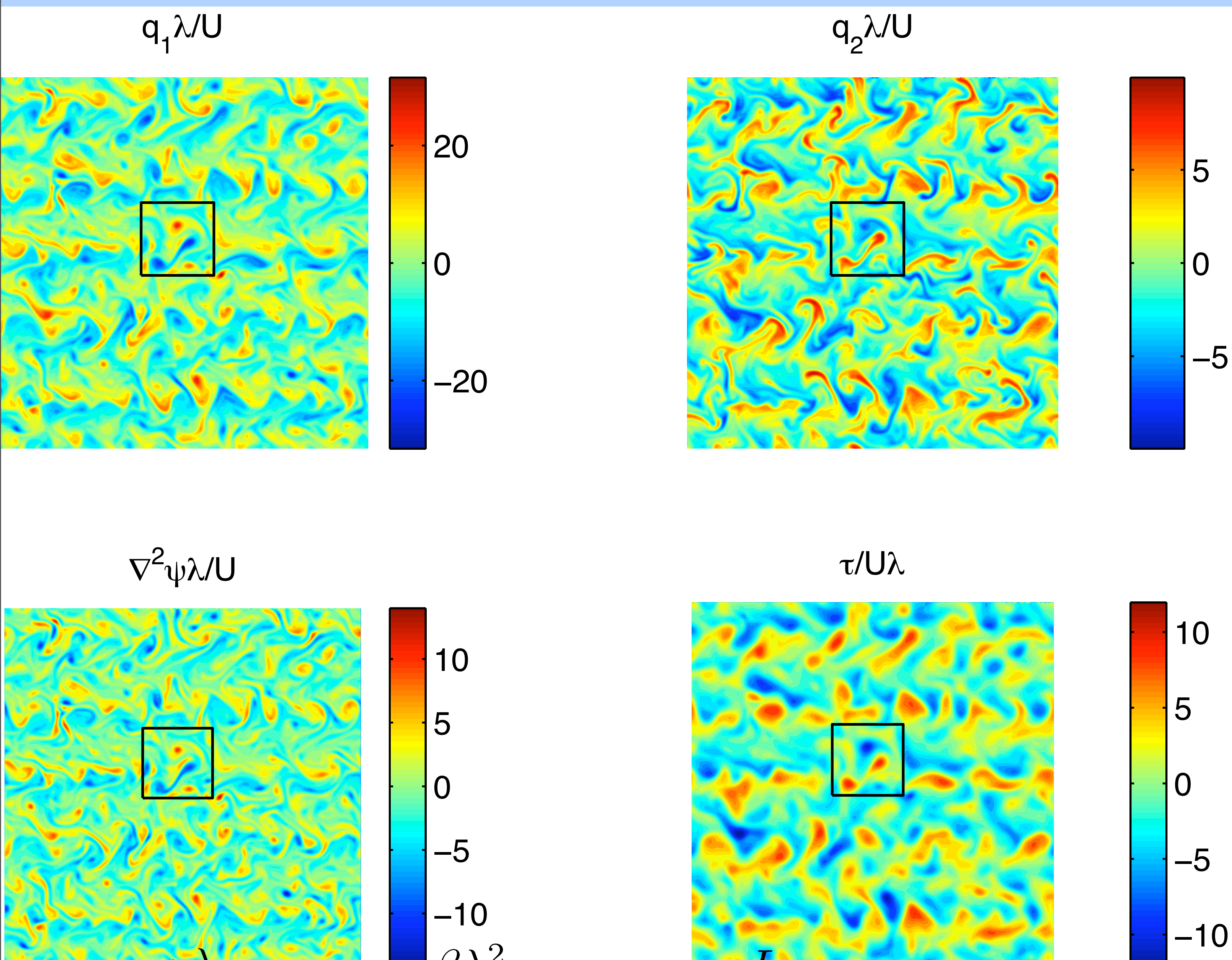
$$D = U\lambda \times D_* \left(\frac{L}{\lambda}, \frac{\kappa\lambda}{U}, 0 \right)$$

The dashed curve is:

$$D_* \approx 2.4 \exp \left(2U/3\kappa\lambda \right)$$

With β=0, D is very sensitive to bottom drag --- the inverse cascade is halted only by bottom drag.

Now the $\boldsymbol{\beta}$-effect



$$\frac{\kappa\lambda}{U} = 0.1 \qquad \frac{\beta\lambda^2}{U} = 0.5 \qquad \frac{L}{\lambda} = 20$$

The inverse cascade of the barotropic mode can be arrested by β (without dissipation).

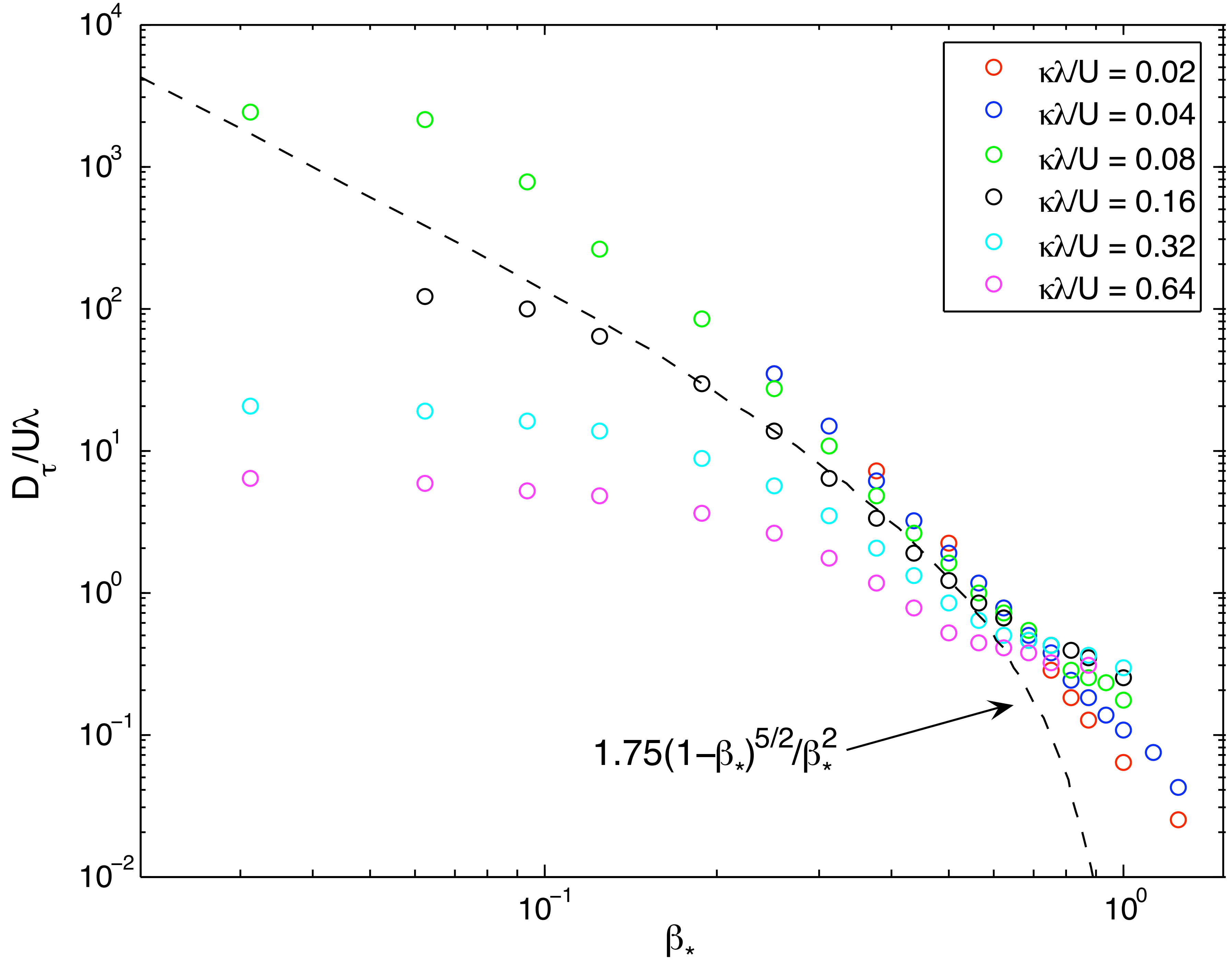
$$\ell_{\text{Rhines}} = \sqrt{\frac{U}{\beta}}$$

Perhaps non-dissipative scaling arguments work?

$$D_\tau^{LH03} = U\lambda \times 1.75\beta_*^{-2}(1-\beta_*)^{5/2}$$

However we still have: $U^2\lambda^{-2}D_\tau \approx \kappa \left\langle |\boldsymbol{\nabla}\psi - \sqrt{2}\boldsymbol{\nabla}\tau|^2 \right\rangle$

QG eddy diffusivity, non-zero $\boldsymbol{\beta}$



L/λ=25

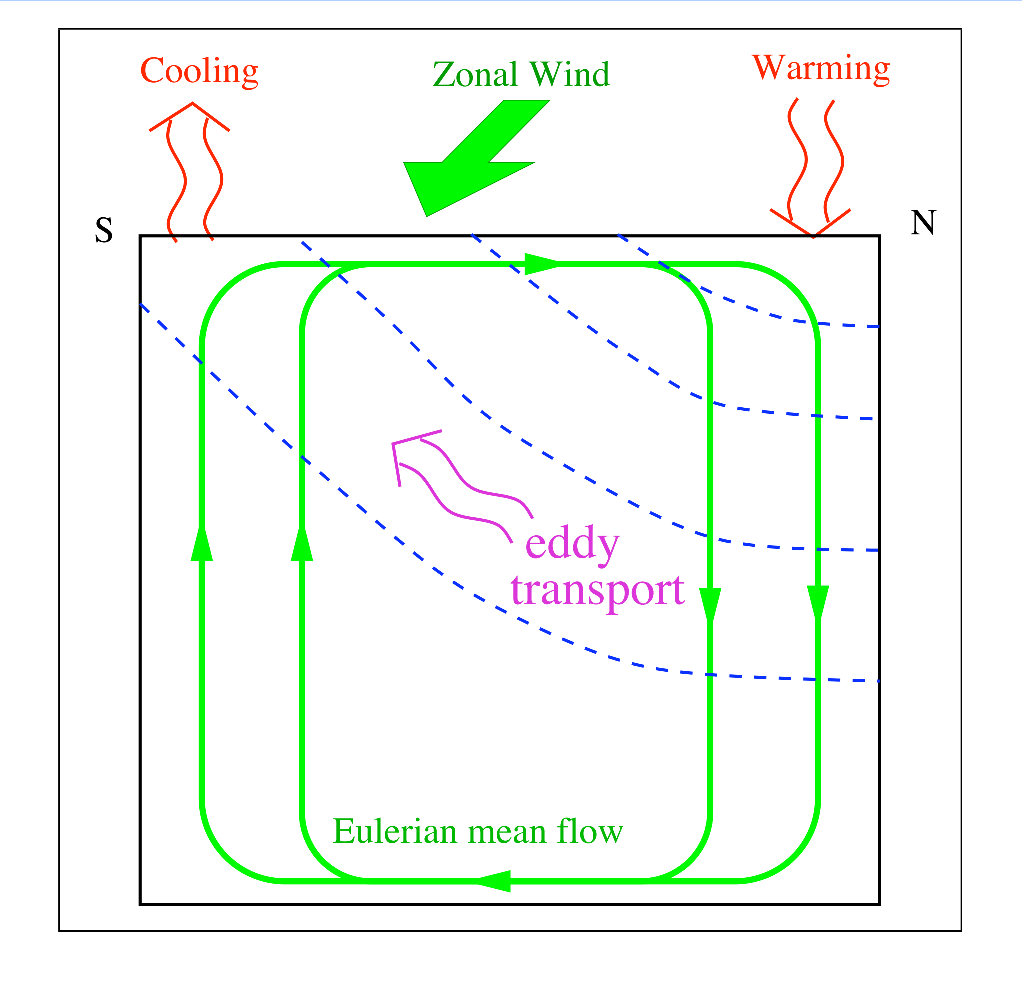
$$D_\tau^{LH03} = U\lambda \times 1.75\beta_*^{-2}(1-\beta_*)^{5/2}$$

QG conclusions

- There is no part of the parameter space where bottom drag is not qualitatively important.
- There are two decisive parameters, β and κ.
- The power integral is important e.g., the TY05 parametrization of D.
- Is there a useful generalization of the power integral to PE models?

$$U^2\lambda^{-2}D_\tau \approx \kappa\left\langle |\boldsymbol{\nabla}\psi - \sqrt{2}\boldsymbol{\nabla}\tau|^2\right\rangle$$

Idealized models of the ACC



from Karsten, Jones & Marshall (2002)

How does the thermocline depth, h, depend on external parameters? Using TEM arguments:

$$\overline{v'b'} \approx -D_{\text{eddy}}\bar{b}_y \approx \frac{\tau_{\text{s}}}{f_0}\bar{b}_z \qquad \Rightarrow \qquad h \sim \frac{\tau L_y}{f D_{\text{eddy}}}$$

How to find the eddy-D? For example, KJM suggest:

$$D_{\text{eddy}}^{\text{KJM}} = L_y \times \frac{g'h}{f L_y}$$

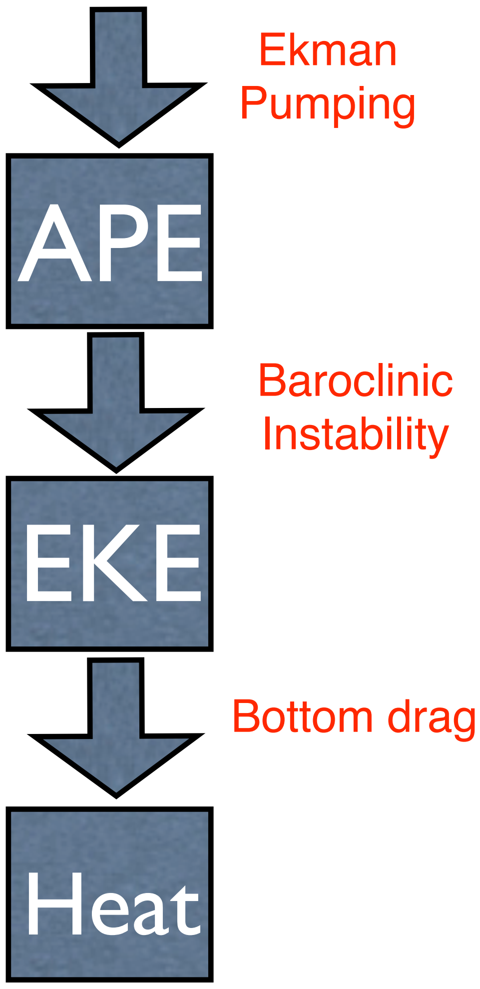
$$\begin{aligned} \frac{D\boldsymbol{u}}{Dt} + \hat{\mathbf{z}} \times f_0 \boldsymbol{u} + \boldsymbol{\nabla} p &= \boldsymbol{\nabla} \cdot \nu \boldsymbol{\nabla} \boldsymbol{u} + b\hat{\mathbf{z}} + \gamma_{\text{s}}(z)\tau_{\text{s}}(z)\hat{\boldsymbol{x}} - r\gamma_{\text{b}}(z)\boldsymbol{u} \ , \\ \frac{Db}{Dt} &= \boldsymbol{\nabla} \cdot \kappa \boldsymbol{\nabla} b \, , \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} &= 0 \, . \end{aligned}$$

Bottom drag is now r,
and the wind stress is τ.

The zonal mean bottom velocity is determined by r.

$$\bar{u}_{\text{b}}(y) = \frac{\tau_{\text{s}}(y)}{rH}$$

The power integral



In this problem the power integral is:

$$\langle w'b' \rangle \approx -\langle \bar{w}\bar{b} \rangle \approx r\langle |\boldsymbol{u}'_{\mathrm{b}}|^2 \rangle$$

PY05 GGS74

TEM arguments give:

$$\langle w'b' \rangle \approx -\left\langle v'b' \frac{\bar{b}_y}{\bar{b}_z} \right\rangle \approx \left\langle D_{\mathrm{eddy}} \frac{\bar{b}_y^2}{\bar{b}_z} \right\rangle$$

Note: $\bar{w} \approx w_{\mathrm{E}} = -\dfrac{1}{f_0} \dfrac{\mathrm{d}\tau_{\mathrm{s}}}{\mathrm{d}y}$ and so: $-\langle \bar{w}\bar{b} \rangle \sim \dfrac{h}{H} \dfrac{\tau g'}{f_0 L_y} \sim r u_{\mathrm{b}}'^2$

CPY05 “eddy transfer velocity”.

The power integral - derive GGS74

$$\langle \gamma_{\mathrm{s}} \tau_{\mathrm{s}} u \rangle + \langle w b \rangle = r \langle \gamma_{\mathrm{b}} |\boldsymbol{u}|^2 \rangle + \langle \nu \|\boldsymbol{\nabla} \boldsymbol{u}\|^2 \rangle$$

Wind work — PE $\Rightarrow$ KE — Bottom drag — viscosity

After some simplifications: $H^{-1} \langle \tau_{\mathrm{s}} \bar{u}_{\mathrm{s}} \rangle \approx r \langle |\boldsymbol{u}_{\mathrm{b}}|^2 \rangle = r \langle \bar{u}_{\mathrm{b}}^2 \rangle + r \langle |\boldsymbol{u}_{\mathrm{b}}'|^2 \rangle$

(N.B. true even with topography)

How about PE $\Rightarrow$ KE??? $\quad \langle w b \rangle = \langle \kappa b_z \rangle$

very small

Now estimate the wind work using: $\bar{u}_{\mathrm{s}} \approx \bar{u}_{\mathrm{b}} - \dfrac{1}{f_0} \displaystyle\int_{-H}^{0} \bar{b}_y(y, z)\, \mathrm{d}z$

Thus:

$$-H^{-1} \left\langle \int_{-H}^{0} \bar{b}(y, z)\, \mathrm{d}z \, w_{\mathrm{E}}(y) \right\rangle \approx r \langle |\boldsymbol{u}_{\mathrm{b}}'|^2 \rangle$$

Note: $\bar{w} \approx w_{\mathrm{E}} = -\dfrac{1}{f_0} \dfrac{\mathrm{d}\tau_{\mathrm{s}}}{\mathrm{d}y}$ and so: $\dfrac{h}{H} \dfrac{\tau g'}{f_0 L_y} \sim r u_{\mathrm{b}}^2$