Long-range propagation of ocean waves

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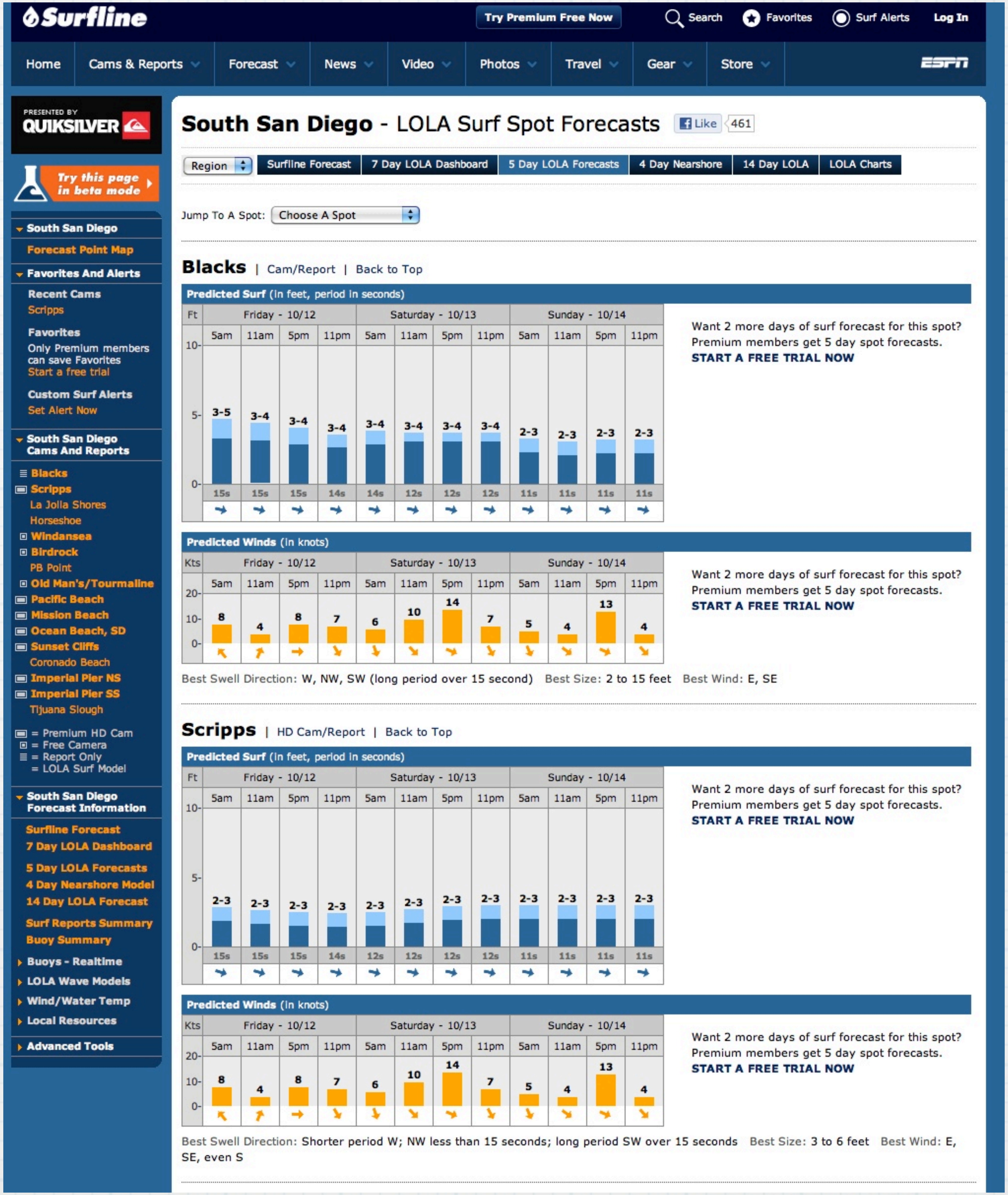
Thanks to Walter Munk, Ryan Abernathey and Falk Feddersen.

Surf and swell

Swell and surf on Sunset Beach, Oahu

http://www.surfline.com/surfdata/lola_surf_model.cfm?id=4812



World War II

- "Bad surf on the Atlantic beaches is a calculated risk." (Roosevelt to Churchill in anticipation of Operation TORCH, 1942.)

- Wave energy incident on beaches can vary by a factor of 100 in 24 hours.

- LCVPs could not operate in more than about 5ft waves.

- During the first two days of TORCH 64% of the 378 LCVPs were stranded or sunk in 6 foot breakers.

Forecasting surf was important....

SIO Digital Archive

Walter Munk and Harald Sverdrup, circa 1940

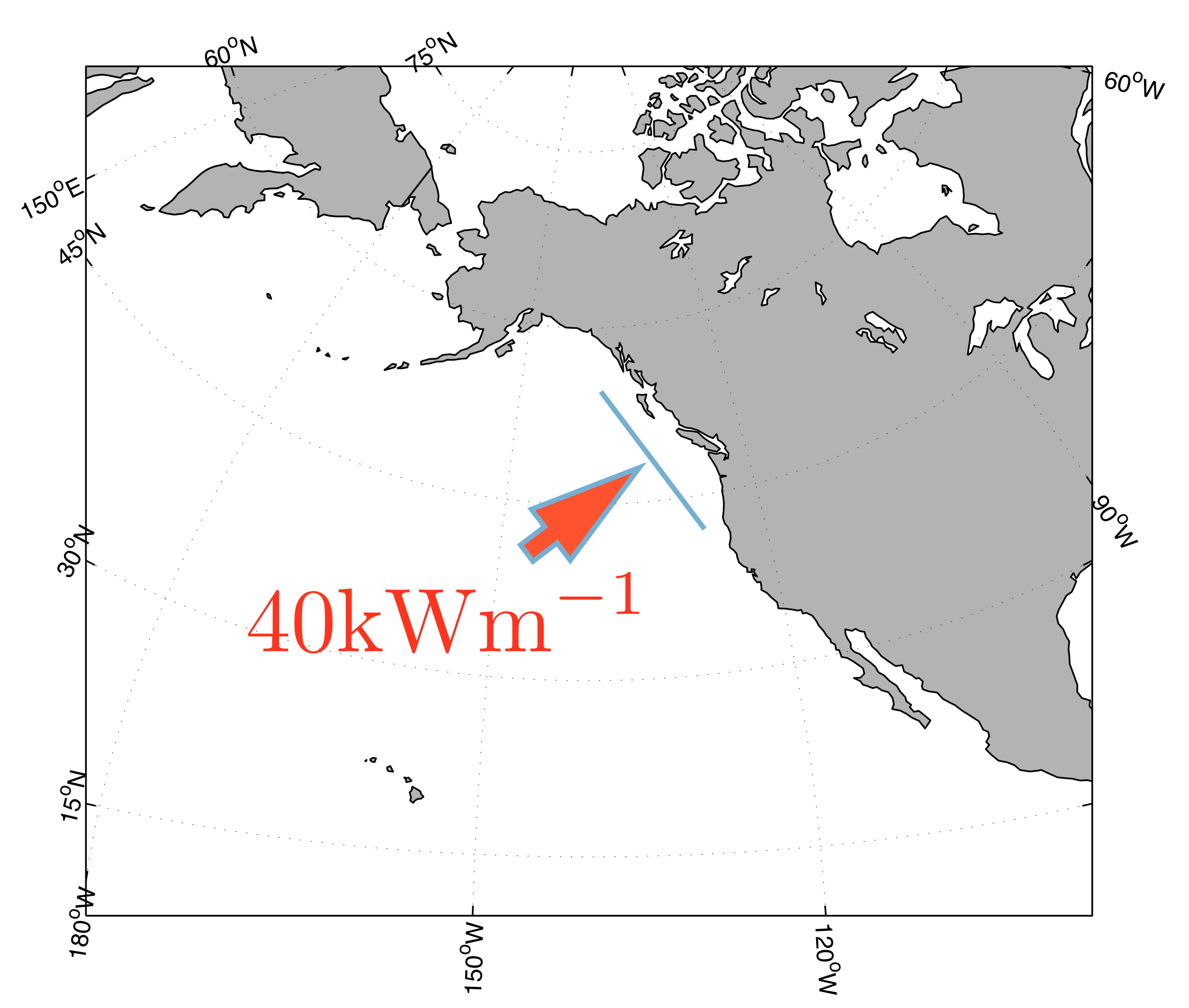
The British also conducted a wave prediction headed by Fritz Ursell and Norman Barber.

Wave Power?

$$\text{Energy Flux} = c_g \times \text{Energy Density}$$
$$= \frac{\rho g^2 T a^2}{8\pi} \quad \text{(kW/meter)}$$

With T=10sec and a = 1 meter, the energy flux is 40kW/meter.

An average 40kW/meter of wave power is typical of good sites.



Pelamis Wave Power

In the best locations, one 700 ton sea-snake delivers 300kW on average.

This natural resource is volatile - fluctuations by a factor of 1000 in 24 hours.

“Survival is more important than further increase in power output.”

Wave Power

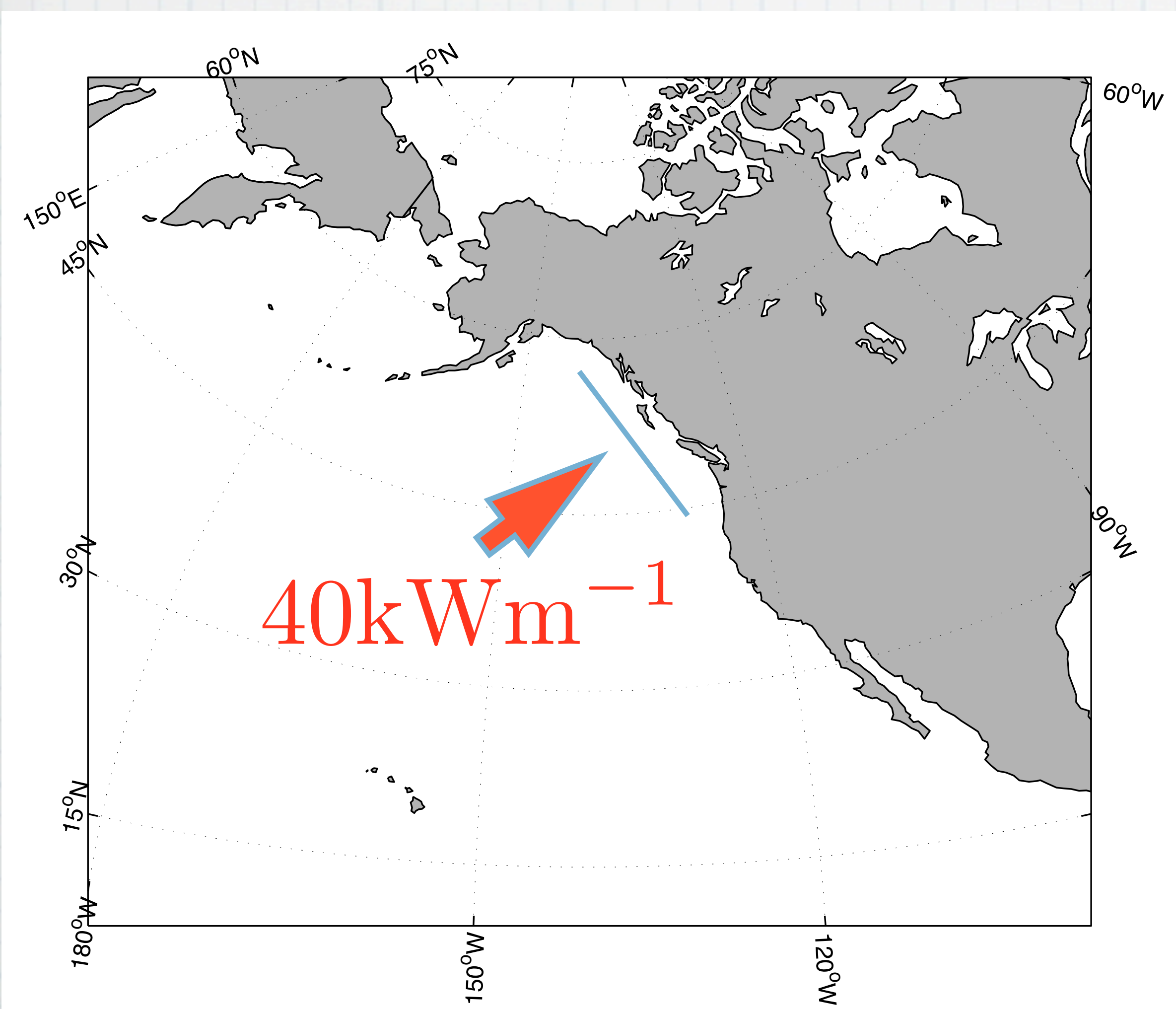
Estimates of total wind-stress forcing are:

- 20TW (Ferrari & Wunsch, 2003)
- 60TW (Wang & Huang, 2004)
- 70TW (Racsale et al., 2008)

About 2TW reaches the shore as surf (Rascale et al. 2008).

Ocean Power Technologies

A 103 foot long, 260ton buoy being tested in Vancouver, Washington. (from the New York Times, September 2012.)



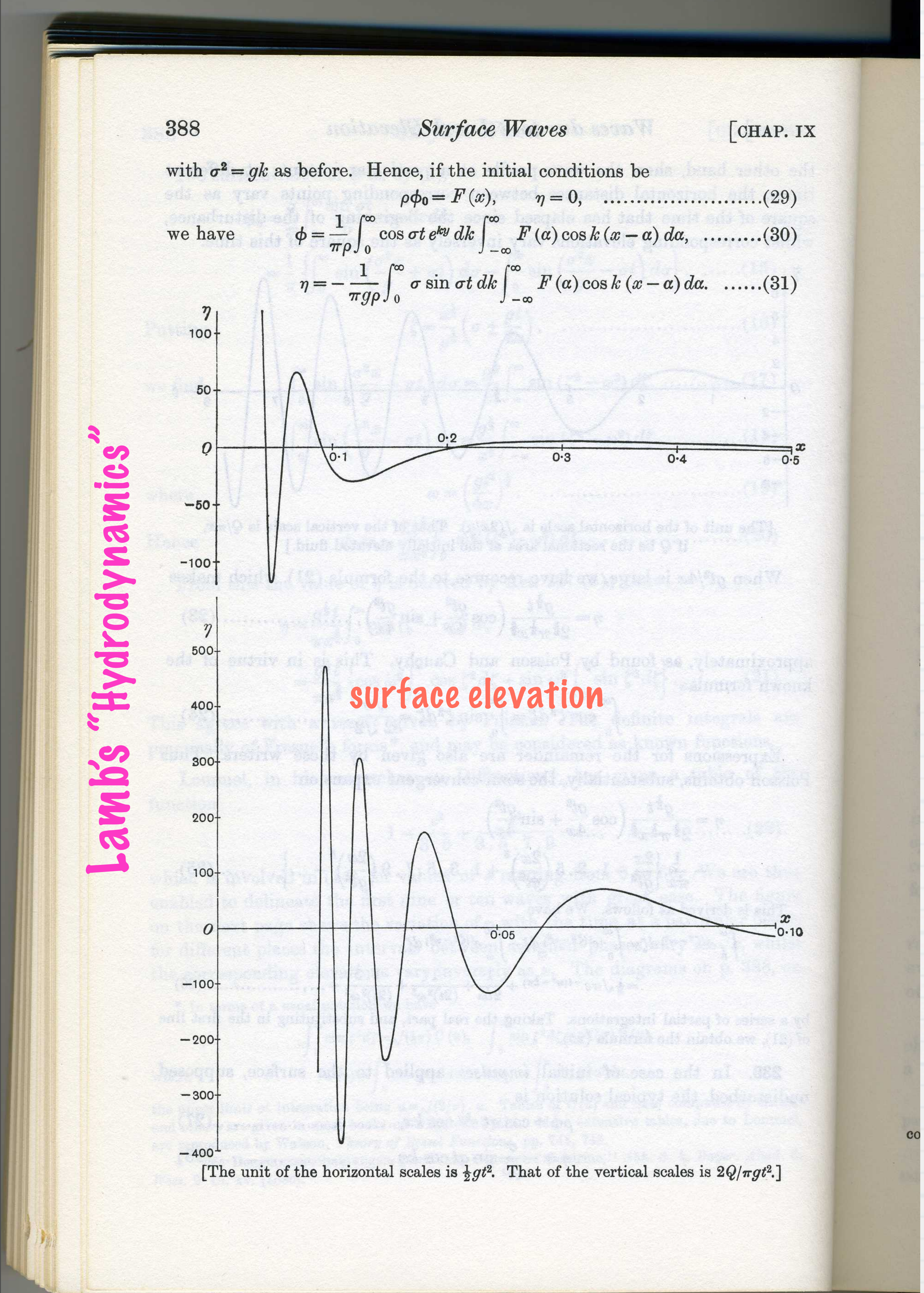
1 TerraWatt =10^{12} Watts
= 1000 nuclear power stations

25km of coastline = 1 nuclear power plant (assuming 100% efficiency)

See "Sustainable Energy - without the hot air" by David McKay

How to make a wave forecast

- **STORM:** a weather map provides U, F and D.
- **SEA:** convert U, F and D into wave height H and period T.
- **SWELL:** wave propagation (decay, dispersion and scattering) between storm and beach.
- **SURF:** the transformation of waves in shallow water.



with $\sigma^2 = gk$ as before. Hence, if the initial conditions be

$$\rho\phi_0 = F(x), \qquad \eta = 0, \qquad \ldots\ldots(29)$$

we have
$$\phi = \frac{1}{\pi\rho}\int_0^\infty \cos\sigma t\, e^{ky}\, dk \int_{-\infty}^{\infty} F(\alpha)\cos k(x-\alpha)\, d\alpha, \qquad \ldots\ldots(30)$$

$$\eta = -\frac{1}{\pi g\rho}\int_0^\infty \sigma \sin\sigma t\, dk \int_{-\infty}^{\infty} F(\alpha)\cos k(x-\alpha)\, d\alpha. \qquad \ldots\ldots(31)$$

[The unit of the horizontal scales is $\frac{1}{2}gt^2$. That of the vertical scales is $2Q/\pi g t^2$.]

The Cauchy-Poisson solution

After the STORM, or the SPLASH, dispersion sorts the waves into a slowly varying train. Long and fast waves out-run the STORM. This is SWELL.

The dispersion relation:
$$\omega = \sqrt{gk}$$

and therefore
$$\frac{\partial\omega}{\partial k} = \frac{1}{2}\frac{\omega}{k} = \frac{gT}{4\pi}$$

$$T = \text{wave period}$$

The wave propagation diagram

Barber & Ursell (1947)

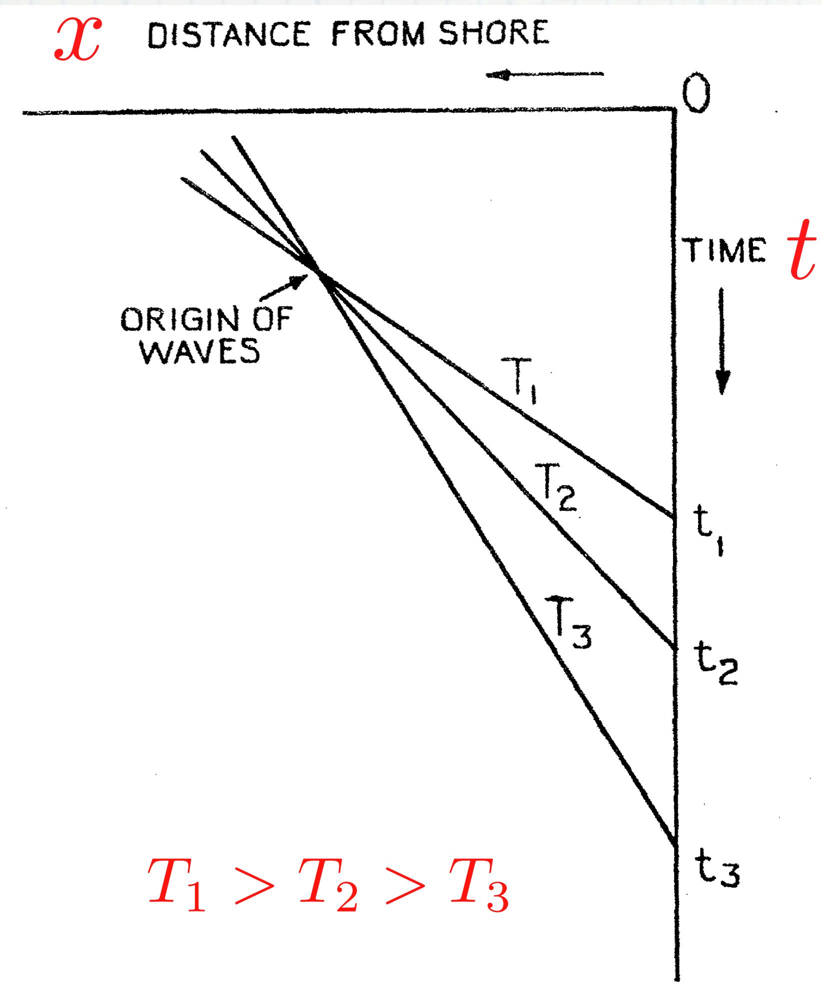


FIGURE 1. Wave propagation diagram.

$$\frac{x}{t} = \frac{gT}{4\pi}$$

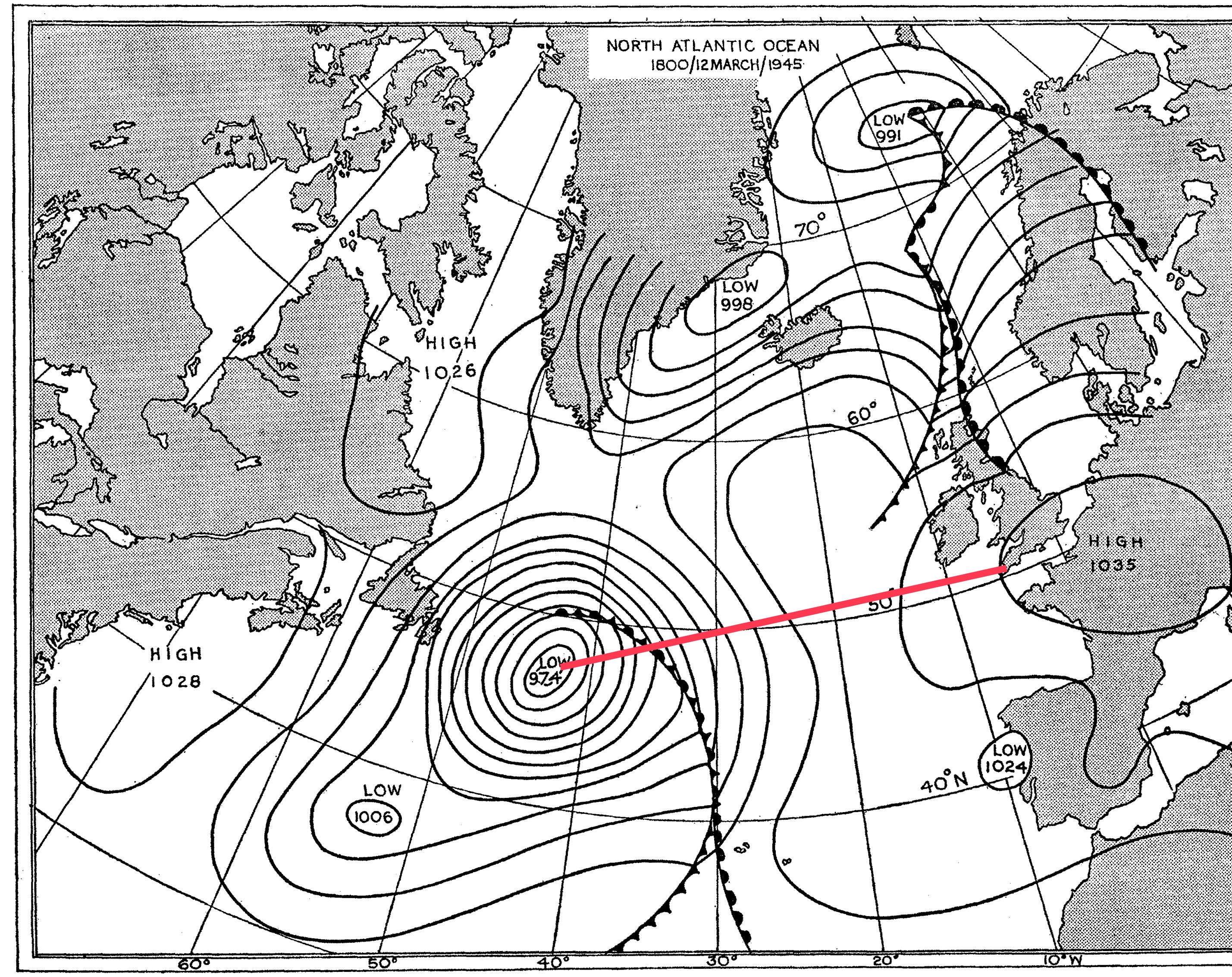


FIGURE 4. Meteorological chart of the North Atlantic Ocean 18.00 hr. 12 March 1945.

Use linear wave theory:

$$\omega = \sqrt{gk} \qquad \text{and} \qquad c_g = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{gT}{4\pi}$$

A 500 meter wave has a period of 18 seconds and a group speed of 14m/s. The wave takes 44 hours to travel 1200 nautical miles.

Linear theory is "reasonably accurate"

- Barber & Ursell (1947)

In this example the area of the storm is too great and it is too close to the observer.

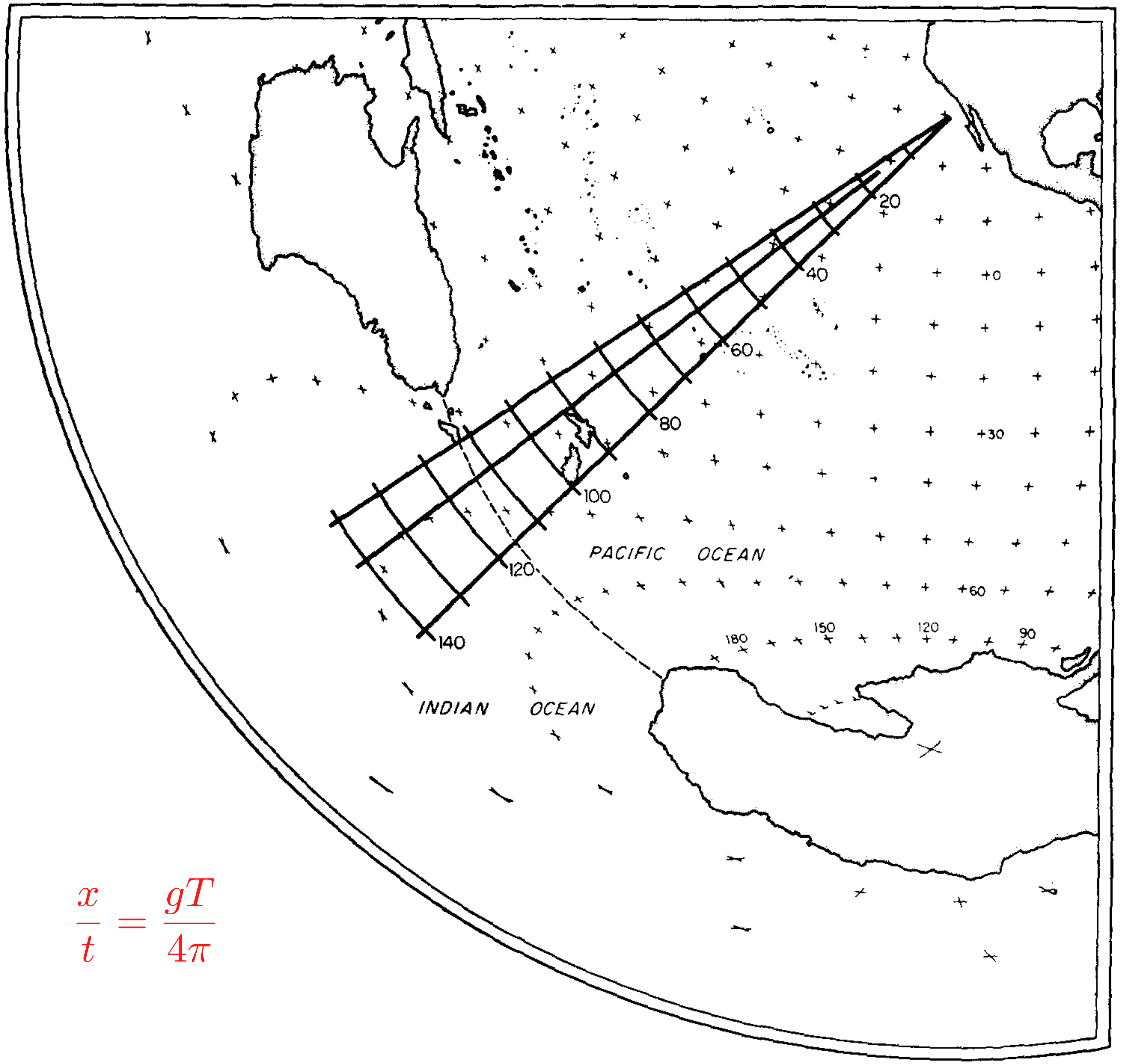
High frequency waves are generated first, and get a head start before being overtaken by longer faster waves.

One should look for examples of
very distant storms.

Enter Munk &Snodgrass (1957)

"The observed frequency shift is consistent only with very distant areas of generation in the Indian Ocean, south of Western Australia."

There are clear observations of a source with a range of 14,800 kilometers.



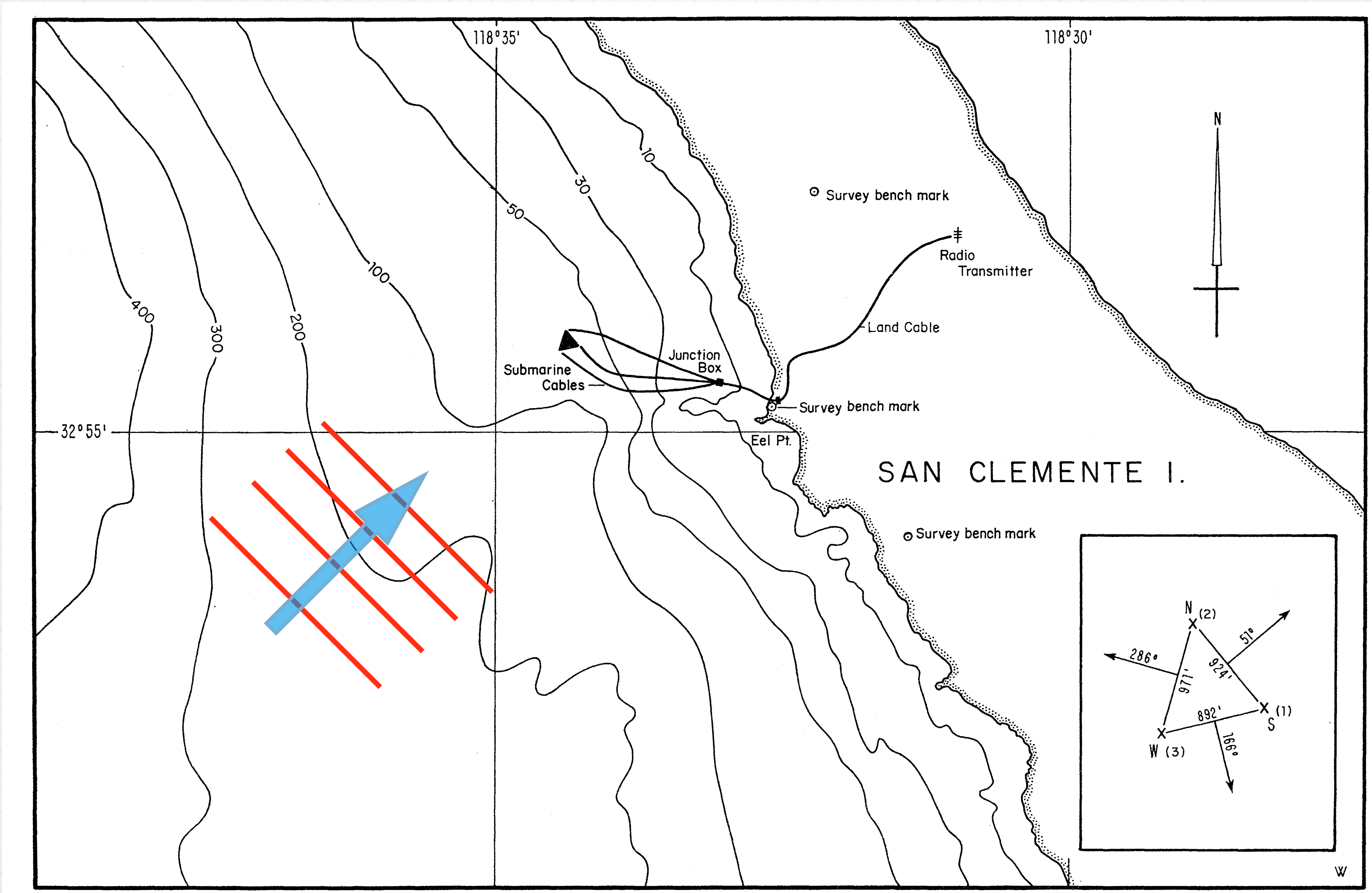
$$\frac{x}{t} = \frac{gT}{4\pi}$$

Fig. 6. Azimuthal-equidistant projection centred on San Diego. Distances from Guadalupe are given in units of degrees (1° equals 111 km).

Azimuthal projection centered on San Diego

Munk, Snodgrass and others, in 1963 (and again in 1966)

Munk, Miller, Snodgrass, and Barber (1963)

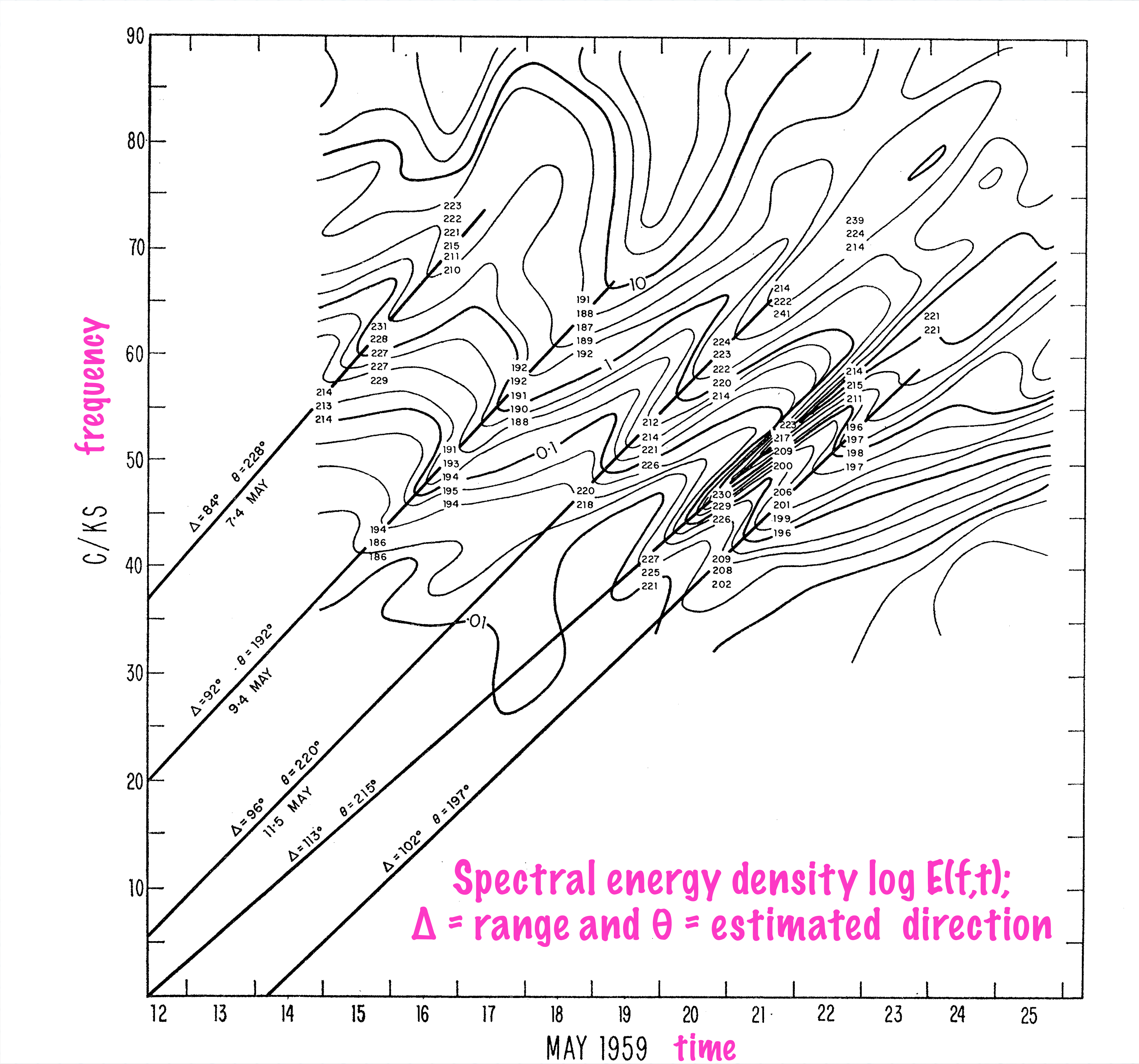


Swell incident on a triangular array of bottom pressure sensors

A glorious victory for linear wave theory

“Energy peaks associated with a given event appear as slanting ridges. The dispersion relation predicts that these ridges should be straight, and they are.”

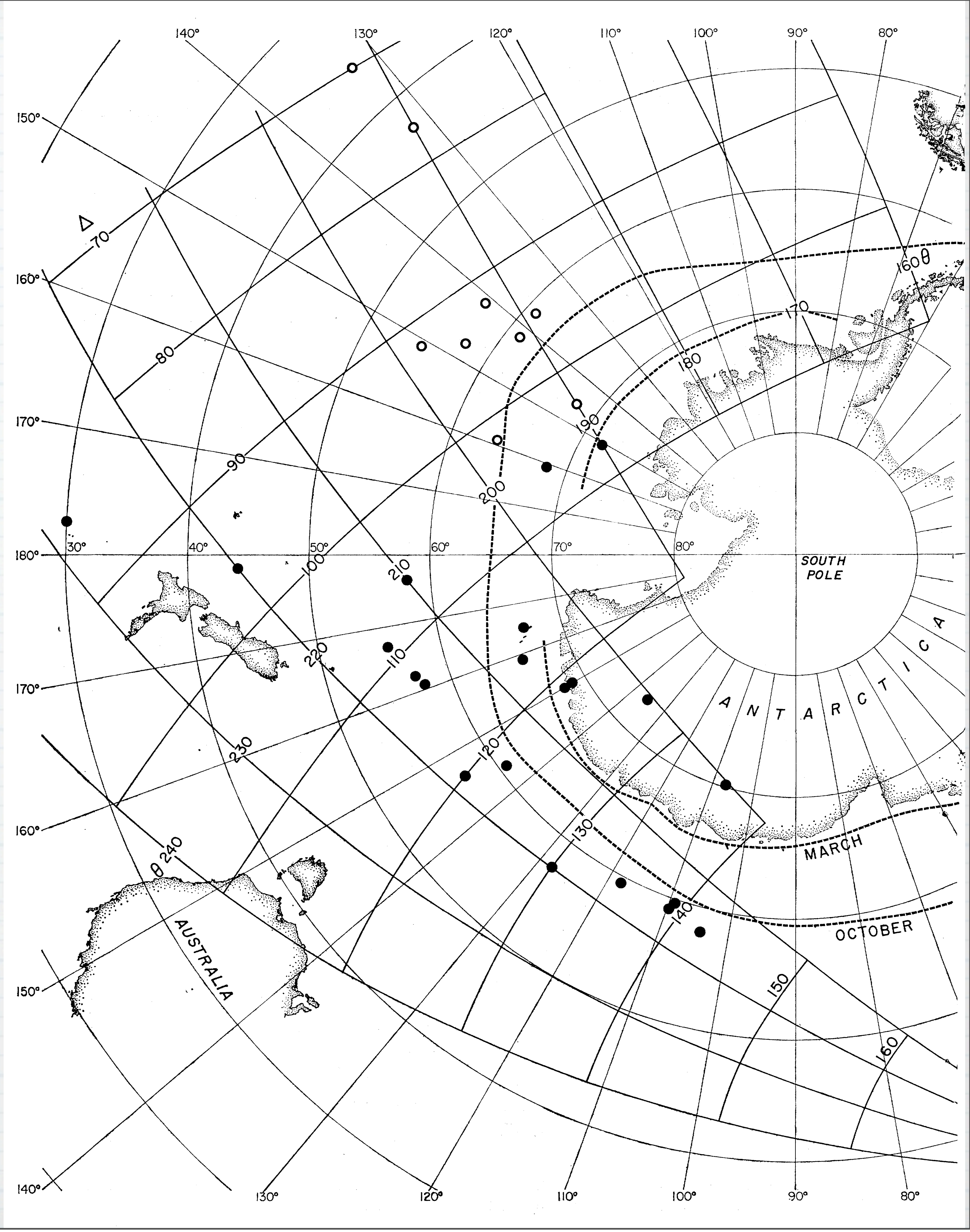
Another version of the wave propagation diagram



Munk, Miller, Snodgrass, and Barber (1963)

Sources on Antarctica!?

There is a **problem:** the inferred sources are displaced by as much as 10 degrees from storms on weather maps. Some of the inferred sources were on the Antarctic Continent (and some were under sea ice).



○ = no meteorological observations

● = supporting meteorology

Modern data

Southern californian buoys

Pitch and roll buoy: a receiver in deep water

NOAA buoy 46086

Located near San Clemente island

- Local depth of 1890 m: **deep-water measurements.**
- 3D-accelerometer: directional recording of swell.
- Height spectrum measured every hour since 2004.
- Mean direction for each frequency bin.

Back-tracking the swell

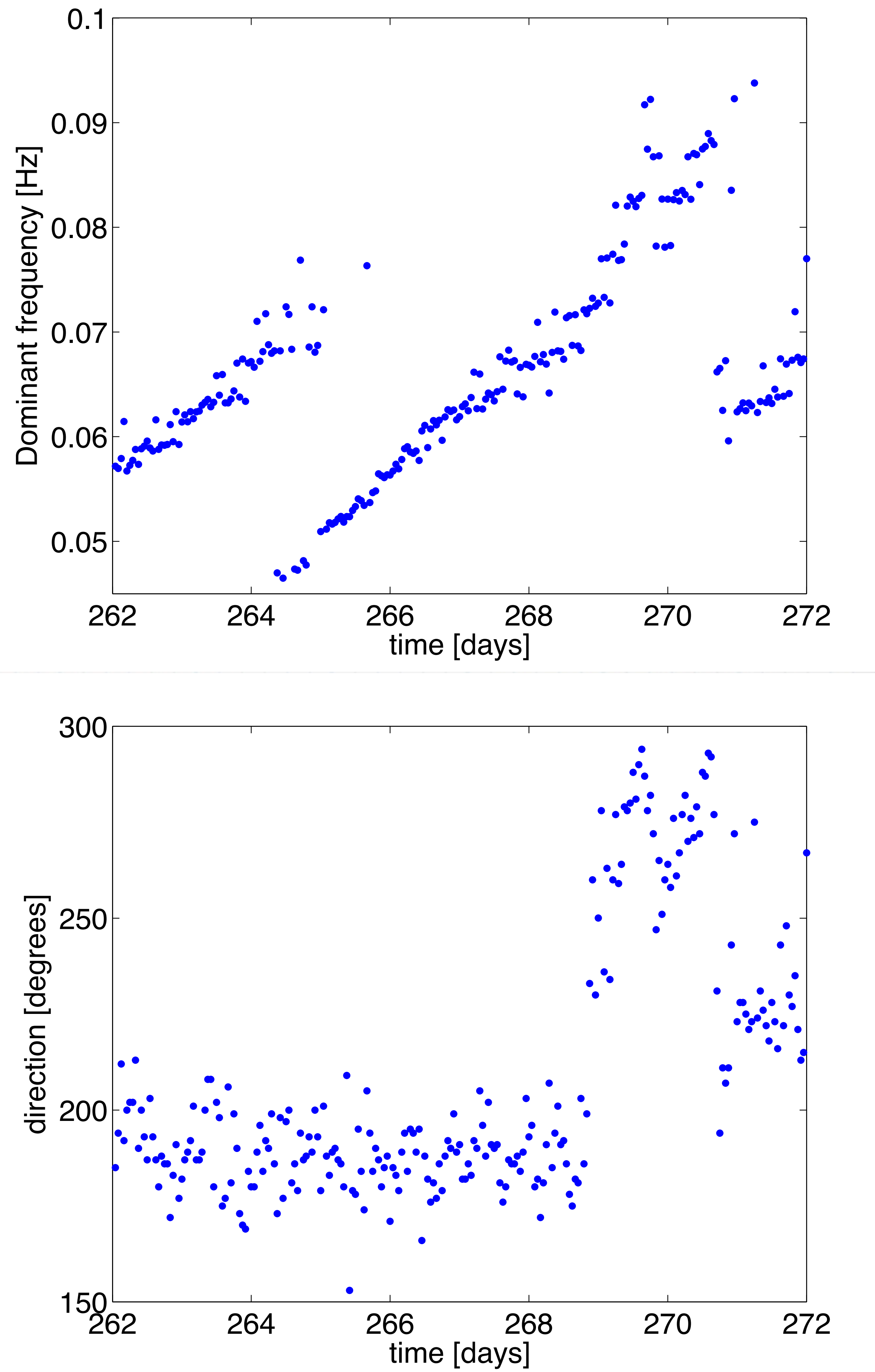
We measure the peak-frequency of the low-frequency part of the spectrum.

This maximum increases linearly in time for swell from a distant storm:

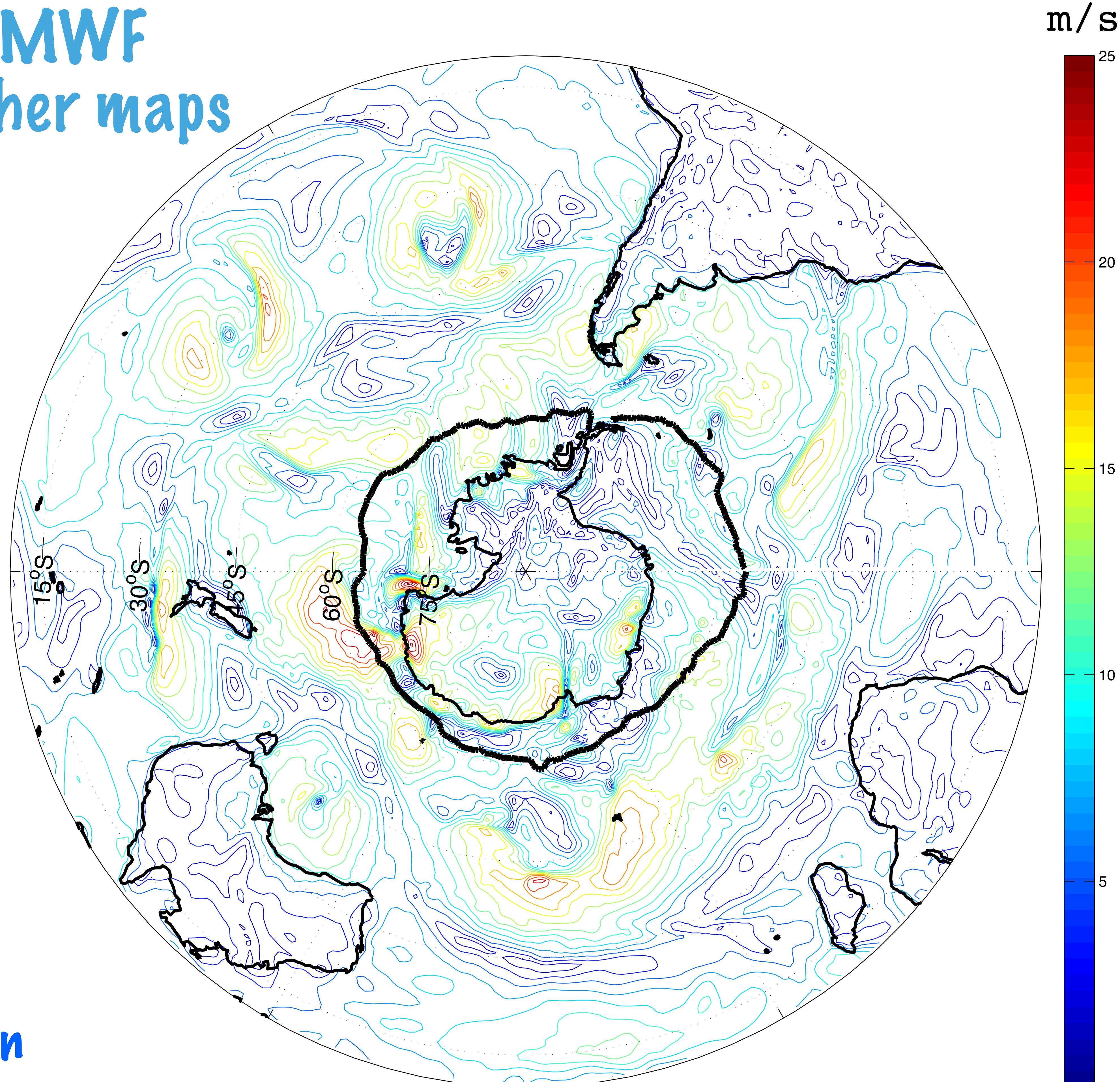
$$f(t) = \left(\frac{g}{4\pi R}\right) \times t$$

The slope gives the range R, and the birth date of the storm.

We average the direction of the incoming swell for this peak frequency. Thus we infer:
Range and direction of the storm



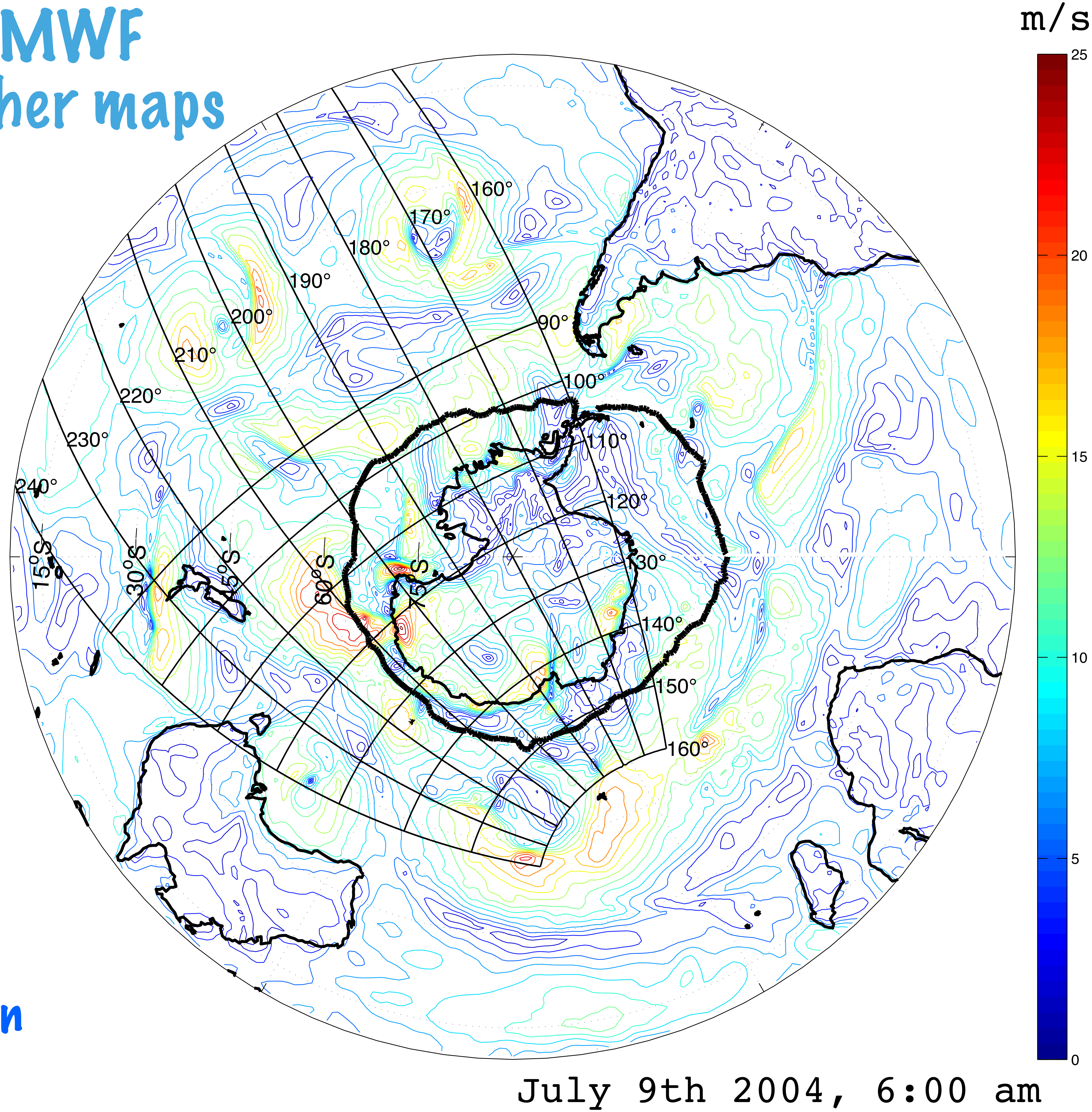
ECMWF weather maps



July 9th 2004, 6:00 am

- 10 m wind speed
- Ice cover
- Range and direction from buoy 46086

ECMWF weather maps



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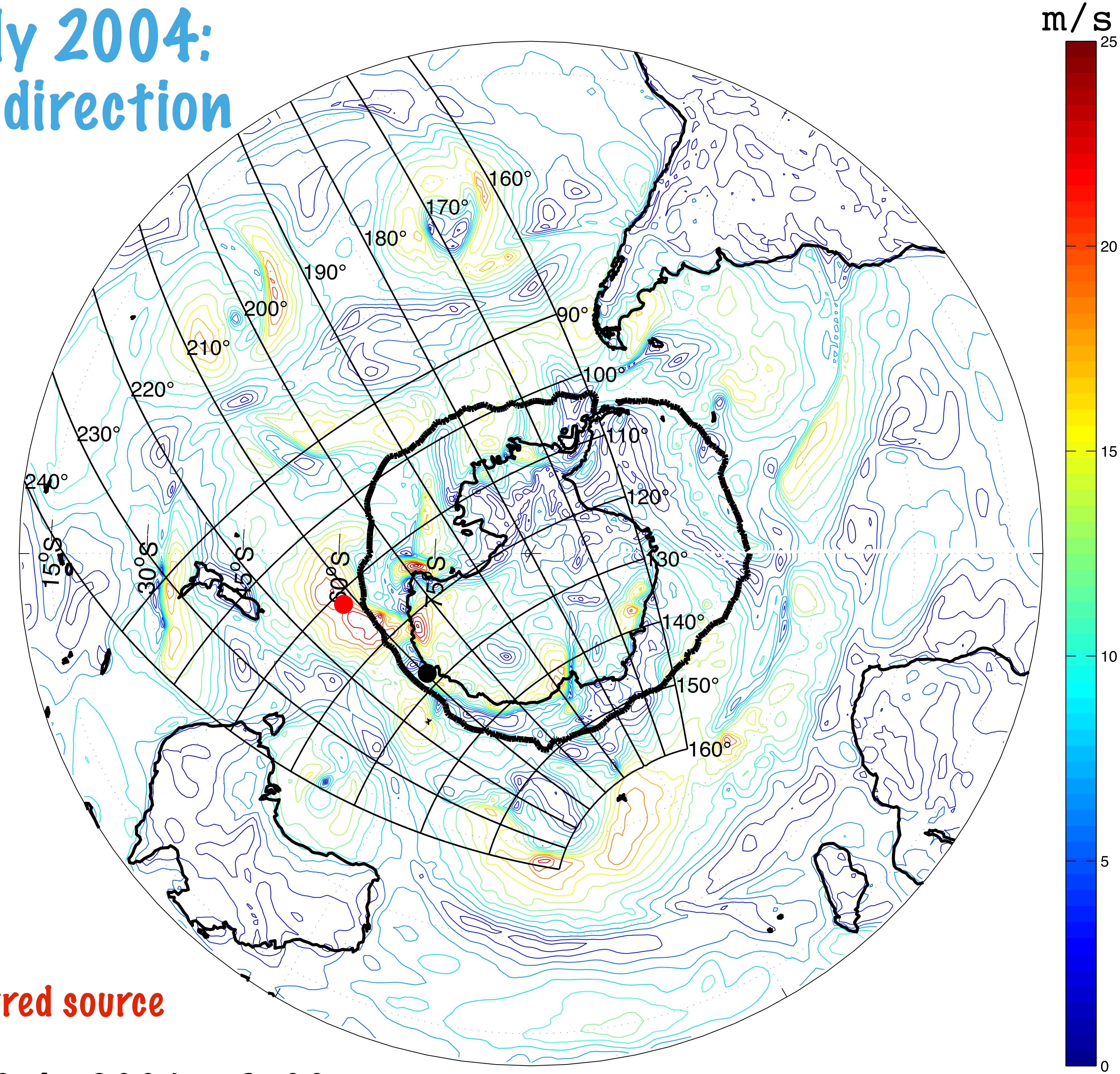
A storm in July 2004: good inferred direction



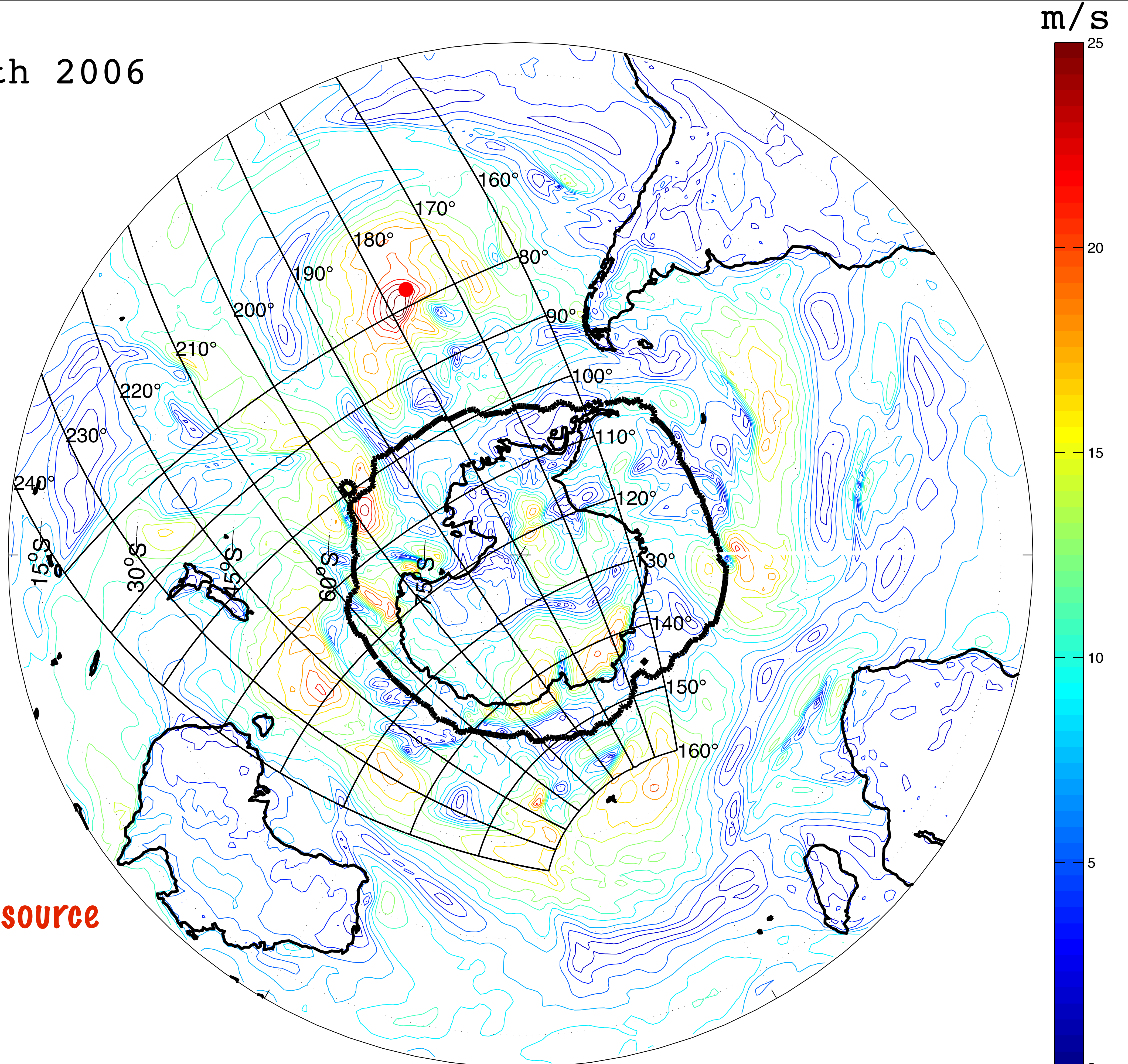
● Inferred source

July 9th 2004, 6:00 am

More examples of well-inferred sources

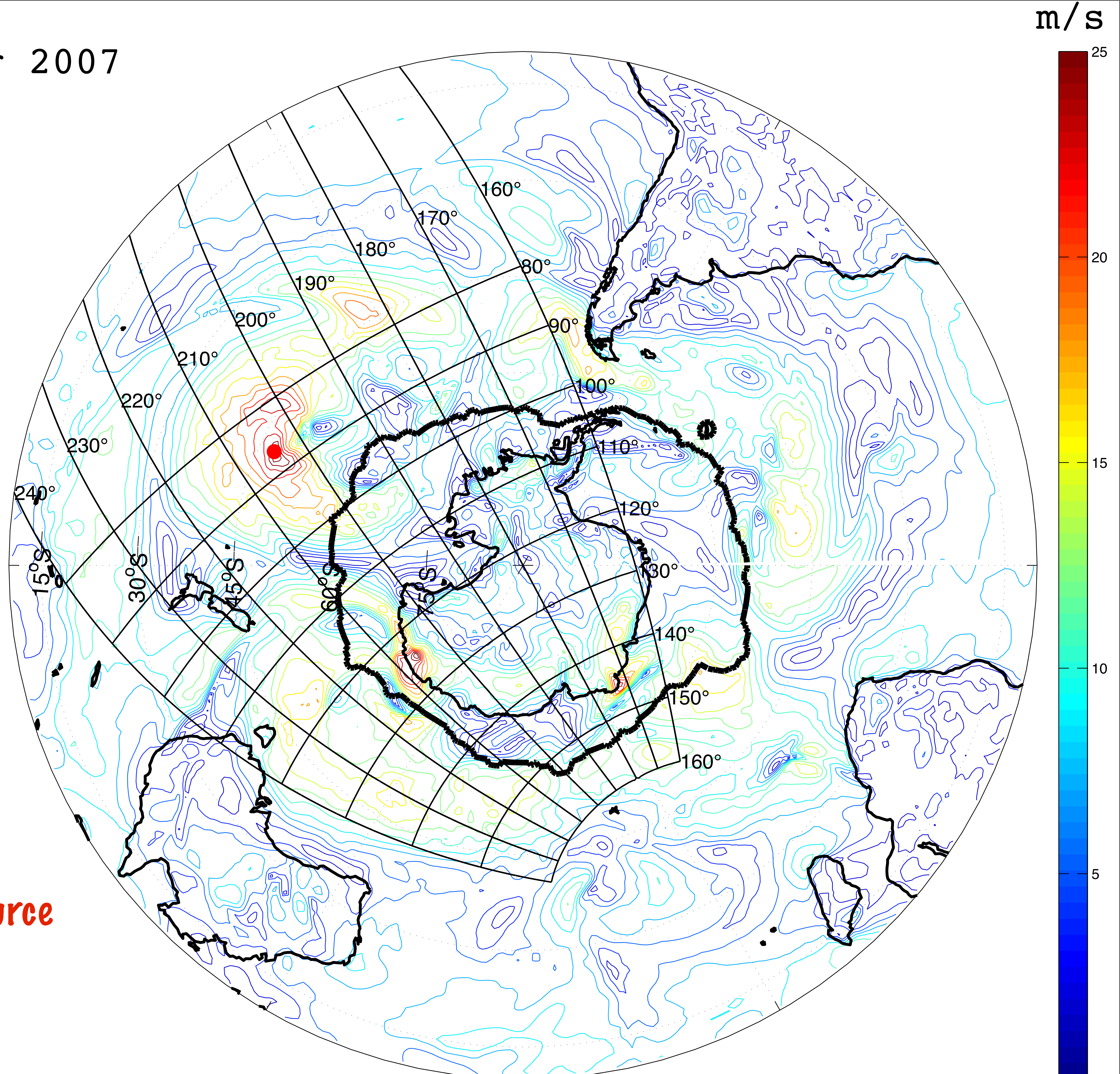
(most of the time it works)

June 13th 2006



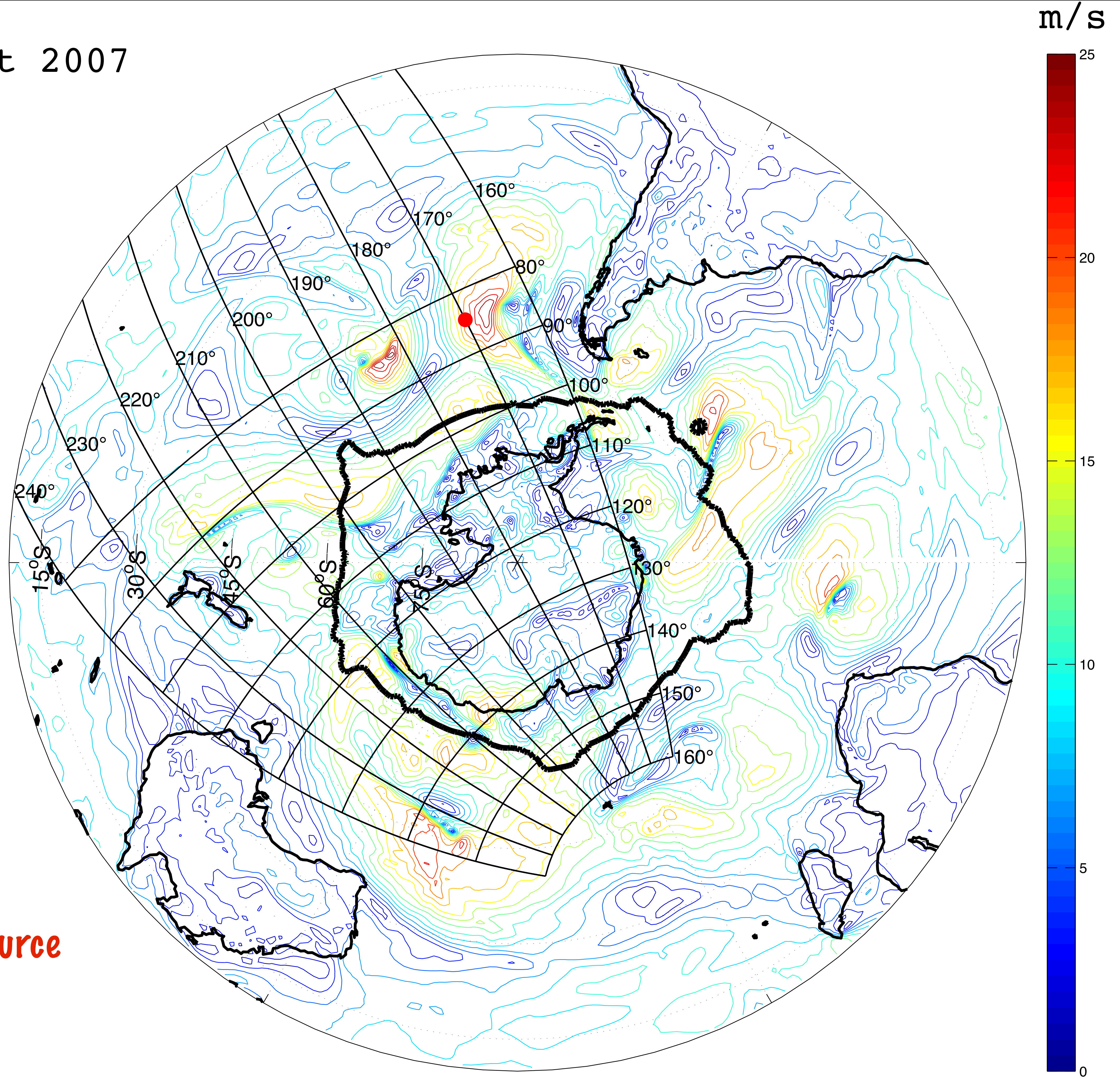
● Inferred source

30 October 2007



● Inferred source

12 August 2007

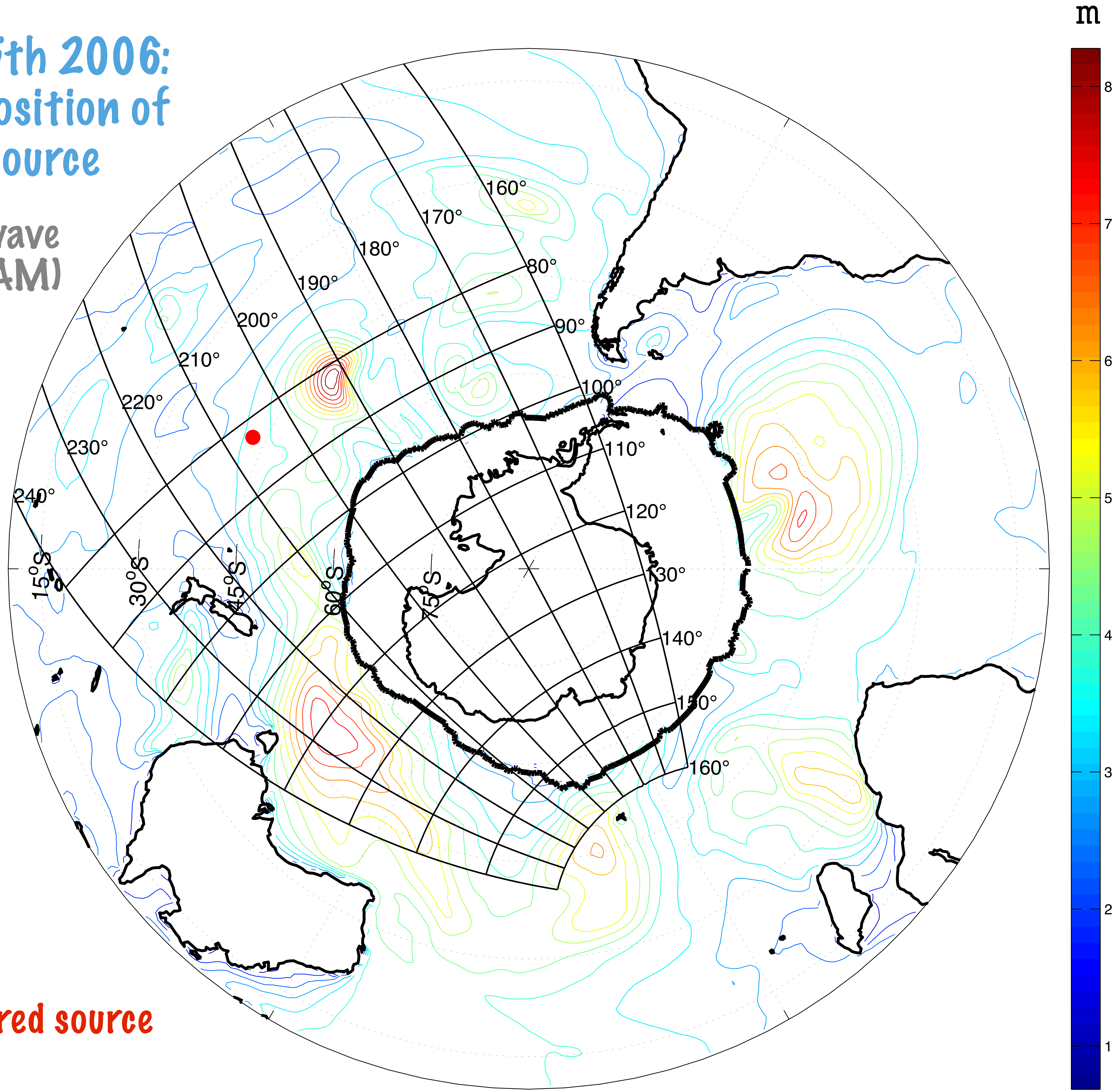


● Inferred source

But a few clean swells come from the "wrong" direction

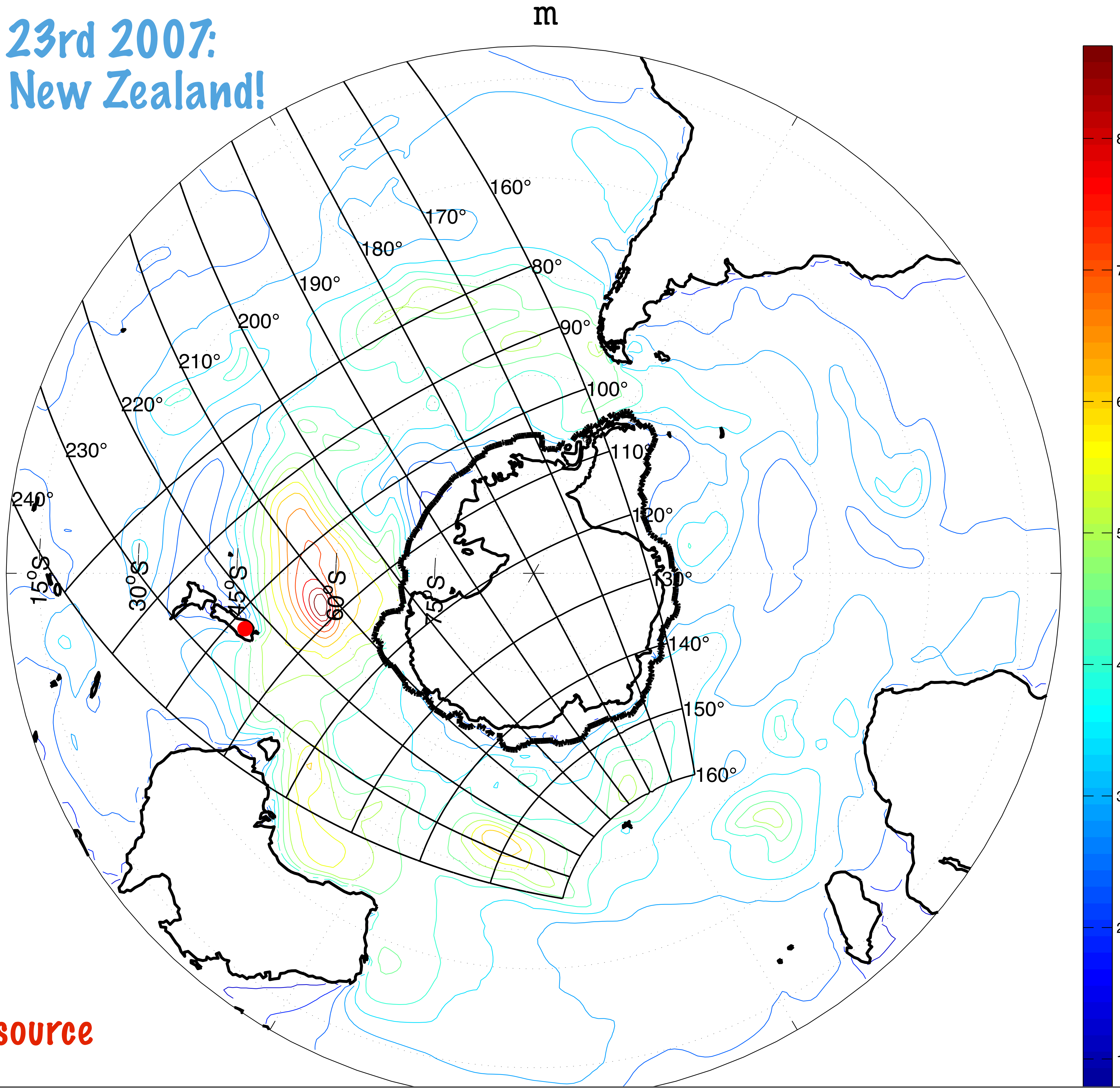
August 5th 2006: wrong position of the source

Surface-wave height (WAM)



● Inferred source

March 23rd 2007:
Source on New Zealand!



● Inferred source

Why don't waves always follow great circles?

- Rotation of the Earth. But this effect is far too small (Backus 1962).

- The Earth is not a perfect sphere. This is even less important than rotation.

- The Coriolis force acts on Stokes drift and sends wave packets round in inertial circles.

- Refraction by bottom topography near the three-sensor array. This is important - Munk (2012).

- The remainder of **this talk**: refraction by surface currents between the source and the receiver.

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Swell travels at over 10m/s, while surface currents are usually much slower than 0.5m/s.

The ray equations (a.k.a. geometric optics)

Waves are shorter than the scale of ocean surface currents and we can use "geometric optics".

$$\eta(\boldsymbol{r}, t) \propto \mathrm{e}^{iS(\boldsymbol{r},t)}$$

$$\omega = -S_t\,, \qquad p = S_x \qquad q = S_y$$

The Doppler-shifted dispersion relation is:

$$\omega = up + vq + \sqrt{g}\left(p^2 + q^2\right)^{1/4}$$

We trace rays by integrating four differential equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial \omega}{\partial p} \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\partial \omega}{\partial q}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial \omega}{\partial x} \qquad \frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{\partial \omega}{\partial y}$$

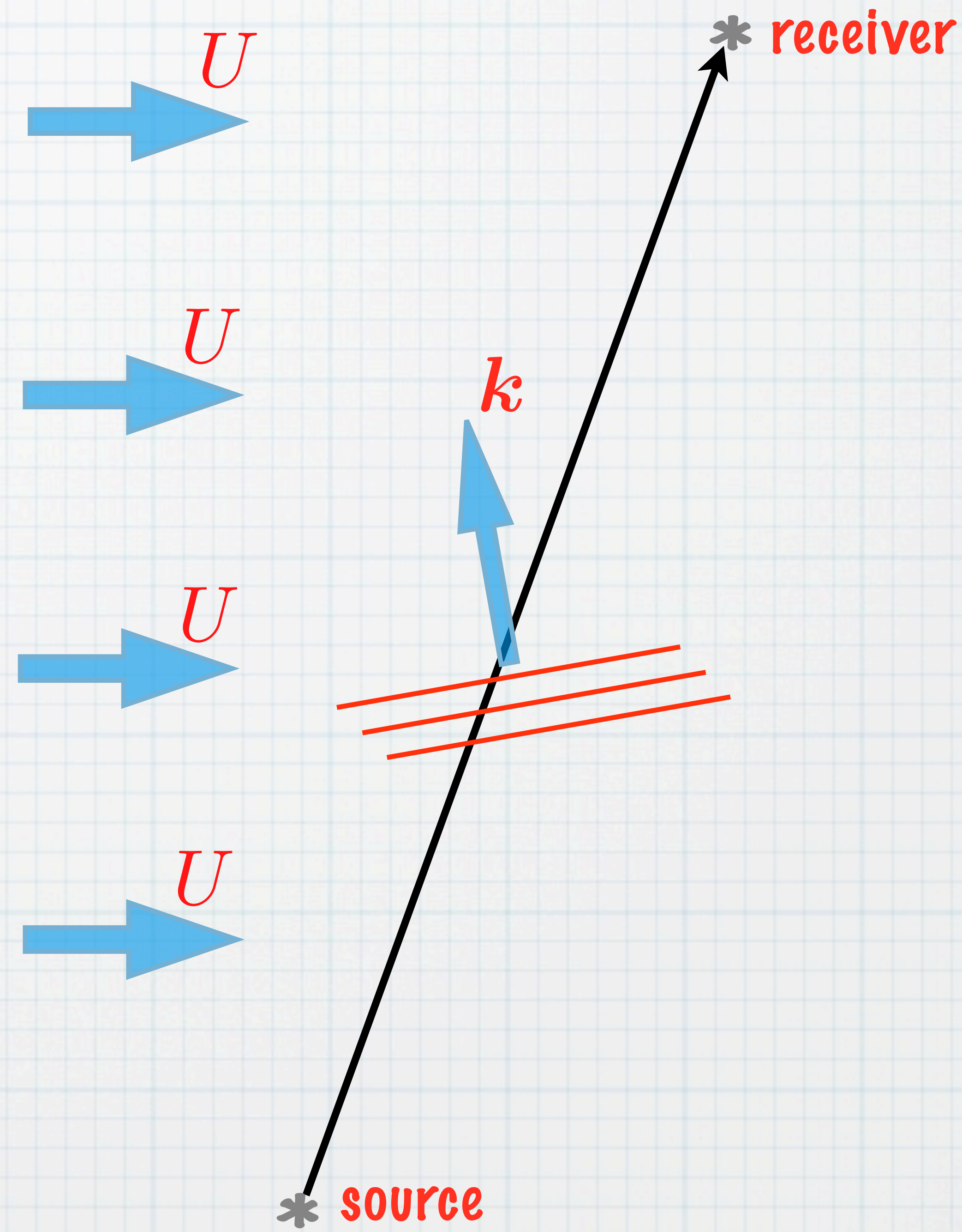
Let's consider a simple, preliminary example

A uniform current does not deflect a ray from a great-circle trajectory.

The wave vector is not tangent to the ray, and the rays are straight, despite the Doppler shift.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = U + \frac{p}{k}\sqrt{\frac{g}{k}} \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{q}{k}\sqrt{\frac{g}{k}}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = 0 \qquad \frac{\mathrm{d}q}{\mathrm{d}t} = 0$$



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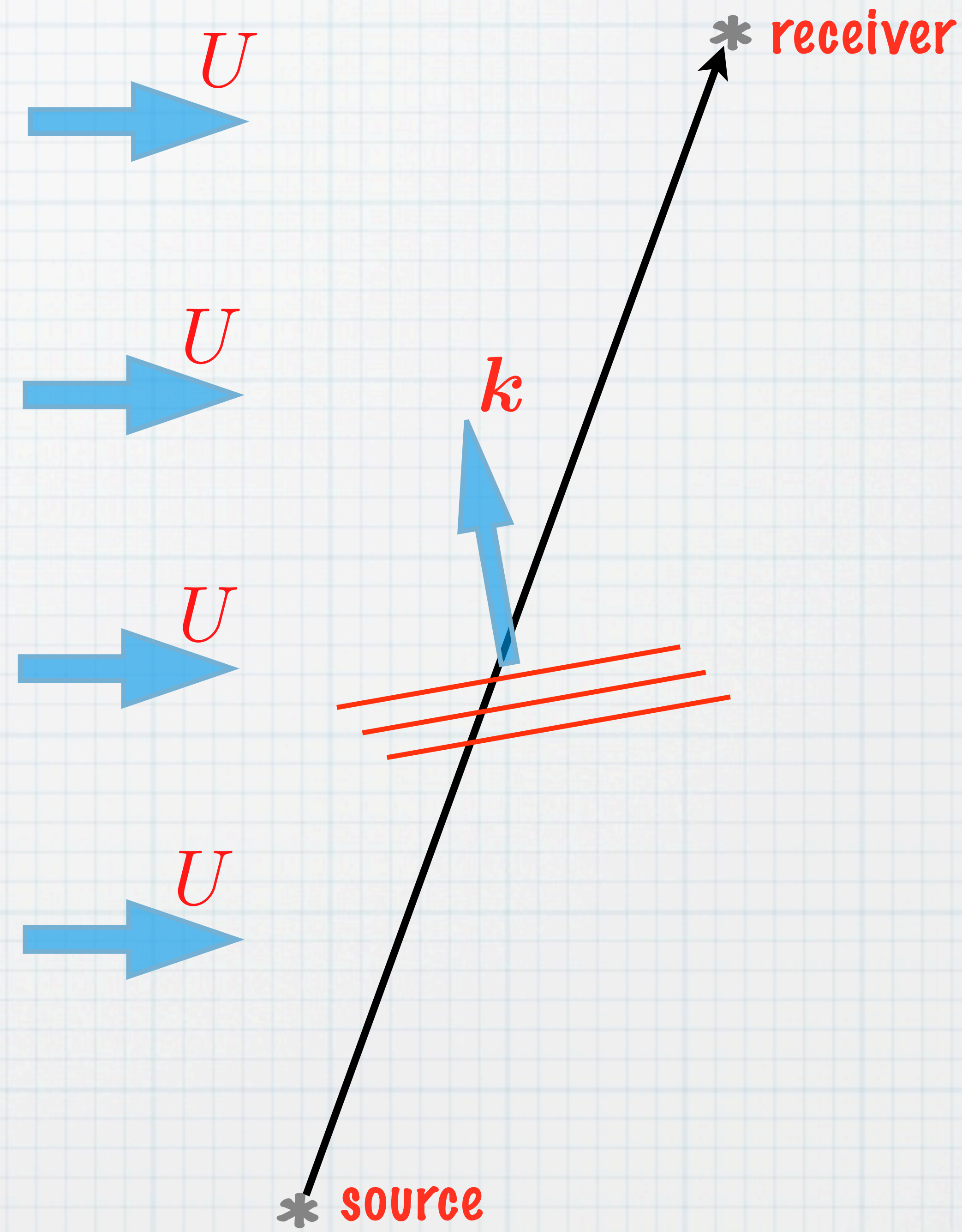


Thus the speed of surface currents is irrelevant for ray bending.

Vorticity (not velocity) bends rays - see L&L

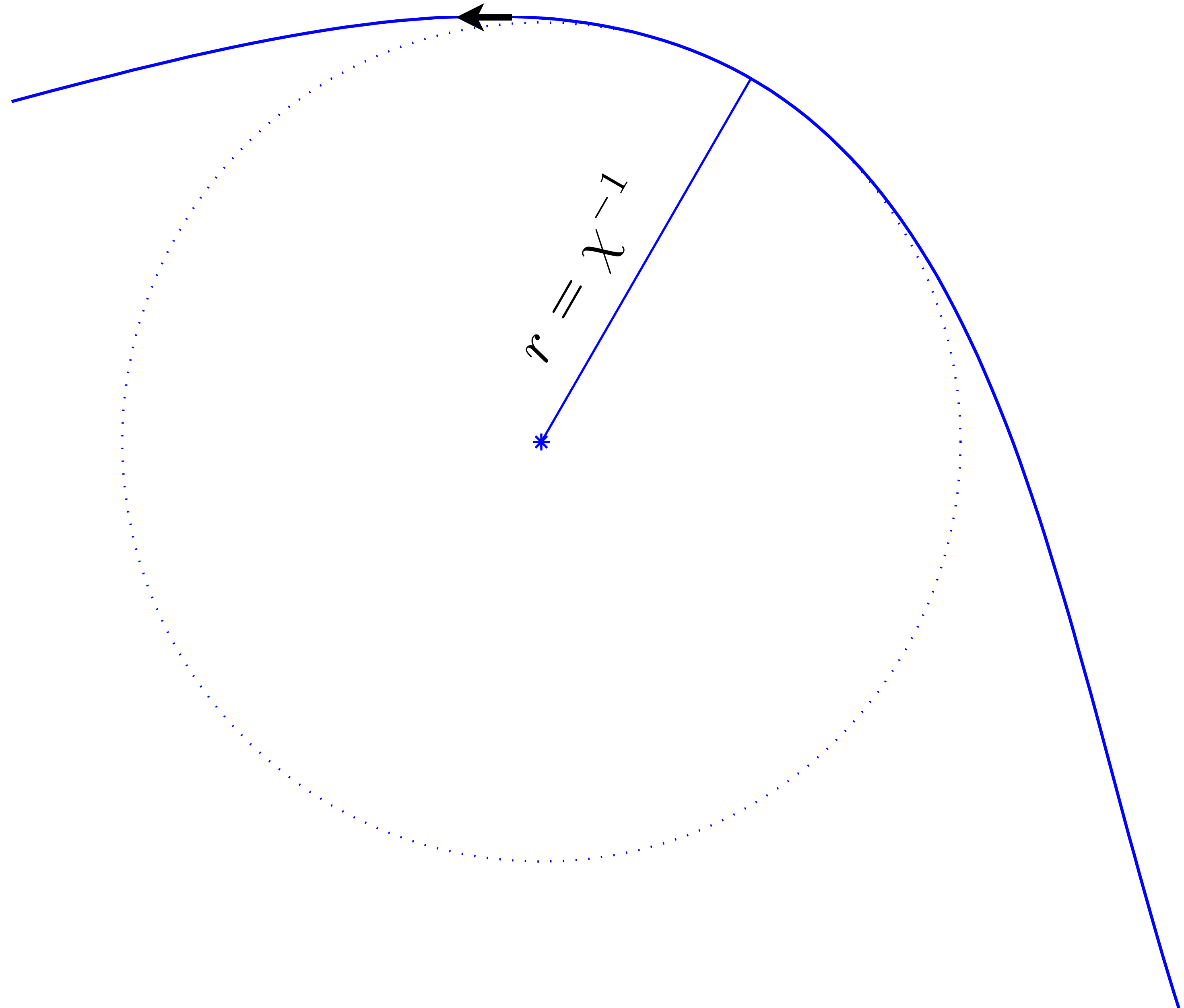
- The key result is: $\chi = \dfrac{\zeta}{c_{\mathrm{g}}}$

- Valid for isotropic waves, Doppler shifted by weak-currents:

$$\frac{|\boldsymbol{u}|}{c_{\mathrm{g}}} \ll 1$$

- Ray curvature is independent of ray direction, and of current direction. There is no anisotropy, despite the direction suggested by velocity.

- An irrotational velocity produces no refraction.



$$c_{\mathrm{g}} = \frac{1}{2}\sqrt{\frac{g}{k}} \qquad \zeta = \boldsymbol{\nabla} \times \boldsymbol{u} \cdot \hat{\boldsymbol{z}}$$

Can the vorticity of realistic surface currents bend rays?

The Doppler-shifted dispersion relation is:

$$\omega = up + vq + \sqrt{g}\left(p^2 + q^2\right)^{1/4}$$

The ray equations are:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial \omega}{\partial p} \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\partial \omega}{\partial q}$$

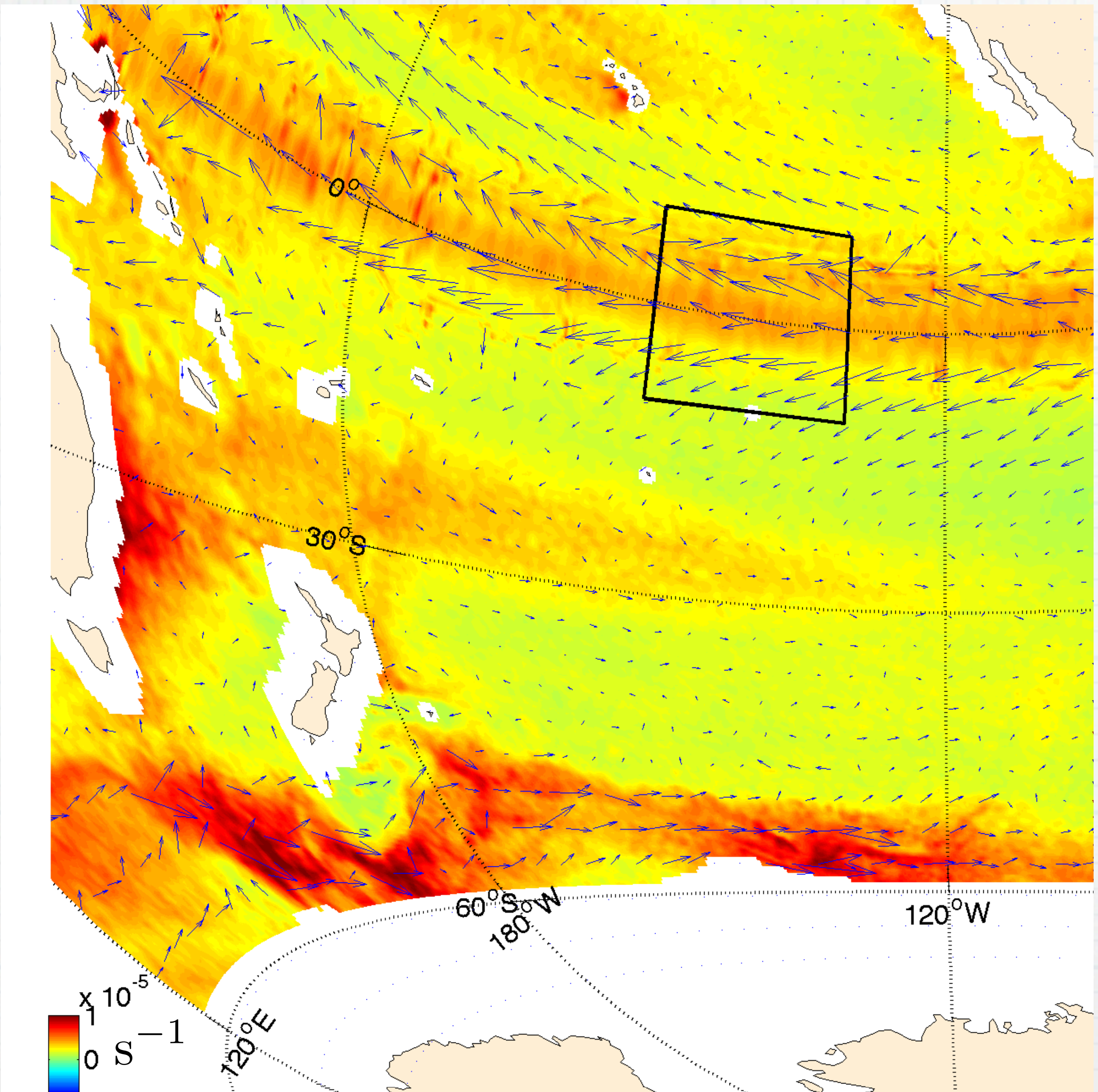
$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial \omega}{\partial x} \qquad \frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{\partial \omega}{\partial y}$$

We need an estimate of the sea-surface velocity, and vorticity.

$u(x, y, t)$ and $v(x, y, t)$

OSCAR sea-surface velocity

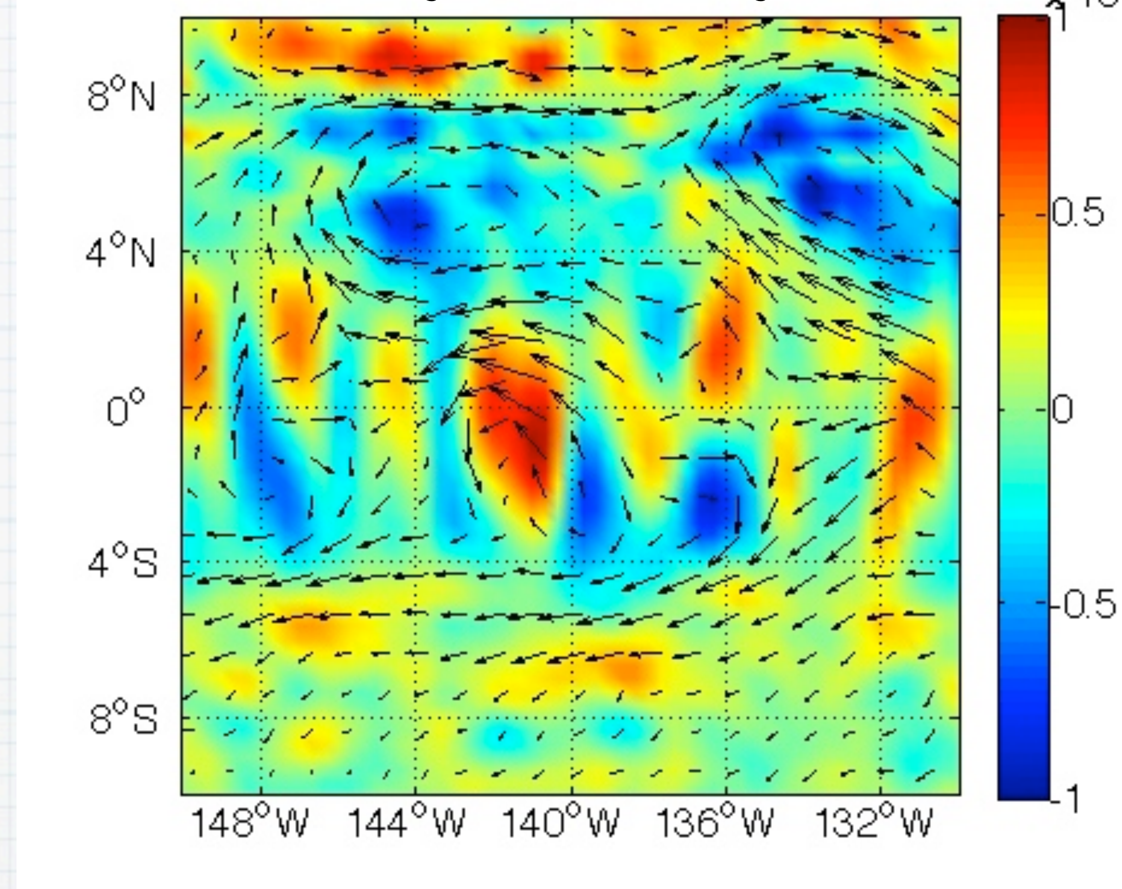
$u(x, y, t)$ and $v(x, y, t)$



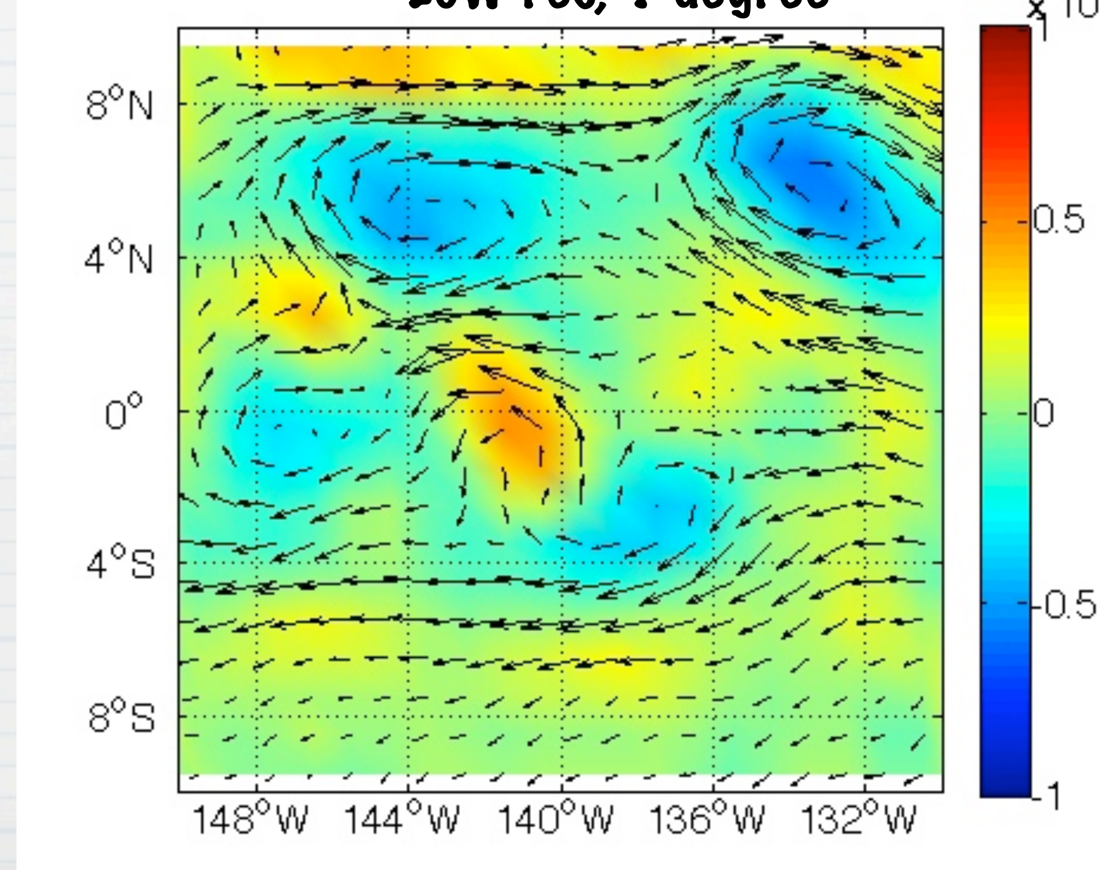
Twenty-year mean

Color is vorticity

High res, 1/3 degree



Low res, 1 degree



Assessment of sea-surface currents

Even very long swell (e.g., 1km) is still much smaller scale than mesoscale ocean eddies: we use geometric optics.

The Doppler-shifted gravity-wave dispersion relation is:

$$\omega(\boldsymbol{r},\boldsymbol{k}) = \boldsymbol{u}(\boldsymbol{r})\cdot\boldsymbol{k} + \sqrt{g|\boldsymbol{k}|}$$

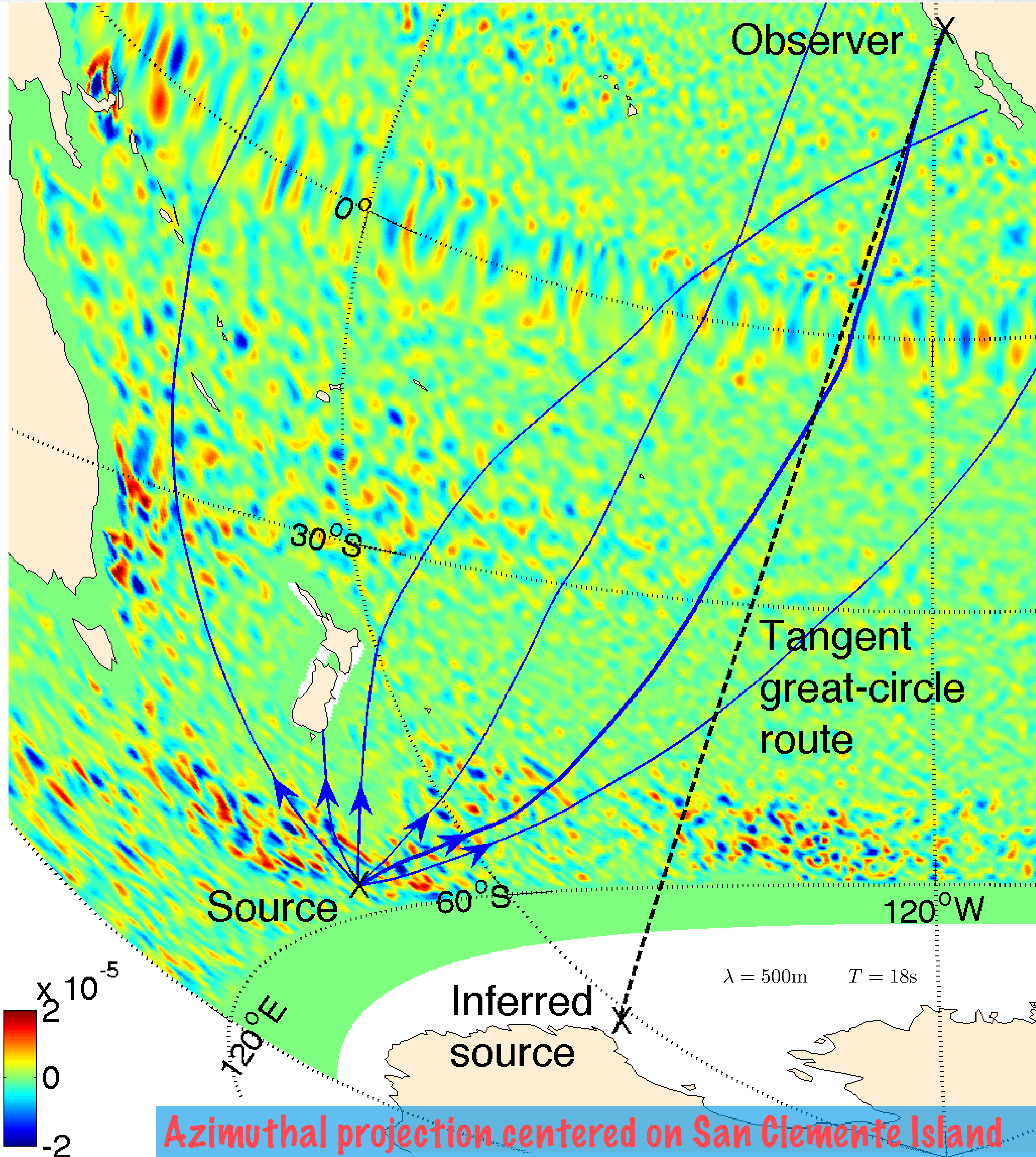
$$\boldsymbol{r} = (x,y) \qquad \boldsymbol{k} = (p,q)$$

We integrate the ray equations

$$\dot{x} = \omega_p\,, \qquad \dot{y} = \omega_q$$

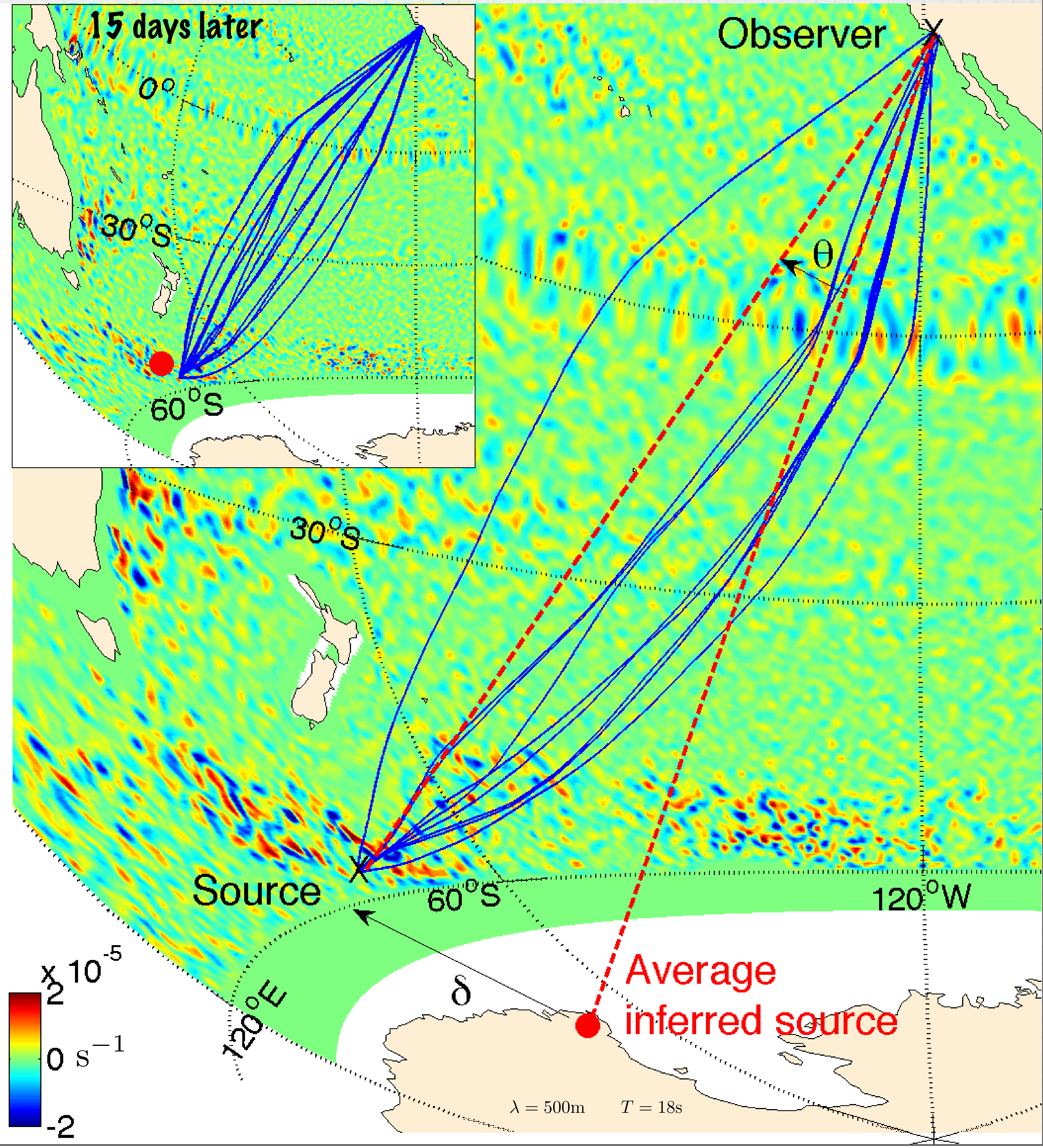
$$\dot{p} = -\omega_x\,, \qquad \dot{q} = -\omega_y$$

The sea-surface velocity is taken from OSCAR.



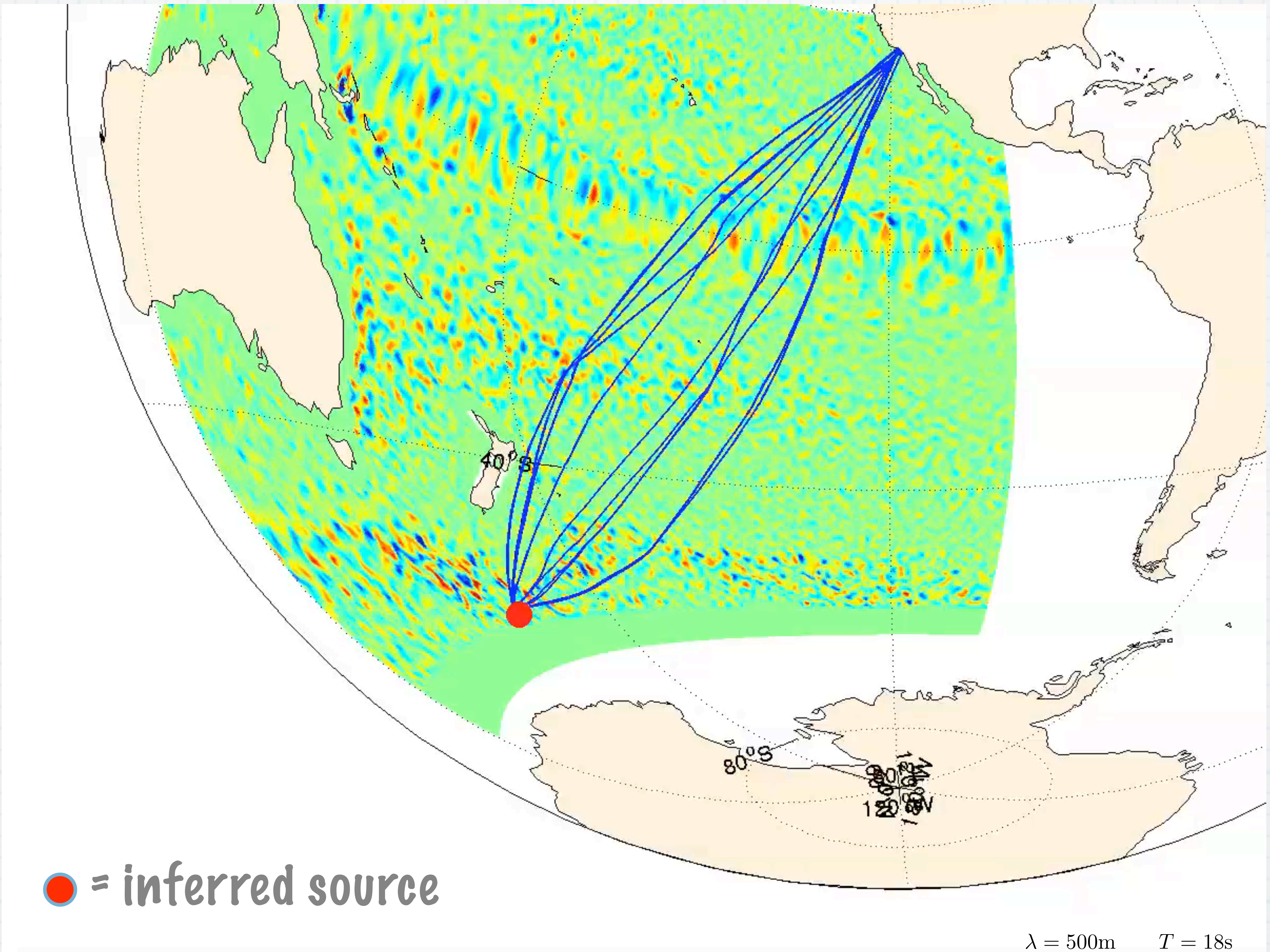
The observer infers the source location by **great-circle backtracking**.

But the rays do not closely follow great circles....



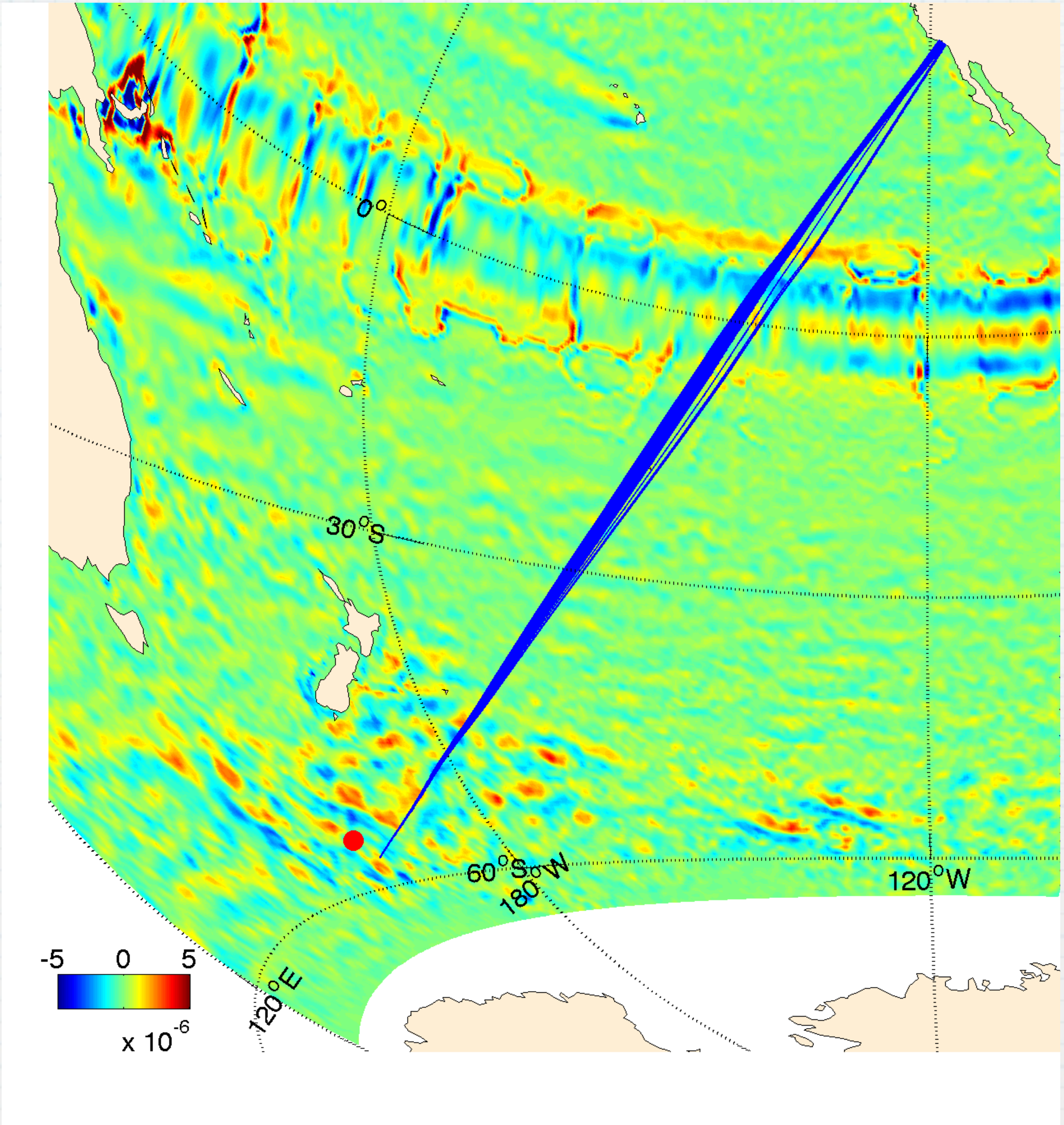
Twenty years of OSCAR data

One frame every five days for twenty years.



Propagation through the twenty-year mean

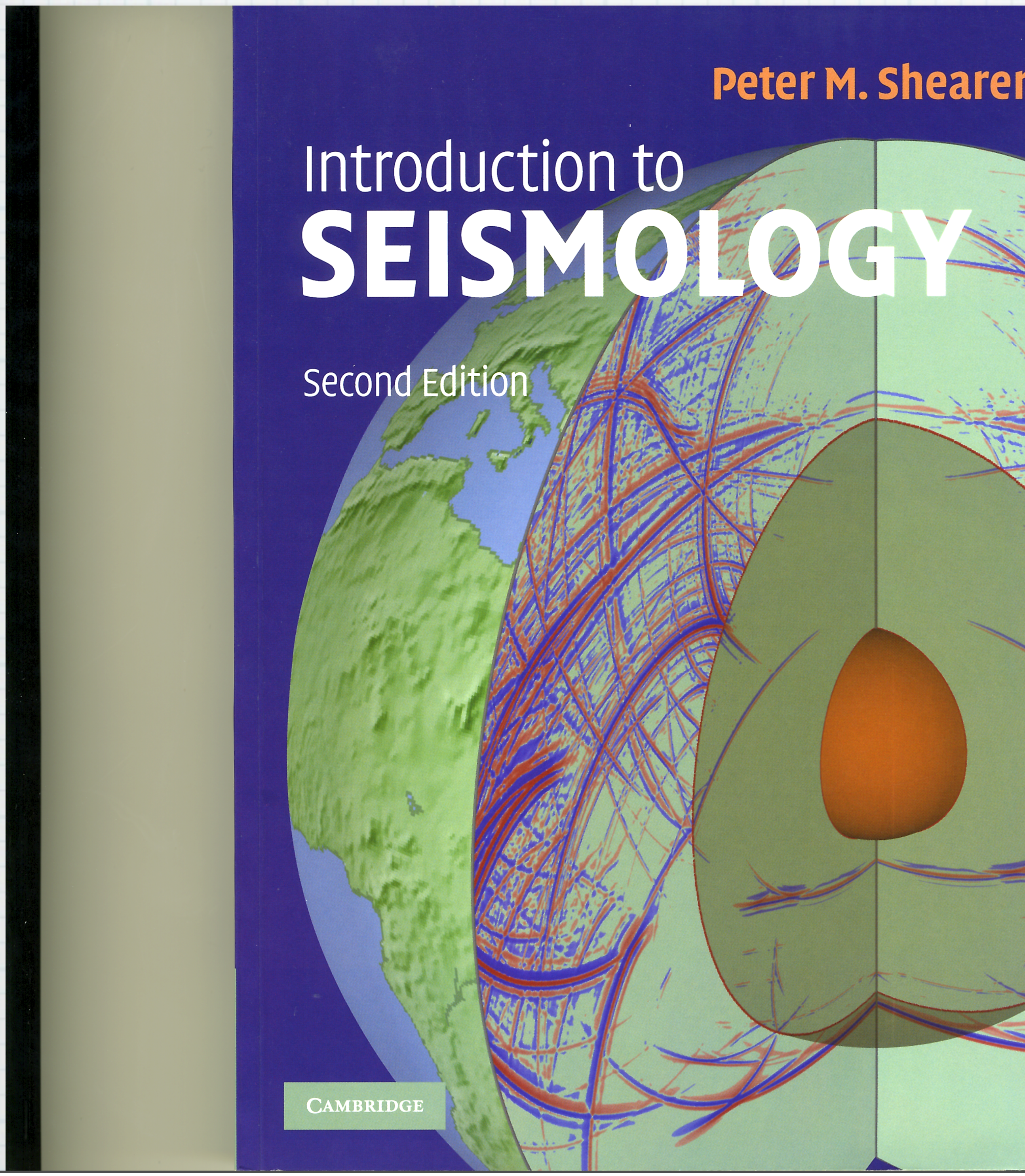
Eddies are crucial.



$$\lambda = 500m \qquad T = 18s$$

How do we understand these numerical solutions?
We do not have analogs of Snell's law, Fermat's principle and the index of refraction.

$$\omega(\boldsymbol{r},\boldsymbol{k}) = c(\boldsymbol{r})|\boldsymbol{k}| \qquad \text{versus} \qquad \omega(\boldsymbol{r},\boldsymbol{k}) = \boldsymbol{u}(\boldsymbol{r})\cdot\boldsymbol{k} + \sqrt{g|\boldsymbol{k}|}$$



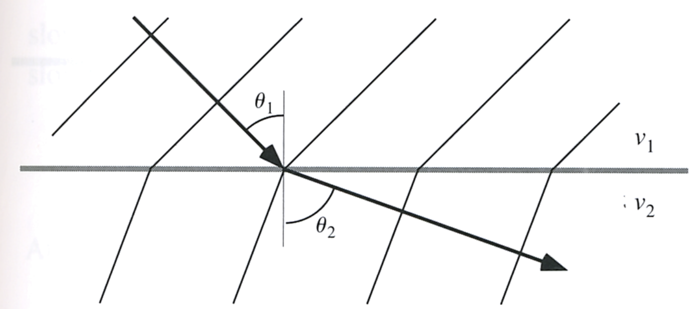
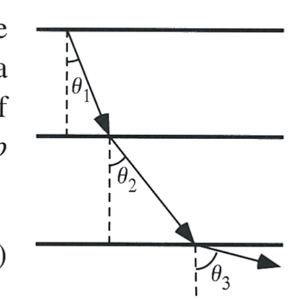


Figure 4.2 A plane wave crossing a horizontal interface between two homogeneous half-spaces. The higher velocity in the bottom layer causes the wavefronts to be spaced further apart.

Notice that this is simply the seismic version of Snell's law in geometrical optics. Equation (4.4) may also be obtained from *Fermat's principle*, which states that the travel time between two points must be stationary (usually, but not always, the minimum time) with respect to small variations in the ray path. Fermat's principle itself can be derived from applying variational calculus to the eikonal equation (e.g., Aki and Richards, 2002, pp. 89–90).

4.2 Ray paths for laterally homogeneous models

In most cases the compressional and shear velocities increase as a function of depth in the Earth. Suppose we examine a ray traveling downward through a series of layers, each of which is faster than the layer above. The ray parameter p remains constant and we have



$$p = u_1 \sin\theta_1 = u_2 \sin\theta_2 = u_3 \sin\theta_3. \qquad (4.5)$$

If the velocity continues to increase, θ will eventually equal 90° and the ray will be traveling horizontally.

This is also true for continuous velocity gradients (Fig. 4.3). If we let the slowness at the surface be u_0 and the *takeoff angle* be θ_0, we have

Derivation of the L&L ray curvature formula: $\chi = \dfrac{\zeta}{c_{\mathrm{g}}}$

The doppler shifted dispersion relation is:

$$\omega(\boldsymbol{r},\boldsymbol{k}) = \boldsymbol{u}\cdot\boldsymbol{k} + \bar{\omega}\,(k)$$

The corresponding ray equations are:

$$\dot{\boldsymbol{r}} = +\omega_{\boldsymbol{k}} = \boldsymbol{u} + \frac{\bar{\omega}_k}{k}\boldsymbol{k}\,,$$

$$\dot{\boldsymbol{k}} = -\omega_{\boldsymbol{r}} = \zeta\,\hat{\boldsymbol{z}}\times\boldsymbol{k} - (\boldsymbol{k}\cdot\nabla)\boldsymbol{u}$$

The "ray acceleration" is:

$$\ddot{\boldsymbol{r}} = \nabla\left(\tfrac{1}{2}|\boldsymbol{u}|^2\right) + \zeta\hat{\boldsymbol{z}}\times\dot{\boldsymbol{r}} + \dot{\beta}\,(\dot{\boldsymbol{r}} - \boldsymbol{u})$$

where: $\qquad \beta(k) \stackrel{\mathrm{def}}{=} \ln\left(\dfrac{\bar{\omega}_k}{k}\right)$

Now use intrinsic coordinates.

Project on the normal to the ray:

$$\chi\dot{s}^2 = \zeta\dot{s} + \hat{\boldsymbol{n}}\cdot\nabla\left(\tfrac{1}{2}|\boldsymbol{u}|^2\right) - \dot{\beta}\,\hat{\boldsymbol{n}}\cdot\boldsymbol{u}$$

With the weak-current approximation:

$$\chi = \frac{\zeta}{\bar{\omega}_k} + O\left(\frac{\boldsymbol{u}}{\bar{\omega}_k}\right)^2$$

and: $\qquad \dot{s} = \bar{\omega}_k + O\left(\dfrac{\boldsymbol{u}}{\bar{\omega}_k}\right)$

Intrinsic Coordinates

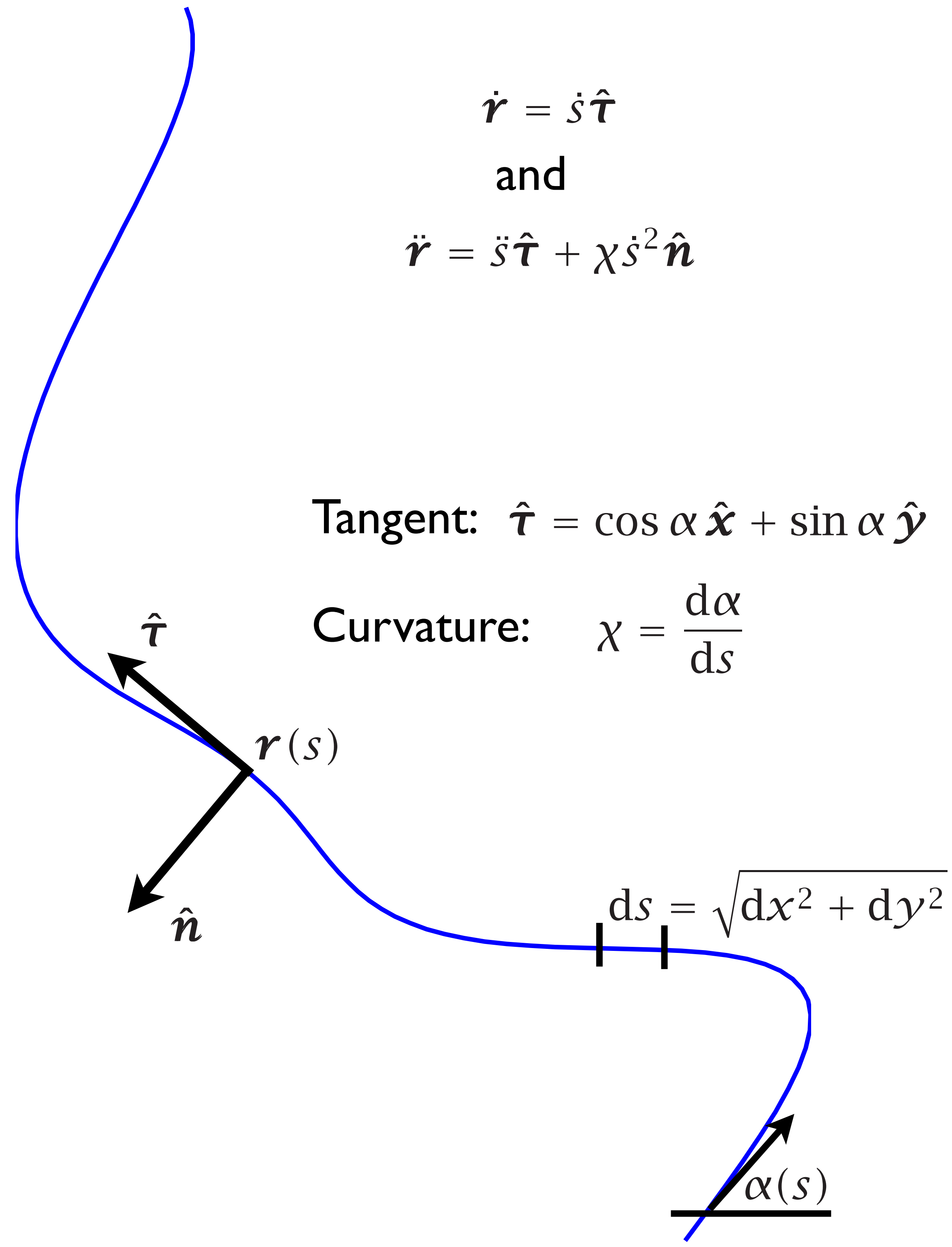
$$\dot{\boldsymbol{r}} = \dot{s}\hat{\boldsymbol{\tau}}$$

and

$$\ddot{\boldsymbol{r}} = \ddot{s}\hat{\boldsymbol{\tau}} + \chi\dot{s}^2\hat{\boldsymbol{n}}$$

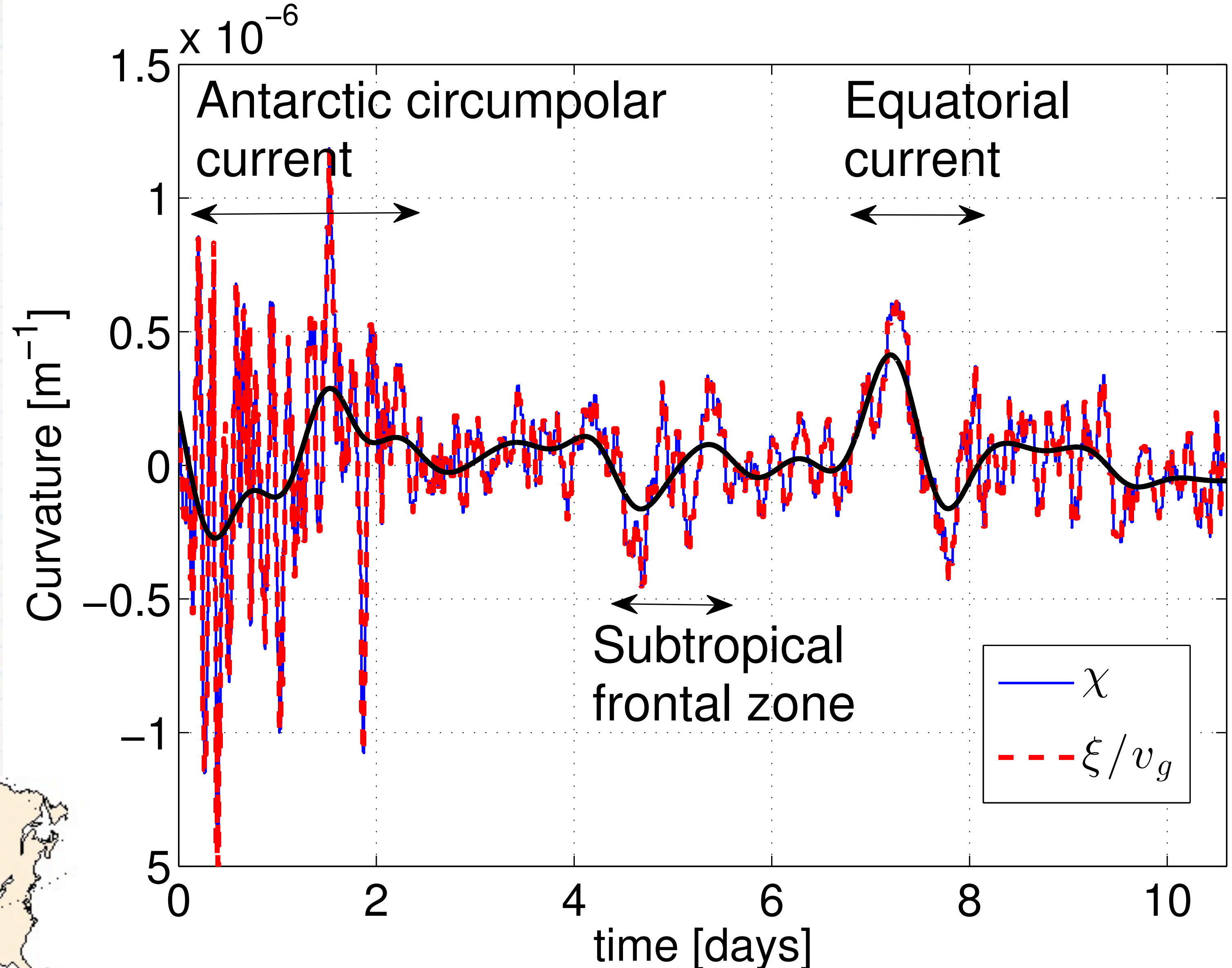
Tangent: $\hat{\boldsymbol{\tau}} = \cos\alpha\,\hat{\boldsymbol{x}} + \sin\alpha\,\hat{\boldsymbol{y}}$

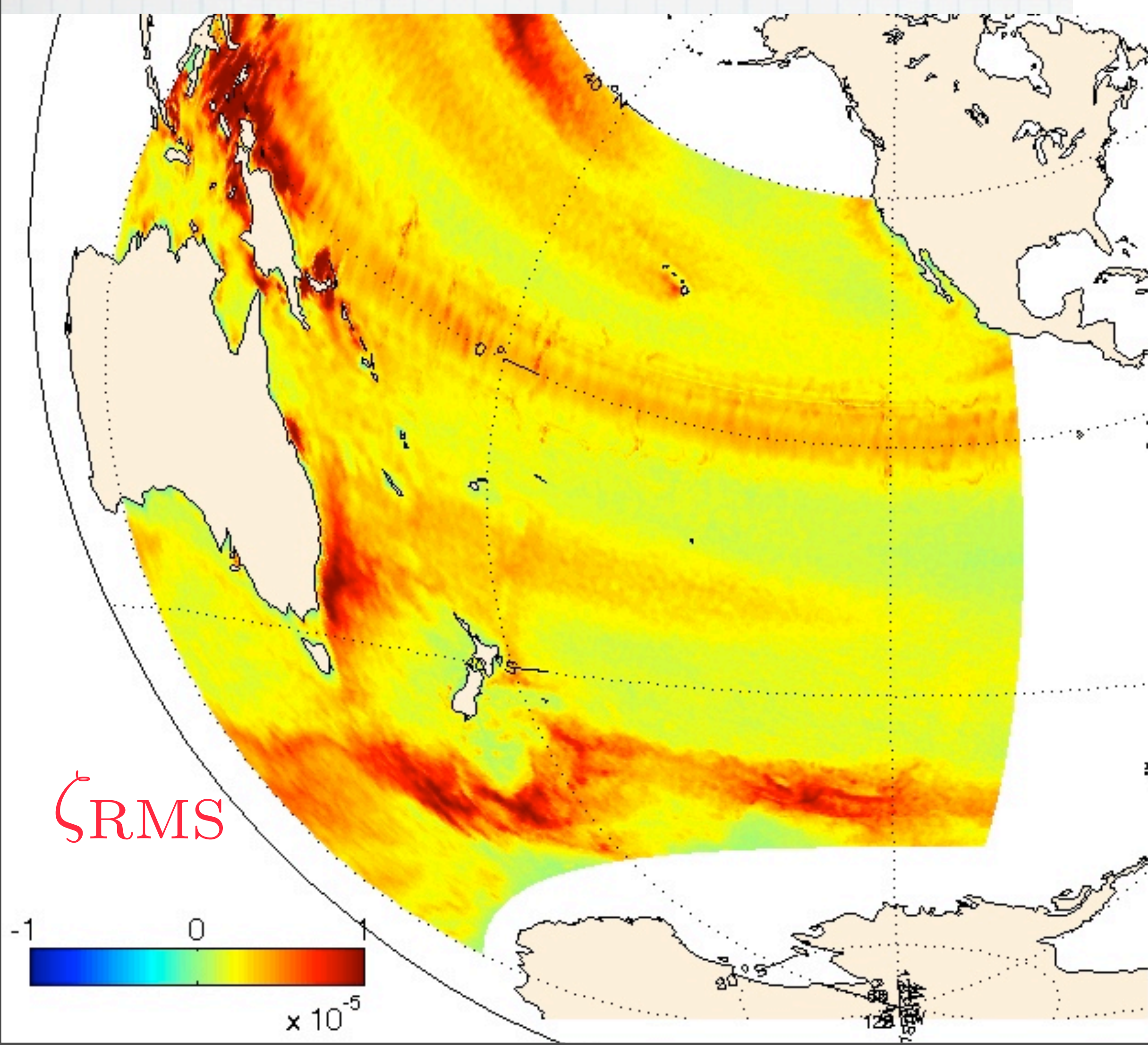
Curvature: $\quad \chi = \dfrac{\mathrm{d}\alpha}{\mathrm{d}s}$



Validation of:

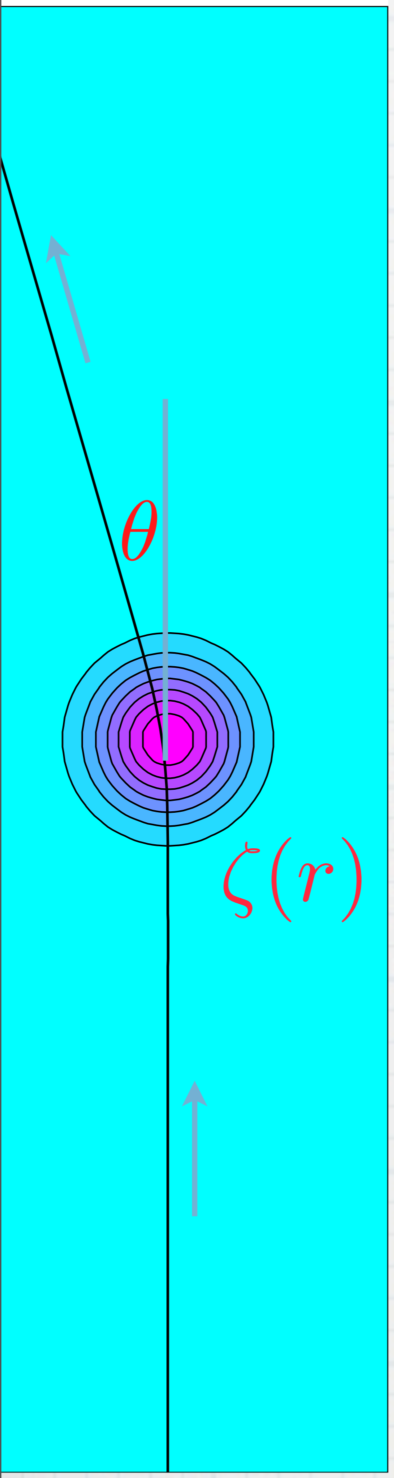
$$\chi = \frac{\zeta}{c_g}$$





- Refraction is due to the vorticity in **three current systems** between the source and the receiver.
- This is **not a random walk**, nor is it the accumulation of many small errors over great distances.
- Most of the errors are due to refractive events in the **equatorial current system**.

Order of magnitude of ray deflections

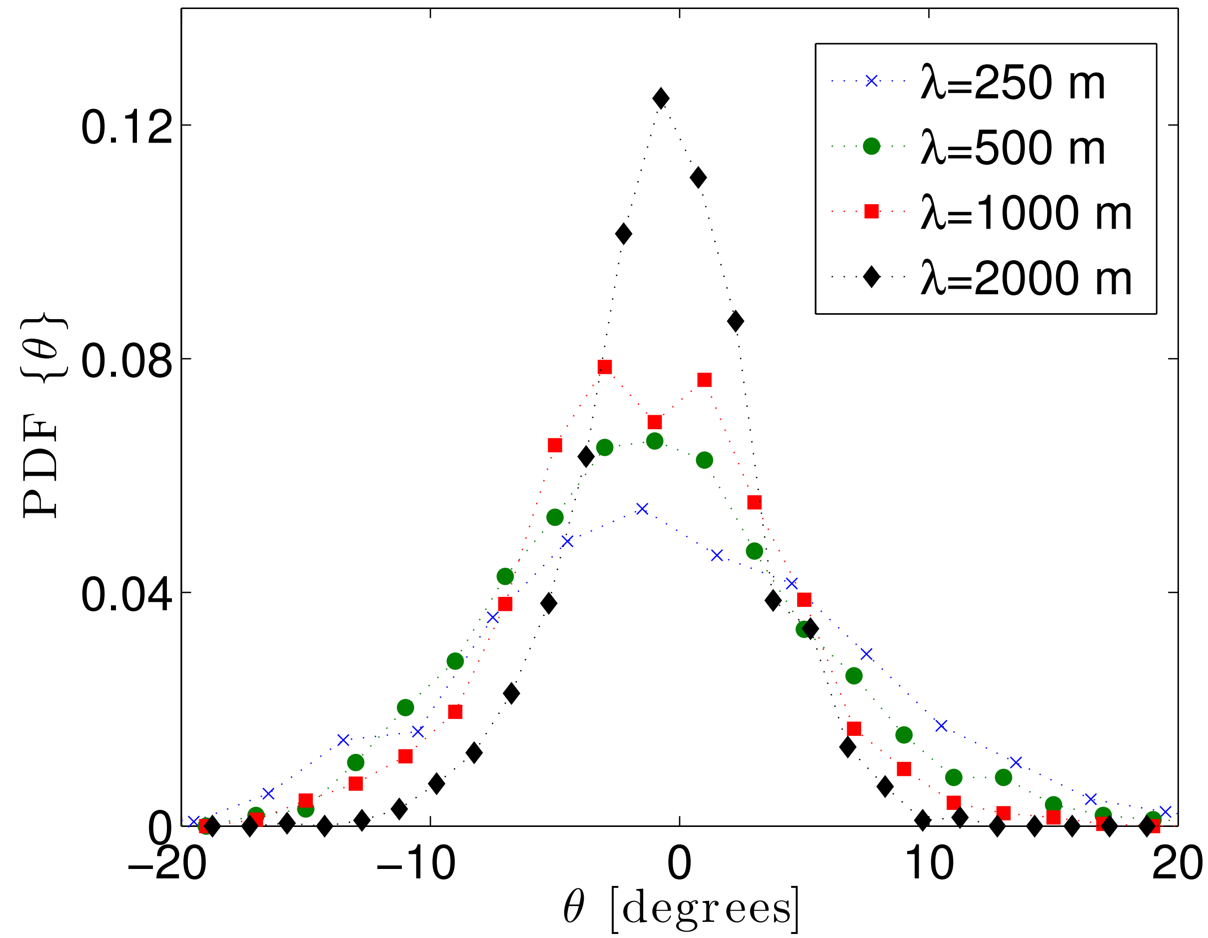


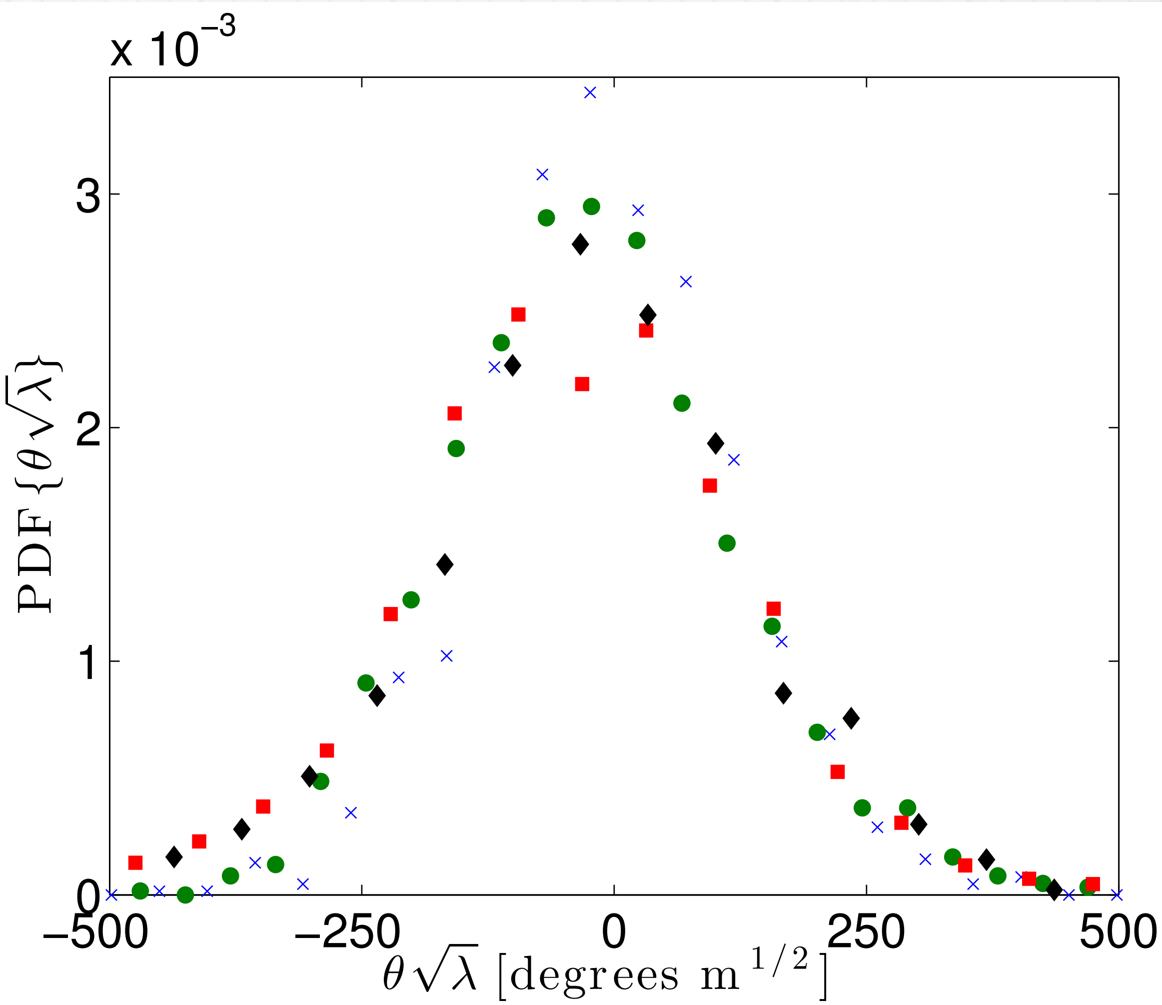
$$\theta \sim \frac{\zeta\ell}{c_{\mathrm{g}}} \sim \frac{\zeta\ell}{\sqrt{g\lambda}}$$

$$\lambda = 500\mathrm{m}\,, \qquad \ell = 400\mathrm{km}$$

$$\zeta = 10^{-5}\mathrm{s}^{-1} \qquad \Rightarrow \theta \sim 3^{\circ}$$

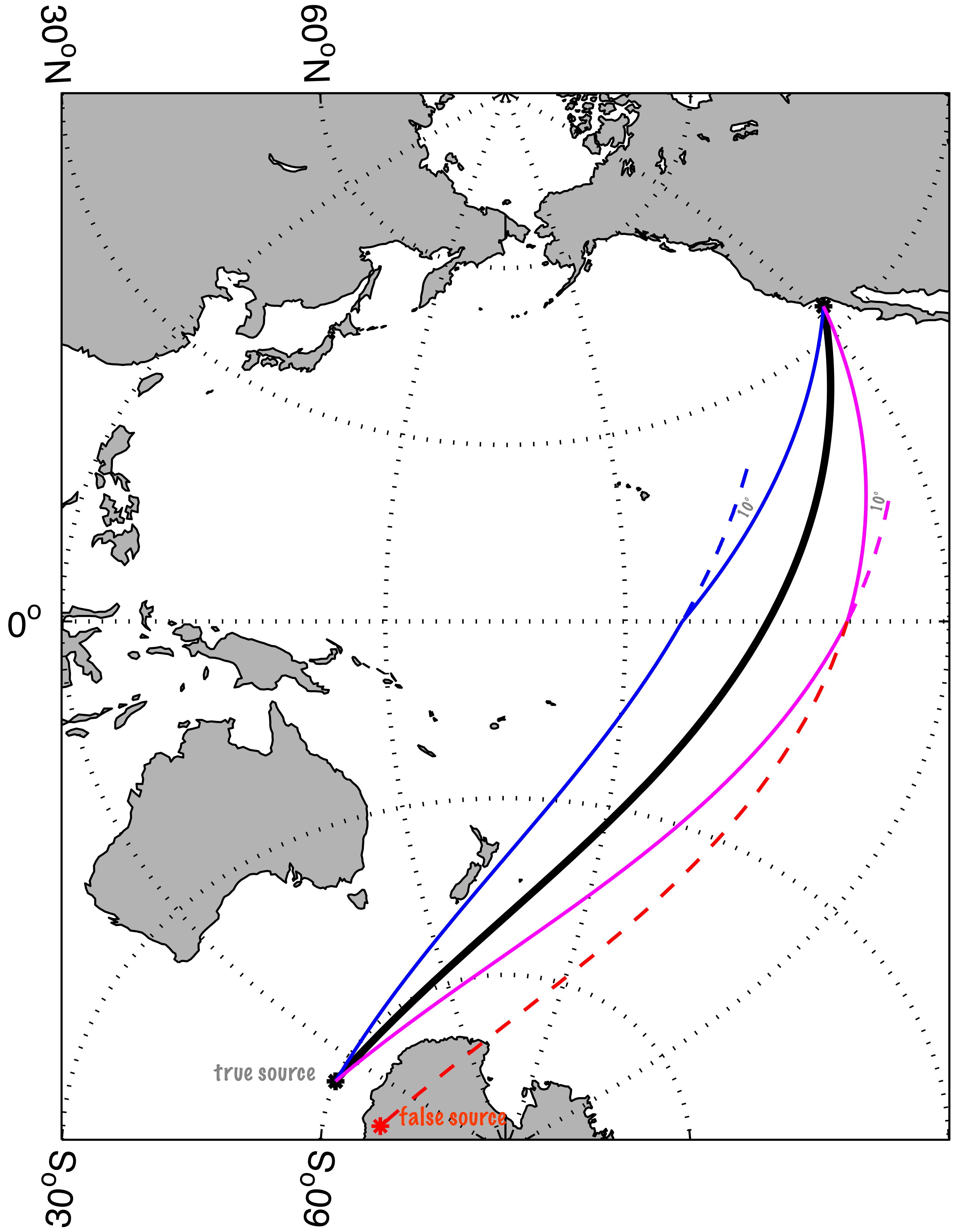
A few events (big eddies) can produce the observed deflections.





Summary

Scattering near the receiver is more important than scattering near the source.



Sunset photo by Peter Sutherland

Really the End

Wikimedia Commons

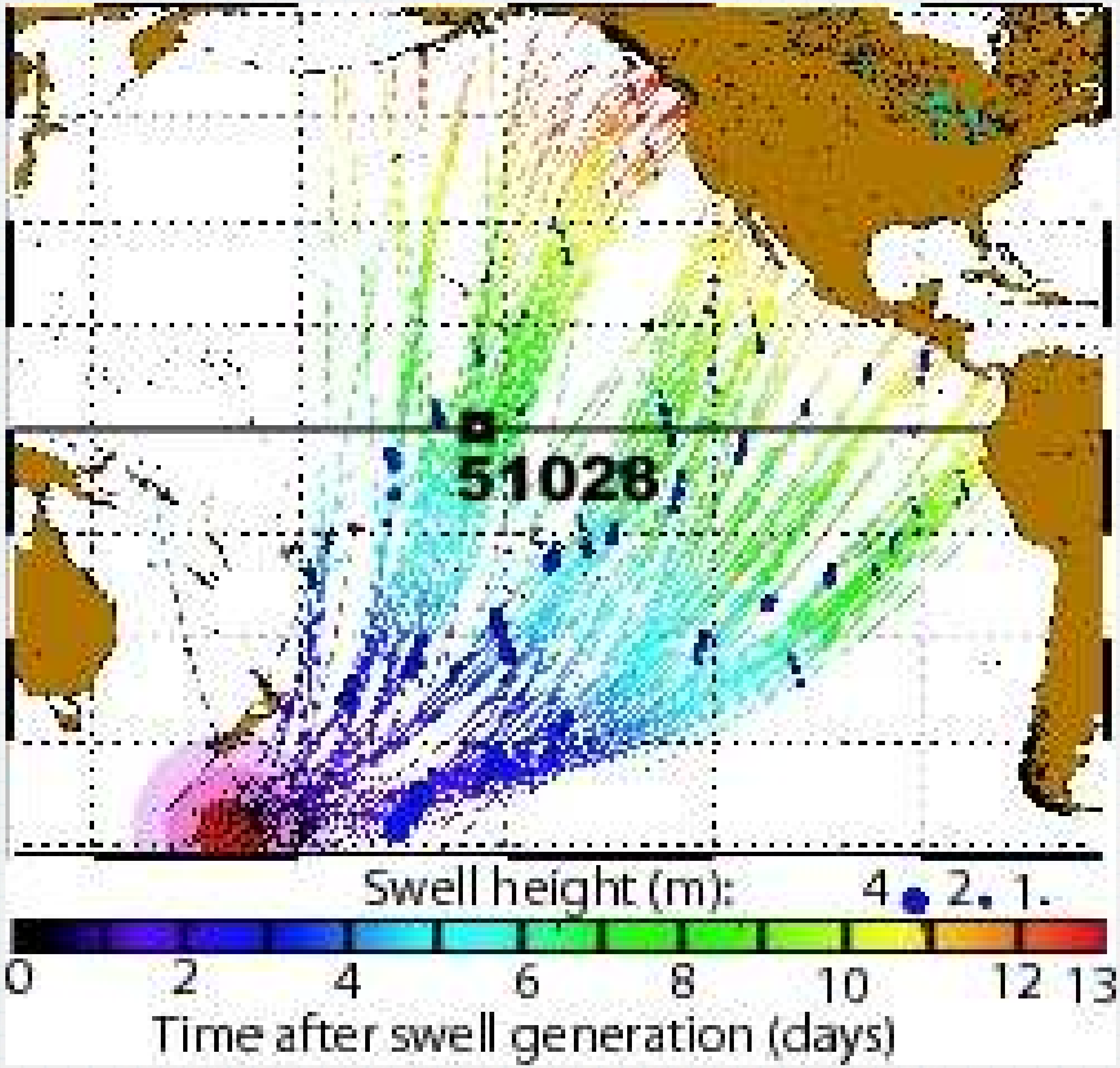


Figure 4. Finding the source storm. All swells with a 17 ± 0.5 s period that were identified in 13 days of ENVISAT synthetic aperture radar data over the Pacific, are re-focussed from their location of observation (filled dots) following their direction of arrival at the theoretical group speed for 17 s waves. This focussing reveals a single swell generation event, well defined in space and time (pink to red disks). The back-tracking trajectories are color-dated from black (July 9 2004 18:00 UTC) to

Munk & Snodgrass, 1957

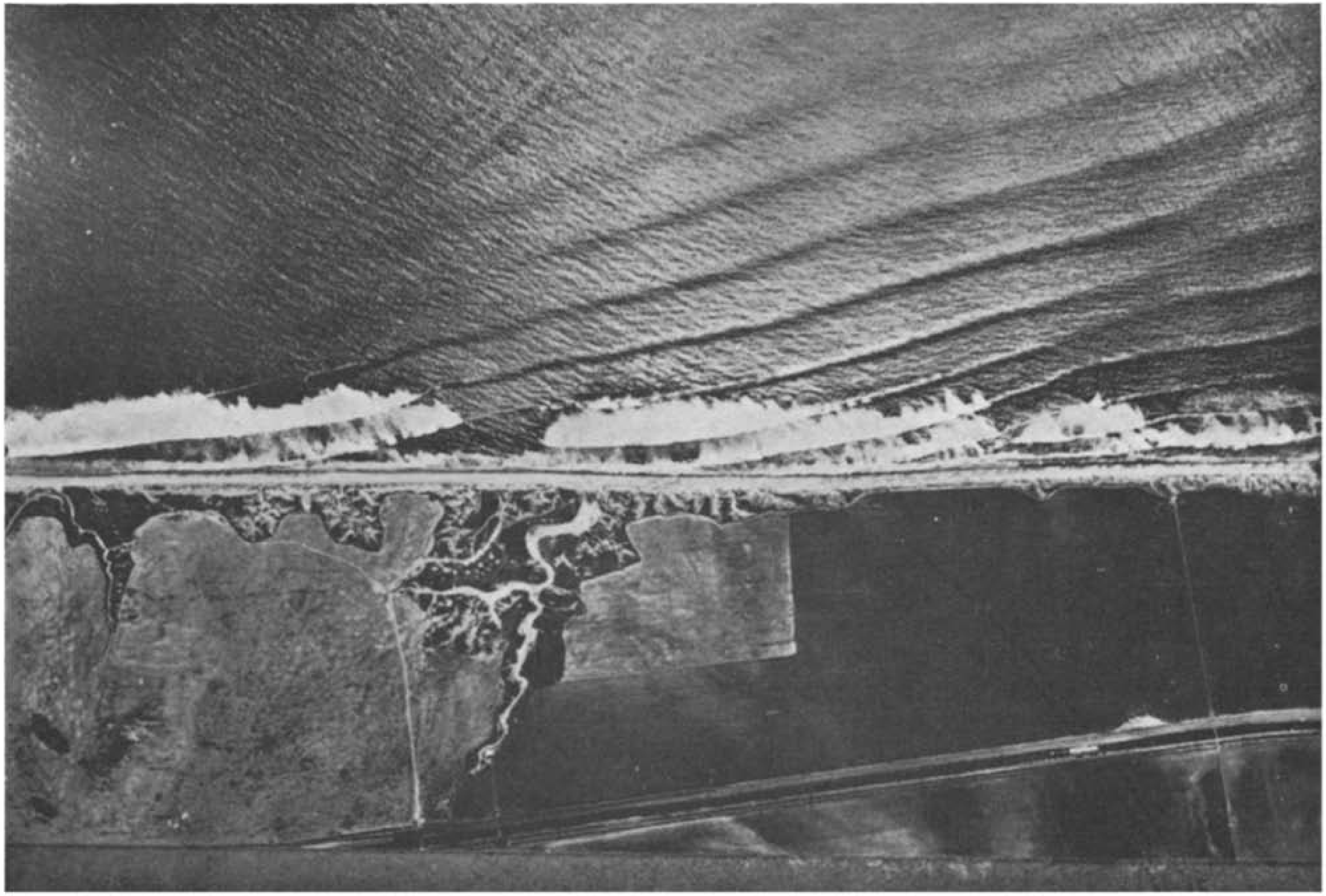
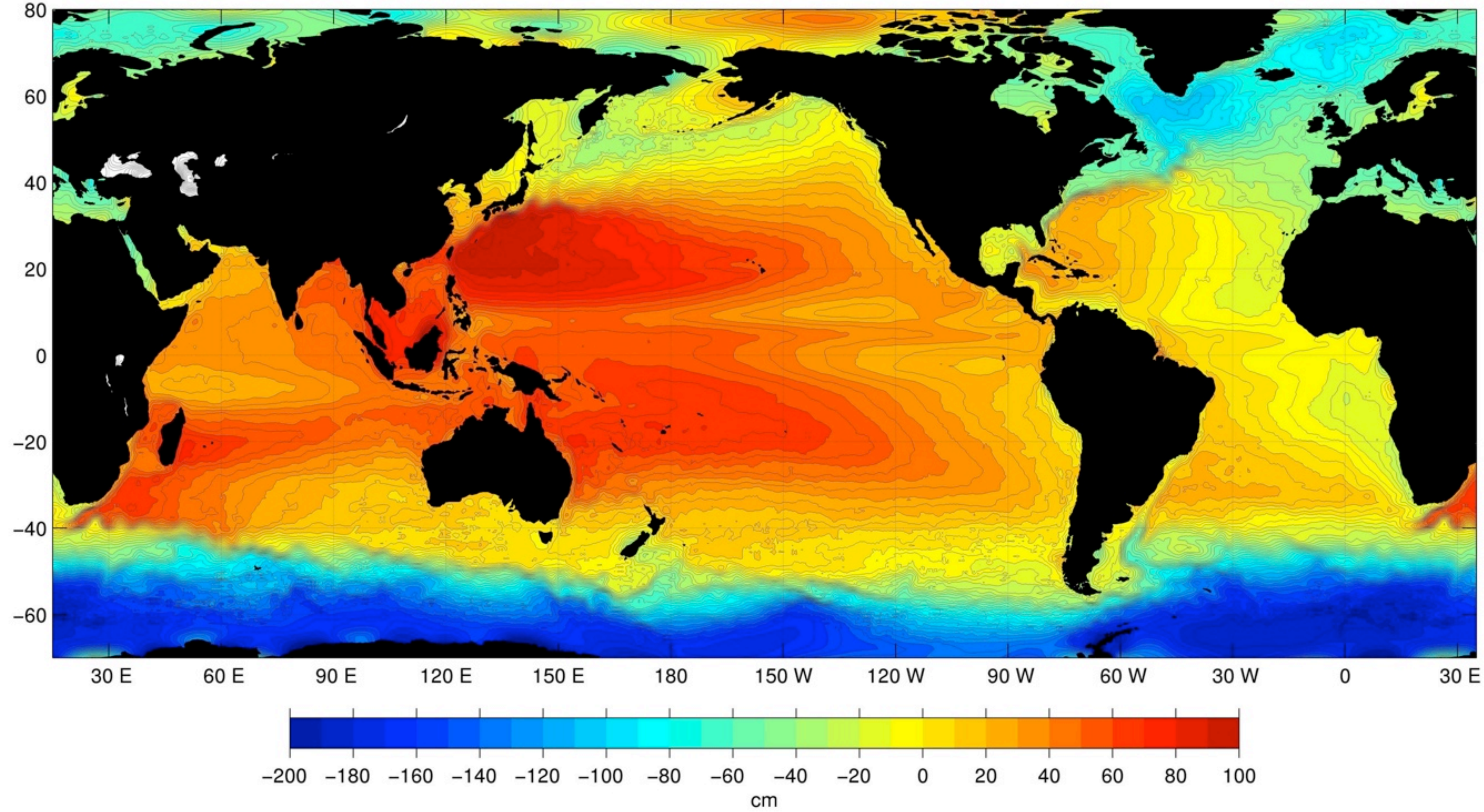
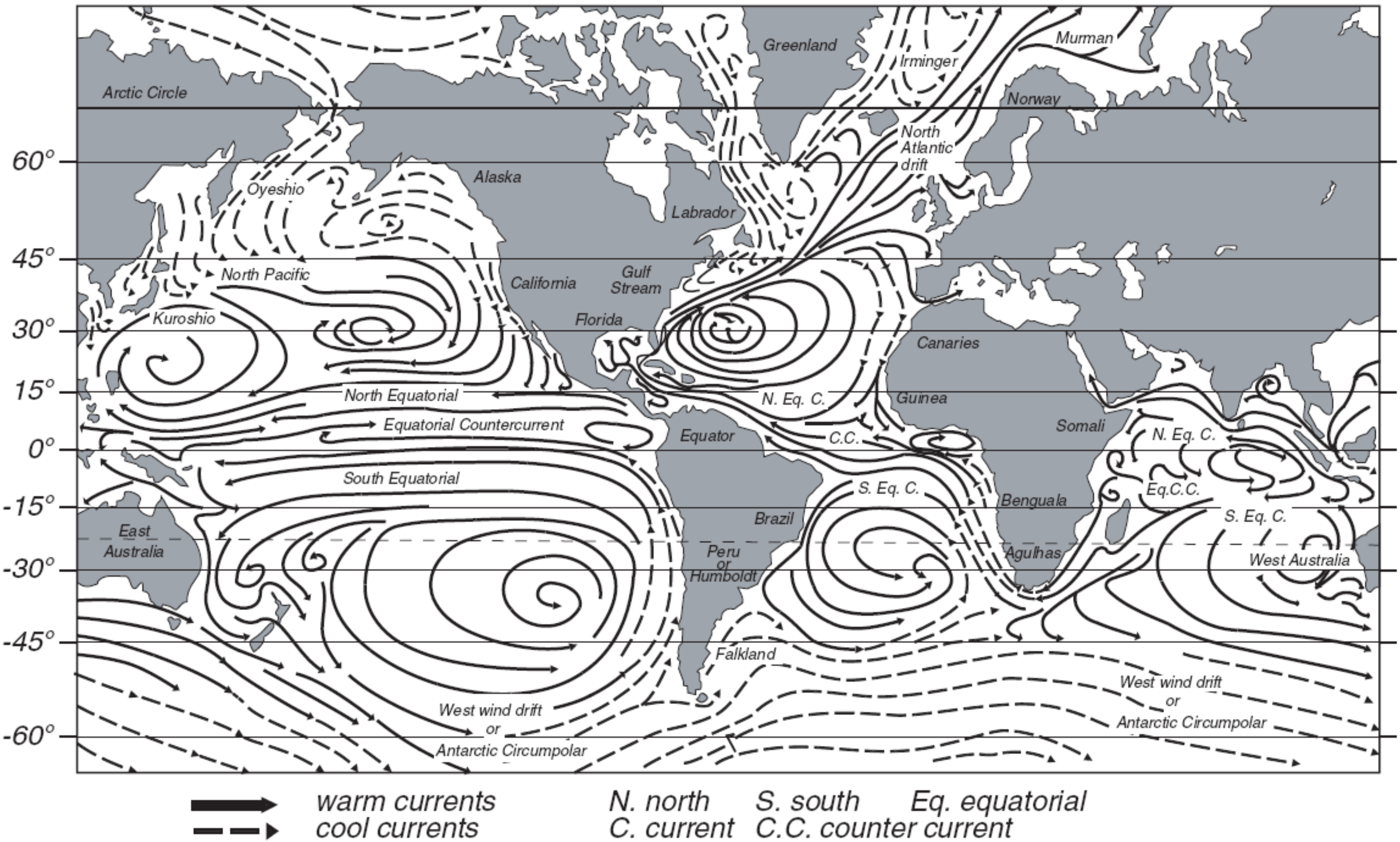


Fig. 1. Aerial photograph of waves at Camp Pendleton, California, on 16 January, 1944. The coast trends north-west – south-east.

Fritz Ursell


80
60
40
20
0
−20
−40
−60
30 E
60 E
90 E
120 E
150 E
180
150 W
120 W
90 W
60 W
30 W
0
30 E
−200
−180
−160
−140
−120
−100
−80
−60
−40
−20
0
20
40
60
80
100
cm

Time-average surface currents based on a century of oceanographic observations (after Tolmazin 1985).



La Jolla Cove, February 1986

El Nino storm strikes the Marine Room