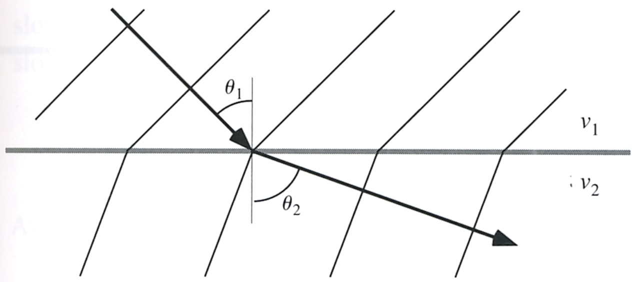
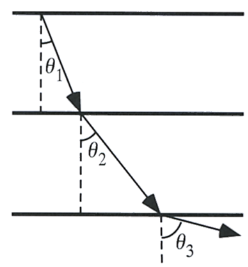


Figure 4.2 A plane wave crossing a horizontal interface between two homogeneous half-spaces. The higher velocity in the bottom layer causes the wavefronts to be spaced further apart.

Notice that this is simply the seismic version of Snell's law in geometrical optics. Equation (4.4) may also be obtained from *Fermat's principle*, which states that the travel time between two points must be stationary (usually, but not always, the minimum time) with respect to small variations in the ray path. Fermat's principle itself can be derived from applying variational calculus to the eikonal equation (e.g., Aki and Richards, 2002, pp. 89–90).

4.2 Ray paths for laterally homogeneous models

In most cases the compressional and shear velocities increase as a function of depth in the Earth. Suppose we examine a ray traveling downward through a series of layers, each of which is faster than the layer above. The ray parameter p remains constant and we have



$$p = u_1 \sin\theta_1 = u_2 \sin\theta_2 = u_3 \sin\theta_3. \tag{4.5}$$

If the velocity continues to increase, θ will eventually equal 90° and the ray will be traveling horizontally.

This is also true for continuous velocity gradients (Fig. 4.3). If we let the slowness at the surface be u_0 and the *takeoff angle* be θ_0, we have

$$u_0 \sin\theta_0 = p = u \sin\theta. \tag{4.6}$$

When $\theta = 90°$ we say that the ray is at its *turning point* and $p = u_{tp}$, where u_{tp} is the slowness at the turning point. Since velocity generally increases with depth in Earth, the slowness *decreases* with depth. Smaller ray parameters are more steeply dipping at the surface, will turn deeper in Earth, and generally travel farther. In these examples with horizontal layers or vertical velocity gradients, p remains