

$$\omega^2 = gm = \frac{f^2}{2} + \sqrt{(gk)^2 + \frac{f^4}{4}}, \quad \text{and} \quad \left(\frac{f}{\omega}\right)^2 + \left(\frac{k}{m}\right)^2 = 1$$