Problem 1

(i) Find the \(x \to \infty\) leading-order behavior of the function \(K(x) \equiv \int_x^\infty e^{-t^4/4} \, dt\). There is a bonus for showing that the remainder is asymptotically smaller than the leading-order term as \(x \to \infty\).

(ii) Evaluate \(K(0)\) in terms of

\[ \Gamma(z) \overset{\text{def}}{=} \int_0^\infty e^{-v} v^{z-1} \, dv. \] 

Problem 2

Find a two-term \(\epsilon \to 0\) expansion of all roots of the polynomial

\[ \epsilon x^4 - x^2 - (2 + \epsilon)x - 1 = 0. \] 

Problem 4

Find the leading order behaviour of

\[ f(n) \overset{\text{def}}{=} \int_0^{\pi/2} e^t \sin^n t \, dt, \quad \text{as } n \to \infty. \] 

Problem 4

Determine the leading-order dependence on \(\epsilon\) of the largest root of

\[ x^{1/\epsilon} = e^x, \quad \text{as } \epsilon \to 0. \]