First assignment SIO203B/MAE294B, 2017

For discussion in the recitation on Thursday April 6th

Problem 1.2

Find a two-term approximation to all five roots of

$$x^5 - x + \epsilon = 0\,. \tag{1}$$

Take $\epsilon = 1/4$ and compare your approximation to a numerical solution.

Problem 1.3

Consider the transcendental equation

$$x^2 - 1 = \epsilon e^{x^2}\,. \tag{2}$$

If $\epsilon = 0$ there is a root $x = 1$. Find the first three terms in the $\epsilon \to 0$ regular perturbation expansion of this root.

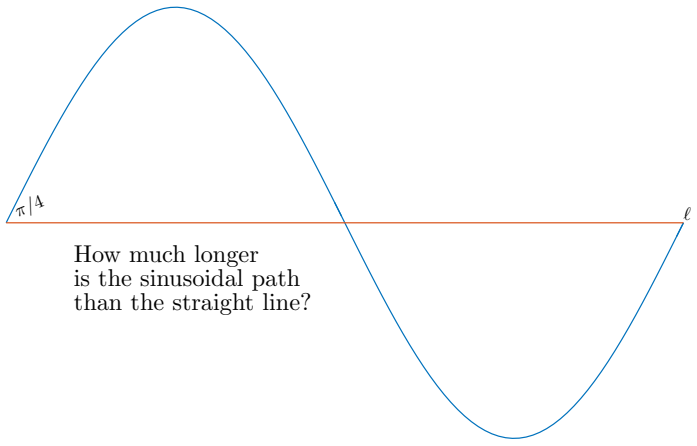

Figure 1: A tipsy walk.

Problem 1.7

Figure 1 shows the path followed by a tipsy sailor from a bar at the origin of the (x, y)-plane to home at $(x, y) = (\ell, 0)$. The path is a sinusoid leaving the bar at an angle α; in figure 1 $\alpha = \pi/4$. How much longer is the sinusoidal path than the straight line? Answer this question by: *(i)* eyeballing the curve in figure 1 and guessing; *(ii)* constructing the integral that gives the arclength and evaluating it numerically; *(iii)* devising an approximation to the arc-length integral based on $\alpha \ll 1$, and then pressing your luck by using this approximation with $\alpha = \pi/4$.

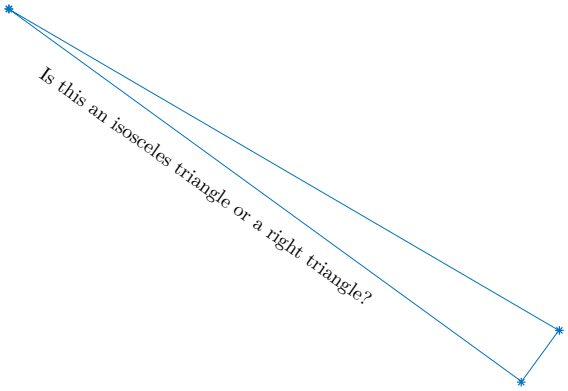

Figure 2: The small angle is about 5.71°.

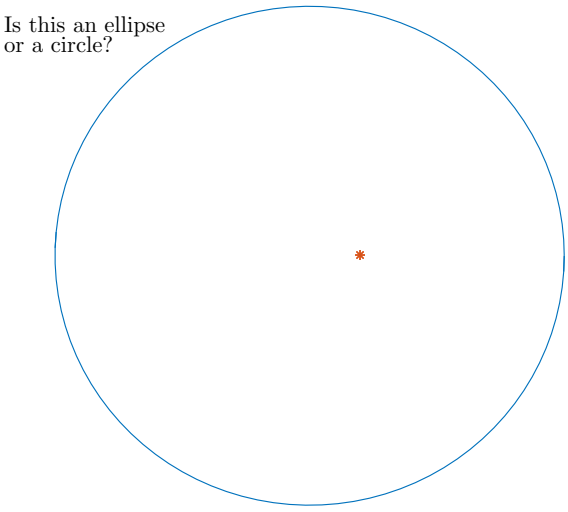

Figure 3: An ellipse with eccentricity $e = 0.2$; it certainly looks like a circle doesn't it? The point $*$ is at a focus.