Fourth assignment SIO203B/MAE294B, 2017

For recitation on Friday April 28st

Problem 4.8 part A

Find an approximate solution of the boundary value problem:

$$10^{-12} v_{xx} = \mathrm{e}^{x^4}\, v\,, \qquad \text{with BCs} \qquad v(\pm 1) = 1\,. \tag{1}$$

Problem 2.1 part B

(i) Find a leading-order $x \to \infty$ asymptotic approximation to

$$A(x;p,q) \overset{\text{def}}{=} \int_x^\infty \mathrm{e}^{-pt^q}\,\mathrm{d}t\,. \tag{2}$$

Show that the remainder is asymptotically negligible as $x \to \infty$. Above, p and q are both positive real numbers.

Problem 2.9 part B

True or false as $x \to \infty$

$$(i)\ x + \frac{1}{x} \overset{?}{\sim} x\,, \qquad (ii)\ x + \sqrt{x} \overset{?}{\sim} x\,, \qquad (iii)\ \exp\left(x + \frac{1}{x}\right) \overset{?}{\sim} \exp(x)\,, \tag{3}$$

$$(iv)\ \exp\left(x + \sqrt{x}\right) \overset{?}{\sim} \exp(x)\,, \qquad (v)\ \cos\left(x + \frac{1}{x}\right) \overset{?}{\sim} \cos x\,, \qquad (v)\ \frac{1}{x} \overset{?}{\sim} 0\,? \tag{4}$$

Problem 3.1 part B

(i) Obtain the leading-order asymptotic approximation for the integral

$$J(x) \overset{\text{def}}{=} \int_{-1}^{1} \mathrm{e}^{xt^3}\,\mathrm{d}t\,, \qquad \text{as } x \to \infty. \tag{5}$$

(ii) Find the leading-order asymptotic approximation for $x \to -\infty$.

Hand-in problems due in class on Tuesday May 2nd

Problem 2.3 part B

(i) Use integration by parts once to find the leading-order term in the $x \to \infty$ asymptotic expansion of the *exponential integral:*

$$\mathrm{E}_1(x) \stackrel{\mathrm{def}}{=} \int_x^{\infty} \frac{\mathrm{e}^{-v}}{v}\,\mathrm{d}v\,. \tag{6}$$

Show that this approximation is asymptotic i.e., prove that the remainder is asymptotically less than the leading term as $x \to \infty$. *(ii)* With some further integration by parts, find a few more terms and try to guess the full expansion.

Problem 2.4 part B

Consider the first-order differential equation:

$$y' - y = -\frac{1}{x}\,, \qquad \text{with the condition} \qquad \lim_{x\to\infty} y(x) = 0\,. \tag{7}$$

(i) For $x \to \infty$, find a valid two-term dominant balance in the differential equation and thus deduce the leading-order asymptotic approximation to $y(x)$ *(ii)* Use an iterative procedure on (7) to deduce the full asymptotic expansion of $y(x)$. *(iii)* Is the expansion convergent? *(iv)* Use the integrating function method to solve the differential equation exactly in terms of the exponential integral in (6). Use MATLAB (`help expint`) to compare the exact solution of (7) with asymptotic expansions of different order. Summarize your study as in Figure 2.3 ofs part B.