Fifth assignment SIO203B/MAE294B, 2017

For recitation on Friday May 12th

Problem 7.2 part A

Analyze the variable-speed Stommel problem

$$\epsilon h'' + (x^a h)_x = -1 \,, \qquad \text{with BCs} \quad h(0) = h(1) = 0 \,, \tag{1}$$

using boundary layer theory. (The case $a = 1/2$ was discussed in the lecture.) How thick is the boundary layer at $x = 0$, and how large is the solution in the boundary layer? Check your reasoning by constructing the leading-order uniformly valid solution when $a = -1$, $a = 1$ and $a = 2$.

Problem 8.1 part A

In an lecture 4 we compared the exact solution of the initial value problem

$$\ddot{f} + (1+\epsilon) f = 0 \,, \qquad \text{with ICs} \quad f(0) = 1 \,, \quad \text{and} \quad \dot{f}(0) = 0 \,, \tag{2}$$

with an approximation based on a regular perturbation expansion — see the discussion following (4.69). Redo this problem with a two-time expansion. Compare your answer with the exact solution and explain the limitations of the two-time expansion.

Problem 8.2 part A

Consider

$$\frac{\mathrm{d}^2 g}{\mathrm{d}t^2} + \left[1 + \epsilon \left(\frac{\mathrm{d}g}{\mathrm{d}t}\right)^2\right] g = 0 \,, \qquad \text{with ICs} \qquad g(0) = 1 \,, \quad \text{and} \quad \frac{\mathrm{d}g}{\mathrm{d}t}(0) = 0 \,. \tag{3}$$

(i) Show that a regular perturbation series fails once $t \sim \epsilon^{-1}$. *(ii)* Use the two-timing method to obtain the solution on the long time scale.

Hand-in problems due in class on Tuesday May 16th

Problem 7.8 part A

Find a leading order, uniformly valid solution of

$$\epsilon y'' + \sqrt{x} y' + y^2 = 0 \,, \tag{4}$$

posed on $0 < x < 1$ with boundary conditions $y(0;\epsilon) = 2$ and $y(1;\epsilon) = 1/3$.

Problem 8.3 part A

Consider the initial value problem:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}t^2} + u = 2 + 2\epsilon u^2 \,, \qquad \text{with ICs} \qquad u(0) = \frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0 \,. \tag{5}$$

(i) Supposing that $\epsilon \ll 1$, use the method of multiple time scales ($s = \epsilon t$) to obtain an approximate solution valid on times of order ϵ^{-1}. *(ii)* Consider

$$\frac{\mathrm{d}^2 v}{\mathrm{d}t^2} + v = u \,, \qquad \text{with ICs} \qquad v(0) = \frac{\mathrm{d}v}{\mathrm{d}t}(0) = 0 \,, \tag{6}$$

where $u(t,\epsilon)$ on the right is the solution from part *(i)*. Find a leading-order approximation to $v(t,\epsilon)$, valid on the long time scale $t \sim \epsilon^{-1}$.