Sixth assignment SIO203B/MAE294B, 2017

For recitation on Friday May 19th

Problem 4.7 part A

Let's make a small change to the formulation of the belligerent-drunks example in section **4.2**. Suppose that we model the bars using a Neumann boundary condition. This means that the flux of drunks, rather than the concentration, is prescribed at $x = 0$ and ℓ. Using dimensional variables, the problem is

$$\kappa u_{xx} = \mu u^2 \,, \tag{1}$$

with the boundary condition

$$\kappa u_x(0,t) = -F \,, \qquad \text{and} \qquad \kappa u_x(\ell,t) = F \,. \tag{2}$$

F, with dimensions drunks per second, is the flux entering the domain from the bars. Repeat *all calculations* in section **4.2**, including the analog of the $\beta \ll 1$ perturbation expansion — find the first two terms of the expansion. (The most difficult part is finding the "correct" scaling for u so that the perturbation expansion is straightforward.)

Problem 8.4 part A

Consider the initial value problem:

$$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} + w = 2\cos(\epsilon t) + 2\epsilon w^2 \,, \qquad \text{with ICs} \qquad w(0) = \frac{\mathrm{d}w}{\mathrm{d}t}(0) = 0 \,. \tag{3}$$

Supposing that $\epsilon \ll 1$, use the method of multiple time scales ($s = \epsilon t$) to obtain an approximate solution valid on times of order ϵ^{-1}.

Problem 7.9 part A

Find a leading order, uniformly valid solution of

$$\epsilon y'' - (1 + 3x^2)y = x \,, \qquad \text{with BCs} \qquad y(0,\epsilon) = y(1;\epsilon) = 1 \,. \tag{4}$$

Problem 6.5 part A

Use boundary layer theory to find leading-order solution of

$$h_x = \epsilon \left(\tfrac{1}{3}h^3\right)_{xx} + 1 \,, \tag{5}$$

on the domain $0 < x < 1$ with boundary conditions $h(0) = h(1) = 0$. You can check your answer by showing that $h = 1/2$ at $x \approx 1 - 0.057\epsilon$. (This problem is similar to the problem in section **6.4**.)

Hand-in problems due in class on Tuesday May 30th

Problem 8.10 part A

The equation of motion of a pendulum with length ℓ in a gravitational field g is

$$\ddot{\theta} + \omega^2 \sin\theta = 0\,, \qquad \text{with} \qquad \omega^2 \stackrel{\text{def}}{=} \frac{g}{\ell}\,. \tag{6}$$

Suppose that the maximum displacement is $\theta_{\max} = \phi$. *(a)* Show that the period P of the oscillation is

$$\omega P = 2\sqrt{2}\int_0^{\phi} \frac{\mathrm{d}\theta}{\sqrt{\cos\theta - \cos\phi}}\,.$$

(b) Suppose that $\phi \ll 1$. By approximating the integral above, obtain the coefficient of ϕ^2 in the expansion:

$$\omega P = 2\pi\left[1 + ?\phi^2 + O(\phi^3)\right]$$

(c) Check this result by re-derving it via a multiple scale expansion applied to (6). *(d)* A grandfather clock swings to a maximum angle $\phi = 5^\circ$ from the vertical. How many seconds does the clock lose or gain each day if the clock is adjusted to keep perfect time when the swing is $\phi = 2^\circ$?

Problem 9.1 part A

Consider the *nonlinear* inverted pendulum

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} - \left[1 + \frac{\alpha}{\epsilon}\cos\left(\frac{t}{\epsilon}\right)\right]\sin\theta = 0\,. \tag{7}$$

Apply the two-time method to this nonlinear equation and find the effective potential resulting from averaging the rapid oscillations. Calculate the phase-plane orbits in the effective potential and use MATLAB to plot the orbits.