SIO203B/MAE294B Mid-term 2014

This exam is open notes, but no computers, iPhones or electronic assistance.

Problem 1

$x(t)$ is defined via the initial value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 1.005 - x - \mathrm{e}^{-x}\,, \qquad \text{with IC} \qquad x(0) = 0\,. \tag{1}$$

Find $\lim_{t\to\infty} x(t)$ to two significant figures.

Problem 2

Find the leading-order, $x \to \infty$, asymptotic expansion of

$$E(x) \stackrel{\text{def}}{=} \int_0^\infty \mathrm{e}^{-xt-t^4/4}\,\mathrm{d}t\,, \qquad \text{and} \qquad F(x) \stackrel{\text{def}}{=} \int_0^\infty \mathrm{e}^{xt-t^4/4}\,\mathrm{d}t\,. \tag{2}$$

Problem 3

Find a two-term $\epsilon \to 0$ expansion of all roots of the polynomial

$$\epsilon x^4 - x^2 - (2+\epsilon)x - 1 = 0\,. \tag{3}$$

Problem 4

Use multiple scale theory to find an approximate solution of

$$u_{tt} + u = \mathrm{e}^{\epsilon t} + \epsilon \mathrm{e}^{-\epsilon t} u^2\,, \qquad \text{with ICs} \qquad u(0) = u_t(0) = 0\,. \tag{4}$$

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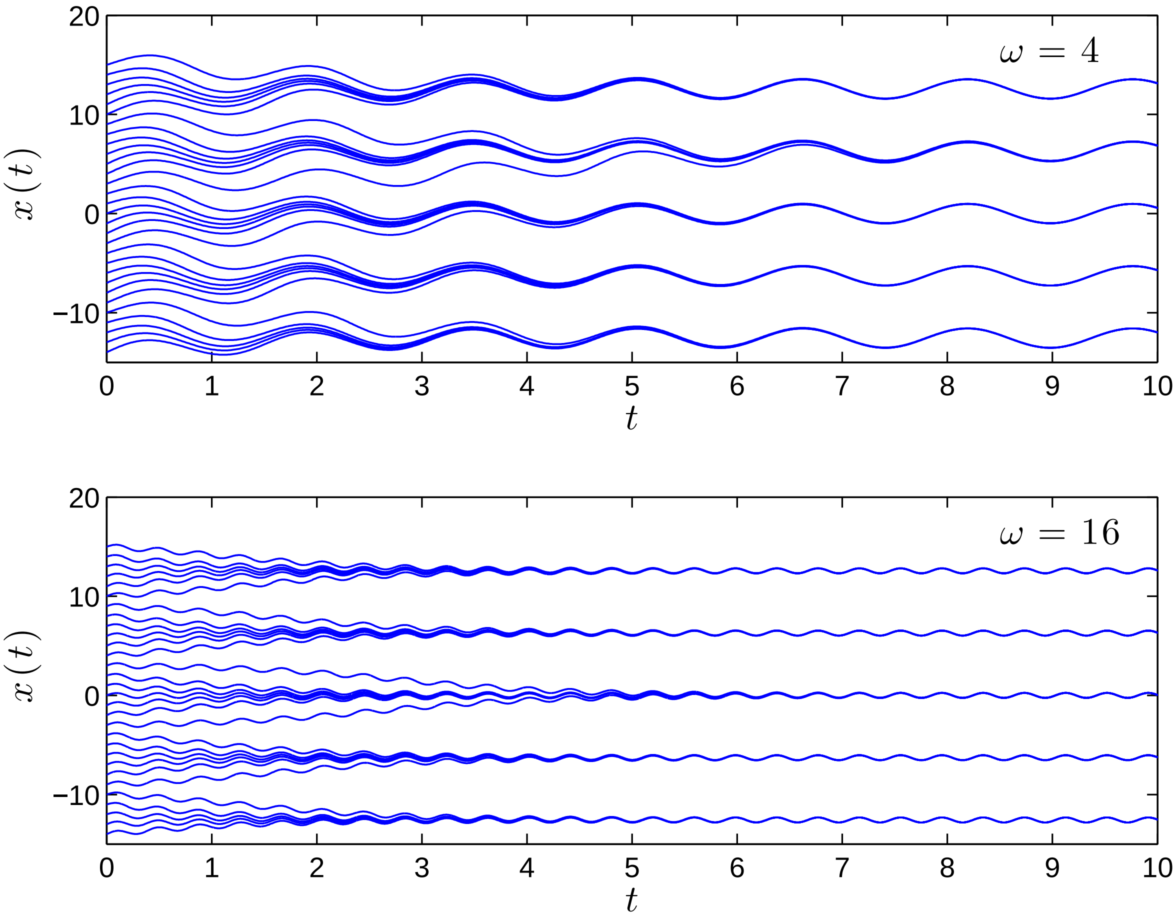

Figure 1: Numerical integration of (5) using `ode45`. In the upper panel $\omega = 4$, and in the lower panel $\omega = 16$.

Problem 5

Figure 1 shows solutions of

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\cos\omega t - \sin x\,. \tag{5}$$

Explain these computations as quantitatively as you can. For example, the solutions ultimately oscillate around particular locations — what are those locations? Why are the oscillations small, and how does their amplitude depend on ω?