

SIO203B/MAE294B Mid-term 2015

This exam is open notes, but no computers, iPhones or electronic assistance.

Problem 1

Find two terms in the $x \rightarrow \infty$ asymptotic expansion of

$$A(x) \stackrel{\text{def}}{=} \int_0^x e^{-t^3} dt. \quad (1)$$

Express all integrals in your answer in terms of $\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t} dt$. Justify the asymptoticness of your two-term expansion by showing that remainder is negligible relative to the second term as $x \rightarrow \infty$.

Problem 2

Find the leading-order, $x \rightarrow \infty$, asymptotic expansion of

$$R(x) \stackrel{\text{def}}{=} \int_0^\infty \exp(-xt - e^t) dt, \quad \text{and} \quad S(x) \stackrel{\text{def}}{=} \int_0^\infty \exp(xt - e^t) dt. \quad (2)$$

Problem 3

Find a two-term $\epsilon \rightarrow 0$ approximation to all roots of the polynomial $y^3 - y^2 + \epsilon = 0$.

Problem 4

$x(t)$ is defined via the initial value problem

$$\frac{dx}{dt} = \exp\left(\frac{x}{10}\right) - x, \quad \text{with IC} \quad x(0) = 0. \quad (3)$$

Find $\lim_{t \rightarrow \infty} x(t)$ to three significant figures.

Problem 5

(i) Use multiple scale theory to find an approximate solution of

$$u_{tt} + u = e^{\epsilon t} + \epsilon e^{-\epsilon t} u^2, \quad \text{with ICs} \quad u(0) = u_t(0) = 0, \quad (4)$$

valid on the long time scale $t \sim \epsilon^{-1}$. (ii) Consider

$$v_{tt} + v = u, \quad \text{with ICs} \quad v(0) = v_t(0) = 0. \quad (5)$$

On the right of (5), $u(t, \epsilon)$ is the solution from part (i). Find a leading-order approximation to $v(t, \epsilon)$ valid on the long time scale $t \sim \epsilon^{-1}$.

Hint: The forcing, $u(t, \epsilon)$, on the right of the oscillator equation (5) is nearly resonant. So you should anticipate that $v(t, \epsilon)$ is much larger than $u(t, \epsilon)$.