

## SIO203B/MAE294B Mid-term 2016

This exam is open notes, but no computers, iPhones or electronic assistance.

### Problem 1

Find two terms in the  $x \rightarrow \infty$  asymptotic expansion of

$$A(x) \stackrel{\text{def}}{=} \int_0^x t^{-1/3} e^{-t} dt. \quad (1)$$

Justify the asymptoticness of your two-term expansion by showing that remainder is negligible relative to the second term as  $x \rightarrow \infty$ .

### Problem 2

Find the leading order uniformly valid  $\epsilon \rightarrow 0$  approximation to the solution of

$$-(e^x g)_x = \epsilon g_{xx} + e^x, \quad \text{with BCs } g(0) = g(1) = 0. \quad (2)$$

### Problem 3

Find two terms in the  $\epsilon \rightarrow 0$  approximation to all roots of the polynomial  $\epsilon z^4 - z + 1 = 0$ .

### Problem 4

$x(t)$  is defined via the initial value problem

$$\frac{dx}{dt} = x^2 - 4 + \frac{1}{23} \ln \frac{1}{x}, \quad \text{with IC } x(0) = 1. \quad (3)$$

Estimate  $\lim_{t \rightarrow \infty} x(t)$  with one significant figure.

### Problem 5

Find a leading order  $x \rightarrow \infty$  approximation to

$$F(x) = \int_{-\infty}^{\infty} e^{+xt-t^4/4} dt. \quad (4)$$

(There is no need to justify asymptoticness.)