THE GENERATION OF LANGMUIR CIRCULATIONS BY THE EDDY PRESSURE OF SURFACE WAVES[1]

Alan J. Faller

Institute for Fluid Dynamics and Applied Mathematics and Graduate Meteorology Program, University of Maryland, College Park 20740

ABSTRACT

The Langmuir circulations in lakes and oceans are shown, by theory and by the extension of laboratory experiments, to be the result of wave action. Specifically, the exponential decay of vertical oscillations that are a maximum near the free surface gives rise to an instability that is analogous to thermal convective instability but which may be two orders of magnitude or more greater than strong thermal convection. Laboratory experiments in a wind-wave tank support the predictions of the theory. Recognition of this mechanism as a primary cause of Langmuir circulations explains apparent differences between observations in lakes and oceans.

INTRODUCTION

Having reviewed the paper "On the mechanism of Langmuir circulations and their role in epilimnion mixing" by Scott et al. (1969), I had asked for the opportunity to comment directly on that paper with the intent of extending the interpretation of the results. Since that time, careful consideration of the possible mechanisms of generation of Langmuir circulations has indicated an entirely new and previously unsuspected process that involves surface waves and turbulence. On the basis of this theory, some of the apparent inconsistencies that the above authors infer in the mechanism of generation of Langmuir circulations can be rationalized without difficulty. In this article I will outline the elements of the new theory, present a series of laboratory experiments designed specifically to test the new mechanism, and discuss the rationalization of observations in the oceans and lakes.

A NEW MECHANISM FOR THE GENERATION OF LANGMUIR CIRCULATIONS

If the equations of fluid motion, heat conduction, and continuity are formulated for the investigation of longitudinal vortices as perturbations on a basic laminar flow, the equations for the vorticity and the energy of longitudinal rolls may be readily derived. The vorticity equation and the equation for longitudinal flow are, respectively:

$$\xi^*_t = -(g/\rho_0)\rho^*_y + \nu(\xi^*_{yy} + \xi^*_{zz}), \quad (1)$$

and

$$u^*_t = -w^*\bar{u}_z + \nu(u^*_{yy} + u^*_{zz}), \quad (2)$$

where subscripts t, x, y, z denote partial differentiation and where asterisks indicate departures from the mean flow $\bar{u} = \bar{u}(z)$, $\bar{v} = 0$, $\bar{w} = 0$. ξ^* is the perturbation x-component of vorticity (in a right-handed Cartesian coordinate system with z vertically upward) and is assumed to be independent of x. It is given in terms of the velocity components u, v, w, by $\xi^* = w^*_y - v^*_z$. The fluid density is denoted by ρ with the constant basic value ρ_0 and with the mean field $\bar{\rho} = \bar{\rho}(z)$, g is the acceleration of gravity, ν is the (constant) kinematic viscosity, and the Boussinesq approximation has been applied. The effects of the earth's rotation have been deliberately omitted so that instability of the Ekman boundary layer (Faller 1966) is not considered here. While the instability of the Ekman layer may sometimes act in the ocean, it cannot explain the rapid formation of Langmuir circulations in lakes.

Energy equations can be obtained from equations (1) and (2) by multiplying these

[1] The research reported here was supported by National Science Foundation Grant GA-4388.

by $-\phi^*$ (the perturbation stream function for the longitudinal rolls) and by u^*, respectively, and then integrating by parts over y and z. For these integrations periodicity is assumed in the y direction, and ϕ^* is taken to be zero at the upper and lower boundaries of the fluid layer. The resultant energy equations are then:

$$K^*_t \equiv \left[\int \frac{v^{*2}+w^{*2}}{2}\,dA\right]_t = -\frac{g}{\rho_0}\int w^*\rho^*\,dA + \frac{1}{\rho_0}\int_{z=0} v^*\tau^*_{zy}\,dy - \nu\int \xi^{*2}\,dA, \qquad (3)$$

and

$$\left[\int \frac{u^{*2}}{2}\,dA\right]_t = -\int u^*w^*\bar{u}_z\,dA + \frac{1}{\rho_0}\int_{z=0} u^*\tau^*_{zx}\,dy - \nu\int[(u^*_y)^2+(u^*_z)^2]\,dA, \qquad (4)$$

where the upper boundary has been assumed to be a smooth level surface. The three terms on the right of equation (3) are called, starting from the left: 1) the thermal convective term, 2) the surface forcing term, and 3) the viscous dissipation term. The corresponding terms in equation (4) are called: 1) the shear generation term, 2) the surface forcing term, and 3) the viscous dissipation term. In both cases the viscous dissipation integrals are positive definite and with the negative signs they clearly dissipate kinetic energy. The surface forcing terms depend on correlations across wind at the free surface ($z = 0$) between the velocities v^* and u^* and their corresponding surface perturbation stresses $\tau^*_{zy} = \rho_0\nu v^*_z$ and $\tau^*_{zx} = \rho_0\nu u^*_z$. These terms represent the driving mechanism suggested by Welander (1963) in which vortices in the air (aided and abetted by surface film effects) can directly drive vortices in the water. If the water is entirely covered by a stationary film, the correlations $v^*\tau^*_{zy}$ and $u^*\tau^*_{zx}$ will be zero. Scott et al. (1969) observed that vortices in the air do not seem to be related to vortices in the water, therefore this possible mechanism is not considered further here. The remaining generation terms involve the thermal stratification and the mean shear flow. However, note that neither $\bar{u}$ nor u^* occurs in equation (3) so that there is no way by which the shear of the longitudinal flow can directly affect the overturning energy, K^*, of the rolls. This conclusion is a direct result of restrictions previously applied, namely: 1) that the basic flow is given entirely by $\bar{u} = \bar{u}(z)$, 2) that the rolls are oriented in the x direction, 3) that the effect of the earth's rotation may be neglected, and 4) that the basic flow is laminar with a level free surface. Within these restrictions the only process that can generate longitudinal rolls is thermal convection.

It has long been known (Jeffries 1928) that thermal convection in the presence of shear flow will tend to take the form of longitudinal rolls oriented along the direction of the shear vector, and the Langmuir circulations have often been attributed to convection of this type. But the data of Scott et al. (1969) clearly show that Langmuir circulations often exist in stable stratification and convect heat downward. Moreover, it has been stated by Ichiye (1967, p. S271), with respect to oceanic observations, that: "These features [striations in dye] were observed on the condition of slight vertical stability in the surface layer except for strong winds above 8 m/sec which produced neutral stability, and except for calm seas in the summer with rather strong vertical stability in the upper 1-meter depth." Similar evidence was reported to me in 1962 by E. R. Baylor and W. H. Sutcliffe (personal communication) of the Woods Hole Oceanographic Institution. They found from thermistor observations at the ocean surface that on days of warm air advection over the ocean, the lines of surface convergence were relatively warm—a clear indication of the downward advection of heat. Thus, *it* must be concluded that the conditions imposed on equations (1–4) are overly restrictive if we wish to explain Langmuir circulations.

Restrictions 1) and 2), that the basic flow is given by $\bar{u}=\bar{u}(z)$, and that the cells are oriented along the direction of $\bar{u}$, will generally fail in an Ekman boundary layer, if the wind direction has a trend, or if for any other reason there is an appreciable variation of the flow direction with depth. However, there appears to be no evidence that this is an important factor for Langmuir circulations (even though it may at times contribute) since in lakes with steady winds the windrows are generally observed to be parallel to the wind and to the shear in the water. With respect to restriction 3), it is easily shown that if the earth's rotation is included in equations (1–4) there is a mechanism by which the energy of the shear flow can be transferred into the overturning energy K. This mechanism does not require the spiral part of the Ekman boundary layer but only the linear shear. It corresponds to the type II instability of the Ekman layer (Faller and Kaylor 1966; Lilly 1966), but once again this instability cannot be the mechanism that operates in lakes over short intervals of time. We can conclude, therefore, that the fourth restriction on equations (1–4) must be relaxed if we are to explain Langmuir circulations, and we must take into account either turbulence or surface waves or both. To this end, I now consider the modification of the vorticity and energy equations when waves and turbulence are included.

In the development of the new perturbation equations, it is necessary to make two separate expansions: one to isolate the turbulent fluctuations, to be denoted by a karat, and another expansion to separate out the infinitesimal amplitude longitudinal rolls, indicated as before by an asterisk. The equations are then averaged in the x direction. The methods of perturbation expansion and of averaging through the wavy free surface are somewhat intricate and are not given in detail here. When carried through to completion the resultant equations necessarily contain all of the possible surface wave mechanisms that have been suggested. However, the new suggested mechanism depends only on the presence of anisotropic fluctuations near the surface. It does not depend on the fact that the surface waves are generated by wind nor that there may be wave–wave interactions of various types. Therefore, to illustrate the principal point clearly, it is assumed that the surface waves are generated away from the region of interest so that there is no local wind effect, and the effects of wave–wave or wave–film interactions are omitted. The vorticity equation then simplifies to

$$\xi^*_t = -[(\hat{w}^2-\hat{v}^2)^*_z + (g/\rho_0)\rho^*]_y - (\hat{v}\hat{w})^*_{yy} + (\hat{w}\hat{v})^*_{zz} + \nu(\xi^*_{yy}+\xi^*_{zz}), \quad (5)$$

where the extra terms (*see* Eq. 1) arise from the Reynolds stresses. The asterisks beside the Reynolds stress terms denote (as with the other variables) that these are perturbations from the mean Reynolds stresses. For example, $(\hat{w}^2)^*$ is a perturbation from the mean turbulent intensity $\overline{\hat{w}^2}$. The mean Reynolds stress $\overline{\hat{v}\hat{w}}$ is identically 0, but the perturbation $(\hat{v}\hat{w})^*$ does not vanish, and the perturbation longitudinal vortices will tend to be dissipated by the action of turbulent stresses of this type. In the customary way, we can approximate the correlation Reynolds stresses in terms of eddy viscosities, for example $(\hat{v}\hat{w})^*_y = -(\nu_e w^*_y)_y$, thereby intimating that these stresses vanish if the perturbed flow vanishes and that they are dissipative in nature. The modified vorticity equation is then:

$$\xi^*_t = -[(\hat{w}^2-\hat{v}^2)^*_z + (g/\rho_0)\rho^*]_y + (\nu_e w^*_y)_{yy} - (\nu_e v^*_z)_{zz} + \nu(\xi^*_{yy}+\xi^*_{zz}). \quad (6)$$

Note that the Reynolds stresses $(\hat{w}^2)$ and $(\hat{v}^2)$ are positive definite. They should not be represented by eddy viscosities, and it is decidedly incorrect to introduce an all-inclusive eddy viscosity into the equations of motion in place of molecular viscosity (e.g., Neumann and Pierson 1966). It is suggested that the terms $\rho_0\hat{u}^2$, $\rho_0\hat{v}^2$, and $\rho_0\hat{w}^2$ be called components of an eddy pressure since they enter the equations of motion in a manner entirely analogous to kinetic pressure.

Turning attention now to the role of eddy pressure effects in equation (6), note that the term $(\hat{w}^2 - \hat{v}^2)^*_z$ has a role exactly analogous to ρ^*. Therefore, if there is a vertical gradient of the mean difference $(\overline{\hat{w}^2 - \hat{v}^2})$ there also will be a perturbation due to vertical advection by w^* in the same manner that the perturbation ρ^* is produced by vertical advection in the presence of a density stratification $\bar{\rho} = \bar{\rho}(z)$. On the other hand, if the eddy pressure is isotropic so that $\overline{\hat{w}^2} = \overline{\hat{v}^2}$, the eddy pressure effect disappears from equation (6) in exactly the same manner as did kinetic pressure. Thus, anisotropy of the eddy pressure is an essential element of the proposed mechanism.

In general, a wind-driven surface wave field will have oscillatory motions in the crosswind direction (i.e., values of $\hat{v}^2$), but these will be small in comparison to the vertical oscillations. In the extreme case where the waves are generated mechanically and are independent of y, the term analogous to ρ^* is simply $(\hat{w}^2)^*_z$ and the mean gradient analogous to $\bar{\rho}_z$ is $(\overline{\hat{w}^2})_{zz}$. Since the vertical velocities due to a sinusoidal wave-train decrease exponentially with depth, $(\overline{\hat{w}^2})_{zz}$ should also be approximately an exponential function even though there may be additional turbulence involved.

To compare the relative magnitudes of the eddy pressure effect and of the thermal (density) effect, an energy equation is formed from equation (6) in the manner indicated for equation (3). The resultant equation is:

$$K^*_t = -\int w^*[(\hat{w}^2)^*_z + (g/\rho_0)\rho^*]\, dA - \int \nu_e[(w^*_y)^2 + (v^*_z)^2]\, dA - \nu \int \xi^{*2}\, dA. \qquad (7)$$

Equation (7) is now made nondimensional by the transformations:

$t = t_n H^2/\nu$, $\xi^* = \xi^*_n \nu/H^2$, $w^* = w^*_n \nu/H$, $v^* = v^*_n \nu/H$, $\rho^* = \rho^*_n(\bar{\rho}_z H)$, $z = z_n H$, $y = y_n H$, $(\hat{w}^2)^*_z = (\hat{w}^2)^*_{zn}[(\overline{\hat{w}^2})_{zz} H]$; and where H is the depth of the fluid layer, and the subscripts n indicate the nondimensional values. Then, dropping n, the resultant equation is

$$K^*_t = -\int w^*[Q(\hat{w}^2)^*_z + \mathcal{R}P^{-1}\rho^*]\, dA - \int (\nu_e/\nu)[(w^*_y)^2 + (v^*_z)^2]\, dA - \int \xi^{*2}\, dA, \qquad (8)$$

where $\mathcal{R} = g\bar{\rho}_z H^4/(\rho_0 \kappa \nu)$ is the Rayleigh number, $P = \nu/\kappa$ is the Prandtl number, $Q = (\overline{\hat{w}^2})_{zz} H^4/\nu^2$ is a new nondimensional number analogous to $\mathcal{R}$, and κ is the thermometric conductivity.

A comparison of the thermal and eddy pressure effects in equation (8) now rests on the assumption that the nondimensionalization has been carried out so that ρ^* and $(\hat{w}^2)^*_z$ are of the same magnitude. If this is true, it is then possible to take the ratio $S = \mathcal{R}P^{-1}/Q$ as the ratio of thermal effects to eddy pressure effects. However, the above assumption is not easily justified because it depends on a knowledge of the manner by which the perturbations $(\hat{w}^2)^*_z$ and ρ^* are dissipated. If only the orbital wave motions are present (i.e., without turbulence), it may be imagined that the perturbations $(\hat{w}^2)^*_z$ are dissipated by some viscous diffusion analogous to the diffusion of ρ^*. If there is turbulence so that molecular diffusive processes are of minor importance, it is suggested that each of the relevant perturbation quantities will be diffused by eddy processes in more or less the same way, but this whole question is rather speculative and requires a more thorough investigation.

The ratio S can be readily estimated if a sinusoidal wave with its exponential z-dependence is assumed, namely, $\hat{w} = Akce^{kz} \sin k(x - ct)$ where A is the wave amplitude, c is its phase speed, and k is its wave number. Then it is easily derived that $(\overline{\hat{w}^2})_z = A^2k^3c^2e^{2kz} = \Delta(\overline{\hat{w}^2})_z e^{2kz}$ where Δ represents a difference between a value at $z = 0$ and at $z = -\infty$. Assume that $\bar{\rho}(z)$ due to surface cooling has the same depth dependence, namely $\bar{\rho} = \Delta\bar{\rho}e^{2kz}$. The ratio S is then $S = g\Delta\bar{\rho}/(\rho_0 A^2k^3c^2)$. To put S in a form that is more easily estimated, let

$c^2 = g/k$ for deep-water waves, $k = 2\pi/L$ where L is the wavelength, and $\Delta\bar{\rho} = \rho_0 \epsilon \Delta\overline{T}$ where ϵ is the thermal expansion coefficient and T is temperature. Then

$$S = \frac{\epsilon \Delta \overline{T}}{(A/L)^2 4\pi^2}.$$

Using $\epsilon = 0.2 \times 10^{-3}$ for water, $\Delta\overline{T} = 1$ degree, and $A/L = 1/20$, $S = 0.002$. This estimate for S shows that the eddy pressure effect can easily be two orders of magnitude greater than the thermal effect. Under either stable or unstable conditions, the oscillations due to orbital wave motions probably will dominate the effects of stratification even though $(\overline{\hat{w}^2})_{zz}$ may be reduced somewhat by crosswind wave components and three-dimensional turbulence.

A more appropriate expression for Q for turbulent flows can be defined by using an eddy viscosity and using the average value $\Delta\overline{(\hat{w}^2 - \hat{v}^2)_z}$ to make $(\hat{w}^2 - \hat{v}^2)^*_z$ nondimensional. Assume that ν_e decays exponentially in z by the relation $\nu_e = \Delta\nu_e e^{2kz}$ and use $\Delta\nu_e$ to nondimensionalize the variables in equation (7). The new nondimensional energy equation is then

$$K^*_t = -\int w^* [Q(\hat{w}^2 - \hat{v}^2)^*_z + \mathcal{R}P^{-1}\rho^*]\, dA - \int e^{2kz}[(w^*_y)^2 + (v^*_z)^2]\, dA - (\nu/\Delta\nu_e) \int \xi^{*2}\, dA,$$

where now

$$Q = \frac{\Delta\overline{(\hat{w}^2 - \hat{v}^2)_z} H^3}{(\Delta\nu_e)^2},$$

and

$$\mathcal{R}P^{-1} = \frac{g\Delta\bar{\rho} H^3}{\rho_0(\Delta\nu_e)^2}.$$

The ratio S reduces to the same relation as before since it does not depend on the diffusion coefficients, but now the values of Q and $\mathcal{R}P^{-1}$ are more appropriate for consideration of convective instabilities.

Because the above theory deals with the complicated subject of turbulence and waves there are a number of loose ends the resolution of which can only be guessed at theoretically. These eventually must be resolved by detailed quantitative experiments. One of these, mentioned above, is the mechanism of decay of perturbations of turbulent quantities such as $(\hat{w}^2)^*_z$. Another is the assumption of an isotropic (although nonhomogeneous) value for the eddy viscosity in the presence of a definitely anisotropic wave field. Nevertheless, the estimated relative magnitude of the newly recognized effect appears to be sufficiently large to suggest that it is important for the generation of Langmuir circulations.

LABORATORY STUDIES OF LANGMUIR CIRCULATIONS

The wind-wave tunnel here is 9 m long, 91 cm wide, and 60 cm deep. It is equipped with a rigid cover, an overhead exhaust fan at one end, and an intake area at the other with no special provision for the smooth introduction of the air stream. Experiments are conducted in the following manner: Water is introduced to a depth of 2 to 30 cm and allowed to come to rest. Crystals of $KMnO_4$ dye are sprinkled uniformly across the tunnel through a slot in the cover on the downwind side of the glass-covered test sections. Three test sections have been used, one near either end of the tank and one near the middle. Essentially the same results are found at each section although the waves become more fully developed near the downwind end of the tank. About 2 min after introducing the dye, the crystals at the bottom have dissolved sufficiently to leave a pool of dye across the tank. The wind is then started with speeds up to about 5 m/sec. Detailed values of the wind profiles and the stress are not yet available. After the wind is started, the surface film is blown to the downwind end of the tank and the remainder of the water is film free. Before the film has passed the test section there are no capillary waves and only weak gravity waves that have propagated faster than the edge of the film. No Langmuir circulations are observed until active wave

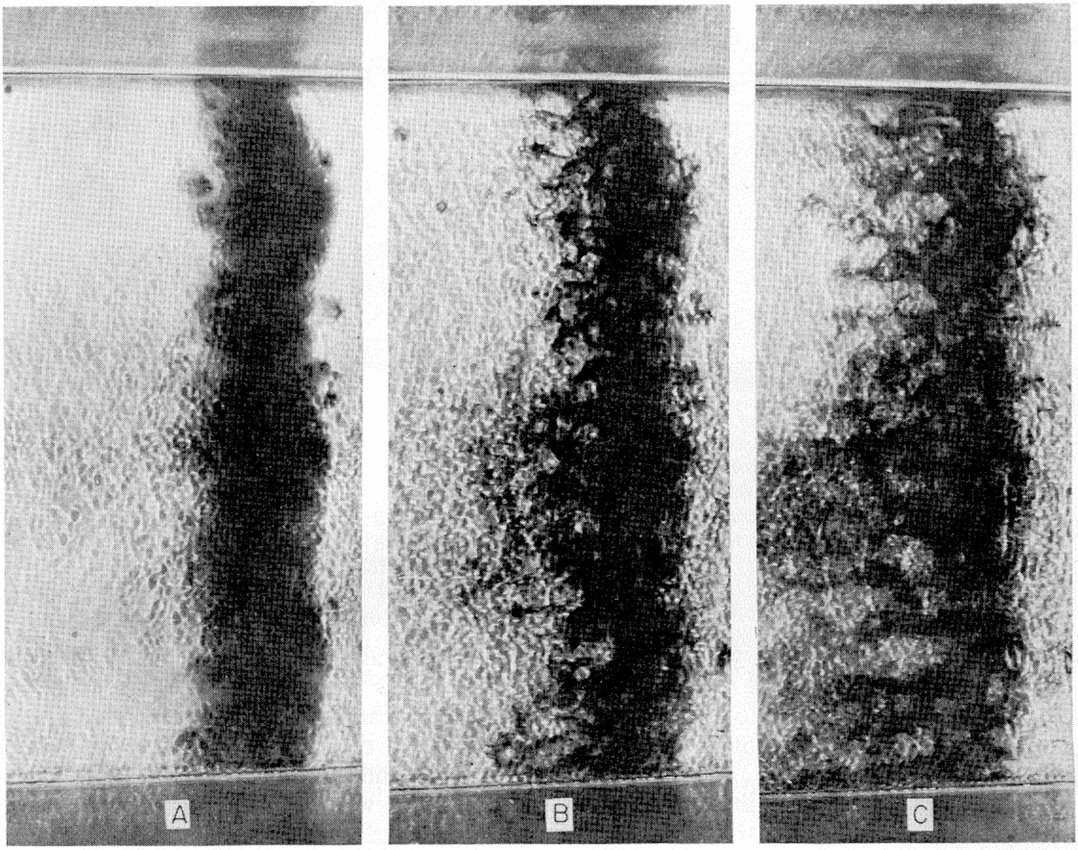

FIG. 1. Patterns of dye at the bottom of the wind-wave tank. Wind was from left to right, but dye flowed from right to left in the return flow at the bottom of the tank. Depth of water was 2.9 cm, width of the tank was 91 cm, and windspeed was approximately 5 m/sec. The test section was 1.5 m from the air intake. Photographs were taken at 10-sec intervals. A. The pool of $KMnO_4$ at the bottom of the tank just before the beginning of convection. B. The dye indicates clear patches where descending columns of water impinged on the bottom boundary. C. The dye clearly shows banded confluence of dye at the bottom due to longitudinal roll vortices.

motions move into the area of the test section. Soon after the arrival of waves at the test section intense convection becomes apparent in the pattern of dye at the bottom. This dye of course drifts counter to the direction of the wind because of the return flow at the bottom of the tank. The intense convection referred to above appears in the form of bursts of descending turbulent fluid which strike the bottom boundary and produce clear spots by rapid divergence of the layer of dye. This development and the concurrent alignment of the dye into bands is illustrated in Fig. 1 which shows a sequence of three photographs taken at 10-sec intervals. In Fig. 1A the effect of the waves has not yet penetrated to the bottom. In 1B round clear spots are evident and the beginning of a banded structure is apparent. In 1C a regular banded structure due to confluence of dye near the bottom is obvious. This confluence indicates the presence of longitudinal vortices—the Langmuir circulations. Figures 2 and 3 are examples of the same phenomenon with deeper layers of water. Despite the turbulence and the waves the helical motions of the longitudinal rolls are easily followed. The spacings of the bands are regularly between 2.5 and

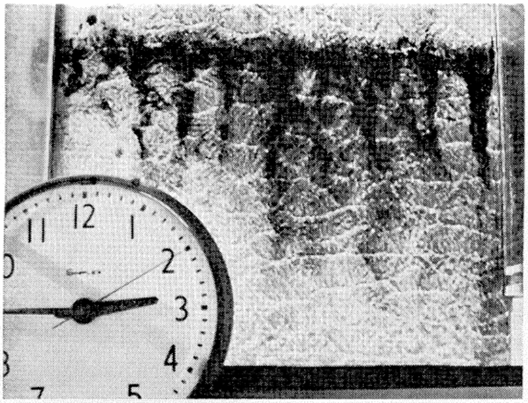

FIG. 2. Longitudinal bands of dye due to wave-excited generation of longitudinal vortices. Wind stress was directed from bottom to top of the figure. Depth of water was 4.0 cm and width of the tank was 91 cm. The test section was 1.5 m from the intake end of the tank.

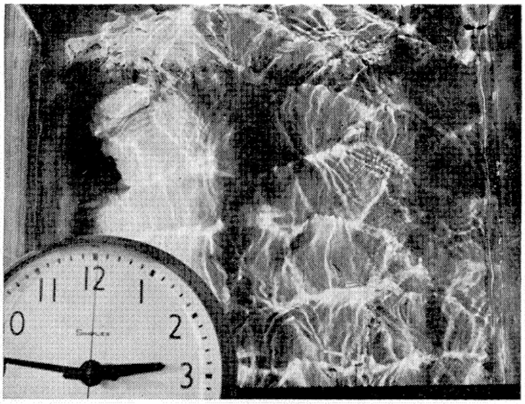

FIG. 3. Longitudinal bands of dye due to wave-excited generation of longitudinal vortices—Langmuir circulations. Wind stress was directed from bottom to top. Waves were much larger than in Figs. 1 and 2 because the test section here was 7 m from the intake end of the tank. Depth of water was 16.2 cm.

3.0 times the depth of the water layer. In a finite tank this ratio must vary somewhat because of the tendency to form an integral number of waves within the given width.

To eliminate the possibility that thermal convection due to evaporative cooling was the dominant mechanism, several experiments were carried out with a cold tank and with water well below the wet-bulb temperature of the laboratory air. The turbulent convection persisted undiminished. The bottom of the tank was lined with plywood, a good insulator, so that thermal convection by heating from below could be ruled out. Moreover, it was evident from the descending plumes of turbulent fluid that the convection was being initiated from above rather than from below. The comparative influence of thermal convection was further examined by filling the tank with warm water (approximately 41C) and ventilating it to provide maximum evaporative cooling. A slow drainage of the tank provided shear for the organization of the convection into longitudinal rolls. The convection due to surface cooling was qualitatively much weaker than that due to surface waves. When (in the surface cooling experiment) the wind was increased sufficiently to generate waves in the test section, vigorous convection and the Langmuir circulations were again observed.

An additional direct test of the wave mechanism was accomplished by producing the longitudinal rolls through the generation of waves mechanically rather than by wind. Again a slight shear flow to organize the convection into rolls was produced by a slow draining of the tank. The draining by itself, even at fairly rapid speeds, produced only laminar flow. A hand-operated plunger was set up a few feet from the test section to provide the waves. The vigorous convection previously described and the Langmuir circulations were again clearly observed when the waves reached the test section. More systematic experiments of this type with careful measurements of the oscillatory motions are needed to provide a quantitative test of the theory. All the experiments were conducted under conditions that produced rather intense convection. While they have supported the general structure of the instability theory as outlined above, they do not clearly distinguish between the possibilities of infinitesimal-amplitude and finite-amplitude instabilities.

LANGMUIR CIRCULATIONS IN LAKES AND OCEANS

The inconsistencies noted by Scott et al. (1969) for observations of Langmuir circulations in the oceans as opposed to those in lakes fall into three categories: 1) the angle of the windrows with respect to the wind, 2) the relation of their spacing to the windspeed and to other parameters, and 3) their apparent rate of formation. But first it is necessary to clearly distinguish between Langmuir circulations and surface streaks. It is clear from the work of several authors, starting with Langmuir himself (1938), that Langmuir circulations do lead to streaks and rows of floating material, but streaks may have other origins as well. Probably Langmuir circulations frequently exist without visible evidence at the surface simply for the lack of a surface tracer. The following remarks, then, apply to Langmuir circulations but not necessarily to surface streaks. These arguments are intended as a rationalization of the apparent inconsistencies mentioned above in the light of the theory and the experiments already described.

1) When wind stress starts to act on a water surface at rest, the shear layer in the water is at first shallow and the waves are small. The Langmuir circulations, being limited by the depth of the turbulent layer and by the scale of the waves, will be correspondingly narrow. Being small, their rate of formation and dissipation will be rapid. As the wavelengths and the depth of the frictional layer increase with time, the spacings of the largest scale rolls should increase in proportion and their persistence should increase accordingly. If there is a wide range of wave numbers k, one may well expect many scales of Langmuir circulations.

2) It is often stated in support of the thermal convective model that the ratio of horizontal wavelength to depth of the convective layer is between 2 and 3. The theory of thermal convection gives cell dimensions within these limits with slight variations according to the boundary conditions that are imposed. However, the factor that basically determines these cell dimensions is the diffusive mechanism at work in the fluid. In the case of cells generated by the eddy pressures due to waves, the relative wavelength should be in the same range of values as for thermal convection. This should be true for any instability mechanism as long as that mechanism itself does not directly force a particular scale. (An example of a forced scale would be a preexisting wind pattern or a pattern in the bottom topography.) Therefore, the ratio of cell wavelength to cell depth cannot be taken as evidence of one mechanism or another.

3) In the case of a shallow layer of water or a shallow thermocline, the depth of penetration of the cells, and therefore, the maximum horizontal wavelength, will be limited by the shallowest limiting feature. In the open ocean with a deep thermocline, the full Ekman layer may eventually develop and the maximum cell size will probably be limited by the depth of the Ekman layer. (In this connection it should be noted that the convection produced by the eddy pressure effects of the waves may well determine the turbulence throughout the depth of the Ekman layer as opposed to turbulence produced by the shear of the Ekman layer itself.) With a shallow epilimnion, as in Lake George (Scott et al. 1969), it is not surprising that the cell spacing correlates better with epilimnion depth than with windspeed. In the cases studied by Faller and Woodcock (1964), the ratio of row spacing to mixed layer depth was 1.1 on the average. Apparently the mixed layer was sufficiently deep that it did not effectively limit the cell size. Hence, there was poor correlation of row spacing with that depth but good correlation with windspeed.

4) With respect to the angles of longitudinal rolls in relation to wind direction, there should be no systematic angle as long as the shear layer has unidirectional flow. It is only when there is a variation of mean flow direction with depth *and* when the cells extend into layers with flow of different directions that one should expect a

systematic angle. Thus, in the open ocean if there is a fully developed Ekman layer with a significant spiral, one should expect the largest scale vortices to be at an angle with respect to the wind stress. However, smaller scale vortices will be shallow and should have the same orientation as the stress. Similarly, if the cells are shallow due to a shallow thermocline or shallow water one should not normally expect an angle. Of course, if the water is stably stratified so that mixing is inhibited and the Ekman layer is shallow, then cells of comparatively small spacing may penetrate through the Ekman layer and may have a measurable angle to the wind. The example of a shallow Ekman spiral provided by the Chesapeake Bay Institute (Faller 1964, Fig. 4) is such a case. Similarly, the asymmetries reported by Woodcock (1944) should not be expected unless the cells penetrate into a layer of different flow direction. It is by this means that the Coriolis effect can influence the orientation and asymmetry of the cells; not by direct action on the cells, but by control of the total Ekman boundary layer in which the cells are imbedded.

5) The time scale of formation and dissipation is related to cell size. For the Ekman instability mechanism to operate starting from a state of rest, the characteristic time will be a matter of hours since it takes this long for an Ekman layer to develop a significant spiral. In the meantime the turbulent and wavy surface layer will have smaller-scale Langmuir circulations. From this point of view it is questionable whether the instability of the Ekman layer [as previously proposed by me (Faller 1966)] has any relevance to the problem except possibly as a determinant of maximum cell size. Or is it the scale of the waves and the attendant maximum cell size and turbulence that determines the characteristic depth of the Ekman layer?

6) Finally, Ichiye (1967, p. S271) has noted that "striations [of dye] are generally superposed on elongated patterns when the wind exceeds 3 m/sec, and they occur even in the calm sea when there are pronounced swells." This observation is an apparent confirmation of the existence of Langmuir circulations without their direct generation by wind, but, rather, by the action of surface waves. Another characteristic of the eddy pressure mechanism that is confirmed in the natural phenomenon is the concentration of the downward current in a narrow band (Scott et al. 1969). Concentrated currents of this type occur in thermal convection when the vertical density gradient is vertically asymmetrical, that is, when there is a stronger gradient at one or the other boundary. Thermal convection due to evaporative cooling generally begins with concentrated downward currents of cold water. In the same manner, due to the exponential character of $\overline{\hat{w}^2}$, it may be expected that regions of downward motion will be narrow and intense compared to regions of upwelling.

CONCLUSIONS

The primary mechanism of formation of Langmuir circulations directly involves the fluctuating motions due to surface waves. These oscillations are represented in the equations of fluid motion by the normal Reynolds stresses that frequently are ignored in fluid dynamics problems. These Reynolds stresses are positive definite and are called components of the eddy pressure because they appear in the dynamical equations in the same way as the kinetic pressure. But whereas kinetic pressure is isotropic, eddy pressure generally is not, and it is the anisotropic characteristic of the fluctuations produced by the surface waves that is important.

The nondimensional number that appears in the energy equation in place of the Rayleigh number (for thermal convection) is, in its simplest form, $Q = \Delta\overline{(\hat{w}^2)}_z \times H^3/\nu^2$. In cases of large H, perhaps a more appropriate stability parameter would be $Q = \Delta\overline{(\hat{w}^2)}_z L^3/\nu^2$ where L, the dominant wavelength, replaces H as the characteristic vertical scale since most of the strong gradients of $\overline{\hat{w}^2}$ will occur within the depth range $z = 0$ to $z = -L$. In either case, if the wind is sufficient to raise capillary waves, a wind greater than 2–3 m/sec, this parameter becomes very large and convection begins immediately.

Recognition of this mechanism's prevalence in natural water bodies may be of the greatest importance for theories of mixing in the surface layer. From the intensity of eddy pressure convection relative to thermal convection, as judged from laboratory experiments and from the theory, it can be stated with some confidence that this process is a primary mechanism by which the thermocline in the oceans is eroded during times of high winds. It appears to be a mechanism by which wave energy is converted into turbulence. Since Langmuir circulations derive their energy from surface waves and since they convect turbulent fluid downward, it is apparent that Langmuir circulations represent an important drain of energy from surface wave motions. Moreover, Langmuir circulations in their ubiquitous relation to surface waves may play an important role in the instability of a simple unidirectional wave train and in the development of crosswind wave components. Still further, the entire turbulent structure of the mixed layer and of the spiral Ekman boundary layer may be largely governed by the processes described here.

It has long been suspected by some investigators who have studied Langmuir circulations that the waves were responsible for these circulations, but it was difficult in the natural environment to separate out direct wave effects from other processes involving wind. The experiments described here show that wave effects dominate over other possible mechanisms. However, much work remains to be done to clarify the details of the interactions of waves and Langmuir circulations. The instability theory presupposes knowledge of the vertical distribution of eddy pressures and tangential Reynolds stresses. In finite-amplitude experiments and in natural cases, it will be difficult to separate out the basic turbulence field from the redistributed turbulence after finite-amplitude eddy pressure convection is underway.

REFERENCES

FALLER, A. J. 1964. The angle of windrows in the ocean. Tellus, **16**: 363–370.

———. 1966. Sources of energy for the ocean circulation and a theory of the mixed layer. Proc. Natl. Congr. Appl. Mech., 5th, Univ. Minn., p. 651–672. *Also* Tech. Note BN 459, Univ. Maryland, Inst. Fluid Mech. Appl. Math.

———, AND R. E. KAYLOR. 1966. A numerical study of the instability of the laminar Ekman boundary layer. J. Atmospheric Sci., **23**: 466–480.

———, AND A. H. WOODCOCK. 1964. The spacing of windrows of *Sargassum* in the ocean. J. Marine Res., **22**: 22–29.

ICHIYE, T. 1967. Upper ocean boundary-layer flow determined by dye diffusion. Phys. Fluids Suppl., **1967**: S270–S277.

JEFFRIES, H. 1928. Some cases of instability in fluid motion. Proc. Roy. Soc. (London), Ser. A, **118**: 195–208.

LANGMUIR, I. 1938. Surface motion of water induced by wind. Science, **87**: 119–123.

LILLY, D. K. 1966. On the instability of Ekman boundary flow. J. Atmospheric Sci., **23**: 481–494.

NEUMANN, G., AND W. J. PIERSON. 1966. Principles of physical oceanography. Prentice-Hall, Englewood Cliffs, N.J. 545 p.

SCOTT, J. T., G. E. MYER, R. STEWART, AND E. G. WALTHER. 1969. On the mechanism of Langmuir circulations and their role in epilimnion mixing. Limnol. Oceanog., **14**: 493–503.

WELANDER, P. 1963. On the generation of wind streaks on the sea surface by action of a surface film. Tellus, **15**: 67–71.

WOODCOCK, A. H. 1944. A theory of surface water motion deduced from the wind-induced motion of the *Physalia*. J. Marine Res., **5**: 196–205.