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THE FORM AND DYNAMICS OF LANGMUIR CIRCULATIONS

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> "*... the water itself is rippled by the wind. I see where the breeze dashes across it by the streaks...*"
>
> Henry Thoreau, *Walden* (1854)

INTRODUCTION

When the wind blows at modest speeds over natural bodies of water, numerous streaks or slicks nearly parallel to the wind direction may appear on the surface. This form of surface streakiness is commonplace, and under favorable conditions it is readily apparent to the casual observer. The streaks result from the collection of floating substances—seaweed, foam from breaking waves, marine organisms, or organic films—into long narrow bands. Flotsam makes the bands visible directly, and compressed films make them visible by the damping of capillary waves, thereby giving the bands a smoother appearance. Naturalists and seafarers often note color variations of the sea due to minute marine organisms. Bainbridge (1957) cited many old descriptions of long narrow "bands," "streaks," or "lanes" including several by Darwin in 1839 during the voyage of the *Beagle*. James Thomson (1862) described observations of streaks made jointly with his brother, Lord Kelvin, in a paper that also indicated increased abundances of marine life below the streaks. The first connection between the wind and streak directions, among the authors cited by Bainbridge, was made by Collingwood (1868): "if a moderate breeze were blowing and the sea

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raised, instead of a uniform pellicle, the dust would be arranged in long parallel lines, bands or streaks, extending unbroken as far as the eye could reach, and always taking the direction of the wind."

In a remarkable series of experiments, Langmuir (1938) showed that these streaks, or "windrows," are the visible manifestations of a parallel series of counterrotating vortices in the surface layers of the water with axes nearly parallel to the wind. The motions strongly resemble thermal convection rolls. In fact, with weak winds and thermally unstable conditions, buoyancy may be the motive force, the resulting thermal convection being organized into rolls aligned with the wind by shear in the water according to experimentally and theoretically established characteristics of thermally unstable Couette flow. Langmuir, and numerous investigators after him, showed that the rolls also form with nearly equal ease under conditions of stable density stratification, and he concluded that the motions were mechanically driven, ultimately by the wind. The motions are now commonly known as Langmuir circulations, regardless of the mechanism responsible for their formation, but those of mechanical origin are peculiarly Langmuir's.

His observations led Langmuir to believe that the vortices he discovered are largely responsible for the formation of thermoclines and the maintenance of mixed layers in lakes (and presumably also in the ocean). If this is true, they assume a special kind of significance. This is partly because of the importance of the mixed layer and the heat, mass, and momentum transport processes within it. Even more intriguing, however, is the attribution of a dominant mixing role to a flow structure whose apparent orderliness and coherence suggests the possibility of a deterministic explanation and treatment. The expectations and hopes raised by this suggestion are clearly shared by those interested in the more recently discovered coherent structures in turbulent flows; it is possible that Langmuir circulations are related to coherent streamwise vortices in turbulent wall layers. At present, while the existence and many features of Langmuir circulations are generally accepted, experimental evidence that they accomplish a large part of the stirring of the mixed layer is ambiguous.

Langmuir circulations have been the subject of previous reviews by Faller (1971) and Pollard (1977); these reviews will be helpful to the reader. Pollard's review provides a valuable summary of observations and an assessment of the state of theories as they existed in 1976. Development has been rapid since that time: new field observations, laboratory experiments, and theoretical work are now available. These are described in what follows, along with the earlier work.

THEORIES

Many mechanisms for Langmuir circulations have been proposed since Langmuir first reported his observations. Most of these easily can be shown to be nonessential to the phenomenon, and have therefore been dismissed. These early ideas are briefly mentioned, followed by theories not so easily discounted, although serious objections can also be raised about some members of this latter group. Before reviewing the theories, however, I first give a sketch of the phenomenon these theories must explain.

Qualitative Features of Observed Langmuir Circulations

A composite of observed characteristics of Langmuir circulations is given here without documentation; the sources from which the picture is drawn are reviewed in a subsequent section.

Langmuir circulations form, aligned within a few degrees of the wind direction, within a few minutes of onset of a wind of 3 m s^{-1} or faster. Circulations form regardless of surface heating or cooling; with surface cooling (thermally unstable conditions), the circulations may form at a lower wind speed and the subsurface motion may be more vigorous, but for wind speeds of 3 m s^{-1} or faster, surface heating or cooling appears to only slightly modify the strength and form of the resulting circulations. The spacing of windrows ranges from a meter or two up to hundreds of meters; the factors determining this spacing have yet to be conclusively determined. A hierarchy of spacings is often observed, with smaller, more irregular, and less well-defined streaks occurring between stronger and more widely spaced streaks. When there is a hierarchy, the evidence suggests that the small scales continually form and are slowly swept up into the more permanent larger scales. The circulations are in the form of parallel vortices with axes aligned with the wind direction, as shown in the sketch in Figure 1*a*. The depth of penetration of the cells appears to be limited to the first significant density gradient in the water body, and there is evidence suggesting that the aspect ratio $(L/2D)$ of the largest cells is not significantly different from unity, where L is the spacing between windrows and D is the depth of penetration.

Surface streaks are lines of surface convergence, and downwelling (vertical) motion takes place below them. Figure 1*b* is a photograph of bands of oil collected in convergences; such collections often occur in oil spills at sea. The downwelling takes the form of powerful jets, with maximum speeds $|w_d|$ of roughly one percent of the wind speed, or one third to one quarter of the maximum wind-induced surface currents. The widths of the jets seem to be quite small compared to streak separations. Surface-sweeping speeds are comparable to the downwelling speeds and probably decay rapidly with depth. The surface-current component (u) in the wind direction is noticeably larger in the streaks above the downwelling jet; the u anomaly is thus also comparable to the downwelling speed $|w_d|$.

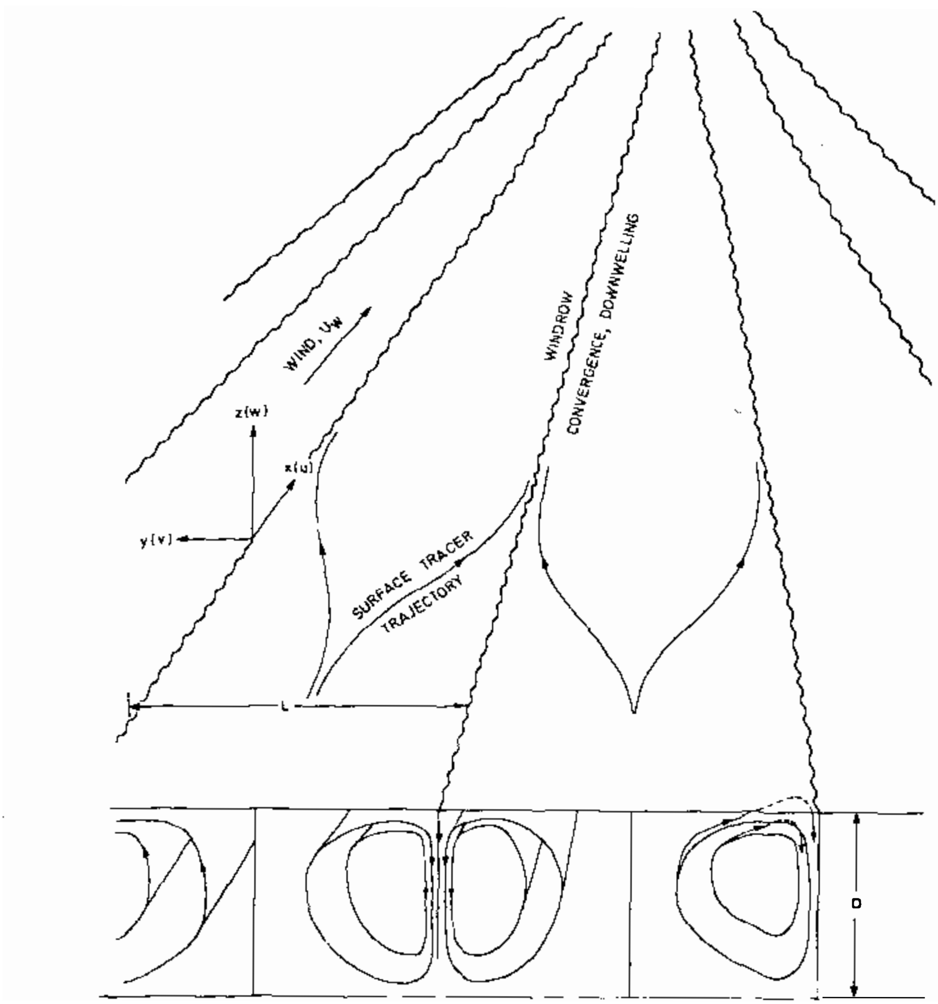

Figure 1a Illustration of Langmuir circulations showing notation used in this review and surface and subsurface motions.

Craik & Leibovich (1976) presented a list that abstracts from this description a minimum of qualitative features that any viable theory of Langmuir circulations must be able to reproduce.

LC (1) A parallel system of vortices aligned with the wind must be predicted.

LC (2) A means must be given by which these vortices are driven by the wind.

LC (3) The resulting cells must have the possibility of an asymmetric structure with downwelling speeds larger than upwelling speeds.

LC (4) Downwelling zones must be under lines where the wind-directed surface current is greatest.

LC (5) The Langmuir circulations must have maximum downwelling speeds comparable to the mean wind-directed surface drift.

These still constitute a minimal checklist against which to compare a theory.

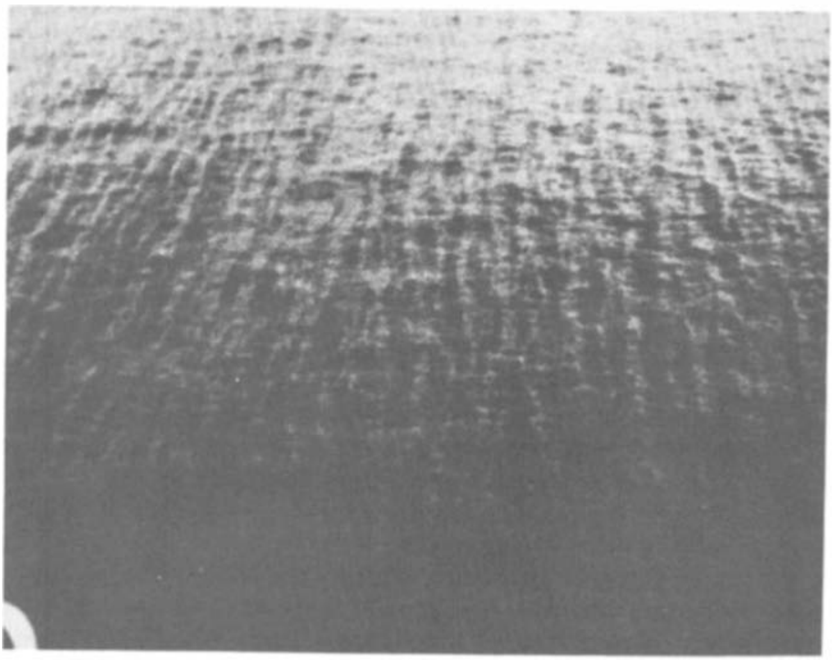

Figure 1b Aerial photograph of oil bands in the Gulf of Mexico collected by Langmuir-circulation sweeping of the surface. The oil originated from the IXTOC I well blowout. The photograph is taken from Atwood et al. (1980).

Early Ideas Concerning Mechanism

Thermal convection has been frequently suggested as the mechanism driving Langmuir circulations, and this idea was discussed at some length by Csanady (1965). Langmuir (1938) himself discounted the idea, clearly stating his belief that the wind was the responsible agent. Thermal convection cannot explain circulations arising under thermally stable or neutral conditions, as Csanady (1965) notes, and has been rejected as a primary mechanism.

Effects produced by surface films have been proposed by Welander (1963) and Kraus (1967) as candidates for a mechanism; neither author reduced his thoughts to mathematical terms. Welander suggested that surface slicks would modify the wind field near the water surface. Damping of capillary waves in slicks produces a smoother surface and presumably a higher wind speed. This in turn is supposed to accelerate the water, producing a secondary inflow into the slick region. This hypothetical sequence, in itself, is hardly clear, since enhancement of wind speed presumably results from a reduction in drag, and therefore the water, if anything, should decelerate at the slicks. Furthermore, no wind-speed anomalies of the sort proposed are known to be correlated with windrows. Consequently, this mechanism has been discounted (see also Pollard 1977). Kraus (1967) suggested that radiation pressure due to the damping of capillary waves in slicks could lead to acceleration of slicks in the wind direction. Several arguments have been advanced to show that this is unlikely (Pollard 1977), but perhaps the most telling is the energy budget given by Myer (1971). It is also worth noting that circulation patterns form in laboratory experiments, such as Faller's (1969), in the absence of films.

The possibility that streaks result from roll vortices in the atmosphere, presumably originating in a paper by Woodcock & Wyman (1946), has been discounted by Stommel (1951), Langmuir (in work cited by Myer 1971), and Myer (1971). While such atmospheric vortices do arise, their positions seem to bear no relation to the streaks, and their effect is too weak to account for the water motion.

Concepts involving purely irrotational wave effects were put forward by Stewart & Schmitt (1968) and Faller (1969), but Langmuir circulations are fundamentally rotational, as Faller (1971) noted in retracting his proposal, and these suggestions make no provision for the necessary streamwise vorticity. It should be noted, however, that Faller's (1969) paper also contains experimental results and a clear statement of the essential physical ingredients required to produce Langmuir circulations, and it therefore remains a key paper in the development of the subject.

Instabilities of the Ekman layer are known to take the form of convective rolls at a small angle to the wind, and were proposed by Faller (1964) as a mechanism for Langmuir circulations. The growth rates of such instabilities are too small to account for Langmuir circulations. More recently, Gammelsrod (1975) again advanced this idea. Although more than one physical situation may correspond to the basic flow he considered, only one seems germane to Langmuir circulations in the ocean. This contemplates a balance between viscous and Coriolis forces in a surface layer bounded below by a strong stable density gradient at a depth H small compared to the Ekman scale depth. The density gradient is assumed to act as an impenetrable boundary to vertical motions, but it does not affect the basic Ekman-layer structure. In this case, Ekman's classic solution (1905) yields a current in the mixed layer that is essentially unidirectional and uniformly sheared. Gammelsrod considered the linearized stability of this current to roll motions parallel to the current direction. The inclusion of the Coriolis acceleration is crucial, since in its absence no instability occurs. When the dimensionless shear rate $f^{-1}\Delta U/H$ is sufficiently large (where ΔU is the basic current variation across the layer with thickness H and f is the Coriolis parameter) Gammelsrod found that small inviscid disturbances will amplify on a time scale consistent with the growth time of Langmuir cells.

This conclusion is at odds with the results of more complete treatments of Ekman layer instability by Lilly (1966) and by Faller & Kaylor (1966). In particular, their analyses show that their "parallel" modes (corresponding to Gammelsrod's solutions) have growth rates of the order of $f^{3/4}$. Gammelsrod's model produces growth rates that have finite limits as $f \rightarrow 0$, or as the Rossby number of the basic flow tends to infinity. A significant Coriolis effect in such a situation is most peculiar and contradicts previous experience. It indicates a discontinuity of the growth rate as f increases from zero, since at $f = 0$ the basic state, plane Couette flow, is known to be stable. There is therefore a puzzle concerning Gammelsrod's stability analysis which bears further examination. Leibovich & Radhakrishnan (1977) and Pollard (1977) have questioned the applicability of Gammelsrod's theory to Langmuir circulation on other grounds.

Wave-Current Interaction Theories

Theories involving the distortion of vortex lines in the current by the action of surface waves have been extensively explored recently, and appear to be capable of predicting the observed features of Langmuir circulations. Two independent but related theories, that of Garrett (1976) and the Craik-Leibovich theory, have been advanced. In both approaches, the motion in the upper layers is assumed to consist of a nearly irrotational wave field and a weaker rotational current, and circulations are driven through nonlinear interactions between the waves and the currents. The theories differ in the mechanisms by which the interactions engender circulations. Common ground shared by these theories is discussed by Leibovich (1980). This paper also discusses a similar model due to Moen (1978) that combines certain features of the Garrett and Craik-Leibovich theories.

Garrett's model depends upon diffraction of surface waves by the u-current anamoly: according to a WKBJ analysis, the surface waves will amplify in the current-anomaly region. Such a picture is consistent with an increase in wave height in streaks reported by Myer (1971) in observations in Lake George. The indicated amplification of waves suggests preferential wave breaking and consequent momentum transfer from the wave field to the currents in streaks. This process is assumed to occur and is modeled in an ad hoc fashion. A wave-current interaction also leads to a force on the water toward the streaks. The combination of this surface convergence and the momentum transfer due to wave breaking leads to an instability below the zone of significant wave activity, provided frictional effects are accounted for there. Thus an infinitesimal anomaly would tend to amplify and produce motions of roll type, with axes presumably aligned with the general direction of wave propagation; this, in turn, will be in the wind direction if the waves are generated by the wind.

Garrett's model thus depends upon the modification of the wave field by weak horizontal variations of current speed, and upon preferential wave breaking. The existence of each effect has recently been cast in doubt, the first on theoretical grounds and the second on experimental grounds. The amplification of waves by a sheared current, predicted by the WKBJ method, may be misleading. Smith (1980) examined the question without appeal to the WKBJ approximation, and was unable to find the amplification predicted by the approximate method. Although his findings cannot be regarded as conclusive, the basis for this principal ingredient of the mechanism is at present without a firm theoretical foundation. The assumption of preferential wave breaking within streaks appears to be contradicted by experiments conducted on Loch Ness by Thorpe & Hall (1980) and on Lake-of-the-Woods by Kenney (1977); these authors find wave breaking to occur with equal frequency between streaks and in them.

The Craik-Leibovich, or "CL," theories have been shown to produce circulatory motions by two distinct theoretical mechanisms, both of which depend upon wave-current interactions. The theories are "rational" in the sense that they are based upon a set of nonlinear equations derived from the Navier-Stokes equations by perturbation procedures that formally indicate the level of error anticipated. The equations defining the theories were first set out by Craik & Leibovich (1976), and evolved from an earlier attempt by Craik (1970) and its critique by Leibovich & Ulrich (1972).

Following Leibovich & Ulrich (1972), the motion in the surface layer is assumed to be dominated by the orbital motion of irrotational surface gravity waves. The rotational currents that are the objects of the investigation are assumed to be smaller, typically comparable to the Stokes mass drift

$$\mathbf{u}_{\mathrm{s}} = \left\langle \int^{t} \mathbf{u}_{\mathrm{w}} \, dt \cdot \nabla \mathbf{u}_{\mathrm{w}} \right\rangle, \tag{1}$$

where $\mathbf{u}_{\mathrm{w}}$ is the velocity vector of the orbital motion induced by the water waves, the angle brackets correspond to an appropriate averaging operation, and $\mathbf{u}_{\mathrm{s}}$ is the resulting Stokes drift (Phillips 1977) associated with the surface waves. Vorticity in the water body may arise from currents whose origins are unspecified, or, as explored by subsequent workers, by diffusion whose source is an applied wind stress at the water surface. Surface waves perturb this configuration, producing a vorticity fluctuation correlated with the waves, and the nonlinear interaction of the fluctuating vorticity with the surface waves produces a stretching and rotation of the vortex lines. By averaging, assuming the waves are either periodic or random but statistically stationary, it is found that the rectified effects of the waves arise from additional advection and stretching of mean vorticity by the wave Stokes drift, as anticipated in the elementary inviscid treatment of Leibovich & Ulrich (1972).

The CL model is extended to allow time development of the currents and is made more systematic in Leibovich (1977a), and a further extension allowing for density stratification (under the Boussinesq approximation) is given by Leibovich (1977b). In these papers, the effects of turbulent fluctuations are crudely represented by constant eddy diffusivities, but more sophisticated turbulence models could be invoked.

The governing equations for the rectified water motion under the Boussinesq approximation and the assumption of constant eddy diffusivities of momentum (ν_T) and heat (α_T) are (Leibovich 1977b)

$$\begin{aligned} \partial \mathbf{v}/\partial t + \mathbf{v}\cdot\nabla\mathbf{v} &= -\nabla\pi + \mathbf{u}_{\mathrm{s}} \times \operatorname{curl} \mathbf{v} + \beta g \theta \mathbf{k} + \nu_T \nabla^2 \mathbf{v} \\ \partial\theta/\partial t + \mathbf{v}\cdot\nabla\theta &= -w\bar{T}' + \alpha_T \nabla^2\theta \\ \nabla\cdot\mathbf{v} &= 0. \end{aligned} \tag{2}$$

Here the temperature distribution in the absence of circulations is $\bar{T}(z)$, θ is the temperature perturbation, β is the coefficient of thermal expansion, g is the acceleration of gravity, $\mathbf{v}$ is the mean velocity vector in the currents, and π is a modified pressure term that includes the mean pressure as well as terms involving wave kinetic energy. These equations reduce to the usual Navier-Stokes equations of a Boussinesq fluid as $\mathbf{u}_s \rightarrow 0$, that is at sufficiently great depth. Near the surface, gravity waves must normally be included. It is possible to modify these equations to include Coriolis forces; for constant-density water, this has been done by Huang (1979), who showed that the Ekman layer is altered by the vortex force, and by Leibovich (1980).

The equations (2) governing the mean flow are the full equations for the instantaneous flow, altered only by the appearance of an apparent "vortex force" (Leibovich 1977b, 1980)

$$\mathbf{f} = \mathbf{u}_s \times \boldsymbol{\omega}, \tag{3}$$

where $\boldsymbol{\omega}$ is the mean vorticity. The waves are essentially unaffected by the currents if the conditions contemplated by the theory are met (as they are generally expected to be in the ocean and in lakes) so that they may be specified and $\mathbf{u}_s$ computed a priori, and the equations are therefore closed.

Heuristic Discussion of Two "Vortex Force" Mechanisms

When the wind blows in a fixed direction over water of unlimited horizontal extent and depth that is otherwise undisturbed, symmetry dictates the development of a surface wave field with unidirectional Stokes drift aligned with the wind or $\mathbf{u}_s = U_s\mathbf{i}$ (see Figure 1*a* for coordinate system). Neglecting Coriolis accelerations, the total momentum imparted to the water will be similarly aligned, and the horizontally averaged velocity will be $\mathbf{u} = U(z,t)\mathbf{i}$, with positive y-vorticity $\boldsymbol{\omega} = (\partial U/\partial z)\mathbf{j}$. The vortex force/mass

$$\mathbf{f}_v = \mathbf{k}U_s(\partial U/\partial z) \tag{4}$$

is oriented vertically upward and is formally analogous to a buoyancy force. If U_s depends only on depth z, then the vortex force can be balanced by the analog of a hydrostatic pressure gradient; this configuration represents a developing unidirectional current. If the Stokes drift varies across the wind (y-direction), however, the vortex force cannot be balanced by pressure, and an overturning is induced. Surface waves in a "short-crested" sea have this character. The directional spectrum of a wind-generated sea is symmetric with respect to the wind (Longuet-Higgins 1962). If it were bimodal, with peaks of wave energy at angles

$\pm\,\theta$ with respect to the wind, and if the wave spatial structure were to remain coherent for times sufficient to carry out the averaging operation inherent in the Stokes drift (many times a typical wave period), then the Stokes drift would be spatially periodic with a crosswind wave number of $2k\,\sin\theta$, where k is a characteristic wave number of the surface waves. This horizontally periodic wave drift produces a torque due to horizontal variations of vortex force that directly drives roll motions; this is the mechanism proposed by Craik & Leibovich (1976) and called the CL 1 mechanism by Faller & Caponi (1978). An alternate kinematic interpretation of the mechanism is that described by Leibovich & Ulrich (1972); vortex lines associated with the current are deformed by the Stokes drift, producing streamwise vorticity periodically (in y) alternating in sign. Both the dynamic and kinematic explanations of the CL 1 mechanism are represented pictorially in Figure 2, which also includes a schematic

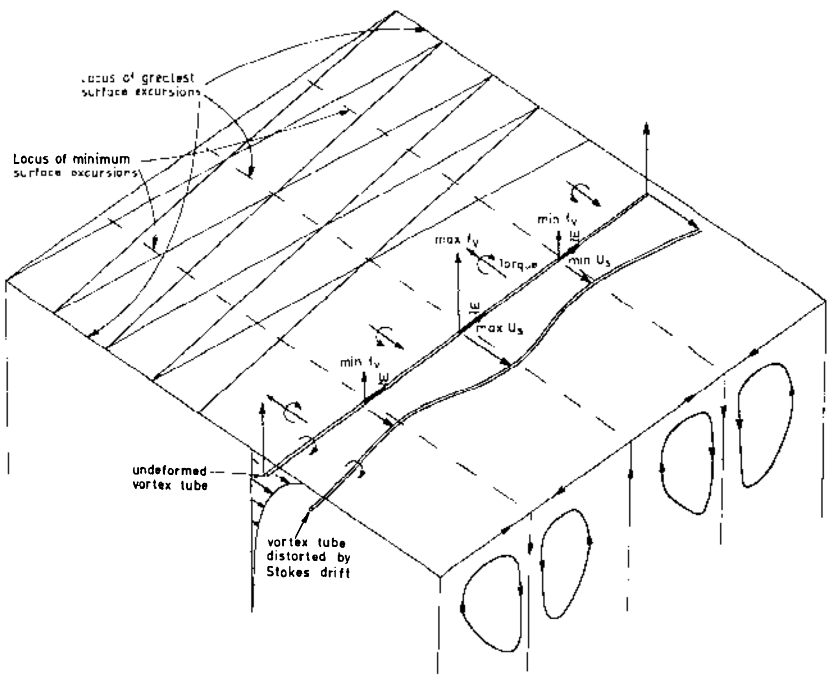

Figure 2 Sketch illustrating the direct-drive mechanism of Langmuir-circulation generation. Wave crests of the assumed crossed-wave pattern are shown; the Stokes drift is higher along lines joining crest intersections and the Stokes-drift variations distort vortex lines of the primary current. The resulting variations of vortex force create a torque leading to overturning.

diagram of idealized "crossed-wavetrains" assumed in calculations with the CL 1 mechanism.

A second mechanism, called CL 2 by Faller & Caponi (1978), was originally suggested by Craik (1977) and further explored by Leibovich (1977b). It requires no coherent surface-wave structure. Circulations are produced via the vortex force as an inviscid instability of the unidirectional current. If the waves lack a coherent spatial structure, $\mathbf{u}_s = U_s(z)\mathbf{i}$; the time development of the waves under the action of the wind is ignored, as it has been in all of the investigations carried out with wave-interaction models. The vortex force (4) can now be balanced by a vertical pressure gradient. Since U_s and U typically decrease monotonically with depth below the surface, the vortex force does as well, and so the joint effects of typical distributions of U_s and U are directly analogous to a statically unstable density distribution. One may therefore anticipate that the rectilinear current is unstable if dissipation is sufficiently weak. (By contrast, if the waves and current are *opposed*, then the vortex force is *stabilizing*.)

Both the dynamics and kinematics of onset of CL 2 mechanisms are traced schematically in Figure 3. Suppose an infinitesimal spanwise

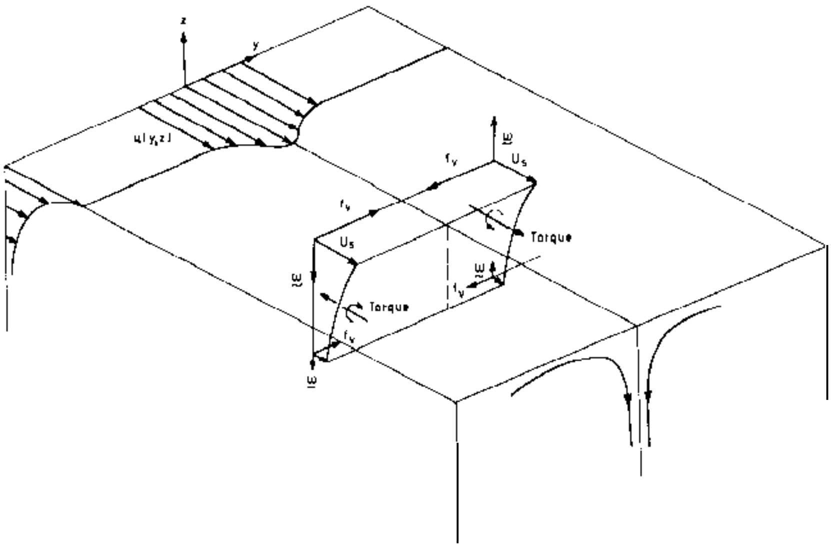

Figure 3 Sketch illustrating the CL 2, or instability mechanism of Langmuir-circulation generation. The Stokes drift is horizontal, but decays in depth. Streamwise vorticity is induced by the Stokes-drift rotation of vertical vorticity associated with spanwise perturbations of the current. Variations of vortex force caused by the current perturbation create torques leading to overturning.

irregularity $u(y, z, t)$ is present in an otherwise horizontally uniform current $U(z)$. This produces vertical vorticity $[\omega_z = -(\partial u/\partial y)]$ and a horizontal vortex-force component $-U_s\omega_z\mathbf{j}$ that is directed toward the planes of maximum u. This causes an acceleration toward these planes, where, by continuity, the fluid must sink. Assuming that $\partial U/\partial z > 0$ and ignoring shear stresses, conservation of x-momentum for a vanishingly thin slab of fluid centered on the convergence plane shows that as the fluid sinks, u must increase. Thus, in the absence of frictional effects, a current anomaly will lead to a convergence and thereby be amplified, which in turn further amplifies the convergence. Kinematically, the vertical vorticity is rotated and stretched by the Stokes drift, leading to convergence and amplification of the anomaly.

DIRECT-DRIVE MECHANISM Craik & Leibovich (1976) set out the theoretical basis for the CL 1 mechanism, explored aspects of the linearized CL equations, and presented numerical results for weakly nonlinear motions in infinitely deep water assuming "crossed-waves" and invariance of the rectified motions along the wind direction (x). The theory was restricted by an assumption of time-independent motion, which led to a need to specify the horizontally averaged drift in an ad hoc manner. (A steady horizontally averaged current corresponding to an applied surface stress is not possible in infinitely deep water.) This restriction precluded the calculation of a complete current system arising from a fully defined physical problem.

These restrictions were removed by Leibovich (1977a), who extended the theory to include time evolution of the coupled (wind-directed) currents and circulations. The formulation of the problem as an initial-value problem, with currents and circulations initially zero and initiated by a step function in surface stress, results in a well-posed mathematical problem. Assuming invariance of the wave field in the x- (wind) direction and its symmetry with respect to the x-axis, the problem is independent of x and any emerging circulations must be in the form of rolls. Assuming a constant wind stress for $t \geqslant 0$ corresponding to a friction velocity u_*, surface waves with characteristic frequency σ, wave number k and characteristic wave amplitude a, and an eddy viscosity ν_T, the initial-value problem was shown to depend upon a single dimensionless parameter

$$\mathrm{La} = \left(\nu_T^3 k^2/\sigma a^2 u_*^2\right)^{1/2} \tag{5}$$

This parameter, which Leibovich called the "Langmuir number," expresses a balance between the rate of diffusion of streamwise vorticity and the rate of production of streamwise vorticity by the vortex stretching accomplished by the Stokes drift: it also has the usual interpretation of an inverse Reynolds number. The reduction of the initial-value problem to dependence on a single dimensionless parameter (plus an angle representing the directional properties of the waves if this feature is invoked) is a consequence of x-invariance and implies adoption of the scalings shown in Table 1. Other scalings can be adopted, but these lead to the greatest simplifications. They emphasize a balance between vortex force and the applied shear stress when x-variations are negligible; they are therefore appropriate also to problems involving the CL 2 instability mechanism when the same set of assumptions is applicable. Leibovich (1977a) assumed the water to be infinitely deep and of constant density, and the Stokes drift to result from an idealized narrow spectrum with energy concentrated in a pair of uniform wave trains propagating at equal and opposite angles to the wind. The fully nonlinear problem was explored numerically by Leibovich & Radhakrishnan (1977). When the Coriolis force is neglected the problem has no steady limit, because in infinitely deep water, there is no bottom friction to balance the applied surface stress: thus the momentum of the water body increases linearly with time. Nevertheless, a definite current structure forms near the surface within about 10 T_d, where T_d is the time scale from Table 1, and changes little thereafter. The continuing momentum increase is accommodated by cellular mixing of momentum to ever-deeper waters. Horizontal spacing of convergence lines is fixed by the Stokes drift and shows its (assumed) periodicity. Downwelling occurs below convergence lines coinciding with lines of *minimum* Stokes drift, and upwelling under lines of *maximum* Stokes drift, as first shown by Leibovich & Ulrich (1972) in their simple inviscid model. All qualitative features [LC (1) to LC (5)] are reproduced. Maximum surface-sweeping speeds are close to the maximum downwelling speeds, and maximum downwelling speeds range from slightly less than twice to more than seven times the maximum upwelling speed, depending upon the wave angle θ and elapsed time. Sharply peaked surface-current anomalies occur over downwelling zones, and exceed the minimum surface speeds midway between streaks by 55–75%. In addition, Leibovich (1977a) shows that the horizontally averaged

Table 1 Scalings for the CL equations

Length	Time (T_d)	Wind-directed component of rectified current	Crosswind velocity component of rectified currents
k^{-1}	$(\nu_T/\sigma)^{1/2}/aku_*$	$u_*^2/\nu_T k$	$(au_*/\nu_T)(\sigma\nu_T)^{1/2}$

currents very near the surface predicted by this deterministic model have the logarithmic profile characteristic of wall-bounded turbulent flows. Logarithmic behavior of wind-drift current profiles is known experimentally to occur (Bye 1965, Wu 1975, Shemdin 1972).

The assumption of a spatial pattern of the Stokes drift coherent with any given line on the water surface is basic to the CL 1 theory and is regarded as its weakest point (see the discussions of Pollard 1977, Leibovich & Radhakrishnan 1977, and Leibovich 1977b). Patterns of the sort contemplated do arise in wind-generated seas (Kinsman 1965, p. 543), and a possible theoretical basis for bimodal directional spectra has been advanced by Longuet-Higgins (1976) and Fox (1976), based on the resonant interaction theory of Davey & Stewartson (1974). The formation of a Stokes-drift pattern requires a given wave pattern to be phase-locked for several wave periods. While this is plausible, the formation of well-developed circulations by the CL 1 mechanism seems to require the Stokes drift pattern to remain fixed for times of the order of 10 T_d, which typically is hundreds of wave periods. Phase-locking for times as long as this apparently is not expected in a wind-generated sea, although it can be achieved in the laboratory (Faller 1978).

Mobley (1977) has attempted to check the CL 1 theory by a direct, three-dimensional numerical simulation, including "crossed" surface waves, using a finite difference form of the full Navier-Stokes equations. He was unable to find evidence of the CL 1 mechanism. The numerical task is clearly formidable, and uncertainities have been expressed (Faller & Caponi 1978) about the reliability of the computations. Putting other numerical questions aside, it appears from the data listed in Mobley (1977) that the finest spatial resolution used leads to grid Reynolds numbers larger than 55 (Mobley cites smaller values, but the formula he invokes for grid Reynolds number is in error and grossly underpredicts it), and is much too coarse to resolve the motion. The numerical scheme is inaccurate as a consequence; it also produces artificial viscosity, and it appears that the computed results may be contaminated by this effect.

INSTABILITY MECHANISMS (CL 2) The linear stability of a unidirectional sheared current in the presence of a parallel, spanwise uniform Stokes drift has been explored for water of constant density by Craik (1977) and for density-stratified water by Leibovich (1977b) using special cases of Equations (2). Both papers deal only with rolls periodic in the crosswise directions and estimate (but do not compute) conditions for marginal stability for constant current shear and constant Stokes-drift gradient based upon an analogy with thermal convection. In addition, Craik

(1977) computed growth rates assuming inviscid fluid for several sample current and Stokes-drift profiles. Leibovich (1977b) also computed growth rates ignoring viscosity and heat conductivity, and discussed stability criteria and characteristics for general temperature, Stokes-drift, and current profiles.

The principal result found by Leibovich (1977b) for an inviscid, nonconducting fluid of infinite depth is this: the system is stable if

$$M(z) = U_s'(z)U'(z) - N^2(z) \tag{6}$$

is everywhere negative, and it is unstable otherwise. Here $N = \sqrt{\beta g \bar{T}'(z)}$ is the Brunt-Väisälä frequency of the basic state, and U_s' and U' are the vertical gradients of the Stokes drift and of the shear currents. If unstable, and if the maximum value of M is obtained at the surface, then the maximum growth rate σ_{max} is given by

$$\sigma_{max} = \sqrt{M(0)} \tag{7}$$

and is obtained for waves of infinitesimal length. In an unstable system with a stable density stratification, $N^2 \geqslant 0$, there will be a characteristic depth below which no disturbances penetrate. The layer above is unstable to waves of all lengths in the crosswind direction, with very short wavelength disturbances felt only in a correspondingly thin layer near the surface and most subjected to the neglected dissipative effects. Disturbances of greater length grow more slowly, but penetrate to greater depths.

According to (6), stability occurs for

$$\mathrm{Ri}^* = \min\left[N^2(z)/U'(z)U_s'(z)\right] > 1 \tag{8}$$

when the minimum is taken over depth. This is in the form of a gradient Richardson number, with the geometric mean of U' and U_s' replacing the usual shear.

Leibovich & Paolucci (1980a,b, 1981) have explored the CL 2 instability with nonzero eddy diffusivities; the basic state considered was constant positive N^2 and the exact time-dependent, unidirectional current solution of (2) in infinitely deep water with a surface stress applied at $t = 0$ and held constant thereafter. The dimensionless problem is characterized by the Langmuir number La of Equation (5), and a characteristic Richardson number

$$\mathrm{Ri} = \beta g \bar{T}' / \left[(a u_* k)^2 \sigma / \nu_T\right]. \tag{9}$$

This is similar to Equation (8), with U' based upon the current shear at the surface and the characteristic Stokes drift gradient based upon $a^2k^2\sigma$. In these papers, the Stokes drift was taken to be $U_s = 2a^2k\sigma\ \exp(2kz)$, twice the Stokes drift of a single wave train with wave parameters (a, k, σ); the factor of 2 was introduced to facilitate comparison with the crossed-wave solution of Leibovich & Radhakrishnan (1977). Thus (9), as used by Leibovich & Paolucci, is 4 times (8), and inviscid stability would be expected for $\mathrm{Ri} \geqslant 4$ in this case. Numerical results for linear-stability limits given in Table 2 were found by Leibovich & Paolucci (1981) for two-dimensional rolls. They found the energy-stability limit $\mathrm{La}_G^{-1} = 1.46$, occurring at $k_G^{-1} = 0.32$; this result is independent of Ri (not, as they indicated, a weak function of Ri). The energy- and linear-stability limits are close; subcritical instability in the narrow gap between them is not ruled out. Consideration of general three-dimensional disturbances (Leibovich & Paolucci 1980b) leads to the conclusion that they are more stable on an energy-stability basis, and that two-dimensional rolls are likely to be preferred, in agreement with observation. One would expect the inverse critical Langmuir number (La_c^{-1}) to be infinite for $\mathrm{Ri} \geqslant 4$, based upon the inviscid theory, but recent unpublished computations by Paolucci indicate otherwise.

Leibovich & Paolucci (1980a) traced the fully nonlinear evolution of the instability mechanism for constant N, with $\mathrm{Ri} = 0.1$ and $\mathrm{La} = 0.01$, using a computational domain with width comparable to the wavelength of the most unstable linear mode. S. K. Lele (private communication) has discovered a programming error in their computer code. The solutions presented are strictly valid only for $\mathrm{Ri} = 0$, and the temperature profiles and mixing computations must be interpreted as those of a passive scalar. The solutions obtained are qualitatively like the direct-drive solutions of Leibovich & Radhakrishnan (1977), but with one interesting difference. Small-scale, very weak perturbations deliberately introduced grew not only in strength, but also cascaded to larger scales until they reached the maximum permitted by the finite size of the computational domain.

Cases with constant N, together with other examples with nonconstant N simulating preexisting mixed layers bounded by "thermoclines," have been computed by Leibovich & Lele (1982) using a corrected version of the Leibovich & Paolucci (1980a) computer code. For constant N (twice the nominal value used by Leibovich & Paolucci 1980a) and $\mathrm{Ri}^* = 0.05$,

Table 2 Critical inverse Langmuir number La_c^{-1} and critical wave number k_c of linear theory at various Richardson numbers

Ri	La_c^{-1}	k_c
0.00	1.52	0.32
0.10	1.58	0.31
0.25	1.66	0.30

the results are qualitatively similar to those reported by Leibovich & Paolucci (1980a), the principal difference being a reduction in the penetration rate as the convective motions mix the upper layers and form a layer of strong temperature gradient. An example of an instantaneous streamline pattern associated with the cellular convective motion is shown in Figure 4, while Figure 5 shows the horizontally averaged temperature that evolves from the initially linear profile. The development of a "thermocline" is of particular interest. The incorrect solution predicted a similar thermocline development, but the error precluded any dynamic effect. For $Ri^* \gtrsim 0.125$, new features emerge; growth rates are smaller and the unstable motions are oscillatory. The motions appear qualitatively to be a mixture of monotonically growing circulations in homogeneous water and internal waves. For larger values of Ri^* ($\gtrsim 0.25$), but well below the inviscid criterion (8), the system appears to be stable at the Langmuir number of 0.01 used in the calculations.

Preexisting thermoclines are found by Leibovich & Lele to act as an impenetrable "bottom" for convective motion, provided the temperature gradient is sufficiently strong and the thermocline is sufficiently thick. This is in accord with Langmuir's (1938) expectations.

FURTHER REMARKS ON THEORY Waves and currents can exist in the absence of wind, of course, and their directions need not then be related. No work has yet been done using the CL theories when $\mathbf{u}_s$ is not parallel to the horizontally averaged current $\mathbf{U}$. Heuristic considerations of the action of the vortex force suggest that instability could occur whenever $\mathbf{u}_s \cdot \mathbf{U} \gtrsim 0$, although there is no longer any reason to believe that rolls would be favored.

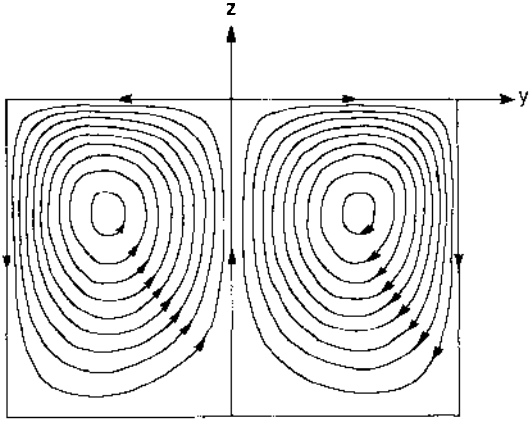

Figure 4 Computed streamlines in a Langmuir convection cell in fluid with a statically stable density stratification.

Other instabilities arising from the density-stratified form of the CL equations are possible, but have yet to be systematically explored. The linearized equations governing stability of a steady current $U(z)$ parallel to a Stokes-drift distribution U_s in a water body with temperature gradient $\bar{T}'$ are given in Leibovich (1977b); disturbances in the form of rolls aligned with U satisfy Equations (18) of that paper. Assuming a strong thermocline exists at a depth d and acts as a stress-free "bottom" to convective motion, and that the vertical gradients $U', U_s', \bar{T}'$ can be regarded as constants above the thermocline, Equations (18) of Leibovich (1977b) can be rescaled and rearranged in the following form

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 w = -R_U u_{yy} + R_T \theta_{yy} && \text{(a)}\\
&\left(\frac{\partial}{\partial t} - \nabla^2\right) u = -w, && \text{(b)}\\
&\left(\frac{\partial}{\partial t} - \frac{1}{\mathrm{Pr}}\nabla^2\right)\theta = -w, && \text{(c)}\\
&\nabla^2 = \left(\partial^2/\partial y^2 + \partial^2/\partial z^2\right) && \text{(d)}
\end{aligned} \qquad (10)$$

$$R_U = U'U_s'd^4/\nu_T^2, \quad R_T = \beta g \bar{T}' d^4/\nu_T^2, \quad \mathrm{Pr} = \nu_T/\alpha_T,$$

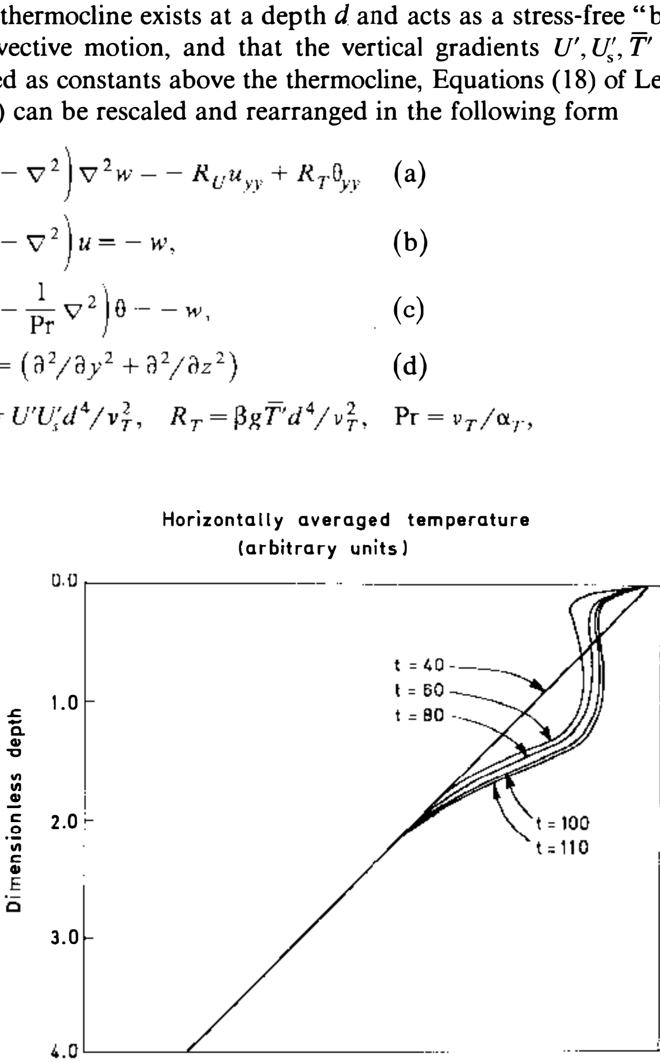

Figure 5 Horizontally averaged temperature at various times after onset of Langmuir circulations by the instability mechanism. The initial state is one of constant temperature gradient. A strong temperature gradient, or thermocline, eventually evolves.

where w is the scaled vertical velocity component, u the scaled perturbation to the basic current U, and θ a scaled perturbation temperature. The scales used to make the equations dimensionless are the layer depth d for length, the momentum diffusion time d^2/ν_T for time, ν_T/d for vertical velocity, $U'd$ for the x-component (u), and $\bar{T}'d$ for temperature. The parameter R_U is in the form of a Rayleigh number based upon a "temperature" gradient ($U'U_s'/\beta g$), and R_T is essentially the negative of a conventional Rayleigh number. These equations are precisely those describing double diffusion of u and θ. The diffusivity of u is typically expected to be larger than that for θ, i.e. one expects $\mathrm{Pr} > 1$. By reference to more familiar thermohaline double-diffusion problems (Turner 1973, Chap. 8), u is seen to act as temperature and θ as salt. The "Richardson number" of Equation (8) is $\mathrm{Ri}^* = R_T/R_U$, and the CL instabilities discussed so far treat only positive R_T and R_U. The literature on double diffusion indicates a number of other possibilities, shown schematically in the diagram of Figure 6 adapted from Baines & Gill (1969). Features in the right-hand quadrant $R_T, R_U > 0$ found in double-diffusion prob-

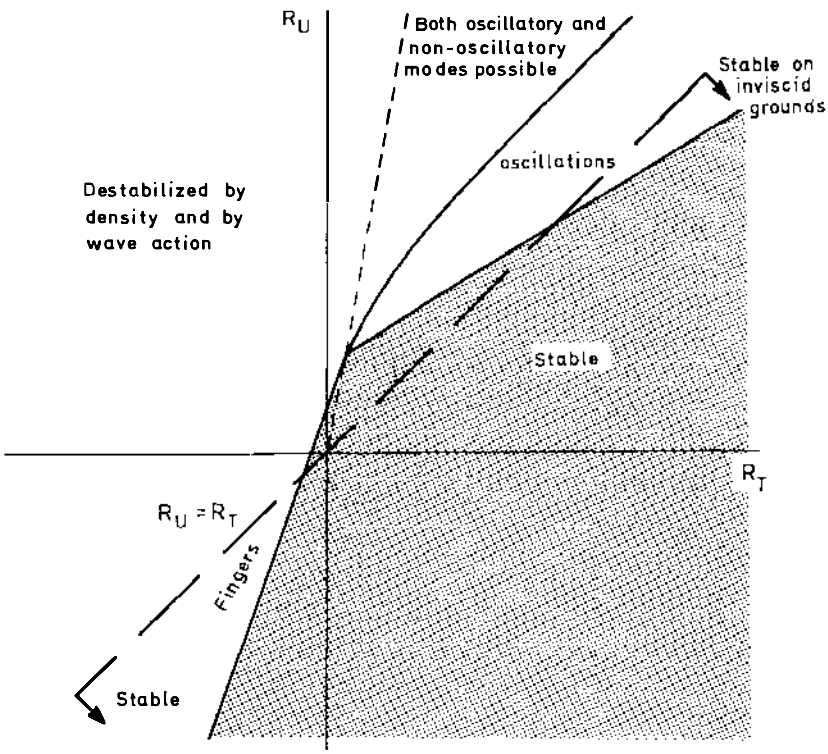

Figure 6 Diagram illustrating parametric regions in which wave-induced instabilities may be expected by analogy with double-diffusive instabilities.

lems have been also found for the Langmuir-circulation problem, including regions of monotonic growth as well as oscillatory growth (Leibovich & Lele 1982). The second quadrant, in which both the vortex force and temperature gradient are destabilizing, is intuitive. The "fingering" regions in the third quadrant correspond to a stabilizing vortex force (U' and U_s' opposed) and destabilizing thermal conditions leading to overall conditions $U'U_s' - \beta g\bar{T}' < 0$, which are stable if diffusion is neglected but are unstable when $\nu_T > \alpha_T$. This represents a new possibility for instability that presumably can exist in the ocean, but has yet to be explored.

Another possibility that has yet to be treated by the CL theories arises when the energy density of the surface waves has a nonzero horizontal divergence. This possibility is anticipated in the depth-averaged surface-convergence force derived by Garrett (1976). In the CL theories, it is represented by the adoption of a Stokes-drift velocity vector that has a nonzero vertical component.

Craik (1982a) has shown that the weak viscous Eulerian current induced by water waves without wind is unstable by the CL 2 mechanism, and that this may be also the case for the similar currents induced near a density interface by internal waves. Craik notes that this instability may be the cause of the scatter found in experiments on the drift current of surface water waves. These possibilities arise in the absence of wind, and are interesting to contemplate.

An alternate derivation of the CL equations was given by Leibovich (1980), using the exact "generalized Lagrangian mean" theory of Andrews & McIntyre (1978). The "GLM" theory presents a general set of equations incorporating the effects of wavy oscillations on the mean flow without the need for assumptions concerning the relative size of irrotational and rotational contributions. Leibovich (1980) suggested that this approach might be the route to a generalization of the CL theories to situations involving strongly sheared mean flows (when the waves can no longer be regarded as irrotational) or when finite wave amplitudes must be accounted for. Craik (1982b) has developed this line of thought farther, and has shown that such theoretical extensions are indeed possible. While the question of coherent structures in turbulent wall layers has not been addressed yet from this point of view, one cannot help but wonder if their physics can be most efficiently expressed by similar formalisms.

FIELD OBSERVATIONS

Measurements at sea are difficult to obtain. These difficulties and the lack of comprehensive theoretical hypotheses have resulted in data that are invariably incomplete in some way. To determine the structural form of a circulation cell, measurements of temperatures and three components of velocity are required at a number of points sufficient to resolve the motion. To educe the structure of a "typical" circulation cell, the conditional sampling of a large number of cells formed under similar environmental conditions is required. To search for a mechanism for the formation of the cells, environmental conditions coexisting with the circulations and conceivably related to them must be determined simultaneously. The experimental task is clearly formidable, and it is not surprising that the available information is fragmentary.

The most significant environmental parameters seem to be wind speed and the distribution of density with depth. Most, but not all, observations of windrows include measurements of wind speed; some also include temperature profiles. Some information (Katz et al. 1965, Myer 1971) suggests the sea state is also important, but as this is strongly coupled to the wind, it is difficult to isolate its independent significance; only a few observers have described surface-wave activity. Observed Langmuir-circulation characteristics include windrow spacing, speeds of surface tracers, time required for realignment of windrows after a shift in wind direction, time required for windrow formation, vertical velocities below convergences and divergences, and the angle between the local wind direction and surface streaks. A directory of observations is given in Table 3. Some of those listed are of an incidental or anecdotal nature, but they substantiate more systematic studies.

Table 3 A chronology of field observations of Langmuir circulations (see Figure 1*a* for notation)

Investigator	Location(s)	Main observables reported
Langmuir (1938)	North Atlantic	L, row width time response
Langmuir (1938)	Lake George, N.Y.	U_w, (u, v, w) currents, L, water temperature profiles
Woodcock (1944)	Gulf of Mexico	v, u anomaly, air and water temperature
Woodcock (1950)	Atlantic	Drawdown of surface sargassum as function of U_w
Van Straaten (1950)	Shallow tidal flats in North Sea	L, v
Stommel (1951)	Ponds on Cape Cod	Reorientation time
Sutcliffe et al. (1963)	Atlantic off Cape Cod; Atlantic off Bermuda	U_w, $\lvert w_d \rvert$

Table 3 (*continued*)

Investigator	Location(s)	Main observables reported
Welander (1963)	Baltic Sea	U_w, u, v, reorientation
Faller & Woodcock (1964)	Atlantic 39°N, 60°W	L, surface heat flux, U_w
Faller (1964)	Atlantic off Provincetown, Mass.	Angle between rows and wind
Williams (1965)	Ocean	L, u, v
Ichiyc (1965)	Atlantic off Long Island	L
Katz et al. (1965)	N.Y. Bight, Pacific off Argentina	L, U_w, u anomaly Sea state, reorientation
Ichiye & Plutchak (1966)	Long Island Sound	L, U_w
Owen (1966)	Pacific	L, air and water temperature
Ichiye (1967)	Ocean	Time required for row formation (measure of v), u anomaly
McLeish (1968)	Pacific off California	Row orientations
Scott et al. (1969)	Lake George	U_w, $\|w_d\|$, temperature perturbations below streaks
Myer (1969, 1971)	Lake George	U_w, air and water temperatures, temperature perturbations below streaks, $\|w_d\|$
Gordon (1970)	Plantagenet Bank off Bermuda	U_w, u anomaly
Assaf et al. (1971)	Atlantic off Bermuda	L, mixed-layer depth
Sutcliffe et al. (1971)	Sargasso Sea	Increased particulates below convergence
Maratos (1971)	Monterey Bay	L, U_w, angle between rows and wind, mixed-layer depth, u, $\|w_d\|$, air temperatures
Harris & Lott (1973)	Lake Ontario	U_w, $\|w_d\|$, air and water temperatures
Kenney (1977)	Lake-of-the-Woods	u, v, w, U_w, L, air and water temperatures
Filatov et al. (1981)	Lake Ladoga, Soviet Union	$\|w_d\|$, U_w
Thorpe & Hall (1982)	Loch Ness	Temperature perturbations below streaks, L, acoustic scattering from bubbles

Threshold Wind Speed

Surface streaking is generally reported to occur only when the wind speed exceeds some threshold value, most frequently placed at 3 m s^{-1} (Walther 1967). This value is not critical however; Myer (1971) observed streaking at lower wind speeds, with thermally unstable conditions requiring a somewhat lower threshold speed than necessary in thermally stable conditions. Ichiye (1967) found streaks "even in the calm sea when there are pronounced swells," a situation that presumably indicates no more than a slight breeze. Since visual detection of streaks is necessarily subjective, a variability in minimum required wind speeds is not surprising. Presumably the strength of surface-sweeping currents in Langmuir circulation scales with wind speed, and there will be some minimum sweeping speed required to overcome the random surface-speed fluctuations that resist organization of surface tracers into rows.

Surface Patterns: Spacing, Orientation, Formation Times, and Persistence

SPACING Streaks have been identified either visually by the collection of surface tracers such as foam, seaweed, or organic films in convergence lines, or by infrared photography, which renders surface-temperature anomalies visible.

Langmuir (1938) reports that streak orientations are parallel to the wind, and spacings in Lake George are between 5 and 25 m with a variability associated with seasonal changes. He also writes of the difficulty in determining spacing: "Quantitative measurements of streak spacings are difficult because between the well-defined streaks there are numerous smaller and less well-defined streaks. Just as large waves have smaller waves upon them, it appears that the surfaces of the larger vortices contain smaller and shallower vortices. The patterns of streaks on the lake surface are slowly changing; some growing, some dying out."

Spacing data in lakes are reported by Scott et al. (1969), Myer (1969, 1971), Csanady & Pade (1969), Kenney (1977), and Thorpe & Hall (1982). The range of cell spacings quoted by Langmuir is generally confirmed in lakes and ponds, although spacings of less than a meter have been reported (Myer 1969).

Reported spacings in the ocean range from 2 to hundreds of meters. Very regular rows of zooplankton 1.5 m apart have been reported by Owen (1966), but he attributed these to thermohaline convection since the pattern was apparently not formed by wind action. The largest scales reported are the massive collections of seaweed in "bands 2 to 6 m wide with spacing from 1100-200 m" seen by Langmuir (1938) in the North

Atlantic in 1927 (it would appear that the 1100 figure is a misprint, and should have read 100), and the 280-m spacing observed by Assaf et al. (1971) near Bermuda.

Spacing hierarchies are reported in the ocean by Assaf et al. (1971), Williams (1965), McLeish (1968), and Katz et al. (1965), and in lakes by Langmuir (1938), Scott et al, (1969), Myer (1969, 1971), and Harris & Lott (1973). The time evolution of a set of marked windrows suggests a dynamical cascade from smaller scales towards some dominant large scale. Harris & Lott (1973), for example, report that the distance between streaks increased with time in Lake Ontario, and that new streaks appeared between streaks already marked. This is supported by oceanic measurements of Williams (1965) and Ichiye (1967). Figure 2 from Williams's report summarizes change in spacing with time. "Young" stripes form with spacings that peak at 2 m and 5 m; as they age, the fraction of small streaks decreases, the number at 5 m grows, and large scales seem to make their appearance. Ichiye's (1967) Figure 4 seems to illustrate the same phenomenon. This kind of consolidation of surface markers suggests a gradual sweeping of the surface (including smaller-scale cells) by the largest scale, and Ichiye's (1967) experiment confirms Harris & Lott's (1973) assumption of the reformation of smaller scales, which can be made visible by periodically reintroducing surface tracers.

The spacing of windrows was presumed by Langmuir (1938) and many others to be proportional to the depth of penetration of the cellular pattern that they mark, in analogy with other types of convective motion. For example, the linearly most unstable Benard roll cells would correspond to a spacing L/D of roughly 2. Langmuir believed, with some support from his measurements, that the largest cells penetrated essentially to the thermocline, which suggests a correlation of windrow spacing and depth to the thermocline.

The hypothetical relationship between spacing of the largest windrow scales and the mixed-layer depth has yet to be conclusively established, but there are no data to dispute it either. Langmuir noted that spacings are generally greater in the fall when the thermocline is deepest than in the spring and early summer when it is shallow. His own "velocity indicator" measurements, however, showed a penetration depth that fell short of the thermocline. Scott et al. (1969) found that a significant maximum in the temperature gradient typically exists between the free surface and the thermocline in Lake George. They found that streak spacing correlates with the depth to this first stable layer, and concluded that heat is readily transported to this depth by Langmuir circulations, but not deeper. Assaf et al. (1971) found spacings of the largest streak scales to be 280 m, with a mixed-layer depth of 200 m or $L/D = 1.4$.

The results of Maratos (1971) on the maximum ratio of spacing to thermocline depth are consistent with this; his (single) extreme realization yields a ratio $L/D = 1.66$, with all other realizations ranging between .66 and 1.42.

The thesis that the thermocline limits the maximum penetration of cells cannot be ruled out by a lack of correlation (found to be the case by Faller & Woodcock and by Maratos) between L and D. Clearly other dynamical processes are at work during the growth phase of cells, and the thermocline has no effect until their penetration depth approaches it. Thus, a significant correlation between spacing and thermocline depth would be expected only if the thermocline depths were comparable to or smaller than the other dynamically important length scales in the problem (which are not obvious a priori). A test of Langmuir's hypothesis is to be had by dealing with an ensemble consisting of pairs of *maximum* spacings and thermocline depths. There is, to date, no evidence to contradict Langmuir's hypothesis; rather, the evidence available must be regarded as supporting it.

A statistically significant correlation between mean spacing and windspeed is reported by Faller & Woodcock (1964). They take this to be $L = (4.8 \text{ s}) \times U_{\mathrm{w}}$, where U_{w} is the windspeed at some unspecified height near the sea surface; the formula is not appropriate for $U_{\mathrm{w}} < 3 \text{ m s}^{-1}$, for which no rows were observed. Katz et al. (1965) present data suggesting the possibility of a positive correlation with smaller slope L/U_{w}, and Maratos (1971) found the correlation $L = 0.1 \text{ m} + (2.8 \text{ s}) \times U_{\mathrm{w}}$. In all of these cases, the number of samples and the range of wind speeds were small, and the data show considerable scatter. Scott et al. (1969) considered the question of spacing-windspeed correlation but did not find it to be statistically significant.

ORIENTATIONS Windrows are always nearly aligned with the wind direction, with maximum angular differences between pairs of streaks being no more than 20°. Whether there is a systematic deviation from the wind direction has been discussed by Faller (1964), who analyzed oceanic data from the Northern Hemisphere and found a systematic deflection of $13° \pm 2°$ to the right of the wind. This conclusion is partially supported by Katz et al. (1965), who found similar angular deviations in 4 out of 6 observations; however, the orientation data of Welander (1963), Maratos (1971), and Williams (1965) in the ocean, of Langmuir (1938) and Walther (1967) in Lake George, and of Kenney (1977) in Lake-of-the-Woods show that windrows are aligned within a few degrees of the wind, with no significant systematic bias to one side or the other. If Langmuir circulations develop in a well-defined Ekman layer, then they will presumably show the marked influence of the skewed velocities of the underlying Ekman spiral. This may explain the occasional angular deviations that have been reported.

If long stretches of streaks are well marked and viewed from a sufficiently distant perspective, they frequently form a network of intersecting lines. This point was emphasized by McLeish (1968) and was also made by Williams (1965) and Harris & Lott (1973). The length that an individual streak line can be traced between intersections is seldom mentioned. Williams (1965) estimates (subjectively) that the ratio of length to a characteristic spacing was of order 20 in his experiments. Kenney (1977) classifies his windrows by this ratio, and defines "very regular" ones as those with length-to-spacing ratios in excess of 100, viewed from the top of a 20-m tower; he also comments that the regularity seldom is perceptible when the windrows are viewed from close to the water surface.

FORMATION TIMES The time scales for formation of Langmuir circulations can be inferred from the times required for surface rows to reorient themselves after a shift in wind direction. All investigators indicate that this is a rapid process, the original pattern being destroyed and a new one reforming, aligned with the new wind direction, within a few minutes of the wind shift. Stommel (1951) found that small-scale windrows on ponds were reoriented within 1 or 2 minutes after a shift in wind direction. Maratos (1971) reports a single instance of a rapid shift in wind direction, followed by a reorientation of streaks within 2 to 4 minutes. Welander (1963) observed that 8-m-wide streaks in the Baltic were reoriented within 10 minutes of a shift in wind direction; the newly formed streaks also had a spacing of 8 m. Langmuir (1938) noted that the reorientation of a large-scale windrow pattern in the Atlantic occurred within 20 minutes of a shift in wind direction. Katz et al. (1965) observed reorientation in one experiment off the Argentine coast: a change in wind direction without change of speed occurring within a half-hour period resulted in a realignment of rows, the new rows being of smaller scale. They found similar results in other experiments.

The extremely rapid reorientation response times seen by Stommel (1951) led him to speculate that the rows marked a process confined to a layer very near the surface ("top few inches"). In the wind-shift observation described by Welander (1963), neutrally buoyant floats placed at depths of 1 and 2 m continued moving in the old wind direction, although surface floats were carried into the reoriented windrows. Faller (1981) has pointed out that the rapid response is presumably due to the smaller scales, which may be expected to be shallower and capable of rapid response. Response times of the order seen in the field are not inconsistent with Faller's (1978) laboratory measurements, or with the CL models described above.

PERSISTENCE Accounts of observations give the impression that individual streaks maintain their identity for substantial periods of time, but this is seldom explicitly discussed. Kenney (1977) used 20-minute reels of film in his study, and found that under steady wind conditions individual streaks remained identifiable for periods in excess of the time required to expose the film. In fact, he observed identifiable streaks for periods in excess of one hour (private communication) during his investigations in Lake-of-the-Woods.

Surface and Subsurface Current Structure

All investigators who report surface-tracer speeds have indicated that the windward (u in Figure 1a) component is larger in streaks than between them. This current anomaly, Δu, has been measured or inferred in a few cases, but no systematic studies of the complete current system have been undertaken. Ichiye (1967) estimates that speeds in streaks were twice those outside; Harris & Lott (1973) measured values of Δu up to 6 cm s^{-1}. Anomalies were estimated to have been 1–3 cm s^{-1} in observations discussed by Gordon (1970), 3 cm s^{-1} by Williams (1965), and 10 cm s^{-1} by Assaf et al. (1971). Unfortunately, these values were reported without reference to other components of the current system or to the prevailing wind speeds.

The order of magnitude of the horizontal surface "sweeping" component v in a system of Langmuir vortices can be inferred from the time required to collect tracers introduced at the surface into rows. This has been estimated in a few cases. Langmuir (1938) found that leaves placed midway between streaks spaced 12–20 m apart took about 5 minutes to reach the streaks, which gives an average sweeping speed of about 3 cm s^{-1}. Since the sweeping both at the streak and at the line midway between is zero, or nearly so, the peak speeds must have been somewhat greater than the average. Ichiye (1967) also noticed that computer cards placed across the wind were swept into rows in a few minutes, and one can infer from his paper that the cards had to travel laterally about 10 m to reach convergence lines. Woodcock (1944) estimated that all of the surface tracers introduced at the surface were swept into convergence zones in 3–5 minutes: one can also infer from his paper that the row spacing was on the order of 20 m, as in Langmuir's case.

Subsurface sweeping motions toward vertical planes marked by surface convergence lines have been observed by Langmuir (1938) and Harris &

Lott (1973) but speeds have not been estimated. Langmuir found that a neutrally buoyant drogue suspended at depths up to 5 m gradually drifted under a streak, but one suspended at 10 m had no such tendency. Since the sweeping velocity presumably must change sign away from the streak plane at some depth, this is consistent with a cellular motion extending to depths of 10 m or more. The picture is seconded by theory (Leibovich & Radhakrishnan 1977, Leibovich & Paolucci 1980a), which shows that maximum sweeping speeds toward convergences very near the surface greatly exceed lateral currents away from streak planes at greater depth. Harris & Lott (1973) also found that drogues suspended at 5 to 6 m drifted toward planes below streaks.

Kenney (1977) observed the sweeping motions of drogues suspended at several depths in the shallow water of Lake-of-the-Woods. All drogues were swept toward planes beneath streaks, including those near the bottom. The return motion anticipated at depth was not seen; this curious observation has yet to be explained and clearly merits further investigation.

The best-documented cellular current component is the vertical velocity below streaks. This has been inferred by Myer (1969, 1971) by timing isotherm displacements in the upper 7 m of Lake George; he found descending currents in narrow "jets" below streaks of about 3 cm s^{-1} under conditions of thermal stability, and 5 cm s^{-1} under thermally unstable conditions. These data are consistent with direct velocity measurements taken by Sutcliffe et al. (1963), Harris & Lott (1973), and Rjanzhin (1980) and Filatov et al. (1981), all of which probably should be considered more reliable. Harris & Lott (1973) traced aluminum plates ballasted for neutral buoyancy; this device was first used by Langmuir (1938), who measured descending currents of 2 to 3 cm s^{-1} below streaks and rising currents of 1 to 1.5 cm s^{-1} midway between adjacent streaks. Rjanzhin and Filatov et al. (1981) used "Sutcliffe" floats, first used by Sutcliffe et al. (1963). This device measures the drag on a circular disk by the buoyancy force on a partially submerged buoyant element; it therefore measures the maximum vertical speed within a distance from the free surface determined by the size of the device. The device used by Sutcliffe et al. (1963) is capable of measuring vertical currents up to 6 cm s^{-1}, after which the device is fully submerged. Filatov et al. (1981) give a nonlinear correlation between wind speed and downwelling speed with coefficients depending upon the gradient Richardson number, a measure of the surface heat flux, in the air near the surface. It appears that they may have forced the correlation to pass through zero at zero wind speed, a refinement that might be relaxed since circulations were observed only for wind speeds larger than a threshold of about 3 m s^{-1}. None of these authors report depths at which the maximum downwelling current is attained.

Data taken by Sutcliffe et al. (1963), Harris & Lott (1973), and Filatov et al. (1981) are shown in Figure 7. Kenney (1977) has pointed out a possible error in the data reduction in Sutcliffe et al. (1963); the drag expression printed in their paper is inconsistent with the aerodynamic form of the drag coefficient used for the disk. If this is not merely a misprint, then downwelling speeds in Figure 7 as taken from their paper should be increased by a factor of $\sqrt{2}$. There is a great deal of scatter in the data shown in Figure 7; clearly the downwelling currents increase with wind speed, but other factors are obviously important. The more vigorous downwelling that occurs with surface heating in the Harris & Lott data is contrary to expectations, as they point out, but they also indicate uncertainties in determining the heat flux. If each data set in Figure 7 is individually considered, with those attributed to thermally stable and unstable conditions treated as distinct, downwelling current data are well fitted by a linear relationship with wind speed. The two straight lines in Figure 7 represent a compromise; the dashed line is a least-squares linear fit to all of the Sutcliffe et al. (1963) and Harris & Lott (1973) data taken together, and the solid line is the corresponding fit to all of the Filatov et al. (1981) data. Both composite data sets are well fitted in this way. The goodness of fit to each data set shows that

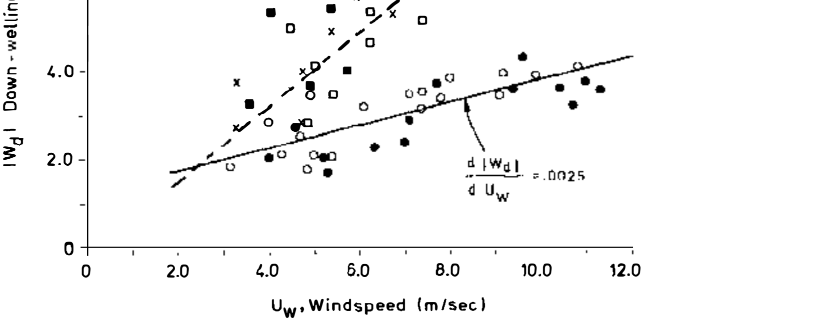

Figure 7 Measured downwelling speeds below streaks as a function of wind speed. The open squares and circles correspond to surface heating, closed symbols to surface cooling.

downwelling increases in an essentially linear fashion with wind speed in the range shown, provided other factors, such as the density structure of the water column, are held more or less fixed.

We also note that Woodcock (1950) inferred downwelling speeds of 3 to 7 cm s^{-1} by observing abundances of surface sargassum, which are submerged by vertical speeds of this order. Maratos (1971) has inferred average upwelling speeds of 0.8 cm s^{-1} by monitoring the sedimentation rate of fine sand in cells driven by a wind speed of 6 m s^{-1}. Additional data taken using unspecified methods are presented in Scott et al. (1969).

Vertical Penetration of Cells

Observations of drogue motions by Langmuir (1938) and Harris & Lott (1973), referred to in the previous section, indicate sweeping motions at depths of 5 to 6 m toward planes below surface streaks. Theoretical models then suggest that weaker cellular motions extend to depths that are a small multiple of these figures, and therefore comparable to the depth of the thermocline and of the streak spacings they observed.

More systematic inferences of the penetration depth have been made by Myer (1969, 1971) and Thorpe & Hall (1982) by measurements of temperature anomalies.

Myer (1969, 1971) found isotherm displacements of up to 3.5 m under stable thermal conditions and significantly larger displacements under unstable thermal conditions. These displacements occurred in narrow "jets" below surface streaks. The depth of penetration was inferred by the deepest isotherm undergoing a displacement beneath streaks, and was estimated to be between 2 to over 7 m under stable conditions. Since Myer's thermistor string was limited to a 7-m length, deeper effects could not be determined.

Harris & Lott (1973) found difficulties in making similar measurements on Lake Ontario; their thermistors did not always show temperature anomalies below streaks in thermally stable conditions. Thorpe & Hall (1982) found similar difficulties, and found that attempts to directly correlate temperature anomalies with surface streaks failed: the expected anomaly signal was weaker than the background "noise" of thermal fluctuations due to other effects. By careful data analysis, however, they were able to extract a statistically significant correlation between surface streaks and temperature anomalies extending throughout the depth of the mixed layer. Isotherm displacements are small, however, amounting to no more than 60 cm, and the temperature anomalies are "very much less than the amplitude of the r.m.s. variation" of the measured temperature. This isotherm displacement is much less than that computed by

Leibovich & Paolucci (1980a) using the CL 2 instability model. They note, however, that the thermal stratification was much larger than that used in the computations. It is also important in this regard to note again the error in Leibovich & Paolucci (1980a) that led to an overprediction of isotherm displacements.

Thorpe & Hall (1982) also measured acoustic scattering from submerged bubbles produced by breaking surface waves, and found that the average acoustic cross section of the bubbles is higher below streaks (at depths between 0.5 and 2.3 m) than between them. Since Thorpe & Hall (1980) found that waves break with equal frequency in windrows and between them, wave breaking provides a uniformly distributed bubble source. The variations in acoustic cross sections are therefore regarded as substantial evidence of downwelling motion below streaks.

LABORATORY EXPERIMENTS

Faller (1969) was the first to demonstrate convincingly that organized rolls could be established mechanically in the laboratory by wind and wave action; McLeish (1968) alludes to possibly similar experiments, but provides little detail. Faller's demonstrations, conducted in a small wind-wave tank, are of particular interest, since they showed that circulations required only current shear and waves. Waves and currents simultaneously produced by wind action led to longitudinal rolls.

Mechanically produced waves by themselves did not lead to circulations: a slight current shear produced by slow draining of the tank also did not, by itself, produce circulations. If mechanically driven waves were then introduced into a slightly sheared flow, however, vigorous convection was observed as soon as the waves reached the test section. Thus, Faller demonstrated that wind stress is not necessary, and that the role of the wind may be simply to introduce the two principal ingredients, current shear and waves, into the water body.

By various experiments Faller (1969) also showed that thermal convection was not necessary, and that the circulations produced by wave and current shear were substantially more vigorous than thermal convection organized by shear.

The generation of Langmuir-like circulations in the laboratory by wind and waves was studied in more detail by Faller & Caponi (1978). Roll motions with axes parallel to the wind readily formed in their wind-wave tank. The crosswind spacings, λ_c, of convergence zones in the cellular motion were determined, and a clear dependence upon the wavelength of the surface waves, λ_w, was demonstrated. Their data, and several oceanic measurements, are fitted reasonably well by the relation

$$\lambda_c/H = 4.8[1-\exp(-0.5\lambda_w/H)], \tag{11}$$

where H is the mean depth of the water in the wind-wave tank, or the estimated depth of the mixed layer for the oceanic cases.

In transient studies, they noted that circulations began within seconds after arrival of the waves. The bands that first formed gradually increased in width by a factor of about 3 before reaching the asymptotic state reported in Equation (11) above. Whether this change in spacing was associated with an increase in λ_w was not made clear. The authors concluded that the mechanism underlying Langmuir circulations is wave-related, and that the scale of the surface waves plays a role in setting the scale of the circulations.

Experiments designed to test the CL 1 mechanism have been reported by Faller (1978), and similar experiments have been performed by S. Mizuno (unpublished) with similar conclusions. Faller generated a crossed-wave pattern using a specially designed mechanical wavemaker, and a current was created by blowing a light wind over the water surface. Circulations did not form with either wind alone or waves alone, but regular, steady, and reproducible Langmuir circulations formed when both were present. The circulations were marked by floats on the surface, which were swept into convergence lines, and by dye crystals on the bottom. In accordance with the CL 1 mechanism, upwelling occurred beneath lines of maximum surface excursions (lines traversed by points of wave intersections) and downwelling occurred midway between. Reversals of the wind direction reversed the direction of circulation in the cells, another feature inherent in the CL 1 model. Direct measurements of most of the parameters occurring in the CL equations could be made, and u_* and ν_T were inferred by indirect means. Using measured and inferred parameters, the time scale T_d (see Table 1) was estimated and found to be 29 s, while La was estimated to be 0.59. The experimental response time was estimated by the increase of the surface-sweeping speed with time after switching on either wind or waves. The exponential time constant found this way was $T = 12$ s. This was compared with the value of 10 T_d quoted by Leibovich & Radhakrishnan (1977) for the time required for essentially steady conditions to be obtained at the surface in numerical experiments at La $= 0.01$. Clearly 10 T_d is considerably larger (24 times) than the observed exponential rise time, but the two values refer to different quantities, and ought not to be directly compared. Furthermore, as the small time solution of Leibovich (1977a) makes clear, the rate at which the surface-sweeping component develops initially is proportional to $\sqrt{\mathrm{La}}$. Thus the surface-sweeping speed at La $= 0.59$

builds up at a rate 7.7 times faster than the corresponding quantity at La = 0.01. It therefore appears that the theory and experiment are probably in rough agreement as far as time scales are concerned.

Further details of Faller's (1978) experiments are given in Faller & Cartwright (1982). This report also mentions preliminary results of experiments to test the CL 2 instability mechanism. A completely stable shear flow is found to be destabilized by the addition of a very small amplitude [O(1 mm)] monochromatic plane wave. The disturbance takes the form of longitudinal rolls, and breakdown is rapid [O(1 min)], with time scale decreasing with increasing wave amplitude. The results are said to be in reasonable agreement with the theoretical stability diagrams of Leibovich & Paolucci (1981), despite some differences between the experimental conditions and those assumed in the theoretical analysis.

Conclusions

The present status of understanding of Langmuir circulations can be summarized as follows:

1. Their existence in lakes and the ocean is well established, although their detailed structure remains to be determined.
2. They are convective in nature and mechanical in origin, and are driven by the wind.
3. Evidence exists to suggest that these convective motions may be an important mixing mechanism in the upper ocean, but the existing experimental case is not strong. In view of the potential significance of identifying this phenomenon as a major contributor to upper-ocean mixing, further work is strongly recommended.
4. Laboratory experiments and recent theoretical concepts show promise of explaining the origin and mechanism of naturally occurring Langmuir circulations.

This natural phenomenon is clearly extremely complex, and cannot be isolated for study in the field from its equally complex environment. As a result, it would appear that further advances in understanding will come most rapidly by a combination of controlled laboratory experiments and theory.

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