Mechanisms and Scales of Langmuir Circulations

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The generation of Langmuir circulations by an instability mechanism

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Laboratory Studies of Wind-Driven Langmuir Circulations

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The Stokes Vortex Force

- Recall that the Craik-Leibovich (CL) equations incorporate wave effects through the Stokes vortex torque

$$\nabla \times [(\nabla \times \mathbf{u}) \times \mathbf{U}^S]$$

- Assuming invariance in the x (down wind/ wave) direction this can be simplified to

$$\underbrace{U_y^S u_z}_{\text{CL1}} - \underbrace{U_z^S u_y}_{\text{CL2}} = F \ \text{(in Craik 1977)}$$

CL1

- Two monochromatic waves at different angles produce horizontal Stokes drift variations.
- Craik 1977: “Since this gives rise to a ‘forcing term’ F with specified y periodicity, the spacing of the cells is predetermined.”

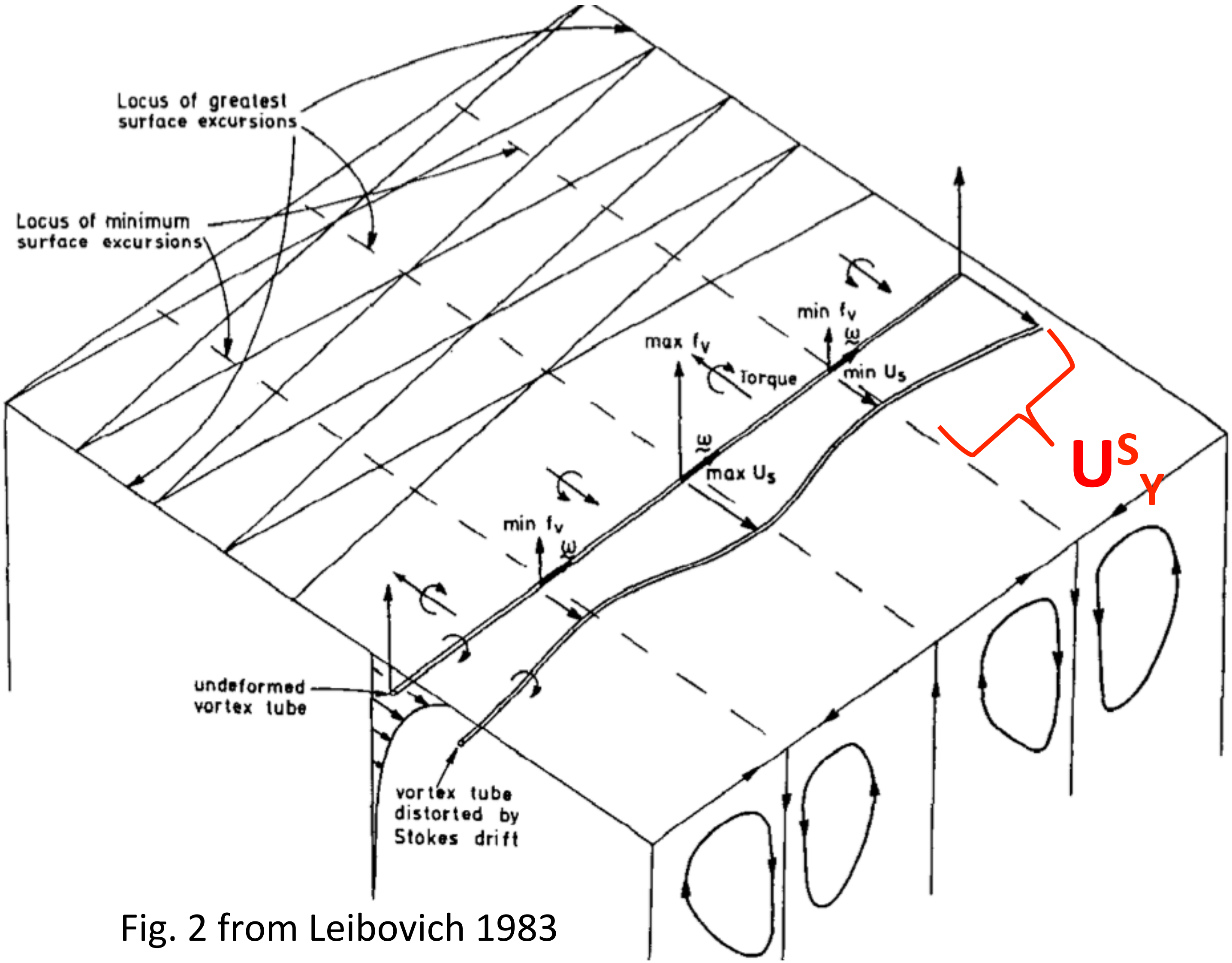


Fig. 2 from Leibovich 1983

CL2

A perturbation u' induces vertical vorticity which is then tilted and stretched by U and U^S.

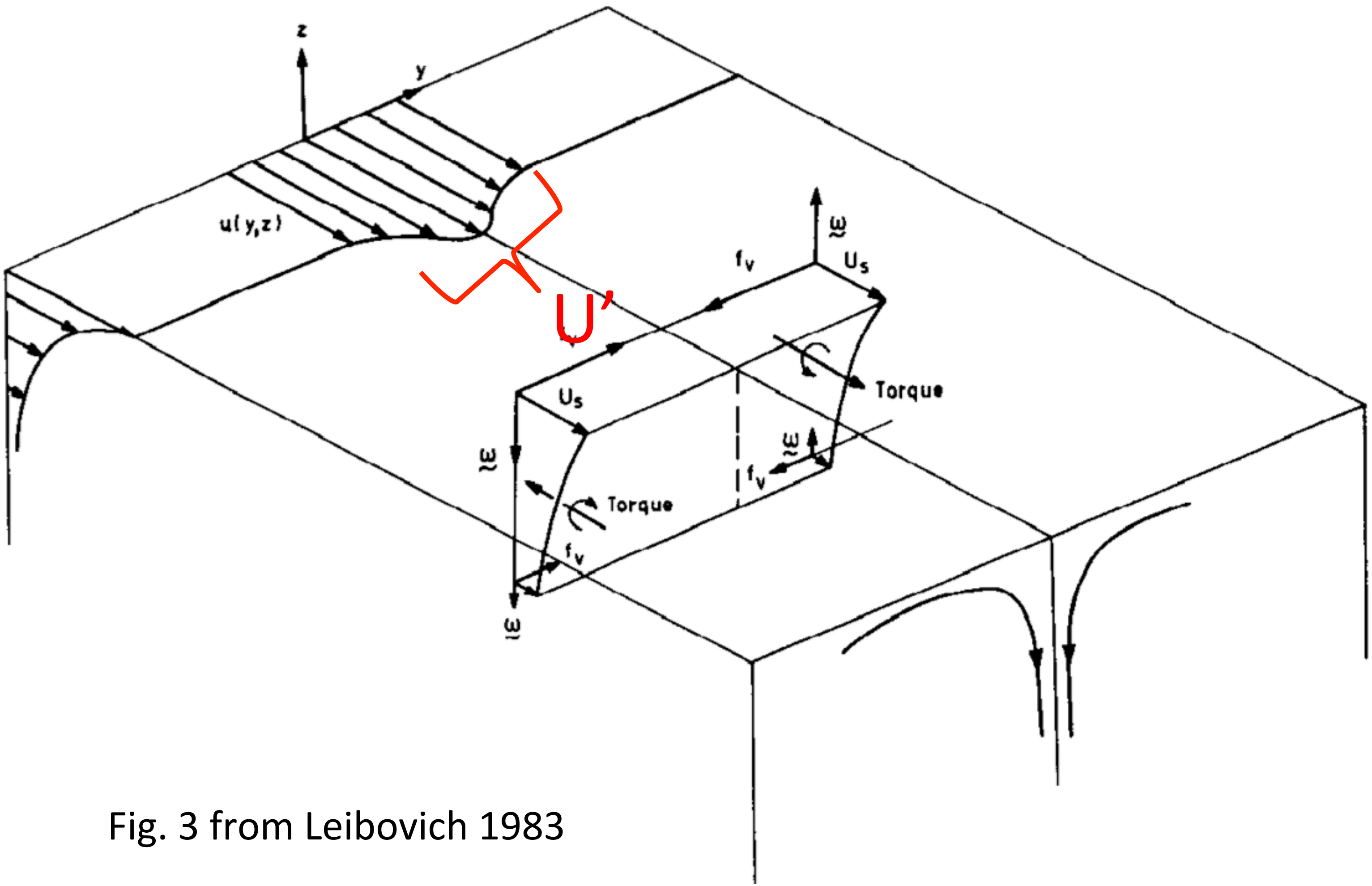


Fig. 3 from Leibovich 1983

How Does this Eulerian Perturbation Evolve?

$$\hat{u}_t + \hat{w}\overline{u}_z = La\nabla^2\hat{u}$$

$$\hat{\omega}_t = La\nabla^2\hat{\omega} + \hat{u}_y U_z^S$$

Assuming a normal mode solution:

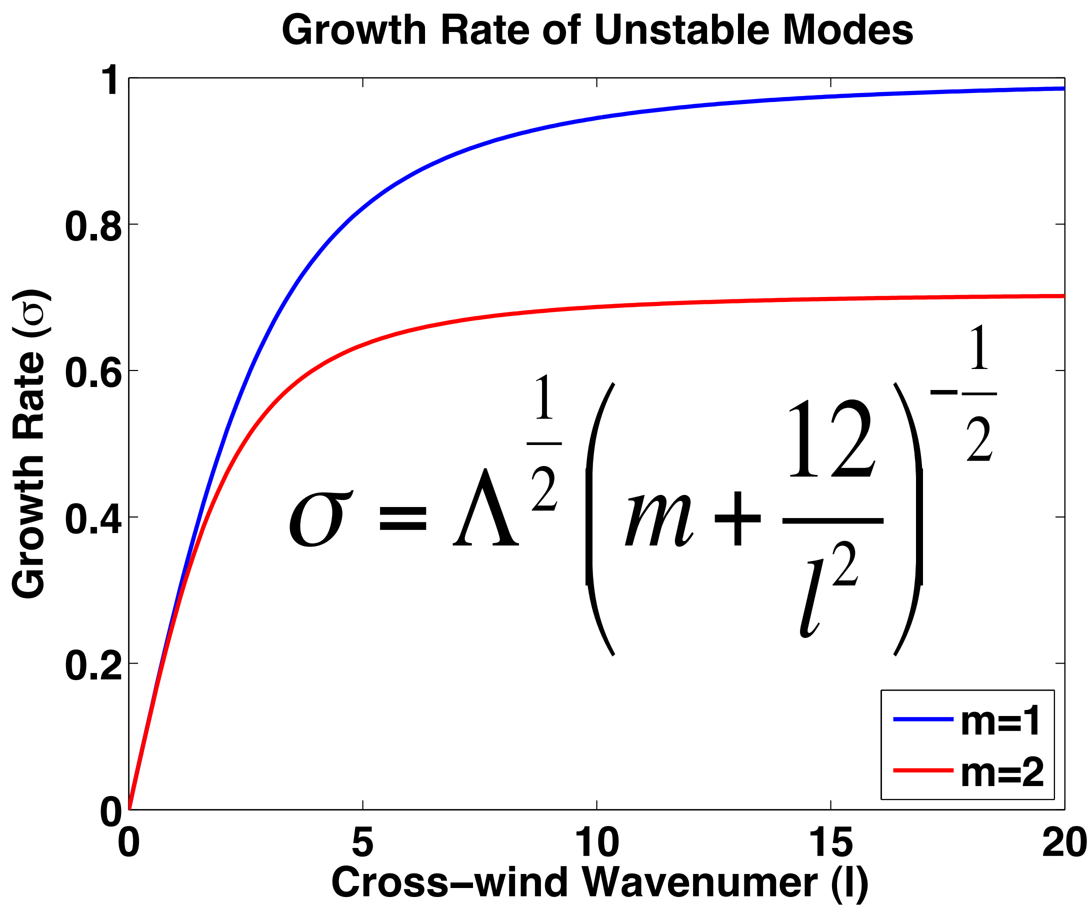
$$(\hat{u},\hat{w}) \sim \mathrm{Re}[(U(z),W(z))e^{ily+\sigma t}],$$

and first considering the inviscid case the problem is simplified into a single equation for the vertical velocity

$$(D^2 - l^2)W = -(l^2\sigma^{-2}\overline{u}_z U_z^S)W$$

$$W(0) = 0, \qquad W \to 0 \text{ as } z \to \infty$$

Constant Stokes and Eulerian Shears



$$\sigma = \Lambda^{\frac{1}{2}}\left(m + \frac{12}{l^2}\right)^{-\frac{1}{2}}$$

Conclusion: YES! CL2 produces an instability.

When Is it Unstable?

- Craik makes the analogy to the Benard convection problem where momentum is being convected instead of heat.
- The critical Rayleigh number for this problem is

$$Ra = La^{-2}H^{4} = (ak)^{2}\left(\frac{du^{*}}{\nu_{T}}\right)^{2}\left(\frac{d^{2}\omega}{\nu_{T}}\right) = 1708$$

Assuming

$$a \sim \frac{0.2U_w^2}{g},\ \omega \sim \frac{g}{U_w},\ k \sim \frac{g}{U_w^2},\ \nu_T \sim 2.3\times10^{-5}\frac{U_w^3}{g}$$

$$Ra \approx 7.6\times10^{6}$$

What is the Cell Spacing?

- Craik says that after the development of the wind driven boundary layer (i.e. the cell width is not limited by the bottom of the pycnocline.) the length scale for a LC-pair is twice the inverse wavenumber of the waves.

$$L = 2k^{-1}$$

- Or for shorter times:

$$L = 0.2k^{-1}$$

Conclusions

(i) Initially weak spanwise-periodic circulations give rise to variations $\hat{u}$ of the downwind velocity by advection of the developing mean Eulerian profile $\overline{u}(z,t)$. This is accomplished by the term $\Psi_y \overline{u}_z$ of (3.2).

(ii) The spanwise variation of $\hat{u}$ implies a periodic distribution of vertical vorticity: $\hat{u}_y$. But vorticity is convected by the Stokes drift $\mathscr{U}(z)$ since, in the absence of viscosity, material lines and vortex lines must coincide. Accordingly, the positive gradient $\mathscr{U}_z$ of the Stokes drift 'tilts' vertical vorticity to generate longitudinal (x) vorticity of a sense which reinforces the initial circulations postulated in (i). This tilting of vertical vorticity is represented by the term $\hat{u}_y \mathscr{U}_z$ of (3.2).

(iii) The diffusive and dissipative roles of (eddy) viscosity will tend to inhibit the inviscid instability mechanism of (i) and (ii), but *La* is usually sufficiently small for instability still to occur. The downwind velocity perturbation $\hat{u}$ according to inviscid theory is precisely zero at the free surface; but the viscous solutions display a structure in qualitative agreement with observed Langmuir circulations.

LC in a Tank

Experimental Setup

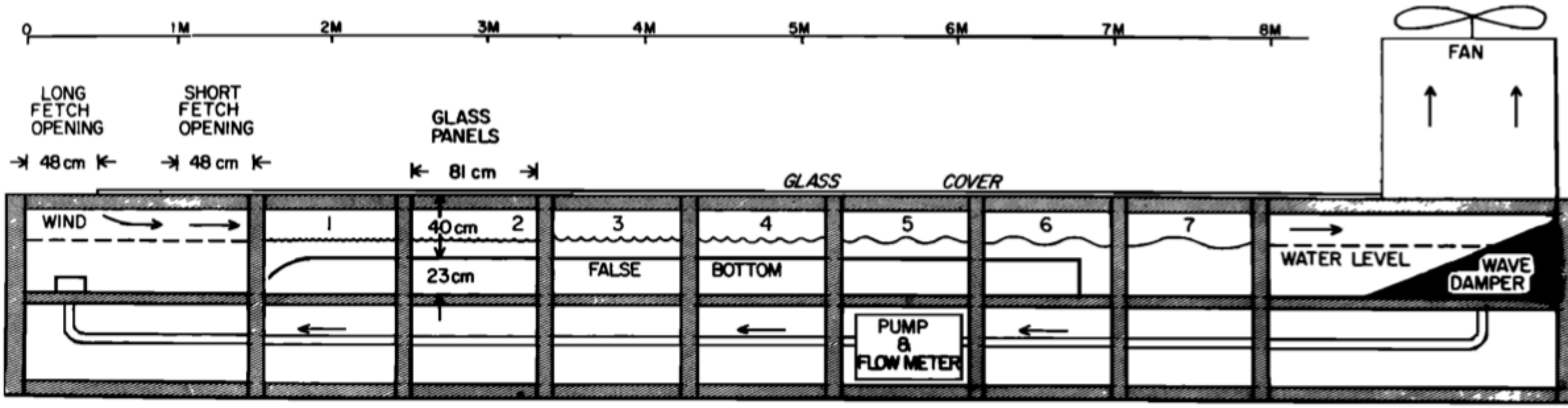
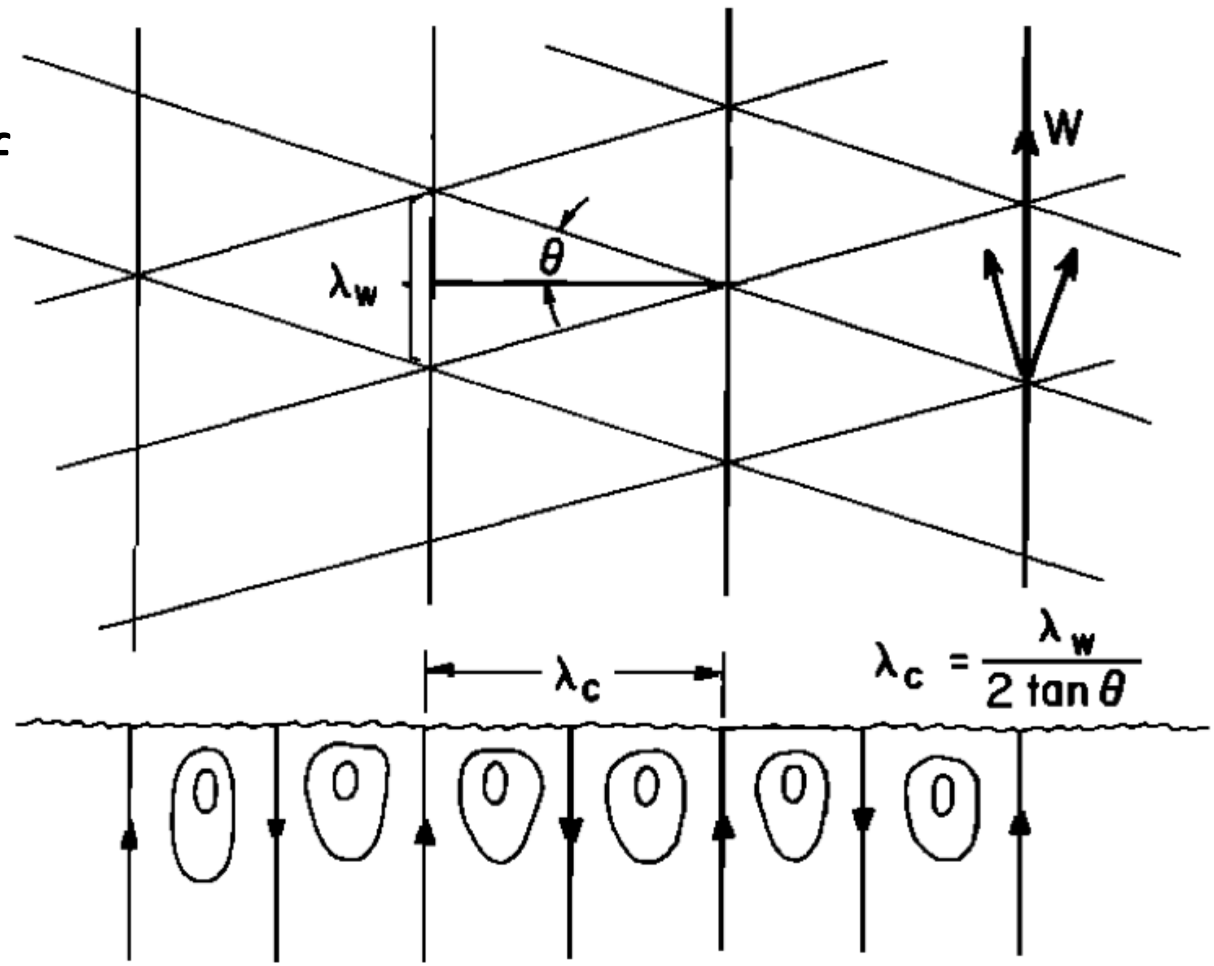


Fig. 3 Faller and Caponi 1978

CL1: Forced Wavelength for LC

- CL1: the wavenumber of the Langmuir cells depends on the wavelength of the waves and the angle between them.



Drifters Align in Rows

- Four cell pairs are found indicating a wavelength of 22cm.
- Maximum wavelength may have been limited by tank width

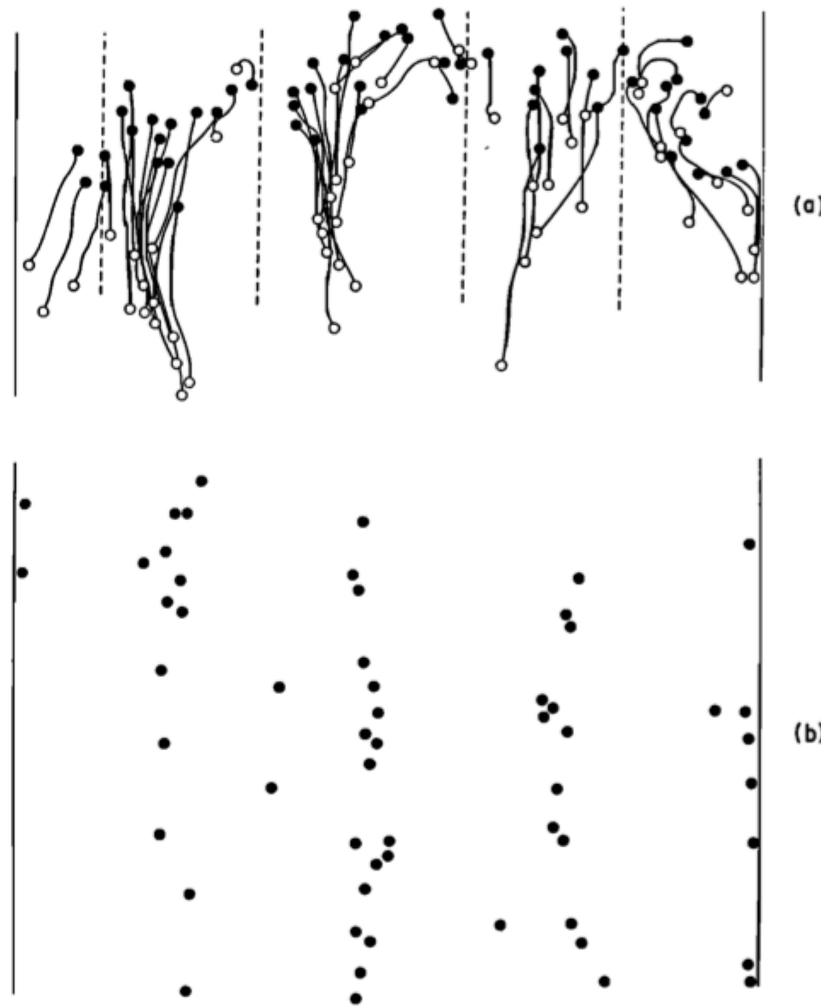


Fig. 8 Faller and Caponi 1978

Vortex Seeding (i.e. Rake Experiments)

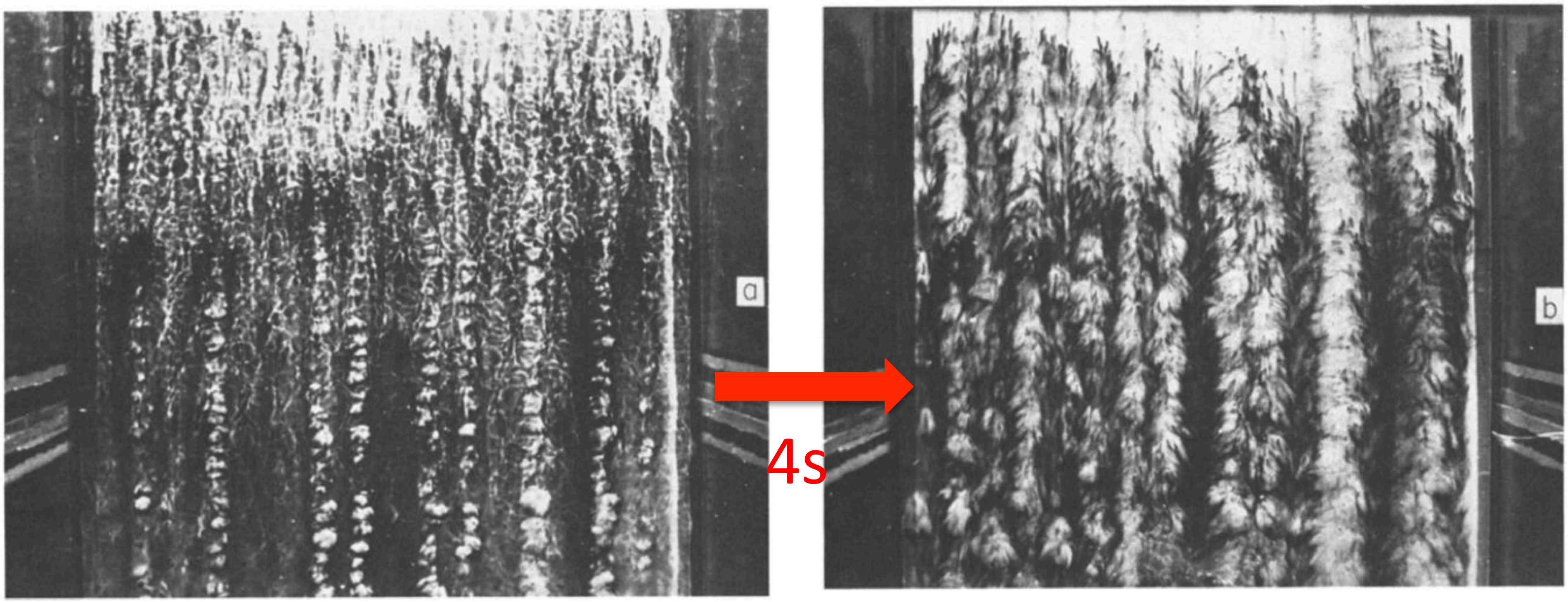


Fig. 13 Faller and Caponi 1978

- Nonlinear cascade to *larger* scales.

My Questions

- Scales
  - Craik suggests that different scales of LC develop during different phases of the boundary layer development, but Faller and Caponi suggest and upscale energy transfer. Which is it? Or is it something else?
  - If it is an upscale cascade, then it seems we should definitely not throw LC into the "sub-grid scale hopefully it goes like $k^{-5/3}$ bin."
  - Are the scales of LC ever limited by the longest wavelength waves?
- CL1 vs. CL2
  - Does it matter which produces LC?
  - Do we see evidence of one or the other in the ocean in places where the wave spectrum is strongly peaked vs. spread?