Observations and Theories of Langmuir Circulation: A Story of Mixing

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Abstract. The study of Langmuir circulation has a strongly interdisciplinary history; the first half of this work is a brief and eclectic review of this. Much of the research has been motivated by interest in the biology and chemistry of the mixed layer. These, in turn, are sensitive to details of typical particle trajectories; i.e., to high-order statistics of the flow such as time and space lagged covariances. In contrast, descriptions of the mixed layer have progressed from means (mixed layer velocity, temperature, and depth) to variances only recently. With the development of new observation techniques, and of complex numerical models, building upon equations for the development of Langmuir circulations developed by A. Craik and S. Leibovich in the late 1970's, it appears that descriptions of these high-order statistics may be attainable. However, systematic discrepancies appear in the observations that remain to be explained: e.g., the magnitude of near-surface velocities associated with Langmuir circulation varies over a factor of 4 between different wind events for apparently similar conditions, yet is well behaved within each individual event. The last part of this work dwells on some recent observations and the nature of this unexplained variability.

1 Introduction

The oceanic surface mixed layer is a crucial link in coupling the air and sea. The form and strength of the mixing motion within this layer can strongly affect fluxes of momentum, heat, and gases across the air-sea interface, and the transport of nutrients and other components up from below. The kinematics of this motion is important to the biota and chemistry as well, gathering seaweed and surfactants into lines, and trapping particles that would otherwise sink or float within "regions of retention." Improved understanding of these processes depends on our understanding of the mechanisms and dynamics involved. An important element of these dynamics, and one which dominates the observed kinematics, is a pattern of alternating horizontal roll-vortices that has come to be known as "Langmuir circulation," after their first description in the scientific literature 61 years ago (*Langmuir* 1938).

This symposium celebrates the 60th birthday of Sid Leibovich, who has contributed significantly to the understanding of Langmuir circulation, and it seems a fitting occasion to review the history of this field of study. An underlying theme here is that the study of Langmuir circulation has been quite interdisciplinary, with cross-fertilization of ideas from physics, chemistry, biology, and applied math. Indeed, the first few decades in the study of Langmuir circulation were motivated largely by the needs of chemists and biologists interested in the oceanic mixed layer. To describe the ecology of organisms in this layer, or to describe the exchanges and fluxes of chemicals and nutrients across the thermocline and air-sea interface, detailed knowledge of typical particle trajectories is needed (among other things!). This knowledge would permit estimates and simulations of light exposure histories, nutrient levels, and the dynamical chemical equilibria appropriate to the oceanic surface mixed layer. In mathematical-physics terms, these requirements correspond to knowledge of high-order statistics of the flow, such as two-point multidimensional time-space correlations of velocity and displacement fields. In contrast, physical oceanographers have been struggling with even the lowest-order statistics: e.g., the mean velocity, thickness, and temperature of the surface layer. From one perspective, then, the study of Langmuir circulation is one of bridging this gap between the need to estimate high-order statistics and the ever-increasing (but still inadequate) ability to measure and parameterize the requisite physical fields.

Rather than an exhaustive review, the following section is an eclectic (and decidedly incomplete) history of the study of Langmuir circulation. It is intended to convey the overall flavor of this interdisciplinary enterprise, rather than to catalog all the contributions. After the review, recent attempts to parameterize the mixing strength and to identify the proper scaling of the surface velocities associated with Langmuir circulation are discussed in somewhat more detail. The paper concludes with a brief discussion of what additional effects might influence the velocity scaling relation, and so help explain some remaining variability.

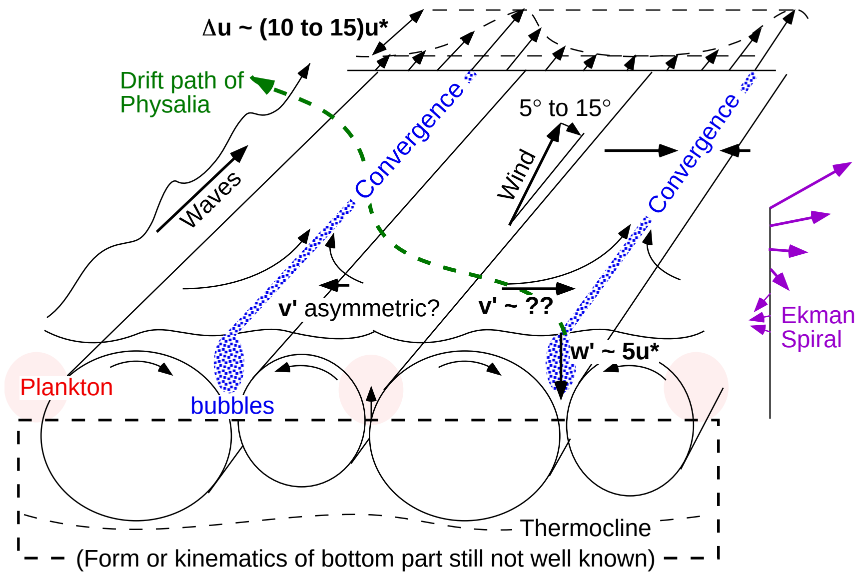

Figure 1. Schematic illustration of Langmuir circulation.

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Some Milestones in the Study of Langmuir Circulation

Inspiration for the investigation into this form of circulation apparently came during a trans-Atlantic passage (*Langmuir* 1938). Irving Langmuir (nominally a chemist employed by Westinghouse) noticed that seaweed aligned into nearly regular rows when the winds exceeded 10 to 20 knots (5 to 10 m/s), and that, when the wind suddenly shifted 90 degrees, the lines reformed within 20 minutes. Not content with these qualitative observations, Langmuir embarked on a systematic and, especially considering the resources available, surprisingly complete investigation into the form of this wind-driven circulation, in a series of experiments conducted on Lake George, NY. His measurements established the essential kinematics of alternating horizontal roll-vortices aligned with the wind (figure 1). They also provided quantitative estimates of upwelling/downwelling velocities and the time it takes for materials (e.g. leaves falling on the lake surface) to advect into rows aligned with the wind and sink. Of note are the suggestions that “The helical vortices set up by the wind apparently constitute the essential mechanism by which the epilimnion is produced,” and that “quantitative measurements of the streak spacings are difficult because between the well-defined streaks there are numerous smaller and less well-defined streaks. Just as large waves have smaller waves upon them, it appears that the surfaces of the larger vortices contain smaller and shallower vortices.”

A few years later, Woodcock suggested that oceanic Langmuir circulation may be asymmetric (*Woodcock* 1944). He noticed that an overwhelming proportion of Portuguese Man-o-War (Physalia) gathered in the N. Atlantic are so constructed that they move about 45° to the left of the wind (looking downwind). He speculated that a competitive advantage would be gained if Langmuir circulation were asymmetric such that these left-tending Physalia would spend a larger proportion of the time over the favorable upwelling region between convergence zones (figure 2). He suggested that the asymmetry arises from the Coriolis effect, and hypothesized that southern-hemisphere Physalia would predominantly go the other way; this was marginally borne out in data collected from Australian beaches. *Munk* (1947) made estimates of the effects of the Coriolis terms in the governing equations, and concluded that (1) the horizontal component of the Coriolis acceleration would produce asymmetry depending on wind direction rather than hemisphere, but (2) linear superposition of an Ekman spiral with a set of otherwise symmetric rolls would produce asymmetry as described by Woodcock.

Stommel (1949) calculated particle trajectories based on idealized roll-vortices, and showed that particles which sink (such as phytoplankton) or rise (such as microbubbles) are trapped within the cores of the vortices. Sinking particles are trapped toward the upwelling region, while rising ones are trapped closer to the downwelling zones.

The following year, *Woodcock* (1950) noted that the amount of sargassum found in net tows near the base of the mixed layer was correlated with the wind speed. He measured the downwelling velocities required to draw sargassum down

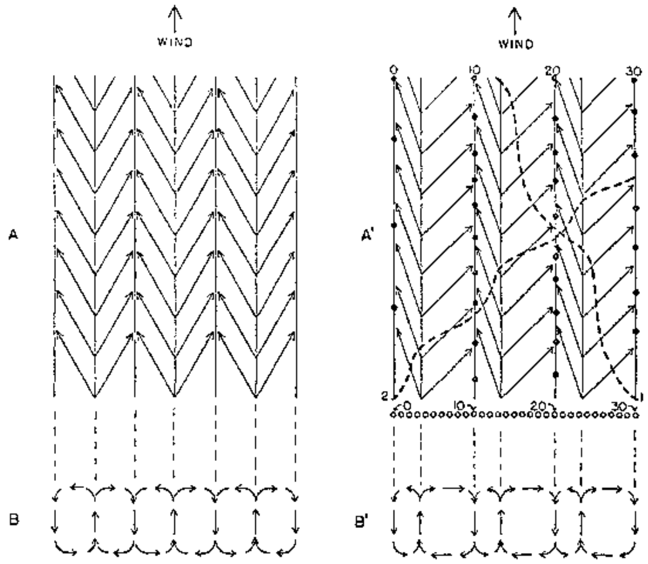

Figure 2. "Idealized drawings of wind-induced helical motions in surface waters, with a illustration of the possible effect of asymmetrical vortices upon the drift of bottles and of Physalia." (from *Woodcock*, 1944).

against buoyancy, and in this way inferred the approximate downwelling velocities as a function of windspeed. A little later, *Sutcliffe et al.* (1963), who were interested in the effects of the surface convergences on surfactants and micro-organisms, developed the "Sutcliffe float," a drag-disk attached to a buoyant pole. These naturally drift to the downwelling zones, where the disk is dragged downward against the buoyancy of the pole. Markings on the pole show the downwelling force applied via drag, and drag calculations are used to estimate the downwelling velocity. This was used in a variety of studies, providing the first data set of downwelling and windspeed large enough to perform statistical regressions (e.g., figure 3).

Through the 1960's, measurements of quantities related to Langmuir circulation were made in many places around the world. Sufficient data were collected to begin doing quantitative statistical analyses, but direct comparisons of results were not possible because the quantities measured varied. On the one hand, few of these data sets were as complete as the original observations of Langmuir (op cit.); on the other hand, they included measurements from the ocean as well as lakes, covered a wide variety of forcing conditions, and included new types of measurements.

In 1969, Faller (as quoted in *Leibovich* 1983) made laboratory measurements establishing the essential requirements for the generation of longitudinal roles of the form described by Langmuir. At the time, the ongoing debate centered on whether these arose as instabilities of the wind-driven shear flow, or from a wave-current interaction; thus he focused on these two effects. He showed that (1)

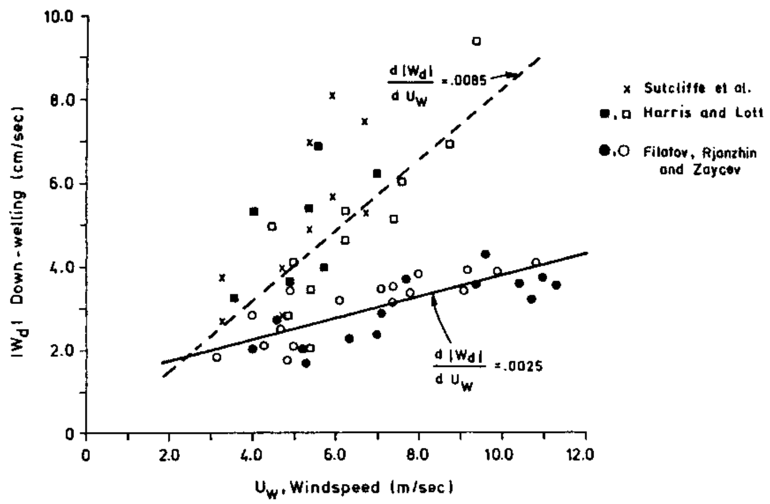

Figure 3. Measured downwelling speeds below streaks as a function of wind speed. The open squares and circles correspond to surface heating, closed symbols to surface cooling. (From *Leibovich*, 1983).

waves with no shear do NOT produce rolls; (2) shear with no waves does NOT produce rolls; and (3) waves and shear together CAN produce rolls. It was further established that the rolls were driven by mechanical, not thermal, instability. Thus, his conclusion was that BOTH shear and waves are required. A couple years later, *Faller* (1971) reviewed the state of the art, showing that it was not encouraging. For example, the strongest correlation with observed streak spacing in oceanic observations was found to be the height of the observer above the water! He also pointed out that a theory (such as one of his own) that depends solely on irrotational waves to produce vortices is necessarily inadequate. Alan Faller's laboratory measurements and review should be regarded as a turning-point in the study of Langmuir circulation: together these inspired a new round of thinking about the underlying mechanism for the generation of such longitudinal rolls.

In the 1970's, new techniques for observation were developed and brought to bear on the problem. For example, *Assaf et al.* (1971) used aerial photography to observe streak patterns, and reconfirmed the existence of multiple scales (as noted originally in *Langmuir* 1938). Three scales were seen in several photos, separated by just under an order of magnitude and ranging from a few to hundreds of meters between streaks.

New mathematical approaches were also developed to explore the possibility of a wave-induced instability. Of note is a critique of a paper by *Craik* (1970), written by *Leibovich and Ulrich* (1972). It is in this critique that the essential ordering of effects necessary to derive "a rational model of Langmuir circulation" (*Craik and Leibovich* 1976) was introduced. The initial analysis ("CL-1") suffered a limitation in that it assumed phase-coherent crossed waves as part of the initial conditions to directly drive the circulations. At the same time, *Garrett* (1976) posed a mechanism based on a "wave force" driving surface water toward the core of any downwave-directed jet. He combined this with hypothesized preferential wave-breaking within the jet to reinforce it, leading to exponential growth of any initial jet-like perturbations. That waves were larger, and might preferentially break within downwind jets, was first suggested by *Myer* (1971), based on observations he made on a lake. It was also suggested by a ray-tracing argument (WKBJ approximation), outlined by *Garrett* (1976): as waves approaching obliquely are refracted by the jet the cross-wind component of their group velocity decreases, so they must become larger to preserve the cross-wind action flux of the waves. As pointed out by Sid Leibovich, however, this analysis is invalid for currents small compared to the waves, and neglects the effects of partial or full wave reflection. Of additional note is that Myer's suggestions were based on observations of only a few waves, providing little statistical confidence.

In the course of discussions among various of these authors, it was soon realized that the model of Craik and Leibovich permitted the growth of Langmuir cells by another instability mechanism, eliminating the need for direct driving by phase-locked crossed waves, and this rigorous approach was promptly applied (*Craik* 1977, *Leibovich* 1977). In this analysis ("CL-2"), the basic state consists of wind-induced shear and surface waves inducing a downwind-directed Stokes' drift that decreases with depth. A perturbation having the form of a downwind directed jet interacts with the waves, inducing a surface convergence along the axis of the jet: the depth-varying Stokes' drift tilts and stretches the vertical vorticity associated with the perturbation jet to produce the longitudinal rolls (no vorticity can be generated by irrotational waves!). The sense of this tilting is to induce a surface convergence along the jet axis. Since the surface water is flowing more rapidly downwind, due to the wind-induced shear, the convergence of surface water toward the axis reinforces the jet, closing a positive feedback loop.

At this point, *Faller and Caponi* (1978) undertook more laboratory studies and, in addition to introducing the terminology "CL-1" and "CL-2" for the first and second editions of the Craik-*Leibovich* analyses, showed that both mechanisms do, in fact, produce rolls in the laboratory.

Parts of the two initial-growth theories can be reconciled. Garrett's "wave force," which drives surface water toward the core of down-wave directed jets, can be compared directly to a vertical integral of the Craik-Leibovich vortex equations describing the bending by the waves' Stokes' drift of the perturbation jet's vertical vorticity (for example, via use of the "Generalized Lagrangian Mean" operators of *Andrews and McIntyre* 1978; see *Leibovich* 1980): the generation of longitudinal vorticity from vertical by the Stokes' drift is analogous to the waves "forcing" a surface layer of water toward the jet maximum. As demonstrated in the CL-2 mechanism, advection of the faster-moving surface water toward convergences is sufficient to close the feedback loop, without the need for direct driving by crossed waves or preferential wave breaking.

A subsequent analysis of waves crossing weak current jets did not corroborate any enhancement of waves within downwind-directed jets, for reasonable directional distributions of surface waves (*Smith* 1983). Indeed, it indicated that waves should be suppressed within such jets by refraction and (especially) reflection effects (*op. cit.*). In any case, being derived from a rigorous expansion based on a sensible and clearly defined ordering of effects, the revised "CL-2" mechanism is preferable to Garrett's, which, in addition to invoking preferential wave breaking, does not resolve the vertical structure of the forcing and is not valid for narrow jets.

At this point, an updated review of the observations and theory was in order. Leibovich provided such an update in his 1983 review (*Leibovich* 1983). The basic description of the mechanism and observations laid out there is still a "must read" for those studying Langmuir circulation. It is safe to say that from the time of its publication to date, the CL-2 mechanism has provided the primary theoretical framework through which results are interpreted– whether from analytical analysis, numerical modeling, laboratory simulations, or field work.

High quality open-ocean time-series of u, v, w near mid-depth in a mixed layer with active Langmuir circulation were at last obtained by *Weller et al.* (1985). Maximum downwelling velocities were found to be larger than had been expected, and this once again sparked increased interest in the circulation.

On the theory side, the effects of horizontal Coriolis on LC development were addressed (*Leibovich and Lele* 1985). While the asymmetry issue raised by *Woodcock* (1944) was not revisited, the time-constant of development was found to depend on wind direction, as suggested originally by *Munk* (1947).

In the 1980's, oceanographers began using upward-looking side-scan sonars to look at the bubble "streaks" produced by Langmuir circulation (*Thorpe and Hall* 1983). It was soon realized that velocities could be estimated from the Doppler shift of the backscattered sound signal, producing quantitative estimates of the near-surface velocity field associated with wind-driven convection (*Smith et al.* 1987). Surface-scattering acoustic intensity and Doppler data both show evidence of spacing proportional to 2 to 3 times the mixed layer depth, particularly as the mixed layer deepens (*Smith* 1992). It is also observed that the convergences, as delineated by lines of bubble clouds, can merge into "Y-junctions (*Thorpe* 1992b, *Thorpe* 1992a). The narrow end of the Y's overwhelmingly point downwind.

In the late '80's and 90's, work was also carried out relating the vertical and horizontal scales of Langmuir circulation, as revealed in up-looking and inverted side-scan sonar images, primarily by D. Farmer and colleagues (IOS, Canada). The reader is strongly encouraged to refer to the companion paper by D. Farmer in this volume for more details.

The first attempt to parameterize LC for use in large-scale modeling of mixed layer behavior was undertaken by *Li et al.* (1995). Although they assumed only fully developed seas, they were able to demonstrate an improved fit to the several-decade-long timeseries from ocean station PAPA.

In the 1990's, more numerical studies of Langmuir circulation were conducted, with increasing emphasis on Large-Eddy Simulations. These simulations have begun to explore aspects of the nonlinear dynamics (chaos and quasi-periodic behavior), and of finite-amplitude stability, etc. For example, *Skyllingstad and Denbo* (1995) explore the difference in response with and without the "CL vortex force" term, showing that the CL force does indeed make a difference (the first such on/off comparison).

New laboratory work on the initial growth of unbounded Langmuir circulation

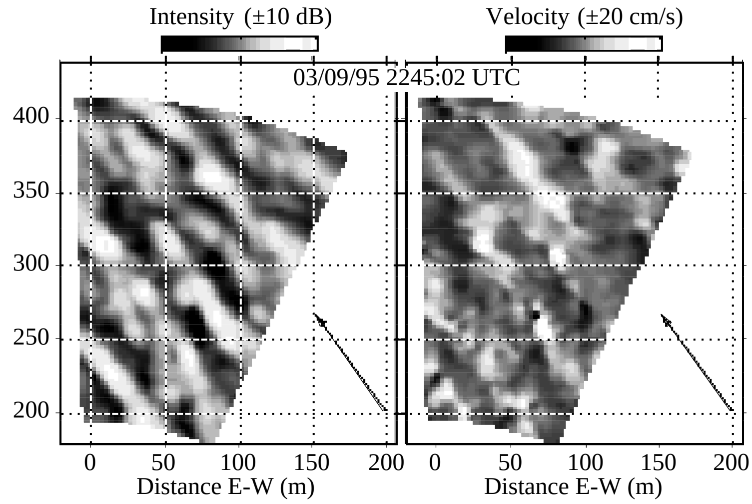

Figure 4. Example of "Phased Array Doppler Sonar" (PADS) data, gathered in the spring of 1995 during MBLEX-1. (Left panel) Acoustic backscatter intensity; (right) Radial velocity. The arrows indicate wind speed and direction; the arrow shown represents a 15 m/s wind. North is up. The data are smoothed to 3 minute averages, modified to "track" the mean flow across the area.

has been undertaken by K. Melville and students (S.I.O., USA), and is described in a paper by Melville and Veron in this volume. Of note is their finding that the time-scale to explosive growth of the waves and the Langmuir-circulation-like turbulence is comparable.

Recently, rms near-surface velocity estimates from Doppler sonar data were shown to be fit better by a term including the waves (Stokes' drift) than by wind alone (*Plueddemann et al.* 1996, *Smith* 1996), using data from the "Surface Wave Processes Program" (SWAPP, 1990). This is a subtle distinction: the wind and waves are themselves strongly correlated, with about 95% of the variance in near-surface Stokes' drift accounted for by direct correlation with windspeed. Based on heuristic arguments, and on the ordering of effects employed in CL-2, the surface velocities associated with Langmuir circulation were expected to scale as either $(u^* U^S)^{1/2}$ or $(u^{*2} U^S)^{1/3}$. To address this issue, *Smith* (1998) attempted to isolate the "non-wind" portion of the dependence, using data from the first leg of the "Marine Boundary Layer Experiment" (MBLEX-1, 1995; see figure 4). He formed a log-log plot of (V^{RMS}/u^*) versus (U^S/u^*), with the expectation that the slope would have a value of either 1/2 or 1/3 for those cases involving Langmuir circulation. However, the surprising result was that, for LC cases alone, the slope is quite tightly estimated as 1.00 +/- 0.034 (figure 5). Subsequently, further analysis of the SWAPP data was found to supports this conclusion (*Smith* 1999a, *Smith* 1999b); however, it also brought to light a significant variation in response: the rms LC velocity can differ by as much as a factor of 4 for similar values of U^S.

It appears that the journey ahead will be long: from estimates and descriptions of the lowest order moments (mean velocities, layer thickness and temperature,

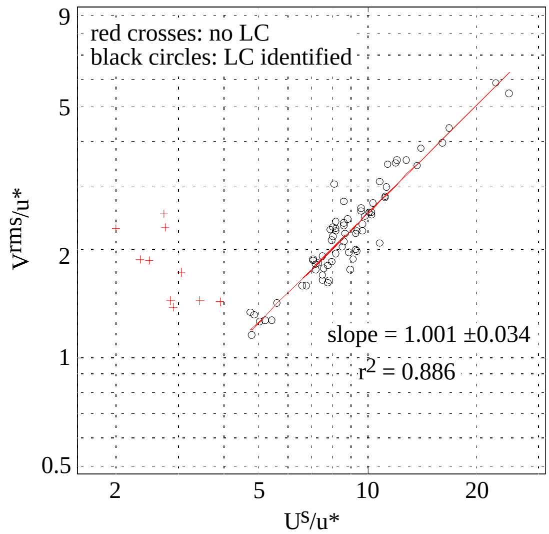

Figure 5. Scaling of the rms measured radial surface velocity takes the general form $V \sim u*(U^S/u*)^n$. The value of n is sought as the slope of $(V/u*)$ versus $(U^S/u*)$ on a log-log plot. Surprisingly, the value n=1 is indicated, with little uncertainty (r^2=0.89; error bounds on the slope are a standard deviation derived by the bootstrap method with 5000 trials, cf. *Diaconis and Efron* (1983); note also that $U^S/u*$ varies over almost an order of magnitude). Thus, $V \sim U^S$, with no dependence on $u*$, once Langmuir circulation is established. Values before year day 67.66, when there were no signs of Langmuir circulation, are shown as red crosses but not included in the fit. (Data from MBLEX-1, using PADS.)

etc.) to the high order statistics needed to describe typical life histories (particle trajectories) and chemical exchange rates. From this perspective, we are just taking the first step: from means to variances. Until we can get the means and variances right, there can be little confidence in estimates of higher-order moments. With this in mind, the balance of this paper discusses some parameterizations and scalings embodying our knowledge to date. Particular attention is paid to the most recent results, on scaling of the rms surface velocities associated with Langmuir circulation. Although these results appear elsewhere (*Smith* 1998, *Smith* 1999a, *Smith* 1999b), it seems appropriate to review them in some detail here, emphasizing what is known and what is not. First this recent data is described, then some "bulk scalings" for Langmuir circulation are summarized, and finally the scaling of the observed surface velocities will be discussed.

3 Recent Field Experiments

Data from two recent field experiments are considered: the "Surface Waves Processes Program" (SWAPP), which took place some 300 km West of Pt. Conception, CA, in March of 1990, and leg 1 of the "Marine Boundary Layer Experiment (MBLEX), which took place some 50 to 100 km WNW of Pt.

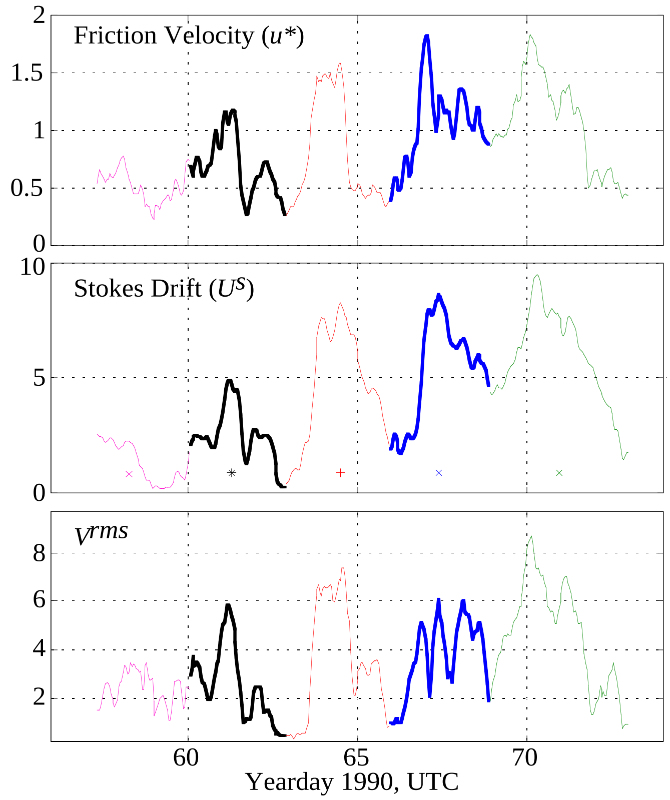

Figure 6. Wind, waves, and rms surface velocity during SWAPP. The first segment has weak variable winds; the last segment contains several directional shifts; the second-to-last segment has little variation in the ratio $U^s/u*$. Thus, only the second and third segments are suitable for testing the relationship between the three parameters.

Conception. In both, surface velocities were estimated from surface-grazing acoustic Doppler sonar systems. In SWAPP, 4 discrete “inverted side-scan” beams were used to trace the time-space evolution of features along 4 directions, distributed at 45° increments. In MBLEX, a newer system (PADS) was used to image a continuous area 35° in bearing by 450 m in range (figure 4). Details concerning the former are found in *Smith* (1992) and concerning the latter in *Smith* (1998). To estimate time series of the velocity associated with Langmuir circulation, data averaged over 1 to 3 minutes were employed. The MBLEX-1 (PADS) data were averaged with a moving window that tracks the mean advection across the imaged area as the average is formed (see *Smith* (1998) for details). The SWAPP data were processed with a dual spatial-temporal lag technique to isolate coherent signals while also tracking advection along the beam (see *Plueddemann, et al.* (1996) or *Smith* (1996) for details). The data were corrected for the spatial response of the instruments, estimated from simulated data.

Wind and Stokes' drift are primary input parameters for models of Langmuir circulation. In both experiments, wind stress was estimated from sonic anemometer data via eddy-correlation methods. Stokes' drift was derived using data from resistance-wire wave arrays, yielding surface elevations and tilts as functions of frequencies up to 0.5 Hz (cf. *Longuet-Higgins et al.* 1963). The results are converted to Stokes' drift via linear theory and integrated over the directional-frequency spectrum to estimate the net drift at the surface. MBLEX-1 provided only one clear storm event. In SWAPP, 5 time segments are identified, but only the second and third segments have both steady wind directions and a wide range of wave age (segments are delineated in figure 6 by different shades of gray; also denoted by the symbols * and + below the peaks in Stokes' drift).

Stratification and the shear across the pycnocline are also primary input parameters to simple mixed layer models. Stratification was monitored with rapid-profiling "Conductivity–Temperature–Depth" (CTD) systems, providing temperature and salinity profiles to 400 m depth every couple minutes. Vertical profiles of horizontal velocity were monitored with additional Doppler sonar systems in a standard Janus configuration. To estimate the bulk shear, the surface velocities estimated from the surface sonar systems were used together with deeper velocity estimates averaged over a sub-thermocline depth interval from the standard Janus-configuration data.

4 Parameterizations of Langmuir Circulation for Mixed-Layer Modeling

In the open ocean, the largest wind-mixing effect is due to the shear across the thermocline. This can be parameterized by a bulk Richardson number,

$$Ri \equiv \frac{\Delta\rho g h}{\rho(\Delta U)^2} \geq 0.64 \text{, or } \Delta\rho \geq 0.64(\Delta U)^2(\rho/gh) \tag{1}$$

(*Pollard et al.* 1973, *Price et al.* 1986), where deepening of the mixing layer continues until the inequality is satisfied. The velocity jump across the thermocline, ΔU, is primarily due to inertial currents, generated by sudden changes in the wind; it generally decreases rapidly after a quarter inertial cycle. For example, the time history of the estimated strength of this term in MBLEX-1 is indicated in figure 7 (thickest line) in terms of the density jump $\Delta\rho$ needed across the thermocline to halt mixing (for the measured ΔU and mixed layer depth h). This term grows rapidly as the wind increases, and then decays almost to zero over the next day. Since the wind rose gradually over the first day, the inertial currents were weak compared to those generated by a sudden "turn-on" of the wind.

After fast deepening by this "PRT mechanism," surface-forced stirring can maintain the mixed layer against restratification and can drive continued slower deepening (*Niiler and Krauss* 1977, *Li, et al.* 1995). The parameterization of *Li et al.* incorporates scaling arguments appropriate to Langmuir circulation (i.e., a

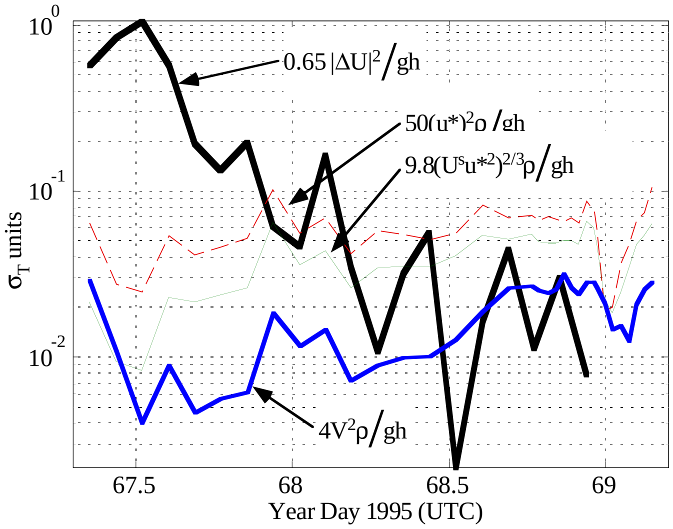

Figure 7. Mixing strength, parameterized by the density jump required to stop mixing, for (1) the bulk Richardson (or PRT) mechanism (thick line); (2) Langmuir circulation, as estimated directly from the rms velocity scale V (medium line); and (3) LC mixing estimated from U^S and v_t via comparison with numerical model results, for developing waves (thin solid line) and for fully developed waves (thin dashed line).

combination of wind and wave velocity scales), although they reduce the result to a simple $u*$ dependence by assuming fully developed seas. This latter parameterization is the one discussed further here. An attempt to extend this to underdeveloped waves was undertaken by *Smith* (1998), and is repeated here, since some of the elements of the argument are useful in the subsequent discussion.

The scaling suggested by *Li et al.* begins with the argument (derived from examining 2D numerical model output) that penetration into the thermocline is stopped if

$$\Delta\rho \geq 1.23\, w_{dn}^2 (\rho/gh)\,, \tag{2}$$

where w_{dn} is the maximum downwelling velocity associated with the Langmuir circulation. This is close to a statement that an entrained parcel must acquire enough turbulent kinetic energy to overcome the increase in potential energy corresponding to its being mixed over the depth of the mixed layer; thus, the factor in front is not likely to be very sensitive to assumptions (such as 2D versus 3D turbulence). Using the same scaling arguments employed in derivation of their model equations, they rewrite this in the form

$$\Delta\rho \geq C\, u*^2\, (\rho/gh) \tag{3}$$

where

$$C \equiv 0.36\, U^s / k v_t\,, \tag{4}$$

in which ν_t is a turbulent kinematic eddy viscosity, k is the wavenumber of the dominant surface waves, and U^S is the Stokes' drift at the surface due to the waves. To fit the long timeseries at ocean station PAPA (which does not include explicit wave data), they set C to about 50 (figure 7, thin dashed line). They argue that this corresponds (more or less) to fully developed seas. Here this entrainment criterion is extended by two methods: (a) evaluating C for underdeveloped waves, and (b) using the measured rms horizontal scale to estimate w_{dn} directly for use in (2), both following *Smith* (1998).

For the underdeveloped case, a significant requirement is estimation of the eddy-viscosity ν_t. Recent dissipation measurements near the surface indicate that the turbulent velocity scale q is described by the energy dissipation rate of the waves (*Terray et al.* 1996) which, in turn, roughly equals the energy input to the waves. The growth rate β of a wave of radian frequency σ and phase speed c is approximately $\beta = 33\sigma(u*/c)^2$ (*Plant* 1982), so the net energy flux through the waves can be written in the form

$$q^3 \propto \rho^{-1}\beta E = 33ga^2\sigma(u*/c)^2 = 33(U^s u*^2)\,, \tag{5}$$

where E is wave energy. A length scale appropriate to wave breaking is the wave amplitude a, yielding an estimate for ν_t of the form $\nu_t \propto a(U^s u*^2)^{1/3}$. Substituting this into (4), and noting that ak in general does not vary significantly from 0.1, we obtain

$$C \propto U^s \big/ k\nu_t \propto (U^s \big/ u*)^{2/3} \tag{6}$$

The values employed by *Li, et al.* (1995) imply that $U^S/u* \to$ 11.5 for fully developed waves. To match the value C = 50 for fully developed waves, as implied by the fit to the many years of ocean station PAPA, the constant of proportionality is set to 9.8. Then (3) becomes

$$\Delta\rho \geq 9.8(U^s\, u*^2)^{2/3}(\rho \big/ gh)\,. \tag{7}$$

This criterion is also shown in figure 7 (thin solid line).

To use the measured horizontal V^{RMS} directly, we need only convert this to an estimate of maximum downwelling. Since the spacing is generally about twice the mixed layer depth, the rolls appear to be roughly isotropic in the crosswind plane, and it's reasonable to set the vertical and crosswind velocity scales equal. Then the rms values must be translated into estimates of maxima. The circulation is not simply sinusoidal (in which case $w_{max} \sim 2^{1/2}V$) but varies somewhat randomly. By analogy to estimating significant wave height ($H_{1/3}$) from rms displacement, we are led to the value $2V$. Hence we substitute $4V^2$ for w_{dn}^2 in (2), providing a fairly direct estimate of the strength of mixing due to the observed Langmuir cells (figure 7, lowest line).

The mixing effect estimated from the surface velocities measured in MBLEX-1 falls below the parametric estimates (3) and (7). However, note that the estimates for the fully-developed case, after *Li, et al.* (1995), is based on a fit to many years' worth of data at ocean station "papa." Further, as indicated below, the SWAPP data fall near these higher values as well. Thus, the discrepancy between these parametric values (and the measurements from SWAPP) versus the velocities

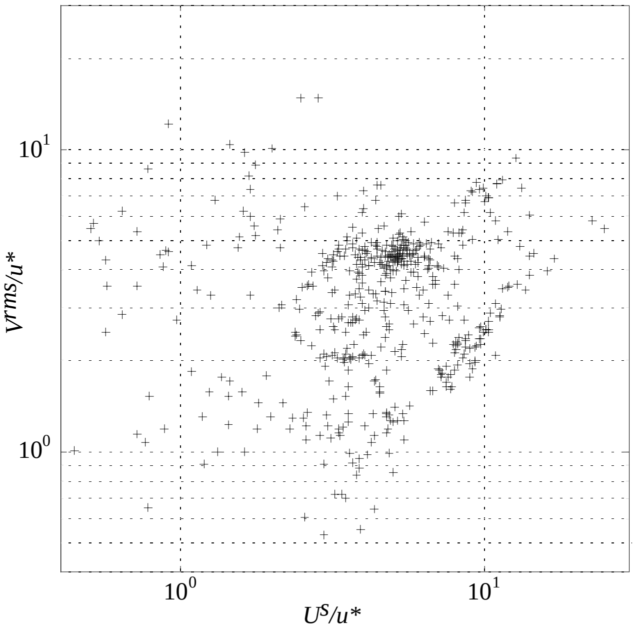

Figure 8. All data points from SWAPP and MBLEX-1, plotted without regard to event or whether LC features were identified. No correlation is seen, and it appears that almost an order of magnitude (residual) uncertainty in V^{rms} must be tolerated.

measured in MBLEX-1 should be taken seriously. Over a period of several days, these differences could lead to significant differences in the mixed layer depth, or, with surface heating, could make the difference between restratification versus remaining well mixed. Clearly, it is important to understand how and why this variance changes from case to case.

5
Scaling of Surface Motion

Having discussed some preliminaries concerning the modeling and parameterization of wind-driven mixing of the upper ocean, we now reexamine the surface velocity observations and the various scaling hypotheses.

In the absence of wave forcing, a relevant velocity scale is provided by the wind W (or friction velocity $u*$; for present purposes, these are taken as proportional). Based on theories for initial growth of Langmuir circulation, it's been suggested that the cross-wind velocity fluctuations should scale with either the geometric mean of the wind and Stokes' drift, $(WU^S)^{1/2}$ (*Plueddemann, et al.* 1996) or with $(W^2U^S)^{1/3}$ (*Smith* 1996). However, data from MBLEX-1 (reviewed in Figure 5) suggest that once Langmuir circulation has developed, $V \sim U^S$, and wind (or windstress) no longer enters directly in scaling the motion. This implies a strong, non-linear influence of the waves on the flow (since a threshold must be applied for the existence of well-developed Langmuir circulation).

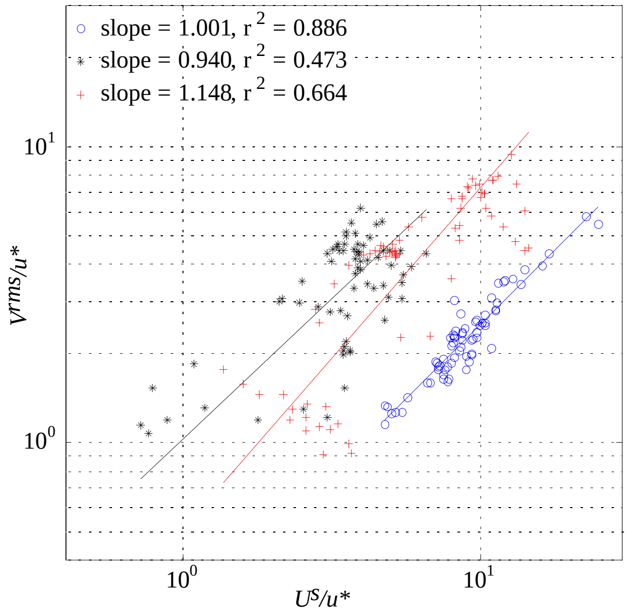

Figure 9. The empirical fits for the exponent n, for each "clean" wind event treated separately. Symbols as follows: MBLEX-1 (o), SWAPP-2 (∗), SWAPP-3 (+). Note that, within each event, the fit is fairly tight. However, the vertical offset of the lines varies significantly over the three events.

A natural question is whether this scaling applies to other data, or is an isolated case. To address this issue, the SWAPP data were similarly analyzed. Figure 8 shows a log-log plot, analogous to figure 5, for all SWAPP and MBLEX data points, without regard for the existence of stripes or non-stationary conditions. It would be easy to dismiss any correlation from this plot! However, it should be recognized that (1) the parameter U^S might be a proxy for another wave parameter, and the relation between these might vary between wind events, and (2) there could be other parameters influencing the relation. It is wise to examine the relation on a case-by-case basis, to see if there is a hidden relation. Unfortunately, as noted above, only events 2 and 3 from SWAPP provide both a "clean" wind event (not confused by wind veering) and a wide range of "wave age," $U^S/u*$. These two events (henceforth "SWAPP-2" and "SWAPP-3") are plotted together with the "MBLEX-1" event in figure 9. The regressions support the value n=1 for the exponent. Although the fits are not as tight for the two SWAPP cases as for the MBLEX-1 data, they are still statistically significant at well over 95%. Intriguingly, there is considerable offset between the lines, especially between SWAPP and MBLEX (by a factor of about 4), but also between the two SWAPP events (by a factor of about 1.5). Some other aspect of the environment must be affecting the relation.

As a brief aside, note again the data shown in figure 3: observed downwelling velocities from the Sutcliffe floats versus wind. As mentioned above, the wind and waves are strongly correlated, so at lowest order they are interchangeable for scaling purposes. The notable aspect of figure 3 is that, for the two separate data sets (different kinds of wind events?), the slopes differ by a factor of about 3.4 (from 0.0025 to 0.0085). This spread is similar to that seen here between the MBLEX and SWAPP events. Perhaps this was an early indication that some additional factor is important in the scaling relation. In any case, it seems clear now that there is an important additional factor involved in the scaling relation between the surface velocity variance and the wind and/or wave velocity scales.

What could be responsible for this further, apparently independent variation in the observed velocity scales V^{rms}? Possible candidates include variations of scaled depth of the mixed layer *kh*, effective viscosity ν_t, a directional effect of the horizontal Coriolis component (*Cox and Leibovich* 1997), or suppression by the buoyancy of the near-surface bubble cloud. A summary of some relevant parameters is given in Table 1. The first 6 parameters summarize the observations for the 3 events; the rest are derived from these. Here, an "effective wave period" T^S is derived from the surface Stokes' drift, assuming the variations in mean-square wave steepness are not very large:

$$U^S \approx \tfrac{1}{2}(ak)^2 c^p = T\,(ak)^2 g/4\pi \approx (const.)T\ . \qquad (8)$$

For the SWAPP-3 event, the peak wave period was estimated in a variety of ways (*Bullard and Smith* 1996), leading to a favored value of about 11.5 s near the end of the event; thus, we set the value of $(ak)^2$ by matching to that value in that event. The corresponding value for $(ak)^2$ is 0.0084, well within reason. The effective wavenumber is then computed from T^S via linear dispersion: $k^s = (2\pi/T^s)^2/g$. Given the uncertainty in defining a "peak period," and given the purported importance of wave Stokes' drift to the generation of Langmuir circulation, this proxy for the wave period and length scales appears to be most appropriate.

Table 1

Additional parameters that may influence the surface velocity scale.

Event Parameter	MBLEX-1	SWAPP-2	SWAPP-3
V^{rms}/U^S	0.24	0.95	0.67
W^{dir}	SE	SSE	NNW
W^{max}	15 m/s	10	13.6
$u*$	1.65 cm/s	1.1	1.5
U^S	7.0 cm/s	4.5	7.5
h	25 m	25	45
T^S	10.7 s	6.9	11.5
k^S	0.035 m^{-1}	0.085	0.031
$k^S h$	0.87	2.12	1.37
F'	0.53	0.77	0.66
$(u*^2 U^S)^{1/3}$	0.124 cm/s	0.082	0.119
ν_t	698 cm^2/s	190	770

As reported in Table 1, the wind directions in MBLEX-1 and SWAPP-2 are similar, but the V-scale differs by the largest amount (a factor of 3) between these two events. This appears to rule out the horizontal Coriolis effect as the cause of the variation in scaling. Since the swell direction and strength was also similar in these two cases, the difference between them is not explained as an effect of opposing swell. The estimated wave-induced viscosity is largest for SWAPP-3, but this event is intermediate in terms of V-scaling, so this too fails to match the observed pattern. Another possibility is suppression of motion by bubble buoyancy. The level of breaking presumably sets the overall density of the near surface bubble-cloud. A likely indicator of this is obtained by matching the rise-rate of the largest bubbles to the wave breaking turbulent velocity scale (as discussed above in connection with the turbulent eddy viscosity). Comparing this with the observed velocity scaling, it is seen that this parameter has the right ordering in magnitude, although the viscosity estimates for MBLEX-1 and SWAPP-3 values are much closer than the V-scales. Finally, there is the "scale depth" *kh* of the mixed layer. It is important to distinguish here between the development of ΔV, V^{rms}, h, U^S, and k over the course of an event versus the differences between events. These all develop in parallel over the course of a wind event; however, what is of interest at the moment is whether they develop either at different rates or from different initial values between events. It is these latter differences between events that might set the ratio of V^{rms} to U^S over the event. Thus it is emphasized that the "scaled depth" employed here refers to final or quasi-equilibrium values of h and k^S.

One way the scaled depth could influence the result is that the layer-averaged convergent force should be subtracted from the surface value, since this works to depress the thermocline rather than drive circulation (the analogy in the vortex-forcing equations is that a portion of the vortex forcing works against the induced "twisting" of the thermocline, which sets up an opposing "buoyancy torque"). For exponential decay of the wave-shear forcing term, with a depth-scale matching that of the Stokes' drift, this leads to a reduction by the (approximate) factor

$$F' = F_0 - \overline{F} \approx 1 - \tfrac{1}{h} \int_{-h}^{0} e^{-2kz} dz = 1 - (2kh)^{-1}(1 - e^{-2kh}) . \tag{9}$$

This varies smoothly from 0 at kh=0 to 1 as kh gets large: the wind-wave forcing mechanism is reduced for thin layers, and reaches full strength as the mixed layer becomes deeper than the wave's scale-depth. As seen in the table, this effect is in the right direction, but again appears too weak to explain the full range of velocity scaling relations observed between the three events. It appears that further investigations are needed to select between alternatives, and to determine why and when suppression or enhancement of the motion occurs.

6
Conclusions

In an overwhelming majority of cases observed the mixed layer deepens rapidly after the onset of wind, slowing significantly within half an inertial day or so. This is consistent with current thinking, that the “bulk dynamics” of shear across the thermocline due to inertial motion is the primary agent for deepening. Surface stirring by the combined action of wind and waves maintains the mixed layer after this, with slower additional deepening. The inertial-current “bulk Richardson number” mechanism is the lowest order term in wind-induced deepening of the surface layer on oceans and large lakes.

The rms velocities associated with Langmuir circulation appear to scale tightly with the Stokes’ drift over the course of individual wind events, once streaks are observed, fitting more tightly to this than to the wind or a combination of wind and waves. This relation is nonlinear in that a threshold must be set for the existence of Langmuir circulation before it holds.

These two essential observations suggest that the velocity scale in the mixed layer approaches a strongly wave-influenced value near the surface, but must also involve the velocity jump across the thermocline as one moves deeper.

Of note is that the “constant of proportionality” between surface velocity variance and Stokes’ drift varies significantly (by a factor of up to 4) between events. A first indication of this may have been provided by the downwelling velocity regressions using “Sutcliffe float” data (see figure 3), however these early observations were not considered reliable enough to spur serious thought. It is suggested that the additional variation may be related to the ratio of surface wave length to mixed layer depth, as parameterized by the “final” or maximal values. Dynamic effects of the near-surface bubble layer or wave-induced viscosity could also play a role.

Our hopes are still aptly described by *Montgomery* (1947):

> “With regard to future work on convective layers, guidance may be obtained from the methods used in the study of mechanical turbulence in boundary layers. By the clever choice of variables and parameters, widely varying problems have been brought under simple empirical laws. It appears probable that properly chosen measurements in convective layers in the ocean and atmosphere can be related to controlled laboratory experiments by means of quantities chosen in a similarly suitable manner, so that greater order will appear out of our scattered knowledge. A step in this direction was made years ago by (*Prandtl* 1932), who suggested a law for the mean temperature distribution in a convective layer.”

Now, 50 years later, we maintain this hope, but still seek that “clever choice of variables and parameters.” It is hoped that with a combination of new observational techniques and increasingly complex and realistic numerical modeling, the choice of variables can be both deduced and verified.

Acknowledgments. Thanks are due to the many people involved in both SWAPP and MBLEX. Regarding the gathering of this data, thanks go especially to Eric Slater, Lloyd Green, Mike Golden, Chris Neely, R. Pinkel, K. Rieder, M. Alford, and to the crew of the research platform *Flip,* captained by E. DeWitt (SWAPP) and T. Golfinas (MBLEX). Thanks also for useful comments from R. Pinkel, C. Garrett, and G. Terray. This work was supported by the Office of Naval Research, grants N00014-93-1-0359 and N00014-96-1-0030.

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