On the size of the Antarctic Circumpolar Current

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Abstract—A model predicted transport of the Antarctic Circumpolar Current (ACC) through Drake Passage compares favorably with measured transport through the passage. The model incorporates the width of the ACC, the strong eddy presence in the region, and the deep penetration of an unstable baroclinic velocity field, all of which are characteristic features of the current. The model involves a downward transfer of wind-imparted zonal momentum by eddy form drag to a depth at which it can be removed by bottom pressure drag. The eddy form drag results from a poleward transport of heat by eddies originating from the baroclinically unstable current.
The problem: what sets the size of the ACC?


Munk and Palmen, 1951: Such models require ‘uncomfortably large’ values of eddy friction coefficients to limit the ACC transport to reasonable values.

Gill 1968: Linear model of the ACC requires a bottom friction coefficient of $10^3 \text{ cm}^2/\text{s}$ and a lateral friction coefficient of $10^8 \text{ cm}^2/\text{s}$.

Fig. 1. 0-1000 db dynamic height field of the Southern Ocean in dynamic meters with 4000 m isobath and station locations. Figure 2 from Gordon et al. (1978).
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Gill 1968: Linear model of the ACC requires a bottom friction coefficient of $10^3$ cm$^2$/s
a lateral friction coefficient of $10^8$ cm$^2$/s.

Even worse: how big *is* the ACC?

Reid and Nowlin, 1971: Eastward transport of 237 Sv through the Drake Passage

Foster, 1972: **Westward** transport of 15 Sv through the Drake Passage (backwards!)

International Southern Ocean Studies (ISOS) program measures Drake Passage transport, structure, dynamics.

Nowlin and Klinck, 1986: 134 Sv +- 10% (eastward), with time variability of 20%. 
Nowlin and Klinck, 1986: 134 Sv +/- 10% (eastward), with time variability of 20%.

ACC characteristics:

- NOT a narrow jet or boundary current with Rossby radius
- Three narrow frontal regions of width two or three times the Rossby radius
- The current penetrates down to the bottom of the water column, with substantial meridional density gradients at depth.

Fig. 2. Section of variability referenced density parameter $\sigma$ across Drake Passage calculated from Melville Section II FDRAKE 75 data. Figure 4 from Nowlin et al. (1977).
McWilliams et al., 1978: A 2-layer, eddy-resolving QG model of the ACC with various configurations of topography and wind forcing.

In all model runs, eddies played a key role in transporting zonal momentum downward.

In model runs without topography, friction was only strong enough to hold model transport values around 400-600 Sv.

In model runs with topography, topographic form drag was strong enough to limit the model transport to 100 Sv.

Bottom and sidewall friction is not enough! Thus:

The goal of this paper is to present a simple model of the dynamics of the ACC that predicts a transport independent of friction coefficients.
Steady state zonal momentum balance:

\[
\begin{aligned}
&u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + f v + \frac{1}{\rho_0} \frac{\partial \tau^x}{\partial z} \\
\text{Noting that:} & \quad u \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (uu) - u \frac{\partial u}{\partial x} \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \\
&v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (uv) - u \frac{\partial v}{\partial x} \\
&w \frac{\partial u}{\partial z} = \frac{\partial}{\partial z} (uw) - u \frac{\partial w}{\partial z}
\end{aligned}
\]  

We can write eqn (1) as

\[
\frac{\partial}{\partial x} (uu) + \frac{\partial}{\partial y} (uv) + \frac{\partial}{\partial z} (uw) = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + f v + \frac{1}{\rho_0} \frac{\partial \tau^x}{\partial z}
\]
Taking the vertical integral:

\[
\int_{z=-H}^{0} \left[ \frac{\partial}{\partial x} (uu) + \frac{\partial}{\partial y} (uv) + \frac{\partial}{\partial z} (uw) \right] \, dz = \int_{z=-H}^{0} \left[ f_v - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tau^x}{\partial z} \right] \, dz
\]

\[
\int_{z=-H}^{0} \left[ \frac{\partial}{\partial x} (uu) + \frac{\partial}{\partial y} (uv) + \frac{\partial}{\partial z} (uw) \right] \, dz =
\]

\[
\frac{\partial}{\partial x} \int_{z=-H}^{0} uu \, dz - (uu) \bigg|_{z=-H} \frac{\partial H}{\partial x}
\]

\[
+ \frac{\partial}{\partial y} \int_{z=-H}^{0} uv \, dz - (uv) \bigg|_{z=-H} \frac{\partial H}{\partial y}
\]

\[
+ (uw) \bigg|_{z=-H}
\]

\[
\int dx \left[ \frac{\partial}{\partial x} \int_{z=-H}^{0} \, dz (uu) + \frac{\partial}{\partial y} \int_{z=-H}^{0} \, dz (uv) \right] = \int_{0}^{H} \, dz \left( f_v - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tau^x}{\partial z} \right)
\]

(1)
Taking the vertical integral:

\[
\int_{z=-H}^{0} \left[ \frac{\partial}{\partial x} (uu) + \frac{\partial}{\partial y} (uv) + \frac{\partial}{\partial z} (uw) \right] \, dz = \int_{z=-H}^{0} \left[ f v - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tau_x}{\partial z} \right] \, dz
\]

\[
\int_{z=-H}^{0} \left[ \frac{\partial}{\partial x} (uu) + \frac{\partial}{\partial y} (uv) + \frac{\partial}{\partial z} (uw) \right] \, dz =
\]

\[
\frac{\partial}{\partial x} \int_{z=-H}^{0} uudz - (uu) \bigg|_{z=-H}^{H} + \frac{\partial H}{\partial x}
\]

\[
+ \frac{\partial}{\partial y} \int_{z=-H}^{0} uvudz - (uv) \bigg|_{z=-H}^{H} + \frac{\partial H}{\partial y}
\]

\[
+ (uu) \bigg|_{z=-H}^{H}
\]

\[
\oint \, dx \left[ \frac{\partial}{\partial x} \int_{z=-H}^{0} dz (uu) + \frac{\partial}{\partial y} \int_{z=-H}^{0} dz (uv) \right] = \int_{0}^{H} dz \left( f v - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tau_x}{\partial z} \right)
\]
In the circumpolar region of the Southern Ocean, integrated zonal momentum balance becomes:

\[
\int dx \left[ \frac{\partial}{\partial y} \int_{z=-H}^{0} dz (uv) + \int_{z=-H}^{0} dz \frac{1}{\rho_0} \frac{\partial p}{\partial x} \right] = \frac{1}{\rho_0} \int dx \tau^x \quad \text{surface to bottom}
\]

Momentum source: Frictional wind stress at the surface of the fluid
In the circumpolar region of the Southern Ocean, integrated zonal momentum balance becomes:

\[
\int dx \left[ \frac{\partial}{\partial y} \int_{z=-H}^{0} dz (uv) + \int_{z=-H}^{0} dz \frac{1}{\rho_0} \frac{\partial p}{\partial x} \right] = \frac{1}{\rho_0} \int dx \tau^x_{\text{surface}}
\]

Momentum source: Frictional wind stress at the surface of the fluid

Potential momentum sinks:
1) Frictional stress at the bottom;
In the circumpolar region of the Southern Ocean, integrated zonal momentum balance becomes:

\[
\int dx \left[ \frac{\partial}{\partial y} \int_{z=-H}^{0} dz (uv) + \int_{z=-H}^{0} dz \frac{1}{\rho_0} \frac{\partial p}{\partial x} \right] = \frac{1}{\rho_0} \int dx \tau^x \tag{2}
\]

Momentum source: Frictional wind stress at the surface of the fluid

Potential momentum sinks:
1) Frictional stress at the bottom;
2) Frictional stress at the sidewalls, via meridional momentum flux divergence;
In the circumpolar region of the Southern Ocean, integrated zonal momentum balance becomes:

\[
\int dx \left[ \frac{\partial}{\partial y} \int_{z=-H}^{0} dz \left( uv \right) + \int_{z=-H}^{0} dz \left( \frac{1}{\rho_0} \frac{\partial p}{\partial x} \right) \right] = \frac{1}{\rho_0} \int dx \tau^x \text{surface bottom}
\]

Momentum source: Frictional wind stress at the surface of the fluid

Potential momentum sinks:
1) Frictional stress at the bottom;
2) Frictional stress at the sidewalls, via meridional momentum flux divergence; or
3) Topographic form stress.
Topographic form stress

\[ \Delta p = \frac{1}{\Delta z} \int \tau^x \, dx \]

\[ v = \frac{1}{\rho_0} \frac{\Delta p}{\Delta x} \]

Fig. 3. Schematic presentation of bottom form drag or mountain drag. Wind stress imparted eastward momentum in the water column is removed by the pressure difference across the ridge. Note that geostrophic balance \( \rho_0 f v = \partial p / \partial x \) demands an equatorward flow (symbolized by \( \Theta \)) along the ridge, evidence of which may be seen in Fig. 1.
How does momentum travel from surface source to seafloor sink?

Interfacial form stress.

![Diagram showing interfacial form drag](image)

**Fig. 4.** Schematic presentation of interfacial form drag. Correlations of perturbations in the interface height, $\zeta'$, and the meridional velocity, $V'$ ($\odot$ indicating poleward flow and $\oslash$ indicating equatorward flow), which are related to pressure perturbations by geostrophy, allow the upper layer to exert an eastward force on the lower layer and the lower layer to exert a westward force on the upper layer; thus effecting a downward flux of zonal momentum.
How does momentum travel from surface source to seafloor sink?

Interfacial form stress.

\[ \tau^x \]

Wind Stress

\[ \eta \]

\[ v' \times \]

Antarctic Circumpolar Current

\[ v' \times > 0 \]

High Pressure

\[ v' \times > 0 \]

Low Pressure

\[ \zeta' = - \frac{T'}{\theta_z} \]

continuous stratification

and

geostrophy

\[ 1 \frac{\partial p'}{\partial x} = f v' \]

\[ \phi p' \frac{\partial \zeta'}{\partial x} \, \text{d}x = - \phi \zeta' \frac{\partial p'}{\partial x} \, \text{d}x = \phi f \frac{v' T'}{\theta_z} \, \text{d}x \]
Interfacial form stress:

Vertical flux of eastward momentum is proportional to eddy heat flux.

Poleward heat flux corresponds to the downward flux of eastward momentum.

\[ \oint p' \frac{\partial \zeta'}{\partial x} \, dx = -\oint \zeta' \frac{\partial p'}{\partial x} \, dx = \oint f \frac{v'T'}{\theta_z} \, dx \]
THE MODEL

\[ \rho_0 f \frac{v'T'}{\theta_z} = \tau^x \quad \forall z \]

downward eddy flux of eastward momentum = wind stress at all depths.

Given a few numbers...

\[ \rho_0 = 1030 \text{ kg/m}^3 \]

\[ f = -1.25 \times 10^{-4} \text{ 1/sec} \]

\[ \tau^x \bigg|_{\text{surface}} = 2 \text{ N/m}^2 \]

...this model can be tested observationally.
Fig. 1. Melville station plan, FDRAKE 75, 19 February–30 March 1975.
Observations used to test \[ \overline{v'\theta'} = \frac{1}{\rho_0 f} \tau^x \overline{\theta_z} \]

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Eddy heat flux ( \overline{\theta' \theta'} ) (°C cm s(^{-1}))</th>
<th>Vertical temperature gradient ( \overline{\theta_z} \times 10^{-4} ) °C cm(^{-1})</th>
<th>Downward momentum flux ( \rho_0 f \overline{v'\theta'} / \overline{\theta_z} ) (dyn cm(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>580*</td>
<td>-0.39</td>
<td>0.066</td>
<td>7.6</td>
</tr>
<tr>
<td>1020†</td>
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<td>7.6</td>
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</tr>
<tr>
<td>1520†</td>
<td>-0.32§</td>
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<tr>
<td>2080*</td>
<td>-0.34§</td>
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<td>5.9</td>
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<td>2520†</td>
<td>-0.24§</td>
<td>0.079</td>
<td>3.9</td>
</tr>
<tr>
<td>2750‡</td>
<td>-0.20</td>
<td>0.074</td>
<td>3.5</td>
</tr>
<tr>
<td>3560*</td>
<td>-0.18</td>
<td>0.054</td>
<td>4.3</td>
</tr>
</tbody>
</table>

* 1977 Drake Passage Cluster Array Moorings.
† FDRAKE 75 mooring 10.
‡ FDRAKE 75 mooring 8.
§ Significantly different from zero at the 95% confidence level.

Table 1. Eddy heat flux and downward momentum flux for current meter records in central Drake Passage. Data used were taken in Drake Passage during the ISOS program. Current meter data used to calculate eddy heat flux values is taken from the 1977 central Drake Passage Cluster Array at 59°S, 64°W (Bryden, 1979b) and moorings 8 (59°S, 64°30′W) and 10 (60°S, 63°W) from the FDRAKE 75 mooring array. Calculation of \( \overline{\theta_z} \) was performed using an arithmetic mean of hydrographic/STD stations 13, 14, 15, 33 and 42 taken near the moorings by the R.V. Melville during FDRAKE 75. Values of \( \rho_0 = 1.03 \text{ g cm}^{-3} \) and \( f = -1.25 \times 10^{-4} \text{ s}^{-1} \) (appropriate for 59°S) were used. See Nowlin et al. (1977) Fig. 1, for locations of R.V. Melville stations and moorings 8 and 10.
Fig. 5. Measured eddy heat flux in central Drake Passage (discrete points with standard error bars) plotted with eddy heat flux predicted from a momentum flux balance argument (equation (4), dashed line) and a large-scale baroclinic field parameterization (equation (5), solid line).

SOLID LINE $\bar{u}'\bar{T}' = \frac{1}{\rho_0 f}\overline{\tau^x}\overline{\theta_z}$
Second test: Can down-gradient eddy heat flux be parameterized in terms of time-averaged baroclinic fields a la Green (1970) and Stone (1972)?

\[ V' \propto H U_z \]
\[ T' \propto \pi R_0 T_y \]
\[ \overline{V'T'} = C H \pi R_0 \overline{U_z} \overline{T_y} \]

where

- \( H \) is the water depth,
- \( R_0 = \frac{1}{\pi f_0} \int_{z=H}^{z=0} N(z)dz \) is the Rossby radius,
- \( \overline{U_z} \) is the vertical shear of the time averaged \( u \) field,
- \( \overline{T_y} \) is the large-scale meridional temperature gradient, and
- \( C \) is the correlation coefficient between velocity and temperature fluctuations.

Employing the thermal wind balance \( \rho_0 f \overline{U_z} = g \alpha \overline{T_y} \), where \( g \) is gravity and \( \alpha = \frac{d \rho}{dT} \), the eddy heat flux term becomes:

\[ \overline{V'T'} = C H \pi R_0 \frac{\rho_0 f_0}{g \alpha} \overline{U_z^2} \]
Observations used to test

$$V'T' = C H \pi R_0 \frac{\rho_0 f_0}{g \alpha} U_z^2.$$ 

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Eddy heat flux $\bar{V}' \bar{T}'$ ($^\circ$C cm s$^{-1}$)</th>
<th>Large-scale scaled baroclinic shear fields $H \pi R_0 \bar{U}_z \bar{T}_y$ ($^\circ$C cm s$^{-1}$)</th>
<th>Correlation coefficient $C = \bar{V}' \bar{T}' / H \pi R_0 \bar{U}_z \bar{T}_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>580*</td>
<td>-0.39</td>
<td>6.35</td>
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<tr>
<td>1020†</td>
<td>-0.33</td>
<td>3.73</td>
<td>0.09</td>
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<tr>
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<td>0.11</td>
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<td>1520†</td>
<td>-0.32§</td>
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<tr>
<td>2080*</td>
<td>-0.34§</td>
<td>1.37</td>
<td>0.25</td>
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<td>2520†</td>
<td>-0.24§</td>
<td>1.12</td>
<td>0.21</td>
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<td>2750‡</td>
<td>-0.20</td>
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<tr>
<td>3560*</td>
<td>-0.18</td>
<td>0.56</td>
<td>0.32</td>
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Estimating transport

SOLID LINE \[ \overline{v'T'} = \frac{1}{\rho_0 f} \overline{\tau^x \theta_z} \]

DOTTED LINE \[ \overline{V'T'} = C \frac{\rho_0 f_0}{g \alpha} \overline{U_z^2} \]
Estimating transport:

Setting eddy heat flux estimates equal to each other:

\[
\frac{1}{\rho_0 f} \tau^x \theta_z = CH\pi R_0 \frac{\rho_0 f_0}{g\alpha} \overline{U_z^2}.
\]

Allows us to calculate time averaged ACC vertical shear:

\[
\overline{U_z(z)} = \frac{N(z)}{f} \sqrt{\frac{\tau_x}{\rho_0 CH\pi R_0}}.
\]

From which we can estimate an ACC transport:

\[
\text{transport} = \frac{W}{f_0} \left[ \frac{\tau_x}{\rho_0 CHR_0 \pi} \right]^{1/2} \int_{z=-H}^{0} \int_{z'=-H}^{0} N(z'')dz''.
\]
\[ \bar{U}_z(z) = \frac{N(z)}{f} \sqrt{\frac{\tau_x}{\rho_0 CH \pi R_0}} \]

\( f = -1.25 \times 10^{-4} \text{ 1/s} \)

\( \tau_x = 0.2 \text{ N/m}^2 \)

\( \rho_0 = 1030 \text{ kg/m}^3 \)

\( C = 0.144 \text{ from Stone (1972)} \)

\( H = 3500 \text{ m} \)

\( R_0 = 12.6 \text{ km (from Drake Passage avg } N^2 \text{ profile)} \)

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**Fig. 6.** Velocity shear observed in Drake Passage (solid line) plotted with velocity shear predicted by equation (7) (dashed line).

**Fig. 7.** Velocity referenced to 3500 m as observed in Drake Passage (solid line) vs that predicted by equation (7) (dashed line).
For discussion: the authors assume that the RHS is just as evenly distributed as the LHS. Is this a reasonable assumption?

\[ \rho_0 f \frac{\nu' T'}{\theta_z} = \tau^x \]

**THE MODEL**


Zoom in to Drake Passage: cDrake Experiment, 2007-2011.
Table 1. Eddy heat flux and downward momentum flux values are taken from the 1977 cent moorings 8 (59°S, 64°30'W) and 10 (60°S) performed using an arithmetic mean of the R.V. Melville during FDRAKE 7 (59°S) were used. See NOWLIN et al. (1989).

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Downward momentum flux $\rho_0 \overline{V'T}/\theta_z$ (dyn cm$^{-2}$)

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Data used to calculate eddy heat $\overline{V'T}$ at 54°W (BRYDEN, 1979b), and $\theta_z$ was taken near the moorings 25 and 42, taken near the moorings 8 and 10.
cDrake spatial and temporal variability.

Vertical, time mean IFS.  Time mean IFS profiles.