On the Obscurantist Physics of "Form Drag" in Theorizing about the Circumpolar Current*

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18 April 1995 and 15 February 1996

Why are you all so obsessed with this form-drag business?

Comments on "On the Obscurantist Physics of 'Form Drag' in Theorizing about the Circumpolar Current"

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29 April 1996

Quick-and-dirty response

Comments on "On the Obscurantist Physics of 'Form Drag' in Theorizing about the Circumpolar Current*

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16 January 1997 and 14 October 1997

I've-been-thinking-about-this-for-a-really-long-time response

So what are these "Obscurantist" physics anyway?

Time-averaged zonal momentum equation

$$fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial z},$$

Integrate vertically over the whole water column

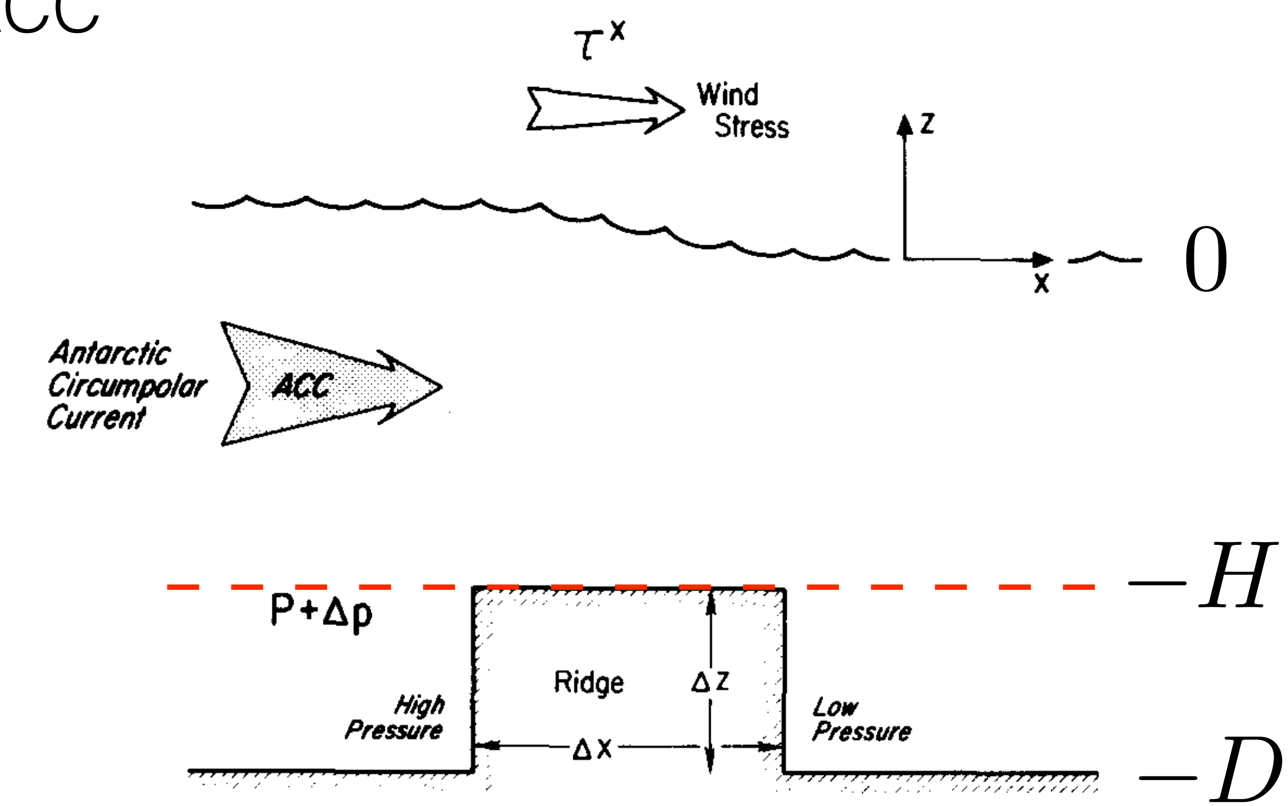
$$f\int_{-D}^{0} v\,dz = -\frac{\partial}{\partial x}\int_{-D}^{0}\frac{1}{\rho}p\,dz - \frac{1}{\rho}p(-D)\frac{\partial D}{\partial x} + \tau_x(0) - \tau_x(-D).$$

Integrate along a streamline around the ACC

$$\int_C \tau_x(0)\,dx = \int_C \frac{1}{\rho}p(-D)\frac{\partial D}{\partial x}\,dx$$

Wind stress = Topographic form stress

Warren et al. treat this as a statement of mass conservation



adapted from Johnson & Bryden (1989)

“The language obscures the more conventional physics”

Time-averaged zonal momentum equation

$$fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial z},$$

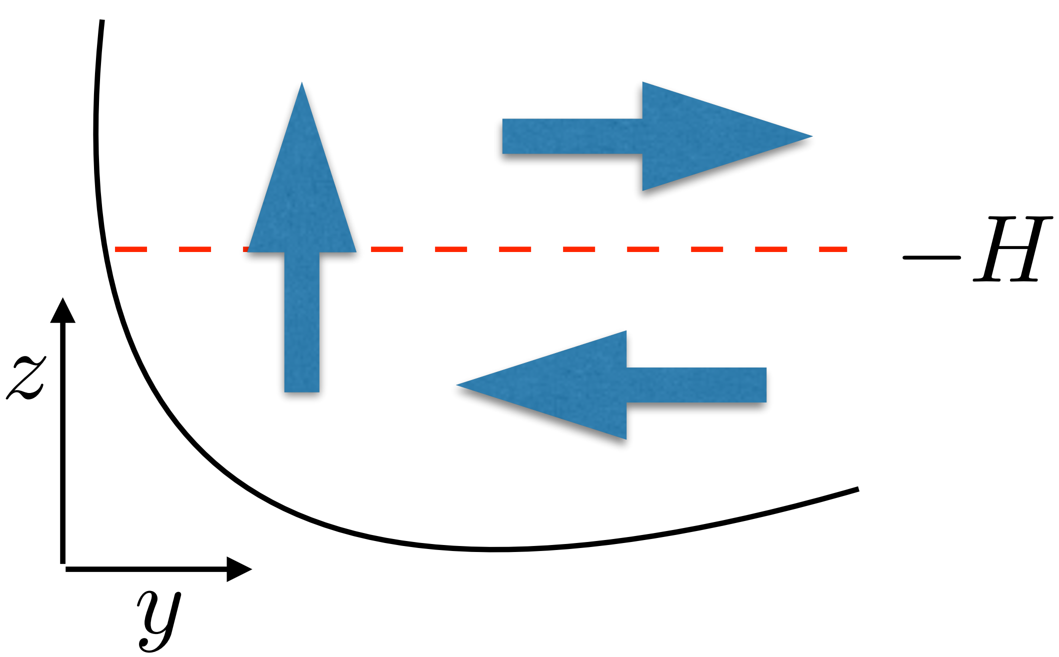
Warren et al. treat this as a statement of mass conservation

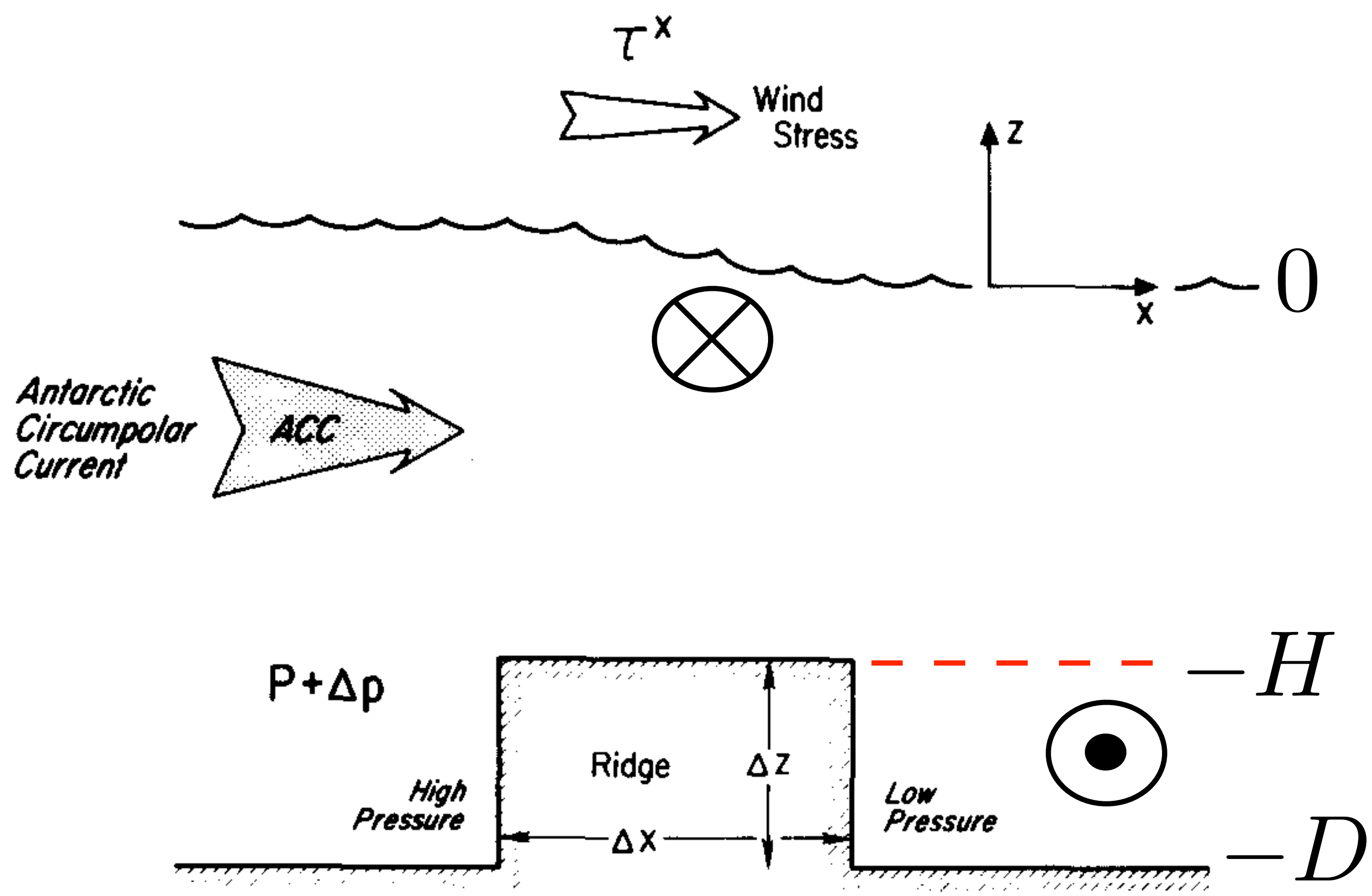
Integrate vertically down to $-H$

$$f\int_C dx\left(\int_{-H}^{0} v\,dz\right) = \int_C \tau_x(0)\,dx$$

Integrate from $-H$ to $-D$

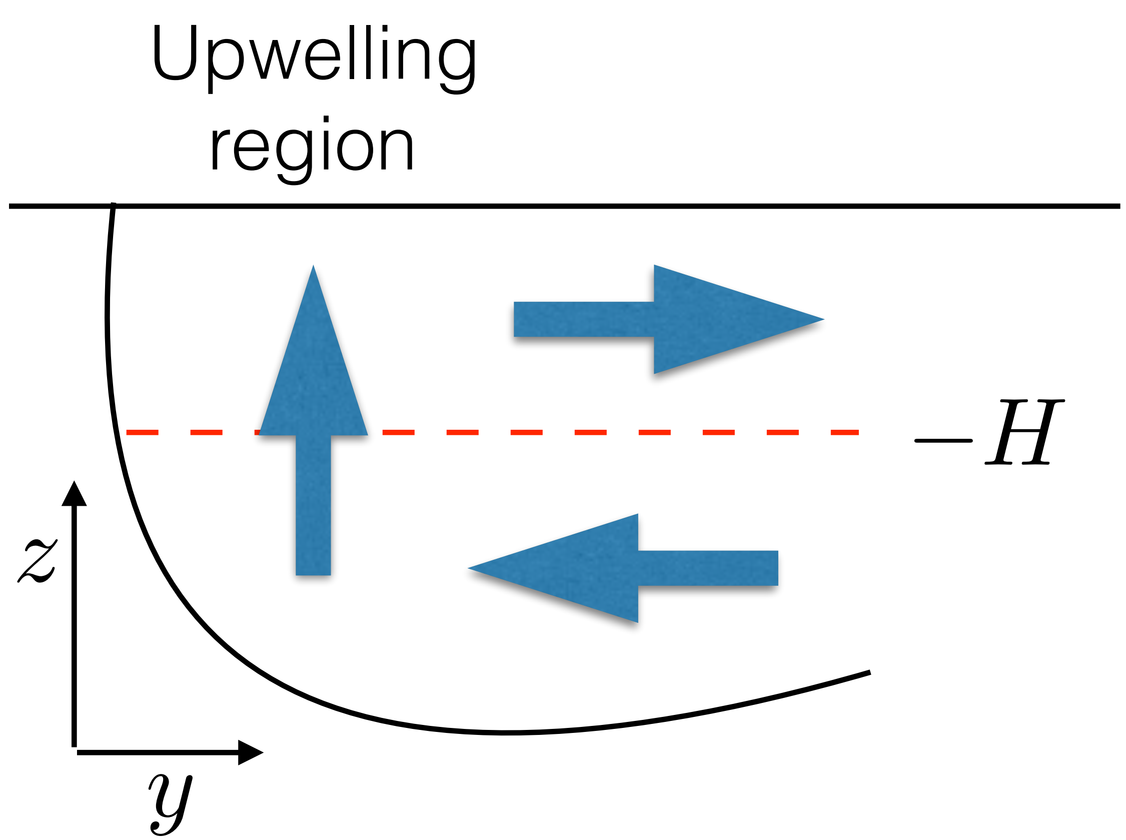
$$f\int_C dx \int_{-D}^{-H} v\,dz = -\int_C dx \int_{-D}^{-H} \frac{1}{\rho}\frac{\partial p}{\partial x}\,dz.$$

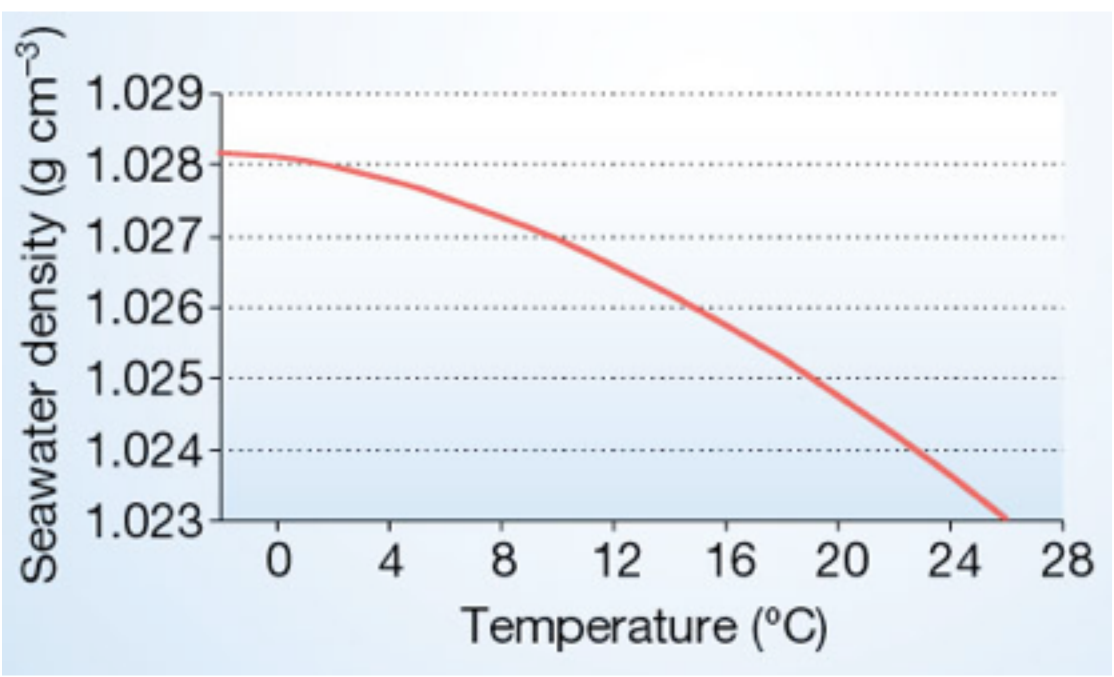




adapted from Johnson & Bryden (1989)

Upwelling and buoyancy





Johnson & Bryden claim that because there is buoyancy loss over the southern ocean, there cannot be upwelling.

Warren et al. argue that this isn't true.

This gives a surface heat loss of 1.4×10^{14} W or buoyancy loss of -0.9×10^{-7} kg m^{-2} s.

However, there is a large freshwater flux with buoyancy flux 2.6×10^{-7} kg m^{-2} s.

So buoyancy is actually **gained**.

Is this still the modern view?

Sverdrup dynamics set the strength of the flow

- Warren et al. suggest that Sverdrup dynamics dictate the strength of the ACC, with southward flow in most of the ACC and northward flow just east of Drake Passage

$$\beta V = \nabla \times \tau$$

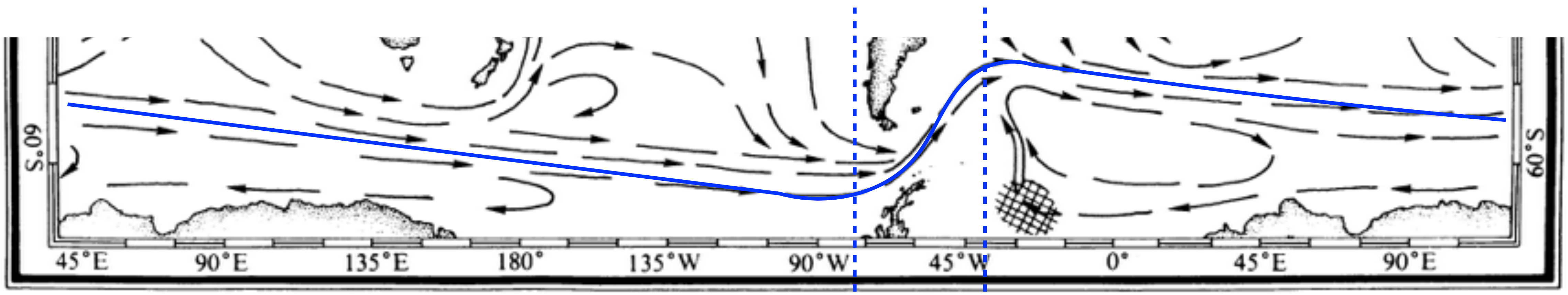


Fig. 2.23 Schematic flow lines for abyssal circulation. The cross-hatched areas indicate regions of production of bottom water. [Adapted from Stommel, H., *Deep Sea Research* (1958).]

- Baker (1982) found that Sverdrup balance predicts a good-ish transport of the ACC (173Sv)

$$\Psi_{ACC} = \frac{1}{\beta} \int_C \nabla \times \tau \, dx$$

Where C is a circle of constant latitude just south of Cape Horn

To recap Warren et al.

- Warren et al. have not assimilated the view that the overturning circulation is along isopycnals (a fairly recent idea at the time), so they assume diabatic flow

Warren et al. assert the following

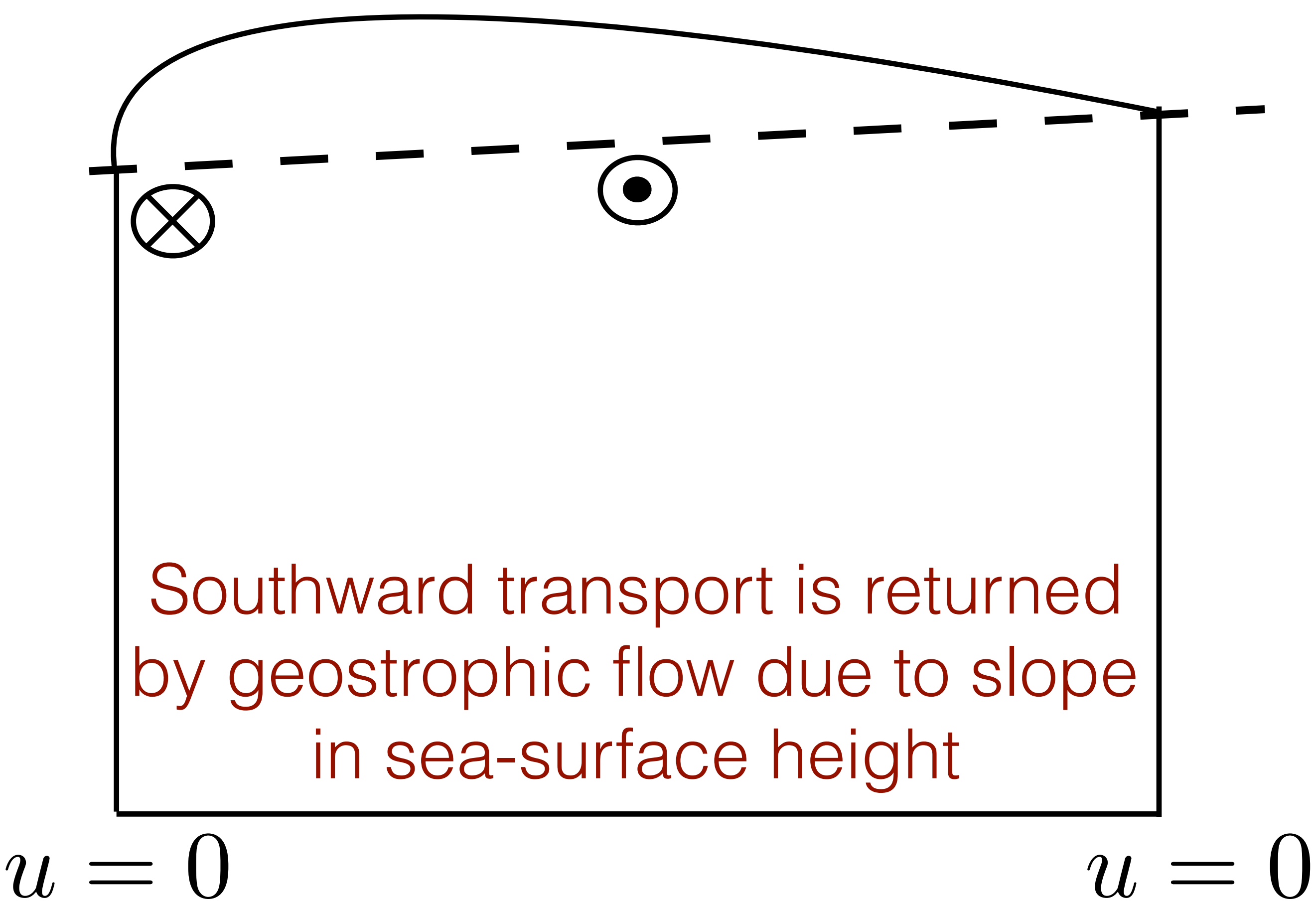
- The wind stress forces Ekman transport in the surface layer and the mass is returned below the topography: this process is not coupled to the transport of the ACC.

- The wind stress is balanced by the Coriolis force in the Ekman layer.

- You can predict the transport of the ACC based on Sverdrup balance alone.

Hughes (1996) response: Sverdrup dynamics do not provide enough constraints

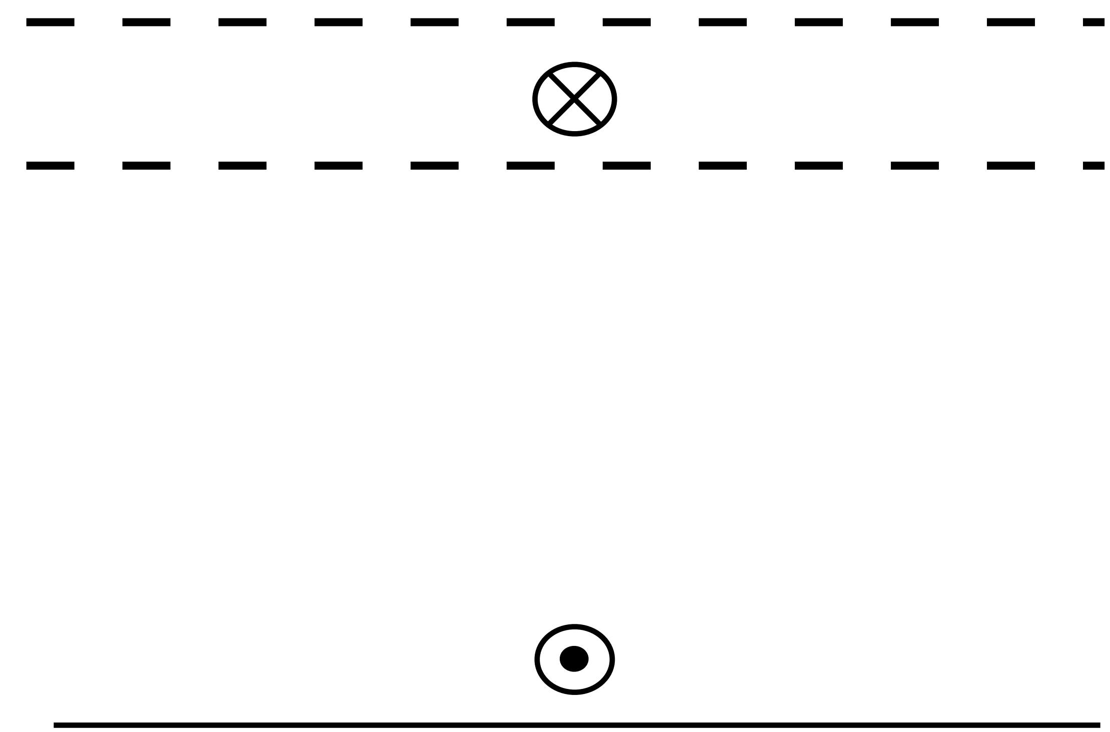
In a closed basin



i.e. we can find u using

$$\beta v = -f w_z$$
$$u_x + v_y + w_z = 0$$

In an open channel



i.e. we can add any zonally-uniform u without changing v

"At latitudes with no boundary, an arbitrary function of latitude can be added to the Sverdrup solution."

Hughes (1996) response: we can't assume that ACC transport is independent of topography

Isopycnals Streamfunction (follows isopycnals)

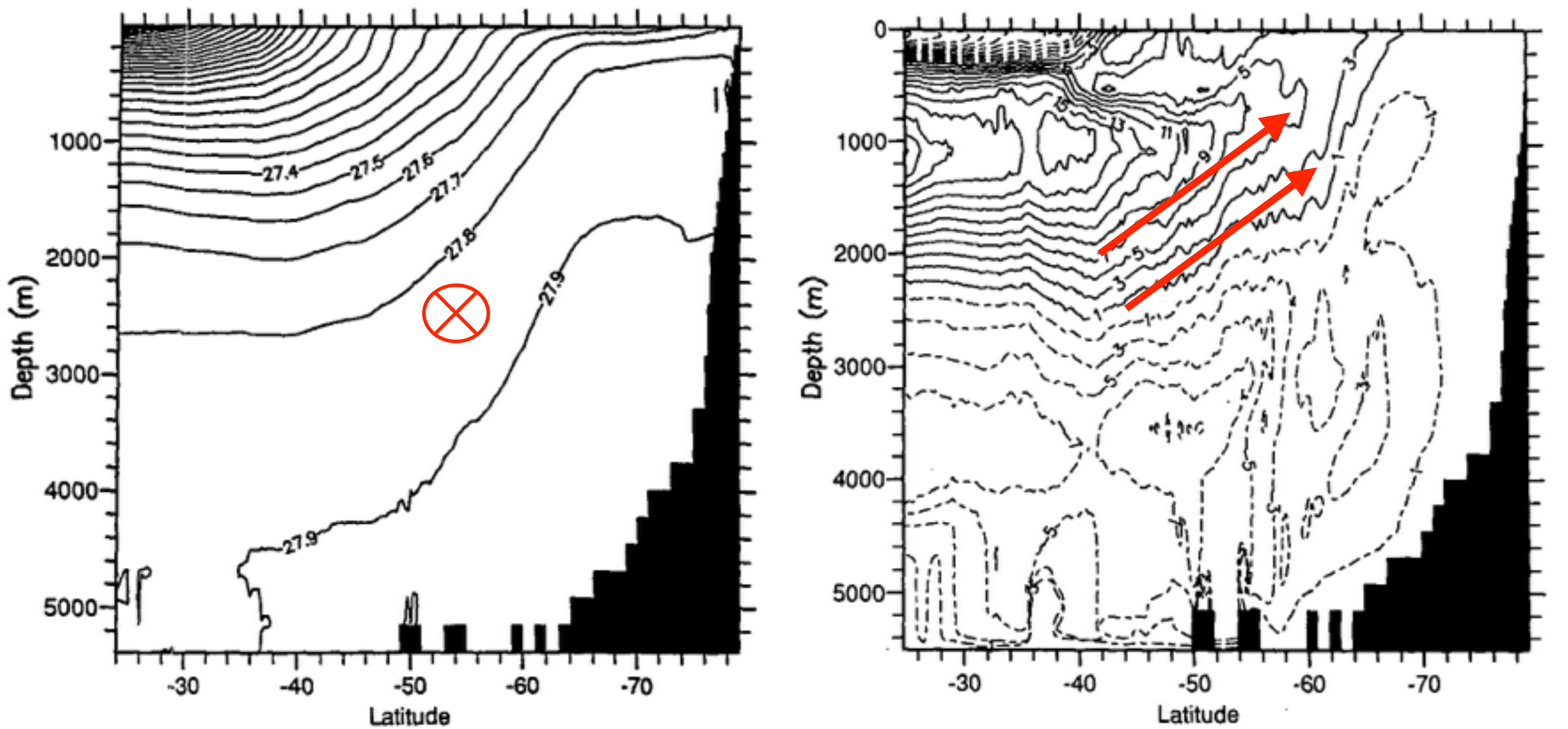


FIG. 5. (a) The zonal-averaged potential density ρ_0, in the FRAM model.
(b) The meridional streamfunction $\psi(\theta, \rho)$ transformed onto the mean density surfaces.

Döös and Webb (1994)

- For return of the Meridional Overturning Circulation, the sloping isopycnals must reach below the topography
- Therefore the ACC must reach below the topography.

Hughes (1996) response: the topography sets the strength of the eastward current

- Some flow appears to be returned at shallower depths than would be permitted at Drake Passage
- This could happen at the Kerguelen plateau
- But in order for this to happen you need a strong eastward current

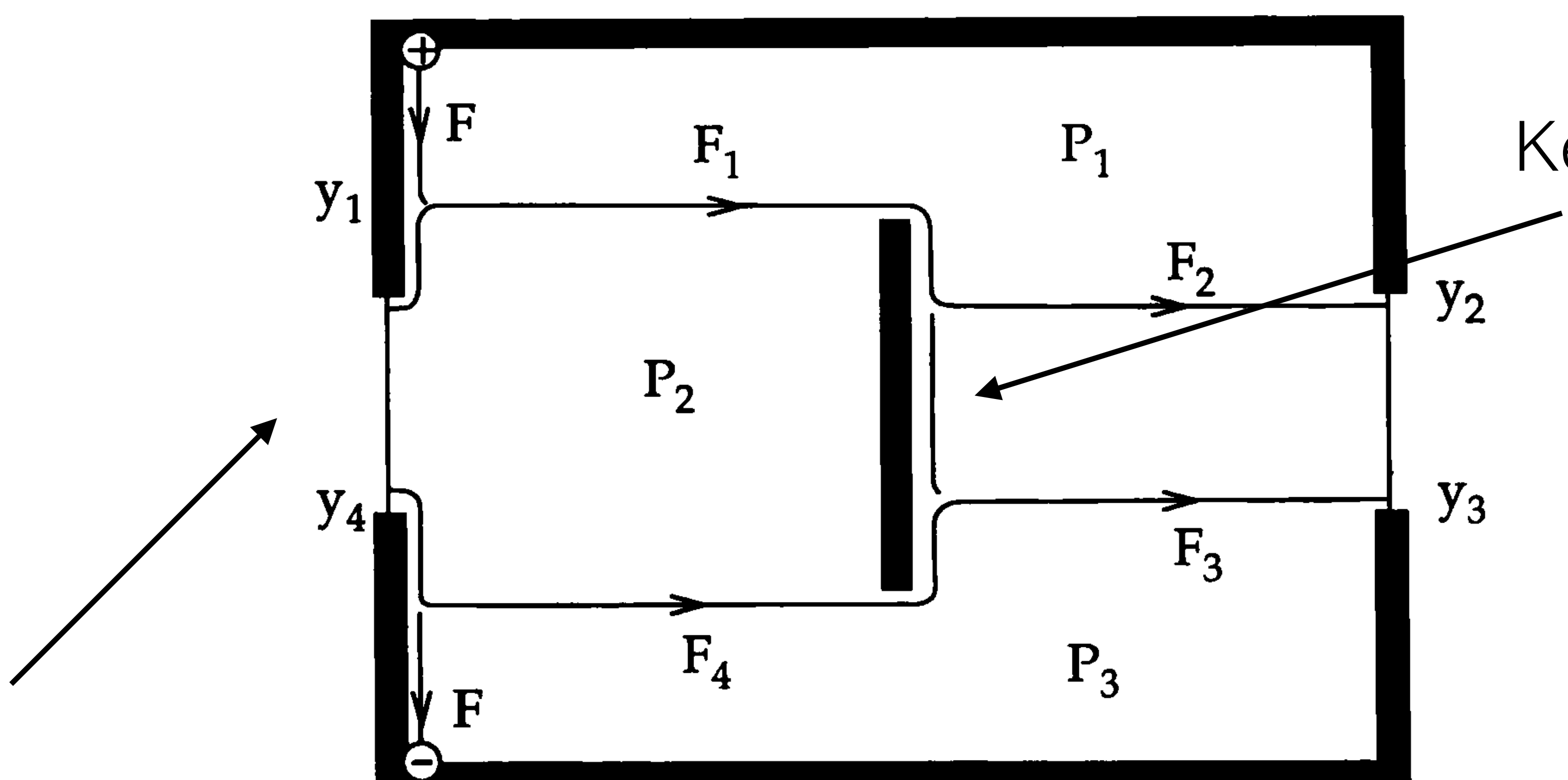


Figure 3 Source and sink flows in a zonal channel with occluding island.

Webb (1994)

Olbers (1998) response: here's what form drag is all about

Time-averaged zonal momentum equation

$$-f\boldsymbol{v} = -\frac{\partial p}{\partial x} + \frac{\partial \boldsymbol{\tau}}{\partial z}$$

Integrate in the x and z directions

$$-f\left\langle\int_{-E}^{0} \boldsymbol{v}\,dz\right\rangle = \langle\boldsymbol{\tau}_0\rangle$$

$$-f\left\langle\int_{-H}^{-E} \boldsymbol{v}\,dz\right\rangle = 0$$

$$-f\left\langle\int_{-D(x)}^{-H} \boldsymbol{v}\,dz\right\rangle = \left\langle p_D\frac{\partial D}{\partial x} - \boldsymbol{\tau}_D\right\rangle,$$

$$\langle\,\cdots\,\rangle := \oint_C dx\,\cdots\,,$$

$$p_D(x) = p(x, z = -D(x))$$

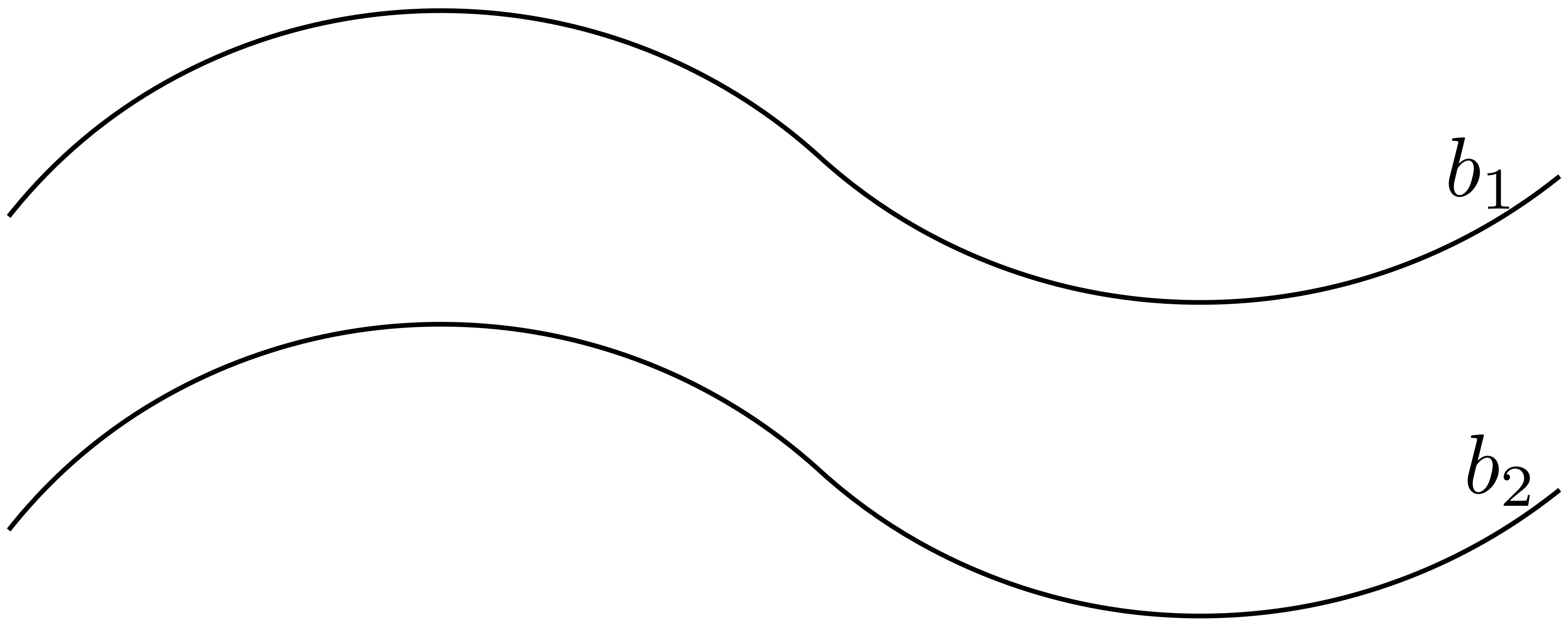
Using mass conservation

$$\left\langle\int_{-D(x)}^{0} \boldsymbol{v}\,dz\right\rangle = 0,$$

$$\left\langle\boldsymbol{\tau}_0 - \boldsymbol{\tau}_D + p_D\frac{\partial D}{\partial x}\right\rangle = 0,$$

How does the momentum get from the surface to the bottom?

Imagine two surfaces below the Ekman layer but above the level of topography

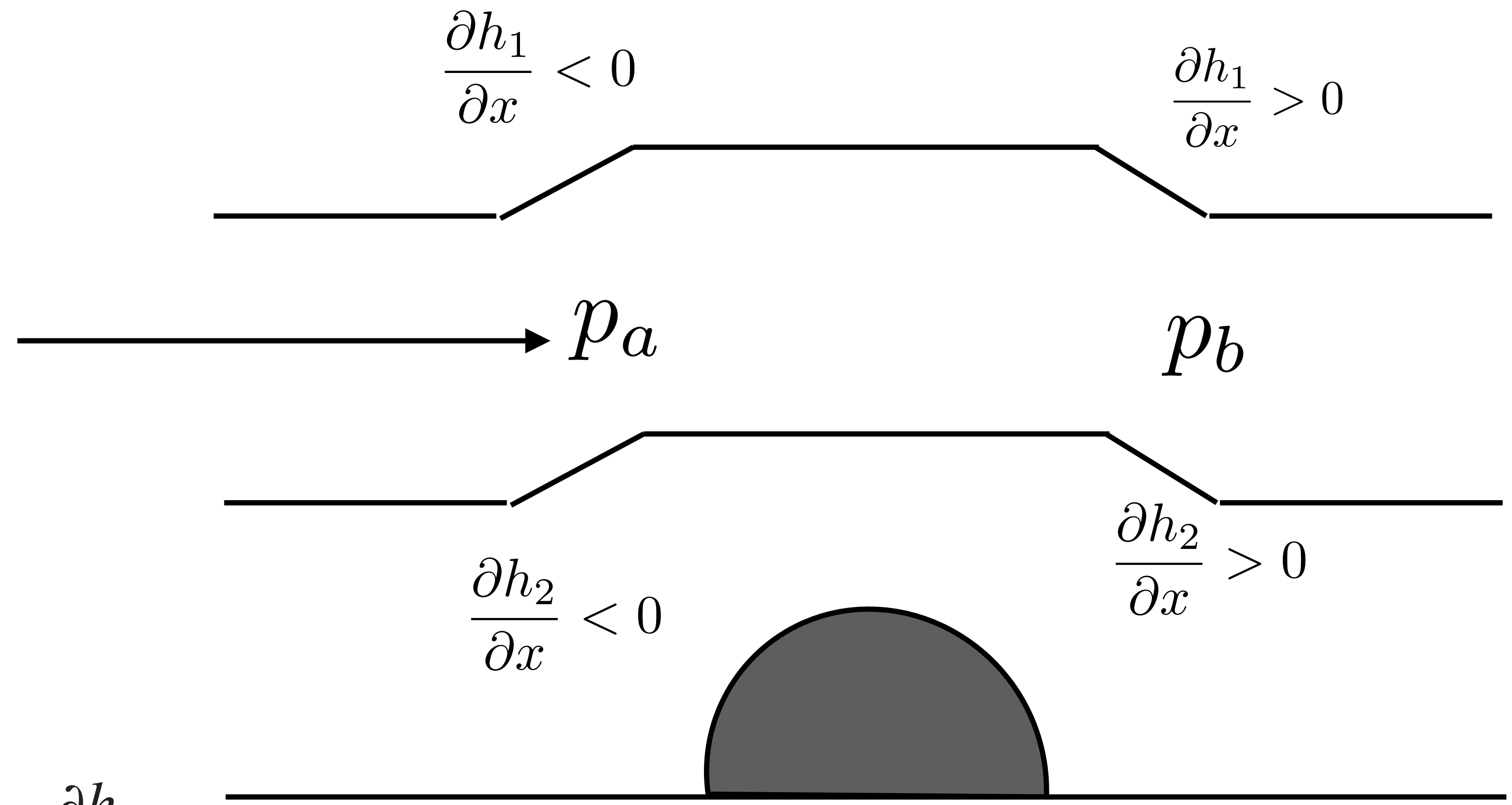


Assume no diabatic transport

$$\oint f \int_{z_2}^{z_1} v \, dz \, dx = 0 = \oint \frac{1}{\rho} \int_{z_2}^{z_1} p_x \, dz \, dx$$

How does the momentum get from the surface to the bottom?

$$\left\langle \tau_0 - \cancel{\tau_D} + p_D \frac{\partial D}{\partial x} \right\rangle = 0,$$



Interfacial form stress

$$\int_{x_l}^{x_r} \frac{\partial p}{\partial x} \int_{-h_2(x)}^{-h_1(x)} dz\; dx$$

$$= \int_{-h_2(x)}^{-h_1(x)} p\; dz \Bigg|_{x_l}^{x_r} + \int_{x_l}^{x_r} \left[p(x,\, -h_1(x)) \frac{\partial h_1}{\partial x} - p(x,\, -h_2(x)) \frac{\partial h_2}{\partial x} \right] dx\,.$$

This term is zero in a re-entrant channel

What is the interfacial form stress?

$$\mathbf{J} = (J^{(x)}, J^{(y)}) \quad J^{(z)}$$

Are heat fluxes

$$\mathbf{J} = \mathbf{u}\theta + \mathbf{I} \qquad J^{(z)} = w\,\theta + I^{(z)}.$$

Conservation of mass & temperature

$$\nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0 \qquad \nabla \cdot \mathbf{J} + \frac{\partial J^{(z)}}{\partial z} + w\Theta_z = 0.$$

$\Theta(z)$ Is the average temperature state

Integrate over area

$$\langle v \rangle + \frac{\partial}{\partial z}\int_{A(z)} w\, dA = 0. \qquad -\Theta_z \int_{A(z)} w\, dA = \langle J^{(y)} \rangle + \frac{dQ}{dz},$$

θ Is the perturbation to this state

Therefore

$$\langle v \rangle = \frac{\partial}{\partial z}\left[(\Theta_z)^{-1}\left(\langle v\theta + I^{(y)} \rangle + \frac{dQ}{dz}\right)\right], \qquad Q(z) = \int_{A(z)} J^{(z)}\, dA$$

Is the diabatic heat flux

Zonal momentum balance

$$-f\langle \overline{v} \rangle = \frac{\partial \mathcal{F}}{\partial z} = \left\langle \frac{\partial \overline{\tau}}{\partial z} \right\rangle - \left\langle \frac{\partial}{\partial y}\overline{uv} \right\rangle + \sum_{\text{ridges}} \delta\overline{p}\,, \qquad \mathcal{F} = -f(\Theta_z)^{-1}\left(\langle \overline{v\theta} + \overline{I}^{(y)} \rangle + \frac{d\overline{Q}}{dz}\right).$$

Compare with Johnson & Bryden

$$\rho_0 f \frac{\overline{v'T'}}{\overline{\theta}_z} = \tau^x.$$

$$\mathcal{F}(f=0) = 0$$

What happens in each layer?

Ekman layer

$$-f\langle\overline{v}\rangle = \frac{\partial \mathcal{F}}{\partial z} \approx \left\langle \frac{\partial \overline{\tau}}{\partial z} \right\rangle.$$

$$-f\int_{-E}^{0} \langle\overline{v}\rangle \, dz = \boxed{-\mathcal{F}(z=-E) \approx \langle\overline{\tau}_0\rangle}$$

Intermediate layer

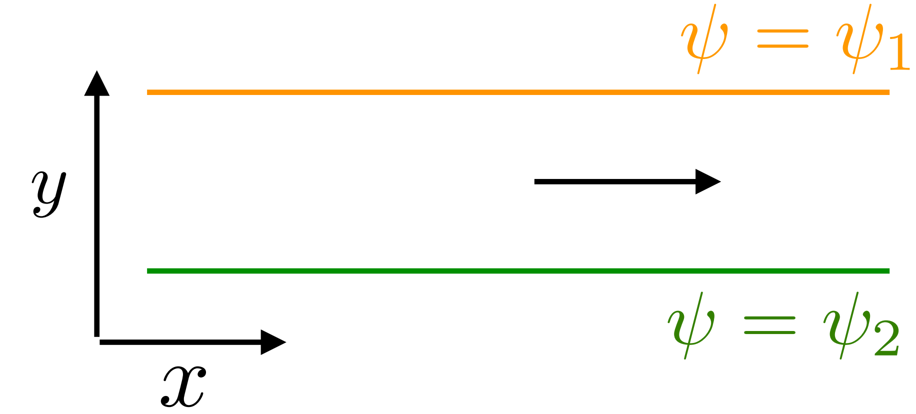
$$\mathcal{F} \approx -\langle\overline{\tau}_0\rangle.$$

Deep layer

$$-f\langle\overline{v}\rangle = \frac{\partial \mathcal{F}}{\partial z} \approx \sum_{\text{ridges}} \delta\overline{p}\,,$$

$$-f\int_{-D_{\max}}^{-H} \langle\overline{v}\rangle \, dz = \mathcal{F}(z=-H) \approx -\left\langle \overline{p}_D \frac{\partial D}{\partial x} \right\rangle.$$

Why is the wind stress curl not enough?



The vorticity balance

$$\epsilon \nabla^2 \psi + \beta \frac{\partial \psi}{\partial x} = \mathbf{J}(p_D, D) + \mathrm{curl}\boldsymbol{\tau}_0,$$

Includes the pressure at depth $p_D(x) = p(x, z = -D(x))$

The momentum equation is a boundary condition on the vorticity equation

$$\epsilon \mathbf{U} + f\mathbf{k} \times \mathbf{U} = -h\boldsymbol{\nabla} p_D - \boldsymbol{\nabla} E + \boldsymbol{\tau}_0.$$

E depends on the windstress, not the wind stress curl:

$$\oint_L d\mathbf{s} \cdot \boldsymbol{\nabla} E$$

$$= \oint_L d\mathbf{s} \cdot [-\epsilon \mathbf{U} - f\mathbf{k} \times \mathbf{U} - h\boldsymbol{\nabla} p_D + \boldsymbol{\tau}_0]$$

$$= \oint_L d\mathbf{s} \cdot [\tau_0 - \tau_D + p_D \boldsymbol{\nabla} h] = 0.$$

Conclusions

- You cannot predict the transport of the ACC based on Sverdrup balance alone

- The momentum imparted to the ocean by the wind is transferred downwards by interfacial form stress and removed from the water column by topographic form stress.

- WLR never mention interfacial stress, and to some extent this s their downfall.

$$-\mathcal{F}(z = -E) \approx \langle \overline{\boldsymbol{\tau}}_0 \rangle$$